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Dai, Yaping
Department of Automatic Control, Beijing Institute of Technology

Hirasawa, Kotaro
Department of Electrical and Electronic Systems Engineering, Graduate School of Information Science and Electrical Engineering, Kyushu University

Zhen, Liu
Department of Earth Resources Engineering, Kyoto University

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A Target Tracking Algorithm with Derivation Measurement

Yaping DAI* , Kotaro HIRASAWA** , and Zhen LIU***

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Abstract: In this paper a new observation model is presented to improve the state estimation and prediction in a target tracking problem. Comparing with conventional approaches, the following are distinguished points of the approach. First, the measurement equation is set up in the polar coordinate and even combines the derivation measurement with the usual position measurements (i.e. there are 6 sensor data now: range, azimuth, elevation angle, range rate, azimuth rate, and elevation rate). Second,the observation noise of sensor data is considered as a colored one and be set up as the model of AR(1), and by means of a pseudo measurement model, the requirement of Kalman filter will be satisfied. As a result, the accuracy of both the observation and the prediction will be increased.

Keyword: Target tracking, Pseudo measurement, Color noise

1. Introduction

In real-time missile and vehicle tracking control systems, many different approaches on the maneuvering target tracking problem have been submitted in recent years. The purpose of the attempts is to track the maneuvering target accurately and effectively. Dr. Singer [1] has proposed a model based on Newton's law, where the maneuvering is considered as zero mean time correlated random process with correlation time τ. Bar-shalom [4] submitted the *Variable Dimension* method, in which a detection scheme has been developed to determine whether a maneuvering is indeed occurring. Chen *et al* [5] used an *Input Estimation* to remove the filter bias which are caused by the target deviating from the assumed constant velocity, straight line motion. T.L.Song *et al* [2] presented a LCCS (line of sight Cartesian coordinate system) method to deal with the coupling problem of Kalman filter, they called the new measurement vectors in the LCCS as pseudo measurement, but it is different from the pseudo measurement proposed in this paper. The original measurement data of target motion is obtained from radar, they are three types of data obtained in the polar coordinate usually expressed by range, azimuth and elevation, and in almost all of theses cases the measurement noise is considered as white, zero mean noise in order to satisfying the requirement of Kalman filter algorithm. How to deal with the Kalman filter with color measurement noise is the main point in this paper. J.-A.Guu *et al* [3] proposed an algorithm to solve the correlated measurement noise. In their paper, an AR(1) model is adopted but the derivation of the sensor data are not considered[3].

Because a new observation vector -pseudo measurement vector is adopted in this paper, a new observation equation with white measurement noise is obtained. Then the Kalman filter can be used to estimate and predict the position of the target. It does not need to add vectors in the state vector space[9] although the color noise model is considered as $AR(1)$. If the derivation measurement is considered together with the usual position measurement, obviously, the accuracy of observation will be improved. Because the observation equation is described in polar coordinate system directly, the correlation problem of measurement noise will be avoid. According to Talor's expansion, a linear observation equation will be obtained, but the Jacobian matrix will become very complex and the computing load will be increased. And use of the gain rotation algorithm will totally reduce about one third of calculation quantity comparing with the common Kalman filter algorithm.

In Chapter 2, the equations of the dynamic motion and the nonlinear measurement equation with color noise observation are described. After using the Taylor expansion and pseudo measurement equation , a new linear measurement equation is obtained, and the noise in the new measurement equation becomes zero mean and white noise. In Chapter 3, how to use Kalman filter to estimate

* Dept. of Automatic Control, Beijing Institute of Technology, P.R.China
** Dept. of Electrical and Electronic Systems Engineering
* * * Dept. of Earth Resources Engineering, Kyoto University

and predict the position of the target is discussed. Because of using the technique of coordination rotation, the quantity of calculation will be reduced. Chapter 4 is the conclusion of this paper.

2. Dynamic & Observation Equation

Let X(t) and W(t) be the target state and the process noise respectively, according to the Singer model[1], the dynamic equation of the target motion is:

$$\dot{X}(t) = \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ 0 & 0 & -\alpha I \end{bmatrix} X(t) + W(t). \tag{1}$$

The discrete state equation of the target motion is:

$$X(k+1) = \Phi(\Delta, \alpha) X(k) + W(k). \tag{2}$$

and

$$\Phi(\Delta,\alpha) = \begin{bmatrix} I & \Delta I & \frac{1}{\alpha^2}[-1+\alpha\Delta+exp(-\alpha\Delta)]I \\ 0 & I & \frac{1}{\alpha}[1-exp(-\alpha\Delta)]I \\ 0 & 0 & [exp(-\alpha\Delta)]I \end{bmatrix} \tag{3}$$

where,

$X(k) = [x, y, z, \dot{x}, \dot{y}, \dot{z}, \ddot{x}, \ddot{y}, \ddot{z}]^T$ is (9×1) dimensional state vector.

The vector $\{x, y, z\}$ is the position of the target,
the vector $\{\dot{x}, \dot{y}, \dot{z}\}$ is its velocity,
the vector $\{\ddot{x}, \ddot{y}, \ddot{z}\}$ is its acceleration.

and,

$W(k) = [w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8, w_9]^T$ is (9×1) dimensional noise with covariance of $Q(k)$, and Q(k) is shown in the Appendix.2.

$\Phi(\Delta, \alpha)$ is discrete state transition matrix,

$\alpha = \frac{1}{\tau}$, τ is correlation time constant, Δ is sample interval, and I is (3×3) identity matrix.

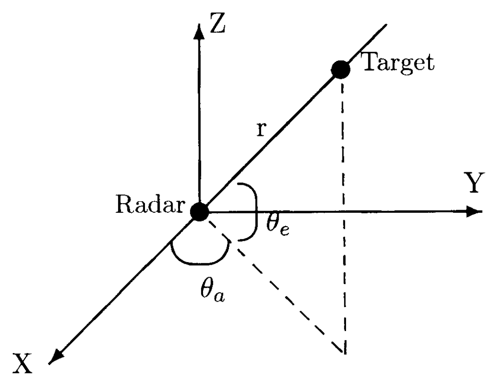

Fig.1 Measurement relation of radar and target

$$Y(k) = h[X(k)] + V(k) = \begin{bmatrix} \sqrt{x^2+y^2+z^2} \\ \tan^{-1}\frac{y}{x} \\ \tan^{-1}\frac{z}{\sqrt{x^2+y^2}} \\ \frac{x\dot{x}+y\dot{y}+z\dot{z}}{\sqrt{x^2+y^2+z^2}} \\ \frac{x\dot{y}-\dot{x}y}{x^2+y^2} \\ \frac{(x^2+y^2)\dot{z}-(x\dot{x}+y\dot{y})z}{\sqrt{x^2+y^2}(x^2+y^2+z^2)} \end{bmatrix} + V(k), \tag{4}$$

where,

$Y[k] = [r, \theta_a, \theta_e, \dot{r}, \dot{\theta}_a, \dot{\theta}_e]^T$

is (6×1) dimensional measurement vector which is obtained from radar,it is shown in **Fig.1**.

$V(k) = [v_r, v_{\theta_a}, v_{\theta_e}, v_{\dot{r}}, v_{\dot{\theta}_a}, v_{\dot{\theta}_e}]^T$

is (6×1) dimensional measurement noise.

It is seen that equation (4) is a nonlinear equation. The noise components in $V(k)$ are commonly considered as statistical and white, mostly it is unreasonable, because the target motion is successive so that the measurement noise is affected by relative of time sequence, then, in this paper the noise is considered as an autoregressive process AR(1).

$$V(k) = \beta V(k-1) + \xi(k), \tag{5}$$

$\beta = \mathrm{diag}(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6)$ are constant matrix, $\xi(k)$ is (6×1) zero-mean, white noise with the following variance

$$E\{\xi(k)\xi^T(j)\} = R(k)\delta_{k,j}, \quad k, j = 1, 2, 3, ... \tag{6}$$

$$\delta_{k,j} = \begin{cases} 1 & k = j \\ 0 & k \neq j \end{cases}$$

R(k) is shown in the appendix 2.

Let us expand the nonlinear function $h[X(k)]$ in Taylor series, and assume that the second and higher-order terms are small and negligible, then equation(4) becomes

$$Y(k) = H_x X(k) + V(k). \tag{7}$$

where

$$H_x = \left.\frac{\partial h[X]}{\partial X}\right|_{X=\hat{X}(k/k-1)}, \tag{8}$$

H_x is (6×9) Jacobian matrix of $h[X(k)]$ and it can be written as below:

$$H_x = \begin{bmatrix} g(X) & c(X) & 0 \end{bmatrix}. \tag{9}$$

$g(X)$ and $c(X)$ denote the coordinate transformations from the inertial rectangular to the polar system which are shown in appendix A.1

As $V(k)$ is 1-*th* order autoregressive process, then pseudo measurement vector $Z(k)$ is adopted to deal with the Kalman filter with the color measurement noise.

$$Z(k) = Y(k) - \beta Y(k-1), \tag{10}$$

According to equations of (2),(5),and (7), a new observation equation will be obtained as below:

$$Z(k) = H_x^* X(k) + V^*(k). \tag{11}$$

where

$$H_x^* = H_x - \beta H_x \Phi^{-1}, \tag{12}$$

and the new measurement noise becomes:

$$V^*(k) = \beta H_x \Phi^{-1} W(k-1) + \xi(k) \tag{13}$$

Equations (2) and (11) are dynamic and observation equations respectively.

3. The equation of Kalman filter

In order to eliminate the correlation in the off diagonal terms of the covariance matrix of the filter error, a rotation Cartesian coordinate system of tracking is introduced based on the line of sight of the measurement sensor[2]. In this system, the target is always on the X_R axis and the measurement of target azimuth and elevation angles are zero.

The new coordinate system rotates with each sample time. The rotation coordinate system (X_R, Y_R, Z_R) is not an inertial coordinate system since the line of sight is in angular motion during the radar tracking mode, but for the time interval $(k-1)\Delta t \leq t < k\Delta t$, the rotation coordinate system is regarded mathematically as an inertial coordinate system which is explained in Berg[6], and Song T.L[2].

From **Fig.2**, it can be seen that the coordinate transformation matrix $m(Y)$ from inertial to rotation coordinate is

$$m(Y) = \begin{bmatrix} \cos\theta_a \cos\theta_e & -\sin\theta_a \cos\theta_e & \sin\theta_e \\ -\sin\theta_a & cos\theta_a & 0 \\ \cos\theta_a \sin\theta_e & -\sin\theta_a \sin\theta_e & \cos\theta_e \end{bmatrix} \tag{14}$$

then, the state vector in the new coordinate system $X_R(k)$ has the following relation with $X(k)$:

$$X_R(k) = M(Y)X(k). \tag{15}$$

and

$$W_R(k) = M(Y)W(k). \tag{16}$$

where

$$M(Y) = \begin{bmatrix} m(Y) & 0 & 0 \\ 0 & m(Y) & 0 \\ 0 & 0 & m(Y) \end{bmatrix} \tag{17}$$

The new state dynamic equation is:

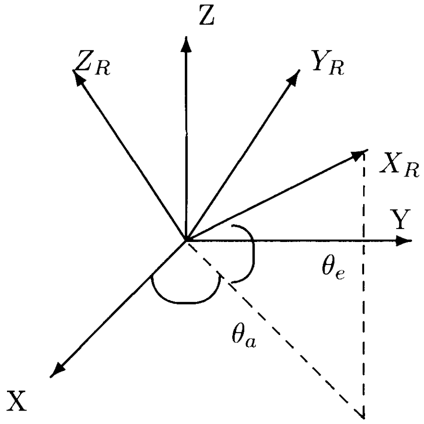

Fig.2 Angular relation between the two coordinate

$$X_R(k+1) = \Phi(\alpha, \Delta)X_R(k) + W_R(k). \tag{18}$$

$X_R(k) = [x_R, y_R, z_R, \dot{x}_R, \dot{y}_R, \dot{z}_R, \ddot{x}_R, \ddot{y}_R, \ddot{z}_R]^T$ is the state vector in the rotation coordinate.

The new measurement vector $Y_R(k)$ has the following relation with $Y(k)$:

$$Y_R(k) = n(Y)Y(k), \tag{19}$$

and

$$Y_R(k) = \begin{bmatrix} r \\ 0 \\ 0 \\ \dot{r} \\ 0 \\ 0 \end{bmatrix} \tag{20}$$

is the measurement vector in the rotation coordinate.

In reference 2), $Y_R(k)$ is called as pseudo measurement, it is different from the pseudo measurement equation (9) described in this paper.

The new pseudo measurement vector $Z_R(k)$ has the following relation with $Z(k)$:

$$Z_R(k) = n(Y)Z(k). \tag{21}$$

The measurement transformation matrix $n(Y)$ is:

$$n(Y) = H_R M(Y) H_x^{-1}, \tag{22}$$

and there is:

$$H_R = \begin{bmatrix} g_0(X) & c_0(X) & 0 \end{bmatrix} \tag{23}$$

$$g_0(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/r & 0 \\ 0 & 0 & 1/r \\ 0 & 0 & 0 \\ 0 & -\dot{r}/r^2 & 0 \\ 0 & 0 & -\dot{r}/r^2 \end{bmatrix}, \tag{24}$$

and

$$c_0(x) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1/r & 0 \\ 0 & 0 & 1/r \end{bmatrix}, \tag{25}$$

The new observation equation in rotation coordinate is :

$$Y_R(k) = H_R X_R(k) + V_R(k). \tag{26}$$

and the new pseudo measurement equation in rotation coordinate is:

$$Z_R(k) = H_R^* X_R(k) + V_R^*(k). \tag{27}$$

where

$$H_R^* = H_R - \beta H_R \Phi^{-1}. \tag{28}$$

and the pseudo measurement noise in the rotation coordinate is:

$$V_R^*(k) = \beta H_R \Phi^{-1} W_R(k-1) + \xi_R(k) \tag{29}$$

There are also the following relations:

$$V_R^*(k) = n(Y)V^*(k). \tag{30}$$

and

$$\xi_R(k) = n(Y)\xi(k). \tag{31}$$

Variance of $\xi_R(k)$ is:

$$E\{\xi_R(k)\xi_R^T(k)\} = R_R(k) \tag{32}$$

Variance of $V_R^*(k)$ is denoted by $R_R^*(k)$ and which is shown as follows:

$$\begin{aligned} R_R^*(k) &= E\{V_R^*(k)V_R^{*T}(k)\} \\ &= DQ_R(k-1)D^T + R_R(k) \end{aligned} \tag{33}$$

and,

$D = \beta H_R \Phi^{-1}$.

Because $W_R(k)$ and $\xi_R(k)$ are white noise, the $V_R^*(k)$ is white also. But the new measurement noise $V_R^*(k)$ is correlated with the process noise $W_R(k-1)$. By reformulating the target dynamic equation, the process noise can be made uncorrelated with the measurement noise[3].It is unnecessary for decorrelating the system. So that formula (26) can be used for the observation equation in the Kalman filter.

The following decouple tracking filter in rotative coordinate can be obtained:

$$\hat{X}_R(k/k-1) = \Phi(\Delta,\alpha)\hat{X}_R(k-1/k-1), \tag{34}$$

$$\begin{aligned} \hat{X}_R(k/k) = \hat{X}_R(k/k-1) + K_R(k)[Z_R(k) \\ - \hat{Z}_R(k/k-1)], \end{aligned} \tag{35}$$

Let the following substitution be made:

$$\begin{aligned} e_R(k/k-1) &= X_R(k) - \hat{X}_R(k/k-1), \\ e_R(k/k) &= X_R(k) - \hat{X}_R(k/k), \end{aligned} \tag{36}$$

then the following equation is obtained:

$$\begin{aligned} e_R(k/k-1) &= \Phi(\Delta,\alpha)e_R(k-1/k-1) \\ &\quad + W_R(k-1), \\ e_R(k/k) &= (I - K_R(k)H_R^*)e_R(k/k-1). \\ &\quad + K_R(k)V_R^*(k) \end{aligned} \tag{37}$$

then the covariance of the prediction error is:

$$\begin{aligned} &P_R(k/k-1) \\ &= E\{e_R(k/k-1)e_R^T(k/k-1)\} \\ &= \Phi(\Delta,\alpha)P_R(k-1/k-1)\Phi^T(\Delta,\alpha) \\ &+ Q_R(k-1), \end{aligned} \tag{38}$$

and the covariance of estimation error is:

$$\begin{aligned} &P_R(k/k) \\ &= E\{e_R(k/k)e_R^T(k/k)\} \\ &= [I - K_R(k)H_R^*]P_R(k/k-1)[I - K_R(k)H_R^*]^T \\ &+ K_R(k)R_R^*(k)K_R^T(k). \end{aligned} \tag{39}$$

In the rotation coordinate system, covariance matrix of estimation error $P(k)$ has the form of:

$$P_R(k) = \begin{bmatrix} p_{xx} & 0 & 0 & p_{x\dot{x}} & 0 & 0 & p_{x\ddot{x}} & 0 & 0 \\ 0 & p_{yy} & 0 & 0 & p_{y\dot{y}} & 0 & 0 & p_{y\ddot{y}} & 0 \\ 0 & 0 & p_{zz} & 0 & 0 & p_{z\dot{z}} & 0 & 0 & p_{z\ddot{z}} \\ p_{\dot{x}x} & 0 & 0 & p_{\dot{x}\dot{x}} & 0 & 0 & p_{\dot{x}\ddot{x}} & 0 & 0 \\ 0 & p_{\dot{y}y} & 0 & 0 & p_{\dot{y}\dot{y}} & 0 & 0 & p_{\dot{y}\ddot{y}} & 0 \\ 0 & 0 & p_{\dot{z}z} & 0 & 0 & p_{\dot{z}\dot{z}} & 0 & 0 & p_{\dot{z}\ddot{z}} \\ p_{\ddot{x}x} & 0 & 0 & p_{\ddot{x}\dot{x}} & 0 & 0 & p_{\ddot{x}\ddot{x}} & 0 & 0 \\ 0 & p_{\ddot{y}y} & 0 & 0 & p_{\ddot{y}\dot{y}} & 0 & 0 & p_{\ddot{y}\ddot{y}} & 0 \\ 0 & 0 & p_{\ddot{z}z} & 0 & 0 & p_{\ddot{z}\dot{z}} & 0 & 0 & p_{\ddot{z}\ddot{z}} \end{bmatrix} \quad (40)$$

$K_R(k)$ is the gain of Kalman filter in the rotation coordinate. It is described as follows:

$$K_R(k) = P_R(k/k-1)H_R^{*T}[H_R^* P_R(k/k-1)H_R^{*T} + R_R^*(k)]^{-1}. \quad (41)$$

The $K_R(k)$ has the form of:

$$K_R(k) = \begin{bmatrix} k_{11} & 0 & 0 & k_{14} & 0 & 0 \\ 0 & K_{22} & 0 & 0 & k_{25} & 0 \\ 0 & 0 & k_{33} & 0 & 0 & k_{36} \\ k_{41} & 0 & 0 & k_{44} & 0 & 0 \\ 0 & k_{52} & 0 & 0 & k_{55} & 0 \\ 0 & 0 & k_{63} & 0 & 0 & k_{66} \\ k_{71} & 0 & 0 & k_{74} & 0 & 0 \\ 0 & k_{82} & 0 & 0 & k_{85} & 0 \\ 0 & 0 & k_{93} & 0 & 0 & k_{96} \end{bmatrix}, \quad (42)$$

According to equation (39) and (41), the off diagonal terms of the filter error covariance matrix is eliminated as zero, so that the calculation quantity of Kalman filter will be reduced. It is of benefit to real time target tracking.

The estimation gain $K(k)$ in inertial rectangular coordinate is:

$$K(k) = M^{-1}(Y)K_R(k)n(Y). \quad (43)$$

So the state estimation in inertial coordinate is:

$$\hat{X}(k/k-1) = \Phi(\Delta, \alpha)\hat{X}(k-1/k-1), \quad (44)$$

$$\hat{X}(k/k) = \hat{X}(k/k-1) + K(k)[Z(k) - \hat{Z}(k/k-1)] \quad (45)$$

The formulas of (33), (34), (37), (38), (40), (42), (43), and (44) are the Kalman recursive prediction and estimation equations. According to the formulas above mentioned, the target position can be estimated and predicted.

4. Conclusion

A pseudo measurement equation to be used in the target tracking is introduced in this paper. By use of the model of pseudo measurement, not only the Kalman filter with color measurement noise will be solved, but also the computing load will be reduced, comparing with the extended state vector method, and it is of benefit to both real time target tracking and dealing with the measurement noise with first -order autoregressive process $AR(1)$, and as the derivation of sensor data are adopted, the precision of measurement is improved, so that the accuracy of prediction will be improved also.

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Appendix

1 The matrices of $g(X)$ and $c(X)$

The two matrices of $g(X)$ and $c(X)$ denote the coordinate transformations from the rectangular to the polar system. It is shown as follows:
assuming

$$d = \sqrt{x^2 + y^2},$$

then

$$c(X) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{x}{r} & \frac{y}{r} & \frac{z}{r} \\ -\frac{y}{d} & \frac{x}{d} & 0 \\ \frac{xz}{dr^2} & \frac{yz}{dr^2} & \frac{d}{r^2} \end{bmatrix}. \quad (A.1.1)$$

$$g(X) = \begin{bmatrix} g_x & g_y & g_z \end{bmatrix}. \quad (A.1.2)$$

$$g_x = \begin{bmatrix} \frac{x}{r} \\ -\frac{y}{d^2} \\ \frac{xz}{r^2 d} \\ \frac{\dot{x}(z^2+y^2) - x(y\dot{y} + z\dot{z})}{r^3} \\ \frac{\dot{y}y^2 - x^2\dot{y} + 2x\dot{x}y}{d^4} \\ g_{x6} \end{bmatrix},$$

where

$$g_{x6} = \frac{(2x\dot{z} - \dot{x}z)r^2 d^2 - x(2d^2 + r^2)[d^2\dot{z} - (x\dot{x} + y\dot{y})z]}{r^4 d^3}$$

$$g_y = \begin{bmatrix} \frac{y}{r} \\ \frac{x}{d^2} \\ \frac{yz}{r^2 d} \\ \frac{\dot{y}(z^2+x^2) - y(x\dot{x}+z\dot{z})}{r^3} \\ \frac{\dot{x}y^2 - x^2\dot{x} + 2xy\dot{y}}{d^4} \\ g_{y6} \end{bmatrix},$$

and

$$g_{y6} = \frac{(2y\dot{z} - \dot{y}z)r^2 d^2 - y(2d^2+r^2)[d^2\dot{z} - (x\dot{x}+y\dot{y})z]}{r^4 d^3}$$

$$g_z = \begin{bmatrix} \frac{z}{r} \\ 0 \\ \frac{d}{r^2} \\ \frac{\dot{z}d^2 - z(x\dot{x}+y\dot{y})}{r^3} \\ 0 \\ g_{z6} \end{bmatrix}.$$

and

$$g_{z6} = \frac{(x\dot{x}+y\dot{y})(z^2-d^2) - 2d^2 z\dot{z}}{r^4 d}$$

2 The means and covariances of process and measurement noise

The means of the state process noise in the inertial coordinate and in the rotation coordinate are:

$E\{W(k)\} = 0,$

$E\{W_R(k)\} = 0,$

and the means of the measurement noise are:

$E\{\xi(k)\} = 0,$

$E\{\xi_R(k)\} = 0.$

The variances of process noise are:

$E\{W(k)W^T(j)\} = Q(k)\delta_{k,j},$

$E\{W_R(k)W_R^T(j)\} = Q_R(k)\delta_{k,j}.$

The relation of $Q_R(k)$ and $Q(k)$ is shown as following:

$Q_R(k) = M(Y)Q(k)M^T(Y).$

According to reference [1], the Q(k) is shown as following:

$$Q(k) = 2\alpha\sigma_m^2 \int_{(k-1)\Delta}^{k\Delta} \Phi(k\Delta-\lambda,\alpha)\Phi^T(k\Delta-\lambda,\alpha)d\lambda,$$

where σ_m^2 is the variance of the target acceleration.

The variances of measurement noise are:

$E\{\xi(k)\xi^T(j)\} = R(k)\delta_{k,j},$

$E\{\xi_R(k)\xi_R(j)\} = R_R(k)\delta_{k,j}.$

$\xi_R(k)$ and $\xi(k)$ have following relation:

$R_R(k) = n(Y)R(k)n^T(Y),$

where

$$R(k) = diag\{\sigma_r^2, \sigma_a^2, \sigma_e^2, \sigma_{\dot{r}}^2, \sigma_{\dot{\theta}_a}^2, \sigma_{\dot{\theta}_e}^2\}.$$

σ_r^2, $\sigma_{\theta_a}^2$, $\sigma_{\theta_e}^2$, $\sigma_{\dot{r}}^2$,$\sigma_{\dot{\theta}_a}^2$, $\sigma_{\dot{\theta}_e}^2$ are the variances of measurement noise v_r,v_{θ_a},v_{θ_e}, $v_{\dot{r}}$, $v_{\dot{\theta}_a}$, $v_{\dot{\theta}_e}$ respectively. The covariances between process noise and measurement noise are:

$E\{W(k)\xi^T(j)\} = 0,$

$E\{W_R(k)\xi_R^T(j)\} = 0.$

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