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出版情報：Algorithmic Game Theory. 9347, pp.3-14, 2015. Springer Berlin Heidelberg
バージョン：
権利関係：

## MI Preprint Series

**Mathematics for Industry**
**Kyushu University**


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**MI 2015-3**

( Received April 23, 2015 )



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# Stable Matchings with Ties, Master Preference Lists, and Matroid Constraints

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**Abstract**

In this paper, we consider a matroid generalization of the hospitals/residents problem with ties and master lists. In this model, the capacity constraints for hospitals are generalized to matroid constraints. By generalizing the algorithms of O'Malley for the hospitals/residents problem with ties and master lists, we give polynomial-time algorithms for deciding whether there exist a super-stable matching and a strongly stable matching in our model, and finding such matchings, if they exist.

## 1 Introduction

The stable matching problem introduced by Gale and Shapley [4] is one of the most popular mathematical models of a matching problem in which agents have preferences. It is known [4] that if preference lists do not contain ties, then there always exists a stable matching and we can find one in polynomial time. However, if there exist ties in preference lists, then the situation dramatically changes. In the stable matching problem with ties, three stability concepts were proposed by Irving [5] (see also Chapter 3 of [11] for a survey of the stable matching problem with ties). The first one is called the weak stability. This stability concept guarantees that there exists no unmatched pair each of whom prefers the other to the current partner. It is known [5] that there always exists a weakly stable matching and we can find one in polynomial time by slightly modifying the algorithm of [4]. The second one is called the strong stability. This stability concept guarantees that there exists no unmatched pair such that (i) there exists an agent $a$ in this pair that prefers the other to the current partner, and (ii) the agent in this pair other than $a$ prefers $a$ to the current partner, or is indifferent between $a$ and the current partner. The last one is called the super-stability. This stability concept guarantees that there exists no unmatched pair each of whom prefers the other to the current partner or is indifferent between the other and the current partner.

One of the most notable differences between the last two concepts and the stability concept in the stable matching problem without ties is that there may exist no stable matching [5]. Thus, from the algorithmic viewpoint, it is important to reveal whether we can check the existence of matchings satisfying such stability concepts in polynomial time. For the one-to-one setting (i.e., the stable matching problem with ties), Irving [5] gave polynomial-time algorithms for finding a super-stable matching and a strongly stable matching (see also [10]). For the many-to-one setting (i.e., the hospitals/residents problem with ties), Irving, Manlove, and Scott [6] gave a polynomial-time algorithm for finding a super-stable matching, and Irving, Manlove, and

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Scott [7] a polynomial-time algorithm for finding a strongly stable matching. It should be noted that Kavitha, Mehlhorn, Michail, and Paluch [9] proposed a faster algorithm for this setting.

In this paper, we consider a matroid generalization of the hospitals/residents problem with ties. In our model, the capacity constraints for hospitals are generalized to matroid constraints. In contrast to the stable matching problem without ties (and the weak stability) [3], to the best of our knowledge, it is open whether we can extend the results about the super-stability and the strong stability to the matroid setting. In this paper, as a step toward the settlement of this question, we focus on the situation in which we are given a master list and the preference list of each hospital over residents is derived from this master list. For the stable matching problem with ties and master lists, Irving, Manlove, and Scott [8] gave simple polynomial-time algorithms for finding a super-stable matching and a strongly stable matching. Furthermore, O'Malley [12] gave polynomial-time algorithms for finding a super-stable matching and a strongly stable matching in the hospitals/residents problem with ties and master lists. In this paper, by generalizing the algorithms of [12], we give polynomial-time algorithms for finding a super-stability matching and a strongly stable matching in a matroid generalization of the hospitals/residents problem with ties and master lists, i.e., a partial positive answer for the above question.

It should be noted that Abraham, Irving, and Manlove [1] posed an open problem related to the super-stability and the strongly stability in the student-project allocation problem that is a variation of the hospitals/residents problem having hierarchical capacity constraints (see also [11, p.268]). As pointed in [1], these hierarchical capacity constraints can be expressed by using a matroid. However, since the definitions of the super-stability and the strong stability in the student-project allocation problem with ties were not explicitly given in [1], we do not discuss about the relationship between their model and our model. Instead, we give a concrete example in the student-project allocation setting that our model can express.

The rest of this paper is organized as follows. In Section 2, we formally define our problem and give basics of matroids. In Sections 3 and 4, we give polynomial-time algorithms for finding a super-stable matching and a strongly stable matching in our model, respectively. In Section 5, we give examples of concrete models that our model can express.

## 2 Preliminaries

For each set $X$ and each element $x$, we define $X + x := X \cup \{x\}$ and $X - x := X \setminus \{x\}$. A pair $\mathbf{M} = (U, \mathcal{I})$ is called a *matroid*, if $U$ is a finite set and $\mathcal{I}$ is a family of subsets of $U$ satisfying the following conditions.

**(I0)** $\emptyset \in \mathcal{I}$.

**(I1)** If $I \in \mathcal{I}$ and $J \subseteq I$, then $J \in \mathcal{I}$.

**(I2)** If $I, J \in \mathcal{I}$ and $|I| < |J|$, then $I + u \in \mathcal{I}$ for some element $u$ in $J \setminus I$.

### 2.1 Problem formulation

In this paper, we are given a simple bipartite graph $G = (R \cup H, E)$ such that its vertex set is partitioned into two disjoint subsets $R$ and $H$, and every edge in $E$ connects a vertex in $R$ and a vertex in $H$. We call a vertex in $R$ (resp., $H$) a *resident* (resp., a *hospital*). For each vertex $v$ in $R \cup H$ and each subset $F$ of $E$, we denote by $F(v)$ the set of edges in $F$ incident to $v$. For each resident $r$ in $R$ and each hospital $h$ in $H$, if there exists an edge in $E$ connecting $r$ and $h$, then we denote by $(r, h)$ this edge. Furthermore, we are given a matroid $\mathbf{N} = (E, \mathcal{F})$ such that

$\{e\} \in \mathcal{F}$ for every edge $e$ in $E$. The matroid $\mathbf{N}$ represents constraints on assignment of residents to hospitals. For example, it can express capacity constraints in the hospitals/residents problem (see Section 5). For each resident $r$ in $R$, we are given a reflexive and transitive binary relation $\succsim_r$ on $E(r) \cup \{\emptyset\}$ such that at least one of $e \succsim_r f$ and $f \succsim_r e$ holds for every edges $e, f$ in $E(r)$, and $e \succsim_r \emptyset$ and $\emptyset \not\succsim_r e$ for every edge $e$ in $E(r)$. For each resident $r$ in $R$, the binary relation $\succsim_r$ represents the preference list of $r$. Namely, for each resident $r$ in $R$ and each edges $e, f$ in $E(r)$, $e \succsim_r f$ means that $r$ preferes $e$ to $f$, or is indifferent between $e$ and $f$. For each resident $r$ in $R$ and each edges $e, f$ in $E(r)$, we write $e \succ_r f$ (resp., $e \sim_r f$), if $e \succsim_r f$ and $e \not\succsim_r f$ (resp., $e \succsim_r f$ and $f \succsim_r e$). Furthermore, we are given a reflexive and transitive binary relation $\succsim_H$ on $R$ such that at least one of $r \succsim_H s$ and $s \succsim_H r$ holds for every residents $r, s$ in $R$. The binary relation $\succsim_H$ represents the master preference list of hospitals in $H$. We define the notations $\succ_H$ and $\sim_H$ in the same way as $\succ_r$ and $\sim_r$ for a resident $r$ in $R$.

A subset $M$ of $E$ is called a *matching* in $G$, if

**(M1)** $|M(r)| \leq 1$ for every resident $r$ in $R$, and

**(M2)** $M \in \mathcal{F}$.

For each matching $M$ in $G$ and each resident $r$ in $R$ such that $M(r) \neq \emptyset$, we do not distinguish between $M(r)$ and its unique element. For each matching $M$ in $G$ and each edge $e = (r, h)$ in $E \setminus M$, we say that $r$ *weakly* (resp., *strictly*) *prefers* $e$ on $M$, if $e \succsim_r M(r)$ (resp., $e \succ_r M(r)$). In addition, for each matching $M$ in $G$ and each edge $e = (r, h)$ in $E \setminus M$, we say that $H$ *weakly* (resp., *strictly*) *prefers* $e$ on $M$, if

**(P1)** $M + e \in \mathcal{F}$, and/or

**(P2)** there exists an edge $f = (s, p)$ in $M$ such that $M + e - f \in \mathcal{F}$ and $r \succsim_H s$ (resp., $r \succ_H s$).

A matching $M$ in $G$ is said to be *super-stable*, if there exists no edge $(r, h)$ in $E \setminus M$ such that $r$ and $H$ weakly prefer $(r, h)$ on $M$. Furthermore, a matching $M$ in $G$ is said to be *strongly stable*, if there exists no edge $(r, h)$ in $E \setminus M$ such that

**(S1)** $r$ and $H$ weakly prefer $(r, h)$ on $M$, and

**(S2)** at least one of $r$ and $H$ strictly prefers $(r, h)$ on $M$.

Unfortunately, it is known [5] that there may exist no matching satisfying the above conditions. Thus, our goal is to give a polynomial-time algorithm for deciding whether there exists a matching satisfying the above conditions, and find such a matching, if one exists.

## 2.2 Basics of matroids

Let $\mathbf{M} = (U, \mathcal{I})$ be a matroid. A subset of $U$ belonging to $\mathcal{I}$ is called an *independent set of* $\mathbf{M}$. A subset $C$ of $U$ is called a *circuit* of $\mathbf{M}$, if $C$ is not an independent set of $\mathbf{M}$, but every proper subset of $C$ is an independent set of $\mathbf{M}$. Notice that the condition (I1) implies that for every subset $X$ of $U$, if there exists a circuit of $\mathbf{M}$ that is a subset of $X$, then $X \notin \mathcal{I}$. Let $I$ and $u$ be an independent set of $\mathbf{M}$ and an element in $U \setminus I$ such that $I + u \notin \mathcal{I}$, respectively. It is known [13, Proposition 1.1.6] that there exists a unique circuit of $\mathbf{M}$ that is a subset of $I + u$, and $u$ belongs to this unique circuit. This circuit is called the *fundamental circuit of* $u$, and denoted by $\mathsf{C}_{\mathbf{M}}(u, I)$. Define

$$\mathsf{C}^-_{\mathbf{M}}(u, I) := \mathsf{C}_{\mathbf{M}}(u, I) - u.$$

It is known [13, p.20, Exercise 5] that we have

$$\mathsf{C}_{\mathbf{M}}(u, I) = \{w \in I + u \mid I + u - w \in \mathcal{I}\}.$$

It should be noted that for each subset $M$ of $E$ and each edge $e$ in $E \setminus M$ such that $M + e \notin \mathcal{F}$, the condition (P2) can be restated as follows.

- There exists an edge $f = (s, p)$ in $\mathsf{C}^-_{\mathbf{N}}(e, M)$ such that $r \succsim_H s$ (resp., $r \succ_H s$).

A maximal independent set of $\mathbf{M}$ is called a *base* of $\mathbf{M}$. The condition (I2) implies that every base of $\mathbf{M}$ has the same size. For each subset $X$ of $U$, we define

$$\mathcal{I}|X := \{J \subseteq X \mid J \in \mathcal{I}\}, \qquad \mathbf{M}|X := (X, \mathcal{I}|X).$$

It is known [13, p.20] that for each subset $X$ of $U$, $\mathbf{M}|X$ is a matroid. For each subset $X$ of $U$, we define $\mathsf{r}_{\mathbf{M}}(X)$ as the size of a base of $\mathbf{M}|X$. It is not difficult to see that for every subset $B$ of $U$, $B$ is a base of $\mathbf{M}$ if and only if $|B| = \mathsf{r}_{\mathbf{M}}(U)$. For each disjoint subsets $X, J$ of $U$, we define

$$\mathsf{p}(J; X) := \mathsf{r}_{\mathbf{M}}(J \cup X) - \mathsf{r}_{\mathbf{M}}(X).$$

For each subset $X$ of $U$, we define

$$\mathcal{I}/X := \{J \subseteq U \setminus X \mid \mathsf{p}(J; X) = |J|\}, \qquad \mathbf{M}/X := (U \setminus X, \mathcal{I}/X).$$

It is known [13, Proposition 3.1.6] that for each subset $X$ of $U$, $\mathbf{M}/X$ is a matroid.

The following property of circuits is used in the sequel.

**Theorem 1** (See, e.g., [13, p.15, Exercise 14])**.** *Assume that we are given a matroid $\mathbf{M} = (U, \mathcal{I})$ and circuits $C_1, C_2$ of $\mathbf{M}$ such that $C_1 \cap C_2 \neq \emptyset$ and $C_1 \setminus C_2 \neq \emptyset$. Then, for every element $u$ in $C_1 \cap C_2$ and every element $w$ in $C_1 \setminus C_2$, there exists a circuit $C$ of $\mathbf{M}$ such that $w \in C$ and $C$ is a subset of $(C_1 \cup C_2) - u$.*

Although the following lemmas easily follow, we give their proofs for completeness.

**Lemma 2.** *Assume that we are given a matroid $\mathbf{M} = (U, \mathcal{I})$ and independent sets $I, J$ of $\mathbf{M}$ such that $I \subseteq J$. Then, for every element $u$ in $U \setminus J$ such that $I + u \notin \mathcal{I}$, we have $J + u \notin \mathcal{I}$ and $\mathsf{C}_{\mathbf{M}}(u, I) = \mathsf{C}_{\mathbf{M}}(u, J)$.*

*Proof.* Let $u$ be an element in $U \setminus J$ such that $I + u \notin \mathcal{I}$. Since $I \subseteq J$, $\mathsf{C}_{\mathbf{M}}(u, I)$ is a subset of $J + u$, which implies that $J + u \notin \mathcal{I}$. Define $C_1 := \mathsf{C}_{\mathbf{M}}(u, I)$ and $C_2 := \mathsf{C}_{\mathbf{M}}(u, J)$. Assume that $C_1 \neq C_2$. Since $u$ belongs to $C_1 \cap C_2$, Theorem 1 implies that there exists a circuit $C$ of $\mathbf{M}$ such that $C$ is a subset of $(C_1 \cup C_2) - u$. In addition, since $C_1 - u \subseteq I \subseteq J$ and $C_2 - u \subseteq J$, we have $C \subseteq J$. This contradicts the fact that $J \in \mathcal{I}$. □

**Lemma 3.** *Assume that we are given a matroid $\mathbf{M} = (U, \mathcal{I})$, circuits $C, C_1, C_2, \ldots, C_k$ of $\mathbf{M}$, and elements $u_1, u_2, \ldots, u_k$ in $U$ such that $u_i \in C \cap C_i$ for every $i = 1, 2, \ldots, k$. In addition, we assume that there exists an element $w$ in $U$ such that $w \in C$ and $w \notin C_i$ for every $i = 1, 2, \ldots, k$. Then, there exists a circuit $C'$ of $\mathbf{N}$ such that $C' \subseteq (C \cup C_1 \cup C_2 \cup \cdots \cup C_k) \setminus \{u_1, u_2, \ldots, u_k\}$.*

*Proof.* We consider the following procedure.

**Step 1.** Set $t := 1$ and $K_0 := C$.

**Step 2.** If $t \leq k$, then do the following steps. Otherwise, go to **Step 3**.

**(2-a)** If $u_t \notin K_{t-1}$, then go to **Step (2-c)**.

**(2-b)** Find a circuit $K_t$ of $\mathbf{N}$ such that $w \in K_t$ and $K_t$ is a subset of $(K_{t-1} \cup C_t) - u_t$.

**(2-c)** Update $t := t + 1$ and go back to the beginning of **Step 2**.

**Step 3.** Output $K_k$ and halt.

Notice that Theorem 1 implies that in **Step (2-b)** there exists a circuit $K_t$ satisfying the above properties. It is not difficult to see that $K_k$ is a subset of $C \cup C_1 \cup C_2 \cup \cdots \cup C_k$ and $u_i \notin K_k$ for every $i = 1, 2, \ldots, k$. This completes the proof. □

**Lemma 4.** *Assume that we are given a matroid $\mathbf{M} = (U, \mathcal{I})$ and a subset $X$ of $U$.*

1. *Let $B$ be an arbitrary base of $\mathbf{M}|X$. For every subset $I$ of $U \setminus X$, $I$ is an independent set of $\mathbf{M}/X$ if and only if $I \cup B$ is an independent set of $\mathbf{M}$.*

2. *For every base $B_1$ of $\mathbf{M}|X$ and every base $B_2$ of $\mathbf{M}/X$, $B_1 \cup B_2$ is a base of $\mathbf{M}$.*

3. *For every subset $I$ of $U$ such that $I \cap X$ is a base of $\mathbf{M}|X$, if $I \setminus X$ is not an independent set of $\mathbf{M}/X$, then $I$ is not an independent set of $\mathbf{M}$.*

*Proof.* The statement (1) follows from Proposition 3.1.7 of [13]. Next we consider the statement (2). Assume that $B_1 \cup B_2$ is not a base of $\mathbf{M}$. Then, there exists an element $u$ in $U \setminus (B_1 \cup B_2)$ such that $(B_1 \cup B_2) + u$ is an independent set of $\mathbf{M}$. If $u$ belongs to $X$, then this contradicts the fact that $B_1$ is a base of $\mathbf{M}|X$. Thus, $u$ does not belong to $X$. Since $B_1$ is a base of $\mathbf{M}$, the statement (1) implies that $B_2 + u$ is an independent set of $\mathbf{M}/X$. This contradicts the fact that $B_2$ is a base of $\mathbf{M}/X$. Lastly we prove the statement (3). Assume that $I \in \mathcal{I}$. Since $I \cap X$ is a base of $\mathbf{M}|X$, the statement (1) implies that $I \setminus X$ is an independent set of $\mathbf{M}/X$. This contradicts the fact that $I \setminus X$ is not an independent set of $\mathbf{M}/X$. □

Let $\mathbf{M}_1 = (U, \mathcal{I}_1)$ and $\mathbf{M}_2 = (U, \mathcal{I}_2)$ be matroids on the same ground set. A subset $I$ of $U$ is called a *common independent set of* $\mathbf{M}_1$ *and* $\mathbf{M}_2$, if $I$ belongs to $\mathcal{I}_1 \cap \mathcal{I}_2$. It is known that we can find a maximum-size common independent set in time bounded by a polynomial in $|U|$ and $\gamma$, where $\gamma$ is the time required to decide whether $X$ is a common independent set of $\mathbf{M}_1$ and $\mathbf{M}_2$ for one subset $X$ of $U$. If we use the algorithm proposed by Cunningham [2], then we can find a maximum-size common independent set in $O(|U|^{2.5}\gamma)$ time.

## 3 Super-Stable Matchings

In this section, we consider the problem of deciding whether there exists a super-stable matching in $G$, and find such a matching, if one exists. We first partition $R$ into non-empty disjoint subsets $R_1, R_2, \ldots, R_n$ such that

1. $r \sim_H s$ for every $i = 1, 2, \ldots, n$ and every residents $r, s$ in $R_i$, and

2. $r \succ_H s$ for every $i, j = 1, 2, \ldots, n$ such that $i < j$ and every residents $r$ in $R_i$ and $s$ in $R_j$.

For each $i, j = 1, 2, \ldots, n$, we define $R_{i,j} := R_i \cup R_{i+1} \cup \cdots \cup R_j$.

Our algorithm `SuperM` is described as follows.

**Algorithm** `SuperM`

**Step 1.** Define $i := 1$, $M_0 := \emptyset$, and $D_0 := \emptyset$.

**Step 2.** If $i \leq n$, then do the following steps. Otherwise, go to **Step 3**.

**(2-a)** For each resident $r$ in $R_i$, we define $T_r$ by

$$T_r := \Big\{ e \in E(r) \setminus D_{i-1} \;\Big|\; \forall f \in E(r) \setminus D_{i-1} \colon e \succsim_r f \Big\}.$$

**(2-b)** If there exists a resident $r$ in $R_i$ such that $|T_r| > 1$, then output **null** and halt (i.e., there exists no super-stable matching in $G$).

**(2-c)** Define $E_i := \cup_{r \in R_i} T_r$. If $M_{i-1} \cup E_i \notin \mathcal{F}$, then output **null** and halt.

**(2-d)** Define $M_i := M_{i-1} \cup E_i$.

**(2-e)** Define

$$L_i := \Big\{ (r,h) \in E \setminus D_{i-1} \;\Big|\; r \in R_{i+1,n},\ M_i + (r,h) \notin \mathcal{F} \Big\}.$$

Furthermore, define $D_i := D_{i-1} \cup L_i$.

**(2-f)** Update $i := i + 1$ and go back to the beginning of **Step 2**.

**Step 3.** Output $M_n$ and halt (i.e., $M_n$ is a super-stable matching in $G$).

**End of Algorithm**

In the sequel, we prove the correctness of the algorithm `SuperM`.

**Lemma 5.** *Assume that the algorithm* `SuperM` *halts when $i = \delta$. Then, for any edge $e$ in $D_{\delta-1}$, there does not exist a super-stable matching $N$ in $G$ such that $e \in N$.*

*Proof.* An edge $e$ in $D_{\delta-1}$ is said to be *bad*, if there exists a super-stable matching $N$ in $G$ such that $e \in N$. We prove this lemma by contradiction. Assume that there exists a bad edge in $D_{\delta-1}$. Define $\Delta$ as the set of integers $\ell$ in $\{1, 2, \dots, \delta - 1\}$ such that there exists a bad edge in $L_\ell$. Let $j$ be the minimum integer in $\Delta$. Let $e = (r,h)$ and $N$ be a bad edge in $L_j$ and a super-stable matching such that $e \in N$, respectively. Since $e$ belongs to $L_j$, $M_j + e \notin \mathcal{F}$. Define $C := \mathsf{C}_{\mathbf{N}}(e, M_j)$. If $C$ is a subset of $N$, then this contradicts the fact that $N \in \mathcal{F}$. Thus, $C \setminus N$ is not empty. For every edge $(s,p)$ in $C \setminus N$, since $r \in R_{j+1,k}$ and $s \in R_{1,j}$, we have $s \succ_H r$.

We first assume that there exists an edge $f = (s,p)$ in $C \setminus N$ such that $N(s) \succ_s f$. Since $e$ belongs to $C \cap N$, we have $e \neq f$. This implies that $f$ belongs to $M_j$, i.e., $f \in T_s$. Thus, the definition of the algorithm `SuperM` implies that $N(s)$ belongs to $D_{j-1}$, i.e., $N(s) \in L_k$ for an integer $k$ in $\{1, 2, \dots, j-1\}$. Furthermore, since $D_{j-1} \subseteq D_{\delta-1}$ and $N$ is super-stable, $N(s)$ is bad. This contradicts the minimality of $j$.

Next we assume that $f \succsim_s N(s)$ for every edge $f = (s,p)$ in $C \setminus N$. If there exists an edge $f$ in $C \setminus N$ such that $N + f \in \mathcal{F}$, then this contradicts the fact that $N$ is super-stable. Thus, we assume that $N + f \notin \mathcal{F}$ for every edge $f$ in $C \setminus N$. For each edge $f$ in $C \setminus N$, we define $C_f$ as $\mathsf{C}_{\mathbf{N}}(f, N)$. Since $N$ is super-stable, $t \succ_H s$ for every edge $f = (s,p)$ in $C \setminus N$ and every edge $g = (t,q)$ in $C_f - f$. For every edge $f = (s,p)$ in $C \setminus N$, since $s \succ_H r$, this implies that $e \notin C_f$. In addition, $f \in C \cap C_f$ for every edge $f$ in $C \setminus N$. Thus, Lemma 3 implies that there exists a circuit $C'$ of $\mathbf{N}$ such that $C'$ is a subset of $(C \cup C^*) \setminus (C \setminus N)$, where $C^*$ is $\cup_{f \in C \setminus N} C_f$. Thus, since $C_f - f$ is a subset of $N$, $C'$ is a subset of $N$. This contradicts the fact that $N \in \mathcal{F}$. $\square$

**Lemma 6.** *Assume that the algorithm* `SuperM` *halts when $i = \delta$, and we are given a super-stable matching $N$ in $G$. Then, for every resident $r$ in $R_{1,\delta}$ such that $T_r = \emptyset$ (resp., $T_r \neq \emptyset$), we have $N(r) = \emptyset$ (resp., $N(r) \in T_r$).*

*Proof.* For every resident $r$ in $R_{1,\delta}$ such that $T_r = \emptyset$, every edge in $E(r)$ belongs to $D_{\delta-1}$. Thus, Lemma 5 implies that $N(r) = \emptyset$ for every resident $r$ in $R_{1,\delta}$ such that $T_r = \emptyset$.

From here, we consider a resident $r$ in $R_{1,\delta}$ such that $T_r \neq \emptyset$. We assume that $r$ belongs to $R_j$ for an integer $j$ in $\{1, 2, \ldots, \delta\}$. If $N(r) \in T_r$, then the proof is done. Thus, we assume that $N(r)$ does not belong to $T_r$, and we prove this lemma by contradiction. If $N(r) \succsim_r e$ for an edge $e$ in $T_r$ (notice that $f \sim_r g$ for every edges $f, g$ in $T_r$), then the definition of $T_r$ implies that $N(r)$ belongs to $D_{\delta-1}$, which contradicts Lemma 5. Thus, we assume that $e \succ_r N(r)$ for an edge $e$ in $T_r$. In addition, we assume that there exists no resident $s$ in $R_{1,j-1}$ such that $T_s \neq \emptyset$ and $e \succ_s N(s)$ for an edge $e$ in $T_s$. Notice that such a resident $r$ clearly exists.

Let $e$ be an edge in $T_r$. Since $e$ belongs to $T_r$, $e$ does not belong to $D_{j-1}$. Since this implies that $e \notin D_{j-2}$ and $e \notin L_{j-1}$, we have $M_{j-1} + e \in \mathcal{F}$. Since $N(r)$ does not belong to $T_r$, $e$ does not belong to $N$. Furthermore, since $N$ is super-stable, $N + e \notin \mathcal{F}$ and $s \succ_H r$ for every edge $(s, p)$ in $\mathsf{C}^-_{\mathbf{N}}(e, N)$. Here we prove that $\mathsf{C}_{\mathbf{N}}(e, N)$ is a subset of $M_{j-1} + e$. This contradicts the fact that $M_{j-1} + e \in \mathcal{F}$. Let $f = (s, p)$ be an edge in $\mathsf{C}^-_{\mathbf{N}}(e, N)$. Since $s \succ_H r$, $s$ belongs to $R_k$ for an integer $k$ in $\{1, 2, \ldots, j-1\}$. This and the definition of the algorithm `superM` imply that $|T_s| \leq 1$. If $T_s = \emptyset$, then in the same way as above we can prove that $N(s) = \emptyset$, which contradict $f = N(s)$. Thus, $|T_s| = 1$. In this case, the unique edge in $T_s$ belongs to $M_{j-1}$. Thus, if $f \in T_s$, then the proof is done. Assume that $f$ does not belong to $T_s$. In this case, since $f = N(s)$, the definition of $r$ implies that $N(s) \succsim_s g$ for an edge $g$ in $T_s$. This implies that $f$ belongs to $D_{\delta-1}$. Since $f$ belongs to a super-stable matching $N$, this contradicts Lemma 5. □

**Lemma 7.** *If the algorithm* `SuperM` *outputs* $M_n$*, then* $M_n$ *is a super-stable matching in* $G$*.*

*Proof.* For every $\ell = 1, 2, \ldots, n$ and every resident $r$ in $R_\ell$, $|E_\ell(r)| \leq 1$. Thus, since $M_\ell \in \mathcal{F}$ for every $\ell = 1, 2, \ldots, n$, $M_n$ is a matching in $G$. What remains is to prove that $M_n$ is super-stable. Let $e = (r, h)$ be an edge in $E \setminus M_n$. We prove that at least one of $r$ and $H$ does not weakly prefer $e$ on $M_n$. If $M_n(r) \succ_r e$, then the proof is done. Thus, we consider the case where $e \succsim_r M_n(r)$. Assume that $r$ belongs to $R_j$ for an integer $j$ in $\{1, 2, \ldots, n\}$. Since $M_n(r)$ belongs to $T_r$ and $|T_r| = 1$, the definition of the algorithm `SuperM` implies that $e$ belongs to $L_k$ for an integer $k$ in $\{1, 2 \ldots, j-1\}$. Thus, $M_k + e \notin \mathcal{F}$ and $s \succ_H r$ for every edge $(s, p)$ in $\mathsf{C}^-_{\mathbf{N}}(e, M_k)$. Since $M_k$ is a subset of $M_n$, Lemma 2 implies that $M_n + e \notin \mathcal{F}$ and $\mathsf{C}_{\mathbf{N}}(e, M_k) = \mathsf{C}_{\mathbf{N}}(e, M_n)$. Thus, $s \succ_H r$ for every edge $f = (s, p)$ in $\mathsf{C}^-_{\mathbf{N}}(e, M_n)$. This completes the proof. □

**Lemma 8.** *If the algorithm* `SuperM` *outputs* **null***, then there exists no super-stable matching in* $G$*.*

*Proof.* We assume that the algorithm `SuperM` outputs **null** when $i = \delta$. We prove this lemma by contradiction. Assume that there exists a super-stable matching $N$ in $G$.

We first consider the case where in **Step (2-b)** the algorithm `SuperM` outputs **null**. In this case, there exists a resident $r$ in $R_\delta$ such that $|T_r| > 1$. Lemma 6 implies that $N(r)$ belongs to $T_r$. Let $e$ be an edge in $T_r$ such that $e \neq N(r)$. Since $e$ and $N(r)$ belong to $T_r$, we have $e \sim_r N(r)$. Thus, what remains is to prove that $N + e \in \mathcal{F}$ and/or there exists an edge $f = (s, p)$ in $N$ such that $N + e - f \in \mathcal{F}$ and $r \succsim_H s$, which contradicts the fact that $N$ is super-stable. If $N + e \in \mathcal{F}$, then proof is done. Thus, we assume that $N + e \notin \mathcal{F}$ and $s \succ_H r$ (i.e., $s$ belongs to $R_{1,\delta-1}$) for every edge $(s, p)$ in $\mathsf{C}^-_{\mathbf{N}}(e, N)$. Since $e \in T_r$, $M_{\delta-1} + e \in \mathcal{F}$. Lemma 6 and the definition of the algorithm `SuperM` imply that $N(s) = M_{\delta-1}(s)$ for every resident $s$ in $R_{1,\delta-1}$. These imply that $\mathsf{C}_{\mathbf{N}}(e, N)$ is a subset of $M_{\delta-1} + e$, which contradicts the fact that $M_{\delta-1} + e \in \mathcal{F}$.

Next we consider the case where in **Step (2-c)** the algorithm `SuperM` outputs **null**. That is, $M_{\delta-1} \cup E_\delta \notin \mathcal{F}$. Lemma 6 and the definition of the algorithm of `SuperM` imply that $M_{\delta-1}(r) =$

$N(r)$ for every resident $r$ in $R_{1,\delta-1}$ and $E_\delta(r) = N(r)$ for every resident $r$ in $R_\delta$. These imply that $M_{\delta-1} \cup E_\delta$ is a subset of $N$, which contradicts the fact that $N \in \mathcal{F}$. □

Lemmas 7 and 8 imply the following theorem.

**Theorem 9.** *The algorithm* `SuperM` *can decide whether there exists a super-stable matching in* $G$*, and find such a matching, if one exists.*

Here we consider the time complexity of `SuperM`. Define $m := |E|$, and we denote by $\gamma$ the time required to decide whether $F \in \mathcal{F}$ for one subset $F$ of $E$. For simplicity, we assume that $\gamma = \Omega(m)$ and $m \geq |R|$. Since the number of iterations of **Step 2** is at most $m$, we consider the time complexity of one iteration of **Step 2**. Clearly, **Step (2-e)** is the bottleneck and the time complexity of this step is $O(m\gamma)$. Thus, the time complexity of the algorithm `SuperM` is $O(m^2\gamma)$.

# 4 Strongly Stable Matchings

In this section, we give an algorithm for the problem of deciding whether there exists a strongly stable matching in $G$, and find such a matching, if one exists. We define subsets $R_1, R_2, \ldots, R_n$ of $R$ and the notation $R_{i,j}$ in the same way as in Section 3.

Our algorithm `StrongM` is described as follows.

**Algorithm `StrongM`**

**Step 1.** Define $i := 1$, $M_0 := \emptyset$, $D_0 := \emptyset$, and $P_0 := \emptyset$.

**Step 2.** If $i \leq n$, then do the following steps. Otherwise, go to **Step 3**.

**(2-a)** For each resident $r$ in $R_i$, we define $T_r$ by

$$T_r := \Big\{ e \in E(r) \setminus D_{i-1} \;\Big|\; \forall f \in E(r) \setminus D_{i-1} \colon e \succsim_r f \Big\}.$$

**(2-b)** Define

$$E_i := \bigcup_{r \in R_i} T_r, \quad P_i := \bigcup_{\ell = 1,2,\ldots,i} E_\ell, \quad \mathbf{N}_i := (\mathbf{N}|P_i)/P_{i-1}, \quad R_i^* := \{a \in R_i \mid T_r \neq \emptyset\}.$$

**(2-c)** If $r_{\mathbf{N}_i}(E_i) > |R_i^*|$, then output **null** and halt.

**(2-d)** Define

$$\mathcal{U}_i := \Big\{ F \subseteq E_i \;\Big|\; \forall r \in R_i \colon |F(r)| \leq 1 \Big\}, \qquad \mathbf{A}_i := (E_i, \mathcal{U}_i).$$

Notice that $\mathbf{A}_i$ is a matroid. Then, find a maximum-size common independent set $F_i$ of $\mathbf{A}_i$ and $\mathbf{N}_i$.

**(2-e)** If $|F_i| < |R_i^*|$, then output **null** and halt. Otherwise, define $M_i := M_{i-1} \cup F_i$.

**(2-f)** Define

$$L_i := \Big\{ (r,h) \in E \setminus D_{i-1} \;\Big|\; r \in R_{i+1,n},\; M_i + (r,h) \notin \mathcal{F} \Big\}.$$

Furthermore, define $D_i := D_{i-1} \cup L_i$.

**(2-g)** Update $i := i + 1$ and go to the beginning of **Step 2**.

**Step 3.** Output $M_n$ and halt.

**End of Algorithm**

In the sequel, we prove the correctness of the algorithm `StrongM`.

**Lemma 10.** *Assume that the algorithm* `StrongM` *halts when* $i = \delta$. *Then,*

1. $F_\ell$ *is a base of* $\mathbf{N}_\ell$ *for every* $\ell = 1, 2, \ldots, \delta - 1$, *and*
2. $M_\ell$ *is a base of* $\mathbf{N}|P_\ell$ *for every* $\ell = 1, 2, \ldots, \delta - 1$.

*Proof.* We first consider the statement (1). Let us fix an integer $\ell$ in $\{1, 2, \ldots, \delta - 1\}$. Since the algorithm `StrongM` does not output **null** when $i = \ell$, we have $r_{\mathbf{N}_\ell}(E_\ell) \leq |R^*_\ell|$ and $|F_\ell| \geq |R^*_\ell|$. These imply that $r_{\mathbf{N}_i}(E_\ell) \leq |F_\ell|$. Furthermore, since $F_\ell$ is an independent set of $\mathbf{N}_\ell$, we have $|F_\ell| \leq r_{\mathbf{N}_\ell}(E_\ell)$. These imply that $|F_\ell| = r_{\mathbf{N}_\ell}(E_\ell)$, i.e., $F_\ell$ is a base of $\mathbf{N}_\ell$.

Next we prove the statement (2) by induction on $\ell$. The statement (2) for $\ell = 1$ is equivalent to the statement (1) for $\ell = 1$. Let $\xi$ be an integer in $\{1, 2, \ldots, \delta - 2\}$. Assume that the statement (2) holds for the case of $\ell = \xi$, and we consider the case of $\ell = \xi + 1$. Since $\mathbf{N}|P_\xi = (\mathbf{N}|P_{\xi+1})|P_\xi$ and $F_{\xi+1}$ is a base of $\mathbf{N}_{\xi+1} = (\mathbf{N}|P_{\xi+1})/P_\xi$, the induction assumption and Lemma 4(2) imply that $M_{\xi+1}$ is a base of $\mathbf{N}|P_{\xi+1}$. □

**Lemma 11.** *Assume that the algorithm* `StrongM` *halts when* $i = \delta$. *Then, for any edge* $e$ *in* $D_{\delta-1}$, *there does not exists a strongly stable matching* $N$ *in* $G$ *such that* $e \in N$.

*Proof.* We say that an edge $e$ in $D_{\delta-1}$ is *bad*, if there exists a strongly stable matching $N$ in $G$ such that $e \in N$. We prove this lemma by contradiction. That is, we assume that there exists a bad edge in $D_{\delta-1}$. Define $\Delta$ as the set of integers $\ell$ in $\{1, 2, \ldots, \delta - 1\}$ such that there exists a bad edge in $L_\ell$. Let $j$ be the minimum integer in $\Delta$. Furthermore, let $e = (r, h)$ and $N$ be a bad edge in $L_j$ and a strongly stable matching such that $e \in N$, respectively. Since $e$ belongs to $L_j$, $M_j + e \notin \mathcal{F}$. Define $C := \mathsf{C}_{\mathbf{N}}(e, M_j)$ (notice that $M_j \in \mathcal{F}$ follows from Lemma 10(2)). If $C$ is a subset of $N$, then this contradicts the fact that $N \in \mathcal{F}$. Thus, $C \setminus N$ is not empty. For every edge $(s, p)$ in $C \setminus N$, since $r \in R_{j+1,k}$ and $s \in R_{1,j}$, we have $s \succ_H r$.

We first assume that there exists an edge $f = (s, p)$ in $C \setminus N$ such that $N(s) \succ_s f$. Since $e$ belongs to $N$, $e \neq f$. This implies that $f$ belongs to $M_j$, i.e., $f \in T_s$. Thus, the definition of the algorithm `StrongM` implies that $N(s)$ belongs to $D_{j-1}$, i.e., $N(s) \in L_k$ for an integer $k$ in $\{1, 2, \ldots, j - 1\}$. Furthermore, since $D_{j-1} \subseteq D_{\delta-1}$ and $N$ is strongly stable, $N(s)$ is bad, which contradicts the minimality of $j$.

Next we assume that $f \succsim_s N(s)$ for every edge $f = (s, p)$ in $C \setminus N$. If there exists an edge $f$ in $C \setminus N$ such that $N + f \in \mathcal{F}$, then this contradicts the fact that $N$ is strongly stable. Thus, we assume that $N + f \notin \mathcal{F}$ for every edge $f$ in $C \setminus N$. For each edge $f$ in $C \setminus N$, we define $C_f$ as $\mathsf{C}_{\mathbf{N}}(f, N)$. Since $N$ is strongly stable, $t \succsim_H s$ for every edge $f = (s, p)$ in $C \setminus N$ and every edge $g = (t, q)$ in $C_f - f$. For every edge $f = (s, p)$ in $C \setminus N$, since $s \succ_H r$, this implies that $e \notin C_f$. In addition, $f \in C \cap C_f$ for every edge $f$ in $C \setminus N$. Thus, Lemma 3 implies that there exists a circuit $C'$ of $\mathbf{N}$ such that $C'$ is a subset of $N$. This contradicts the fact that $N \in \mathcal{F}$. □

**Lemma 12.** *Assume that the algorithm* `StrongM` *halts when* $i = \delta$, *and we are given a strongly stable matching* $N$ *in* $G$. *Then, for every resident* $r$ *in* $R_{1,\delta}$ *such that* $T_r = \emptyset$ (*resp.*, $T_r \neq \emptyset$), *we have* $N(r) = \emptyset$ (*resp.*, $N(r) \in T_r$).

*Proof.* For every resident $r$ in $R_{1,\delta}$ such that $T_r = \emptyset$, every edge in $E(r)$ belongs to $D_{\delta-1}$. Thus, Lemma 11 implies that $N(r) = \emptyset$ for every resident $r$ in $R_{1,\delta}$ such that $T_r = \emptyset$.

From here, we consider a resident $r$ in $R_{1,\delta}$ such that $T_r \neq \emptyset$. We assume that $r$ belongs to $R_j$ for an integer $j$ in $\{1, 2, \ldots, \delta\}$. If $N(r) \in T_r$, then the proof is done. Thus, we assume that $N(r)$ does not belong to $T_r$, and we prove this lemma by contradiction. If $N(r) \succsim_r e$ for an edge $e$ in $T_r$, then the definition of $T_r$ implies that $N(r)$ belongs to $D_{\delta-1}$, which contradicts Lemma 11. Thus, we assume that $e \succ_r N(r)$ for an edge $e$ in $T_r$. Furthermore, we assume that there exists no resident $s$ in $R_{1,j-1}$ such that $T_s \neq \emptyset$ and $e \succ_s N(s)$ for an edge $e$ in $T_s$. Notice that such a resident $r$ clearly exists.

Let $e$ be an edge in $T_r$. Since $e$ belongs to $T_r$, $M_{j-1} + e \in \mathcal{F}$. Since $N(r)$ does not belong to $T_r$, $e$ does not belong to $N$. In addition, since $N$ is strongly stable, $N + e \notin \mathcal{F}$ and $s \succ_H r$ for every edge $(s, p)$ in $\mathsf{C}^-_{\mathbf{N}}(e, N)$. Define $C := \mathsf{C}_{\mathbf{N}}(e, N)$. Since $M_{j-1} + e \in \mathcal{F}$, $C$ is not a subset of $M_{j-1} + e$. Let $f = (s, p)$ be an edge in $C \setminus (M_{j-1} + e)$. Since $s \succ_H r$, $s \in R_k$ for an integer $k$ in $\{1, 2, \ldots, j-1\}$. In the same way as above, we can prove that $f \in T_s$ or $g \succ_s f$ for an edge $g$ in $T_s$. The assumption of $r$ implies that $f$ belongs to $T_s$, i.e., $f \in E_k$. Since $f \notin (M_{j-1} + e)$ implies that $f$ does not belong to $F_k$ and Lemma 10(1) implies that $F_k$ is a base of $\mathbf{N}_k$, $F_k + f$ is not an independent set of $\mathbf{N}_k = (\mathbf{N}|P_k)/P_{k-1}$. Thus, since Lemma 10(2) implies that $M_{k-1}$ is a base of $(\mathbf{N}|P_k)|P_{k-1}$, Lemma 4(3) implies that $M_k + f \notin \mathcal{F}$. Since $M_k \subseteq M_{j-1}$, $M_{j-1} + f \notin \mathcal{F}$. For each edge $f$ in $C \setminus (M_{j-1} + e)$, we define $C_f := \mathsf{C}_{\mathbf{N}}(f, M_{j-1})$. For every edge $f$ in $C \setminus (M_{j-1} + e)$, since $e \notin M_{j-1}$ and $f \neq e$, $e$ does not belong to $C_f$. Furthermore, $f \in C \cap C_f$ for every edge $f$ in $C \setminus (M_{j-1} + e)$. Thus, Lemma 3 implies that there exists a circuit $C'$ of $\mathbf{N}$ that $C'$ is a subset of $M_{j-1} + e$, which contradicts the fact that $M_{j-1} + e \in \mathcal{F}$. □

**Lemma 13.** *Assume that the algorithm* `StrongM` *halts when $i = \delta$, and we are given a strongly stable matching $M$. Then, for every $\ell = 1, 2, \ldots, \delta$, the following statements hold.*

1. *If $\ell > 1$, then $N \cap P_{\ell-1}$ is a base of $\mathbf{N}|P_{\ell-1}$.*

2. *$N \cap E_\ell$ is a base of $\mathbf{N}_\ell$.*

*Proof.* We prove this lemma by induction on $\ell$. We first consider the case of $\ell = 1$, and we prove this case by contradiction. Assume that $N \cap E_1$ is not a base of $\mathbf{N}_1 = \mathbf{N}|E_1$. Then, since $N \in \mathcal{F}$, $N \cap E_1$ is an independent set of $\mathbf{N}_1$. Since $N \cap E_1$ is not a base of $\mathbf{N}_1$, there exists an edge $e = (r, h)$ in $E_1 \setminus N$ such that $(N \cap E_1) + e \in \mathcal{F}$. Since $D_0 = \emptyset$ and $e \in T_r$, we have $e \succsim_r N(r)$. Thus, if $N + e \in \mathcal{F}$, then this contradicts the fact that $N$ is strongly stable. Thus, we assume that $N + e \notin \mathcal{F}$. Then, since $(N \cap E_1) + e \in \mathcal{F}$, $\mathsf{C}_{\mathbf{N}}(e, N)$ is not a subset of $(N \cap E_1) + e$. Let $f = (s, p)$ be an edge in $\mathsf{C}_{\mathbf{N}}(e, N) \setminus ((N \cap E_1) + e)$. If $s \in R_j$ for an integer $j$ in $\{2, 3, \ldots, n\}$, then $r \succ_H s$. This contradicts the fact that $N$ is strongly stable. Thus, we have $s \in R_1$. Since $f \in E(s)$ (i.e., $E(s) \neq \emptyset$) and $D_0 = \emptyset$, we have $T_s \neq \emptyset$. Thus, since $f \in N$, Lemma 12 implies that $f \in T_s$, i.e., $f \in E_1$. This contradicts the fact that $f \notin N \cap E_1$.

Let $\xi$ be an integer in $\{1, 2, \ldots, \delta - 2\}$, and assume that this lemma holds for the case of $\ell = \xi$, and we consider the case of $\ell = \xi + 1$. We first prove the statement (1). If $\xi = 1$, then the statement (1) for $\ell = \xi + 1$ is equivalent to the statement (2) for the case of $\ell = 1$. Assume that $\xi \geq 2$. The induction assumption implies that $N \cap P_{\xi-1}$ is a base of $(\mathbf{N}|P_\xi)|P_{\xi-1}$ and $N \cap E_\xi$ is a base of $(\mathbf{N}|P_\xi)/P_{\xi-1}$. Thus, Lemma 4(2) implies that $N \cap P_\xi$ is a base of $\mathbf{N}|P_\xi$, which completes the proof. Next we prove the statement (2) by contradiction. Assume that $N \cap E_{\xi+1}$ is not a base of $\mathbf{N}_{\xi+1}$. Then, there exists an edge $e = (r, h)$ in $E_{\xi+1} \setminus N$ such that $(N \cap E_{\xi+1}) + e$ is an independent set of $\mathbf{N}_{\xi+1} = (\mathbf{N}|P_{\xi+1})/P_\xi$. Since the statement (1) for the case of $\ell = \xi + 1$ implies that $N \cap P_\xi$ is a base of $(\mathbf{N}|P_{\xi+1})|P_\xi$, Lemma 4(1) implies that $(N \cap P_{\xi+1}) + e \in \mathcal{F}$. Since $e$ belongs to $E_{\xi+1}$ (i.e., $T_r$), Lemma 12 implies that $e \sim_r N(r)$. Thus, if $N + e \in \mathcal{F}$, then this contradicts the fact that $N$ is strongly stable. Thus, we consider the case where $N + e \notin \mathcal{F}$. Since $(N \cap P_{\xi+1}) + e \in \mathcal{F}$, $\mathsf{C}_{\mathbf{N}}(e, N)$ is not a subset of $(N \cap P_{\xi+1}) + e$. Let $f = (s, p)$ be an edge in $\mathsf{C}_{\mathbf{N}}(e, N) \setminus ((N \cap P_{\xi+1}) + e)$. Since $f = N(s)$, Lemma 12 implies that $f$ belongs to $T_s$. This and $f \notin P_{\xi+1}$ imply that $s \in R_j$ for an integer $j$ in $\{\xi + 2, \xi + 3, \ldots, n\}$. Thus, $r \succ_H s$, which contradicts the fact that $N$ is strongly stable. □

**Lemma 14.** *If the algorithm* `StrongM` *outputs $M_n$, then $M_n$ is a strongly stable matching in $G$.*

*Proof.* Since Lemma 10(2) implies that $M_n \in \mathcal{F}$ and $|M_n(r)| \leq 1$ clearly holds for every resident $r$ in $R$, $M_n$ is a matching in $G$. What remains is to prove that $M_n$ is strongly stable.

Let $e = (r, h)$ be an edge in $E \setminus M_n$. If $M_n(r) \succ_r e$, then $r$ does not weakly prefer $e$ on $M$. Thus, we do not need to consider this case. Assume that $e \succsim_r M_n(r)$ and $r \in R_j$ for an integer $j$ in $\{1, 2, \ldots, n\}$. We first consider the case where $e \succ_r M_n(r)$. In this case, since $M_n(r) \in T_r$, the definition of the algorithm `StrongM` implies that $e \in L_k$ for an integer $k$ in $\{1, 2, \ldots, j - 1\}$. Thus, $M_k + e \notin \mathcal{F}$ and $s \succ_H r$ for every edge $(s, p)$ in $\mathsf{C}^{-}_{\mathbf{N}}(e, M_k)$. Since $M_k \subseteq M_n$, Lemma 2 implies that $\mathsf{C}_{\mathbf{N}}(e, M_k) = \mathsf{C}_{\mathbf{N}}(e, M_n)$. Thus, $s \succ_H r$ for every edge $f = (s, p)$ in $\mathsf{C}^{-}_{\mathbf{N}}(e, M_n)$. This implies that $H$ does not weakly prefer $e$ on $M$. Lastly we consider the case where $e \sim_r M_n(r)$. We first assume that $e \notin T_r$. In this case, the definition of the algorithm `StrongM` implies that $e \in L_k$ for an integer $k$ in $\{1, 2, \ldots, j - 1\}$. Thus, we can treat this case in the same way as above. Next we assume that $e \in T_r$. Notice that $e \in E_j \setminus F_j$. Since Lemma 10(1) implies that $F_j$ is a base of $\mathbf{N}_j$, $F_j + e$ is not an independent set of $(\mathbf{N}|P_j)/P_{j-1}$. In addition, Lemma 10(2) implies that $M_{j-1}$ is a base of $(\mathbf{N}|P_j)|P_{j-1}$. Thus, Lemma 4(3) implies that $M_j + e \notin \mathcal{F}$. Since $M_j \subseteq P_j$, we have $s \succsim_H r$ for each edge $(s, p)$ in $\mathsf{C}_{\mathbf{N}}(e, M_j)$. Since $M_j \subseteq M_n$, Lemma 2 implies that $M_n + e \notin \mathcal{F}$ and $\mathsf{C}_{\mathbf{N}}(e, M_j) = \mathsf{C}_{\mathbf{N}}(e, M_n)$. Thus, we have $s \succsim_H r$ for every edge $f = (s, p)$ in $\mathsf{C}^{-}_{\mathbf{N}}(e, M_n)$. This completes the proof. □

**Lemma 15.** *If the algorithm* `StrongM` *outputs* **null***, then there exists no strongly stable matching in $G$.*

*Proof.* We assume that the algorithm `StrongM` outputs **null** when $i = \delta$. We prove this lemma by contradiction. Assume that there exists a strongly stable matching $N$ in $G$.

We first consider the case where in **Step (2-c)** the algorithm `StrongM` outputs **null**. In this case, we have $r_{\mathbf{N}_\delta}(E_\delta) > |R^*_\delta|$. Since Lemma 13(2) implies that $N \cap E_\delta$ is a base of $\mathbf{N}_\delta$, we have $|N \cap E_\delta| = r_{\mathbf{N}_\delta}(E_\delta)$, which implies that $|N \cap E_\delta| > |R^*_\delta|$. However, since $N$ is a matching and $E_\delta$ is a subset of $\cup_{r \in R^*_\delta} E(r)$, $|N \cap E_\delta| \leq |R^*_\delta|$, which contradicts the fact that $|N \cap E_\delta| > |R^*_\delta|$.

Next we consider the case where in **Step (2-e)** the algorithm `StrongM` outputs **null**. In this case, we have $|F_\delta| < |R^*_\delta|$. Since $N$ is a matching in $G$, $|N(r)| \leq 1$ for every resident $r$ in $R_\delta$. Furthermore, Lemma 13(2) implies that $N \cap E_\delta$ is an independent set of $\mathbf{N}_\delta$. Thus, $N \cap E_\delta$ is a common independent set of $\mathbf{A}_\delta$ and $\mathbf{N}_\delta$. Since $F_\delta$ is a maximum-size common independent set of $\mathbf{A}_\delta$ and $\mathbf{N}_\delta$, we have $|N \cap E_\delta| \leq |F_\delta| < |R^*_\delta|$. This implies that there exists a resident $r$ in $R^*_\delta$ such that $N(r) \notin E_\delta$. However, since $T_r \subseteq E_\delta$, this contradicts Lemma 12. □

Lemmas 14 and 15 imply the following theorem.

**Theorem 16.** *The algorithm* `StrongM` *can decide whether there exists a strongly stable matching in $G$, and find such a matching, if one exists.*

Here we consider the time complexity of `StrongM`. Define $m := |E|$, and we denote by $\gamma$ the time required to decide whether $F \in \mathcal{F}$ for one subset $F$ of $E$. For simplicity, we assume that $\gamma = \Omega(m)$ and $m \geq |R|$. Since the number of iterations of **Step 2** is at most $m$, we consider the time complexity of one iteration of **Step 2**. Clearly, **Step (2-d)** is the bottleneck. Lemma 4(1) implies that by computing bases of $\mathbf{N}|P_\ell$ for all $\ell = 1, 2, \ldots, n$ in $O(m^2\gamma)$ time, we can decide in $O(\gamma)$ time whether $F$ is an independent set of $\mathbf{N}_\ell$ for an integer $\ell$ in $\{1, 2, \ldots, n\}$ and one subset $F$ of $E_\ell$. Thus, the time complexity of this step is $O(m^{2.5}\gamma)$, and the time complexity of `SuperM` is $O(m^{3.5}\gamma)$.

# 5 Examples

In this section, we give examples of concrete models that our model can express.

## 5.1 Hospitals/residents setting

Assume that for each hospital $h$ in $H$, we are given a positive integer $q_h$. Then, we define

$$\mathcal{F} := \Big\{F \subseteq E \;\Big|\; \forall h \in H\colon |F(h)| \leq q_h\Big\}.$$

It is not difficult to see that $\mathbf{N} = (E, \mathcal{F})$ is a matroid. When we define the matroid $\mathbf{N}$ as above, a subset $M$ of $E$ is a matching if and only if

1. $|M(r)| \leq 1$ for every resident $r$ in $R$, and
2. $|M(h)| \leq q_h$ for every hospital $h$ in $H$.

Furthermore, for each matching $M$ in $G$ and each edge $e = (r, h)$ in $E \setminus M$, $H$ weakly (strictly) prefers $e$ on $M$ if and only if

1. $|M(h)| < q_h$, and/or
2. there exists an edge $f = (s, h)$ in $M(h)$ such that $r \succsim_H s$ (resp., $r \succ_H s$).

Thus, our problem is equivalent to the hospitals/residents problem with ties and master lists [12].

## 5.2 Student-project allocation setting

Assume that $H$ is partitioned into non-empty subsets $H_1, H_2, \ldots, H_k$, and we are given positive integers $c_h$ and $d_i$ for each hospital $h$ in $H$ and each $i = 1, 2, \ldots, k$, respectively. Then, we define

$$\mathcal{F} := \Big\{F \subseteq E \;\Big|\; \forall h \in H\colon |F(h)| \leq c_h,\; \forall i = 1, 2, \ldots, k\colon \textstyle\sum_{h \in H_i} |F(h)| \leq d_i\Big\}.$$

It is not difficult to see that $\mathbf{N} = (E, \mathcal{F})$ is a matroid. When we define the matroid $\mathbf{N}$ as above, a subset $M$ of $E$ is a matching if and only if

1. $|M(r)| \leq 1$ for every resident $r$ in $R$,
2. $|M(h)| \leq c_h$ for every hospital $h$ in $H$, and
3. $\sum_{h \in H_i} |M(h)| \leq d_i$ for every $i = 1, 2, \ldots, k$.

These constraints are exactly the same as the capacity constraints of the student-project allocation problem [1]. For each matching $M$ in $G$ and each edge $e = (r, h)$ in $E \setminus M$ such that $h \in H_i$ for an integer $i$ in $\{1, 2, \ldots, k\}$, $H$ weakly (strictly) prefers $e$ on $M$ if and only if at least one of the following conditions is satisfied.

1. $|M(h)| < c_h$ and $\sum_{h \in H_i} |M(h)| < d_i$.
2. $|M(h)| < c_h$ and there exists an edge $f = (s, p)$ in $M$ such that $p \in H_i$ and $r \succsim_H s$ (resp., $r \succ_H s$).
3. There exists an edge $f = (s, h)$ in $M(h)$ such that $r \succsim_H s$ (resp., $r \succ_H s$).

It should be noted that in the above definition, if $|M(h)| < c_h$ and $M(r) \in H_i$, then $H$ weakly prefers $e$ on $M$ (i.e., the condition (2) is satisfied), but it does not mean that $H$ strictly prefers $e$ on $M$ since $r \not\succ_H r$.

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