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A TEST OF THE STANDARD VOLUME TABLE BY MULTIPLE REGRESSION

Kenkichi KINASHI and F. X. SCHUMACHER*

1. Introduction

A sampling survey for the Kyushu University Forests, Sanpoo-dake area, Miyazaki prefecture, Japan, was carried out in March, 1959. This area occupied the southern part of the University Forests and the elevation rises from about 700 m to 1476 m, at the summit of Mt. Sanpoo-dake. Total acreages cover about 560 ha and they are divided into five compartments, No. 17, 18, 19, 20 and 21. Tree species are about 80 % hardwood including both evergreen and decidious, and 20 % conifers. Almost all of them are natural forests, except small plantations. Systematic sampling survey was laid out with 84 square plots, whose dimensions were 20 m and the interval between plots is 200 m in either direction. Throughout the 84 plots, sample trees were drawn and cut mechanically with the interval 50 trees. Except conifer trees, the number of hardwood sample trees are 28, including 13 species and mainly limited to small size. The original list of the sample tree is shown as Table 1.

Table 1. List of original sample trees

Tree no.	Species	Dbh (cm)	Total height (m)	Volume (cubic m.)
1	*Carpinus* sp.	12.35	9.08	0.05028
2	*Stewartia monadelpha*	10.50	9.07	0.03995
3	*Quercus cripula*	15.90	9.77	0.09321
4	*Quercus cripula*	14.85	10.23	0.08693
5	*Carpinus* sp.	9.30	7.56	0.02318
6	*Xolisma ellipica*	13.50	9.90	0.05770
7	*Illicium religiosum*	11.25	6.00	0.04024
8	*Stewartia monadelpha*	21.75	10.00	0.16890
9	*Betula grossa*	15.25	12.80	0.11774
10	*Peterocarya rhoijolia*	14.00	15.20	0.12518
11	*Illicium religiosum*	13.20	9.50	0.07088
12	*Kalopanax pictum*	15.50	10.20	0.09359
13	*Acer carpinifolium*	16.00	10.88	0.12905
14	*Carpinus* sp.	18.75	12.00	0.17550
15	*Quercus cripula*	9.76	9.30	0.04331
16	*Stewartia monadelpha*	11.75	11.15	0.05552
17	*Betula* sp.	10.90	11.00	0.04759
18	*Qercu crispula*	12.45	9.70	0.06913
19	*Styrax japonicum*	9.80	8.10	0.03707
20	*Styrax japonicum*	10.20	8.30	0.04013
21	*Cornus controversa*	14.50	13.50	0.10662
22	*Quercus crispula*	16.25	8.00	0.08215
23	*Castania crenata*	23.00	14.10	0.25086
24	*Carpinus* sp.	14.50	10.80	0.09020
25	*Carpinus* sp.	11.90	8.60	0.05247
26	*Stewartia monadelpha*	11.90	13.30	0.07320
27	*Castana crenata*	14.30	11.50	0.09864
28	*Cornus controversa*	15.70	11.40	0.11394

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2. The standard volume table

There is a standard volum table for hardwood for the Kyushu Island, issued by the Kumamoto Regional Forest Bureau, Japanese Forest Service. If x is an actual volume and y is a standard volume corresponding to x, there may be one linear relationship:

$$Y = 0.0923 + 1.0745\ (x - 0.0869).$$

Two t-tests for slop and mean are shown as the followings,

$$t = |\bar{x} - \bar{y}|\ /\ \mathrm{SE}\,(a) = 2.7692^{*}$$
$$t = |b - 1|\ /\ \mathrm{SE}\,(b) = 1.8909$$
for both, DF=26 and $t_{.05} = 2.056$

Accoding to the F test, the same result will appear. Covariance analysis between the standard volume table regression and $y = x$ (45° line) may be shown as the following;

due to	DF	SS	MS	F
standard vol.	26	0.00278306		
$x=y$	26	0		
Within	52	0.00278306	0.00005352	
Reg.	1	0.00019055		3.56
Common	53	0.00297361		
Adj. mean	1	0.00039649		7.41**
Genenal	54	0.00337010		

For this area, the standard volume table may give over-estimation of 0.0054 cubic meters per tree in the average. However, individual deviation from the standard volume table is not clear. This problem will be solved in the following section.

3. Volume equation

A multiple regresson with weight may be used for test of the standard volume table. Each sign must be denoted as the following:

X = actual measured volume (cubic meters)
Y = the volume from the standard table (cubic meters)
D = dbh (m)
H = total height of tree (m)
b = coefficients of multiple regression

The volume equation may be expressed as the following form;

$$X = b_1 + b_2 D + b_3 D^2 + b_4 H + b_5 DH + b_6 D^2 H$$

If the weight is $(1/Y^2)$, the weighted sum of squared residuals is shown as the following:

$$\mathrm{SSR} = S\left[\frac{1}{Y^2}(X - b_1 - b_2 D - b_3 D^2 - b_4 H - b_5 DH - b_6 D^2 H)^2\right]$$

$$= S\left[\left(\frac{X}{Y} - b_1\frac{1}{Y} - b_2\frac{D}{Y} - b_3\frac{D^2}{Y} - b_4\frac{H}{Y} - b_5\frac{DH}{Y} - b_6\frac{D^2 H}{Y}\right)^2\right]$$

The first set of the normal equations are the following:

$1/Y$	D/Y	D^2/Y	H/Y	DH/Y	D^2H/Y	X/Y
8059.6191	912.3703	107.1421	70520.3710	8122.7597	970.1905	404.8706
	106.7833	13.0368	8124.3243	970.0277	120.8236	49.9837
		1.6650	973.1695	121.0865	15.8155	6.4745
			642003.4326	75434.4271	9208.7776	3797.1239
				9204.9524	1173.4849	479.7580
					157.0943	63.5720
						26.0959

In this first set of normal equations, the sum of squares due to each independent variable alone may be compared as the following.

Source	SS due to
1	20.3385
D	23.3966
D^2	25.1767
H	22.4581
DH	25.0048
D^2H	25.7259

Considering each independent variable alone $\left(\frac{D^2H}{Y}\right)$ contributes 25.7259 to the sum of squares, which is more than the contribution of any other variable alone. So the normal equation with this leading term should be put first and one reduction made. Then repeating the process shows that the variable (D/Y) contributes 0.0857, which is more than that of any other variable, over and above $\left(\frac{D^2H}{Y}\right)$, as given below.

Source	SS due to
1	0.0727
D	0.0857
D^2	0.0760
H	0.0487
DH	0.0542

In the systematic pattern of the least square solution, the first line should be reserved as a room to put the first normal equation with the leading term. After deciding the leading term, that is, which term has the largest contribution for the sum of squares of dependent variable, the normal equation with D^2H row must be shifted to the first line from the 6 th line. Consequently the corresponding column and row with D^2H will be eliminated, shown as the following pattern. After the first redudtion, the same procedure will be repeated until no more need.

The above process may be detailed as the following.

D^2H/Y	$1/Y$	D/Y	D^2/Y	H/Y	DH/Y	D^2H/Y	X/Y	ck	SS due to
157.0943	970.1905	120.8236	15.8155	9208.7776	1173.4849		63.5720	11709.7584	
	8059.6191	912.3703	107.1421	70520.3710	8122.7597	970.1905	404 8706	89097.3233	20.3385
		106.7833	13.0368	8124.3243	970.0277	120.8236	49.9837	10297.3497	23.3966
			1.6650	973.1695	121.0865	15.8155	6.4745	1238.3899	25.1767
				642003 4326	5434.4271	9208.7776	3797.1239	810061.6260	22.4581
					9204.9524	1173.4849	479.7580	95506.4963	25.0048
						157.0943	~~63.5720~~	~~11709.7584~~	25.7259
D/Y							26.0959	4827.8786	
13.8560	166.1821		0.8729	1041.7143	67.4827		1.0895	1291.1975	
	2067.8702	166.1821	9.4680	13648.3614	875.4955		12.2596	16779.6368	0.0727
		13.8560	~~0.8729~~	~~1041.7143~~	~~67.4827~~		~~1.0895~~	~~1291.1976~~	0.0857
			0.0728	46.0740	2.9457		0.0744	59.5078	0.0760
				102190.1811	6645.4165		70.5698	123642.3171	0.0487
					439.0913		4.8790	8035.3108	0.0542
							0.3700	89.2423	
	74.7632		−1.0011	1154.5486	66.1409		−0.8073	1293.6443	0.0009
			0.0178	−19.5519	−1 3056		0.0058	−21.8350	0.0019
				23872.5800	1571.9686		−11.3404	26568.2049	0.0054
					110.4312		−0.4272	1746.8079	0.0017
							0.2844	−12.2848	

Thereafter, no variable contributes a significant amount. In summary:

Source	SS	DF	MS	F
D^2H alone	25.7259	1	25.7259	2217.75
Additional effect of D	0.0857	1	0.0857	7.39
Remaining variables	0.0272	4	0.0068	0.59
Residuals	0.2551	22	0.0116	
Total	26.0959	28		

Then, 2 out of the 6 variables, D^2H and D contain all of thc significant effects and the normal equations and systematic solution of the least square are shown as the following;

	D^2H/Y	D/Y	X/Y	ck	(DF)
D^2H/Y	157.0943	120.8236	63.8236	341.4899	
D/Y		106.7833	49.9837	277.5906	
X/Y			26.0959	139.6516	(28)
0.769115111		13.8560	1.0895	14.9456	
0.4046741352			0.3700	1.4995	(27)
0.0786301963			0.2843	0.2843	(26)

From the back solution:

$$b_2 = 0.07863$$
$$b_1 = 0.40467 - 0.76912 \times 0.07863 = 0.34420$$

and the c multiplers are;

$$c_{22} = 1/13.8560 = 0.07217$$
$$c_{12} = -\ 0.07217 \times 0.76912 = -\ 0.05551$$
$$c_{11} = 1/157.0943 - (-\ 0.05551) \times 0.76912 = 0.04906$$

and $$s^2 = \frac{0.2843}{26} = 0.01093462$$

Thence, the volume equation is

$$X = 0.3442\,(D^2H) + 0.0786\,(D).$$

4. Testing equation and test of the standard volume table.

If Y represents the volume from a standard volume table and

$$\frac{X}{Y} = 0.3442\,(D^2H/Y) + 0.0786\,(D/Y)$$

is the calculated ratio of the volume of the sample tree to that of the table volume, then the test

$$\frac{X}{Y} - 1 = 0,\ \text{or}\ \frac{X}{Y} = 1$$

is the appropriate test. Perhaps certain tree sizes will have a significant departure from zero or 1. This will readily become apparent if the confidence band of X/Y will be ploted.

To make the confidence band Var (X/Y) will be needed.

$$\mathrm{Var}\,(X/Y) = 0.01093462\left\{0.04905744\left(\frac{D^2H}{Y}\right)^2 + 0.07217090\,(D/Y)^2 - 2 \times 0.05550773\,(D^3H/Y^2)\right\}$$

Based on this variance equation, the confidence band of X/Y with the probability 95 % and the value of X/Y itself may be shown as the following table:

Table of the value X/Y and it's canfidence band

Total Height (m)	Dbh (cm)								
	6	8	10	12	14	16	18	20	22
8	1.0139 0.1550	0.9917 0.1040	0.9833 0.0723	1.0022 0.0544	1.0156 0.0552	1.0259 0.0432	1.0330 0.0457	1.0397 0.0502	1.0532 0.0559
10	0.9293 0.1063	0.9397* 0.0671	0.9400* 0.0481	0.9516* 0.0406	0.9573* 0.0416	0.9683 0.0469	0.9890 0.0537	0.9961 0.0598	0.0104 0.0662
12		0.8811* 0.0235	0.8945* 0.0186	0.9066* 0.0195	0.9293* 0.0230	0.9388* 0.0265	0.9542* 0.0300	0.9674 0.0332	0.9828 0.0362
14			0.8631* 0.0375	0.8756* 0.0442	0.8932* 0.0526	0.9128* 0.0606	0.9250* 0.0676	0.9434 0.0723	0.9561 0.0795

Stars* represent significant departure from X/Y=1. 11 out of 28 sample trees are situated on the non significant portion but the remaining, about 2/3 of all sample trees are in the significant portion.

5. New Volume Table

New Volume Table may be shown as following; (Cubic meter)

Height (m) \ Dbh	6	8	10	12	14	16	18	20	22 (cm)
8	0.0146	0.0238	0.0354	0.0491	0.0650	0.0831	0.1033	1.0397	1.0532
10	0.0171	0.0282	0.0423	0.0590	0.0785	0.1007	0.1256	0.1534	0.1839
12		0.0326	0.0492	0.0689	0.0920	0.1183	0.1479	0.1809	0.2172
14			0.0561	0.0788	0.1054	0.1360	0.1702	0.2085	0.2505
16			0.0630	0.0887	0.1189	0.1536	0.1925	0.2360	0.2838

Fig. 1 shows the volume curve and Fig. 2 shows X/Y and its 95 % confidence band.

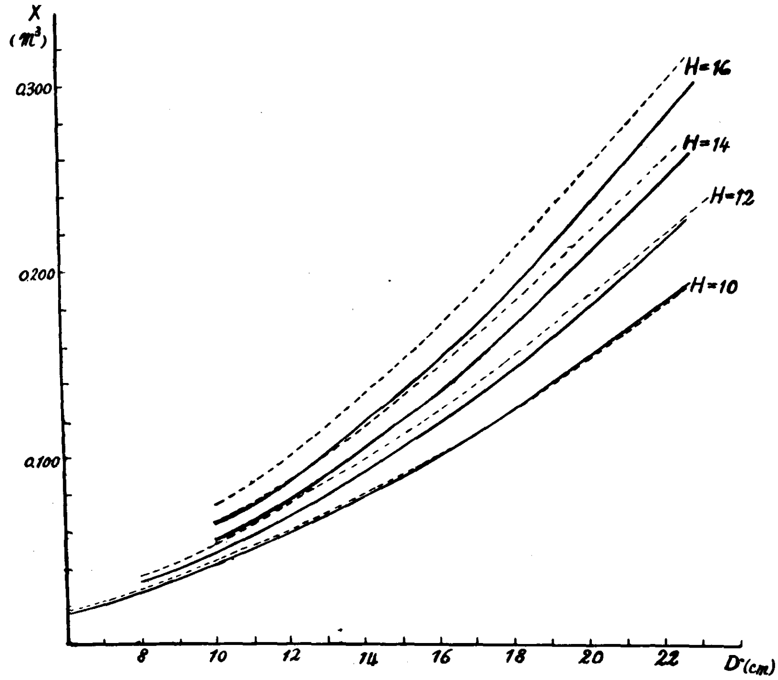

Fig. 1. Voume curve. Solid line : New volume table value
Doted line : Standard volume table value

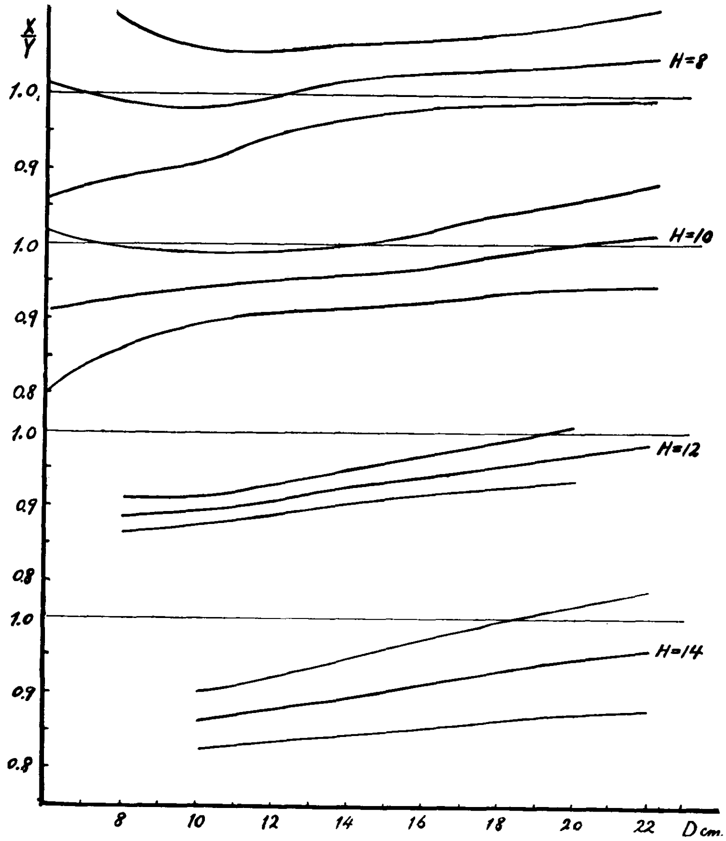

Fig. 2. X/Y and 95% confidence band

摘　　要

本例は九大宮崎演習林三方嶽地区の広葉樹の材積が熊本営林局広葉樹標準材積表に適合するか否かの検定である．得られた資料が比較的小径木に偏しているが，その範囲については十分に適合しているとはいえない．検定と重回帰の最小二乗法における有効な項を決定するための分散分析を中心として取扱われたものである．得られた材積方程式は

$$X = 0.3442\,D^2H + 0.0786\,D$$

であり標準材積表数値を Y とするとき

$$\frac{X}{Y} = 0.3442\,(D^2H/Y) + 0.0786\,(D/Y)$$

から $\frac{X}{Y} = 1$ の検定が行われた．その結果は 28 標本中 17 本が不適合の部分に属している．樹高 8 m 級を除いては大部分が 1 と有意差があるといえる．標準材積表材積に比して一割乃至二割低く従つて標準材積表を用いると過大な推定値となる．