On the cohomological coprimality of Galois representations

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論 文 名　： On the cohomological coprimality of Galois representations
（コホモロジー的に互いに素なガロワ表現について）

区　　分　：甲

論　文　内　容　の　要　旨

In this thesis we study the vanishing of some Galois cohomology groups with coefficients in a p-adic Galois representation V associated with a proper smooth variety over a p-adic field or a number field. In the setting mentioned, it is known due to Coates-Sujatha-Wintenberger and Sujatha that the cohomology groups of the Galois group G of the field of definition of the representation V vanish. In this case, we say that V has vanishing G-cohomology. The same can be said for the subgroup H of G that corresponds to the field extension obtained by adjoining p-power roots of unity. Such vanishing results have been known to be useful in the Iwasawa theory of elliptic curves over general p-adic Lie extensions. In this regard we are led to ask which subgroups of G have corresponding cohomology groups that vanish. Such subgroups correspond to restriction of the representation to the Galois group of some extension of the base field. Of particular interest are subgroups which correspond to a Galois representation associated with another variety over the base field. The symmetry of the latter motivates the notion of “cohomological coprimality” of two representations. As the terminology suggests, cohomological coprimality leads to another notion of “independence” between representations. We present in this thesis some results on the cohomological coprimality of Galois representations associated with some varieties over a p-adic field or a number field.

Chapters 3, 4 and 5 are expositions of topics which are essential in this thesis. First, we recall some necessary background in cohomology of profinite groups and Lie algebras in Chapter 3, as well as in p-adic Hodge theory in Chapter 4. In Chapter 5, results on the vanishing of the cohomology groups of G and of H are recalled. Since these results are essential in this thesis, we reproduce the proofs of Coates, Sujatha, and Wintenberger in the case where the base field is a p-adic field and of Sujatha in the case where the base field is a number field. A sketch of the idea of the proofs is as follows. Consider the cohomology groups of the Lie algebra of G. These Lie algebra cohomology groups are shown to be trivial using a criterion named as “strong Serre criterion”. This criterion requires an element of the Lie algebra of G with some conditions on its eigenvalues. In the local setting, the construction of the element uses Tannakian formalism and the theory of unramified p-adic representations. In the global setting, the desired element is obtained from Sen’s operator by using Bogomolov’s result which states that the Lie algebra of G contains the homotheties. Once this element has been obtained, the vanishing of the

cohomology groups of G is obtained by a theorem of Lazard which allows us to identify cohomology groups of G as vector subspaces of the corresponding Lie algebra cohomology groups. The same idea is applied to prove that the cohomology groups of H are trivial.

Following Serre, we discuss in Chapter 6 the almost independence of systems of representations of a profinite group. We prove some properties of almost independent systems which are used in the proof of a cohomological coprimality result in the global setting.

The proof of our results are given in Chapter 7. We determine some criteria for the vanishing of cohomology. These criteria are special consequences of the results of Coates, Sujatha and Wintenberger mentioned above. We show that a criterion by Ozeki for the vanishing of the 0^{th}-cohomology group for abelian varieties with potential good ordinary reduction over a p-adic field is in fact a criterion for the vanishing of all the cohomology groups. We use these criteria to prove our cohomological coprimality results in the local setting. By showing the cohomological coprimality of p-adic Galois representations that correspond to two "different" elliptic curves over a p-adic field, we are able to extend as well the results of Ozeki on the vanishing of the 0^{th}-cohomology group to the vanishing of all cohomology groups for elliptic curves. In the global setting, we can say more. From the results on almost independence in Chapter 6, we may extend the theorem of Sujatha to systems of representations associated with a proper smooth variety over a number field indexed by a set of primes. This extension, together with the fact that the intersection of the fields of definition of two non-isogenous elliptic curves is of finite degree over the maximal cyclotomic extension of the base field, is used to prove the cohomological coprimality of two systems of Galois representations of a number field associated with non-isogenous elliptic curves.

Finally, we include two appendices. In Appendix A, we recall some definition and essential results on Tannakian formalism which are used in the proof of Coates, Sujatha and Wintenberger. Some definitions and results in the theory of Lie algebras are also listed in Appendix B.