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SETS OF COLLISIONS AND CONNECTED SUBSETS

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1. Introduction

Some dynamical systems such as cellular automata (cf. Wolfram (2002)) and billiard systems (cf. Ito (2010) and Ito et al. (2008)) are regarded as parallel rewriting systems. An essential problem in the rewriting systems is that some of the rewriting (or local) rules conflict when their scope intersect. A set of abstract collisions is a device introduced by Ito (2010) and Ito et al. (2008) so that such problem is avioded by Def. 2.1. The set of collisions is a kind of set systems, namely a subset of the power set of a given set, and its remarkable advantage is to partition any subset of the given set into a disjoint union of subsets in the set of collisions. Although the articles by Ito (2010) and Ito et al. (2008) unfortunately included no mathematical overview of the sets of collisions, it turns out that the set of all connected subsets of a topological space forms a set of collisions. Then a natural question arises. Do all sets of collisions originate with the set of all connected subsets of a toplogical space? This note answers negatively the question to give a counterexample: The set $\mathcal{C}_0$ of all singleton sets and all connected open subsets of the ordinary topological space $\mathbb{R}$ of reals forms a set of collisions, but there exists no topology $\mathcal{O}$ on $\mathbb{R}$ such that the set of all connected subsets with respect to $\mathcal{O}$ coincides with $\mathcal{C}_0$.

2. Sets of abstract collisions

In the section we will recall the definition and the basic properties of sets of (abstract) collisions introduced by Ito (2010) and Ito et al. (2008).

Definition 2.1. Let X be a nonempty set. A subset $\mathcal{C}$ of the power set $\wp(X)$ is called a *set of (abstract) collisions* on X if it satisfies

(a) $\{x\} \in \mathcal{C}$ for all $x \in X$,

(b) If $\mathcal{X}$ is a nonempty subset of $\mathcal{C}$ such that $\cap\mathcal{X} \neq \emptyset$, then $\cup\mathcal{X} \in \mathcal{C}$. □

The set $\{\{x\} \mid x \in X\}$ of all singleton subsets of a set X is the least set of collisions on X, and the power set $\wp(X)$ is the greatest set of collisions on X.

The following two propositions are the basic results by Ito (2010) and Ito et al. (2008).

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Proposition 2.2. (a) *For any family* $\mathfrak{C}$ *of sets of collisions the intersection* $\bigcap_{\mathcal{C}\in\mathfrak{C}} \mathcal{C}$ *is also a set of collisions,*

(b) *For each subset* $\mathcal{D}$ *of* $\wp(X)$ *there exists the least set* $\mathcal{C}(\mathcal{D})$ *of collisions on* X *containing* $\mathcal{D}$. □

Let $\mathcal{C}$ be a set of collisions on a set X. For all $A \in \wp(X)$ and all $a \in A$ define

$$C_A(a) = \cup\{C \in \mathcal{C} \mid (a \in C) \wedge (C \subseteq A)\}.$$

Proposition 2.3. *Let* $\mathcal{C}$ *be a set of collisions on a set* X. *Then*

(a) $a \in C_A(a) \subseteq A$ *for all* $a \in A$, $(\cup_{a\in A} C_A(a) = A)$

(b) $C_A(a) \in \mathcal{C}$ *for all* $a \in A$,

(c) *If* $A \in \mathcal{C}$, *then* $C_A(a) = A$ *for all* $a \in A$,

(d) *If* $C_A(a) \cap C_A(b) \neq \emptyset$ *for* $a, b \in A$, *then* $C_A(a) = C_A(b)$. □

3. Connected Spaces

We now review some fundamentals on connectivity of topological spaces.

Let X be a topological space with topology $\mathcal{O}$. A subset S of X is *connected* if $S \subseteq U \cup V$ and $S \cap U \cap V = \emptyset$ implies $S \cap U = \emptyset$ or $S \cap V = \emptyset$ for all pairs of open sets $U, V \in \mathcal{O}$. In other words, a subset S of X is *disconnected* if there exists a pair of open sets $U, V \in \mathcal{O}$ such that $S \subseteq U \cup V$, $S \cap U \cap V = \emptyset$, $S \cap U \neq \emptyset$ and $S \cap V \neq \emptyset$. A subset of a topological space is often called *clopen* if it is open and closed. A topological space X is disconnected if there exists a clopen subset U such that $U \neq \emptyset$ and $U \neq X$.

Unless otherwise stated, subsets of a topological space will be identified with subspaces. The following two propositions are well-known properties of connectivity (cf. Dudundji (1996)).

$\bullet_1$ Every singleton set $\{x\}$ of a topological space is connected.

$\bullet_2$ If $\mathcal{A}$ is a nonempty set of connected subsets of a topological space such that $\cap\mathcal{A} \neq \emptyset$, then $\cup\mathcal{A}$ is also connected.

Therefore the set of all connected subsets of a toplogical space forms a set of collisions. On the other hand we encounter a natural question: *Is every set of collisions isomorphic to the set of all connected subsets of a topological space?* The aim of the paper is to give a counter example of the question. Now we remark another property about connectivity, which is not imposed on the definition of sets of collisions.

$\bullet_3$ Let E and A be subsets of a topological space. If E is connected and $E \subseteq A \subseteq E^-$, then A is connected, where E^- denotes the closure of E.

To prepare for the later discussion we recall one more simple fact related to a separation axiom and connectivity.

PROPOSITION 3.1. *A topological space X is a T_1-space iff all singleton sets $\{x\}$ of X are closed iff all two-element subsets $\{x, y\}$ of X are disconnected.* □

4. A counter example

It is well-known (cf. Dudundji (1996)) that a connected subsets of $\mathbb{R}$ (with the standard topology) is one of a singleton set, an open interval, a closed interval and a semi-open interval, that is,

$$\emptyset,\ \{a\},\ (a,b),\ (-\infty,a),\ (a,\infty),\ [a,b],\ (-\infty,a],\ [a,\infty),\ [a,b),\ (a,b],\ \mathbb{R}$$

where a and b are reals. Thus a connected open subset of $\mathbb{R}$ is an open interval (including $\emptyset$ and $\mathbb{R}$) :

$$\emptyset,\ (a,b),\ (-\infty,a),\ (a,\infty),\ \mathbb{R}.$$

In what follows let $\mathcal{C}_0$ denote the set of all singleton sets and all connected open subsets of $\mathbb{R}$ (with respect to the ordinary topology). Then it is easy to see the following.

PROPOSITION 4.1. *$\mathcal{C}_0$ forms a set of collisions on $\mathbb{R}$.* □

The next proposition is our main claim of the paper.

PROPOSITION 4.2. *There exists no topology $\mathcal{O}$ on $\mathbb{R}$ such that the set of all connected subsets with respect to $\mathcal{O}$ coincides with $\mathcal{C}_0$.*

Proof. Assume that the set of all connected subsets in a topological space $\langle \mathbb{R}, \mathcal{O} \rangle$ coincides with $\mathcal{C}_0$. By 3.1 $\mathcal{O}$ is a T_1-topology, because two-element sets are disconnected. Consider the complement $\{a\}^c$ of $\{a\}$ for an arbitrary $a \in \mathbb{R}$. Then $\{a\}^c$ is disconnected since $\{a\}^c \notin \mathcal{C}_0$, and so there exist $U, V \in \mathcal{O}$ such that

$$\{a\}^c \subseteq U \cup V,\ \{a\}^c \cap U \cap V = \emptyset,\ \{a\}^c \cap U \neq \emptyset \text{ and } \{a\}^c \cap V \neq \emptyset.$$

It follows from $\{a\}^c = (-\infty, a) \cup (a, \infty)$ that

$$\{a\}^c \cap U \neq \emptyset \leftrightarrow (-\infty,a) \cap U \neq \emptyset \text{ or } (a,\infty) \cap U \neq \emptyset$$

and

$$\{a\}^c \cap V \neq \emptyset \leftrightarrow (-\infty,a) \cap V \neq \emptyset \text{ or } (a,\infty) \cap V \neq \emptyset.$$

Thus we inspect the following four cases.
(1) In the case of $(-\infty,a) \cap U \neq \emptyset$ and $(-\infty,a) \cap V \neq \emptyset$:
$(-\infty,a)$ is disconnected because $(-\infty,a) \subseteq U \cup V$ and $(-\infty,a) \cap U \cap V = \emptyset$. This contradicts $(-\infty,a) \in \mathcal{C}_0$.
(2) In the case of $(a,\infty) \cap U \neq \emptyset$ and $(a,\infty) \cap V \neq \emptyset$:
(a,∞) is disconnected because $(a,\infty) \subseteq U \cup V$ and $(a,\infty) \cap U \cap V = \emptyset$. This contradicts $(a,\infty) \in \mathcal{C}_0$.
(3) In the case of $(-\infty,a) \cap U \neq \emptyset$ and $(a,\infty) \cap V \neq \emptyset$:
The case of $(-\infty,a) \cap V \neq \emptyset$ has been already discussed in (1). So we may assume $(-\infty,a) \cap V = \emptyset$, which is equivalent to $V \subseteq (-\infty,a)^c = [a,\infty)$. Similarly, the case of

$(a,\infty)\cap U \neq \emptyset$ has been already discussed in (2). Again we may assume $(a,\infty)\cap U = \emptyset$ and so $U \subseteq (a,\infty)^c = (-\infty, a]$. On the other hand it holds that

$$\begin{array}{rcll} (-\infty,a) & = & (-\infty,a)\cap(U\cup V) & \{\ \{a\}^c \subseteq U\cup V\ \} \\ & \subseteq & U, & \{\ (-\infty,a)\cap V=\emptyset\ \} \end{array}$$

and

$$\begin{array}{rcll} (a,\infty) & = & (a,\infty)\cap(U\cup V) & \{\ \{a\}^c \subseteq U\cup V\ \} \\ & \subseteq & V, & \{\ (a,\infty)\cap U=\emptyset\ \} \end{array}$$

which shows $(-\infty,a) \subseteq U \subseteq (-\infty,a]$ and $(a,\infty) \subseteq V \subseteq [a,\infty)$. Consequently we have

$$U = (-\infty,a] \quad \text{or} \quad U = (-\infty,a)$$

and

$$V = [a,\infty) \quad \text{or} \quad V = (a,\infty).$$

Again we consider the following four cases.
(3-1) In the case of $U = (-\infty,a]$ and $V = [a,\infty)$: we have $\{a\} = U\cap V \in \mathcal{O}$ and so $\{a\}$ is clopen (for $\mathcal{O}$ is a T_1-topology), which implies $\mathbb{R} \notin \mathcal{C}_0$, the absurdity.
(3-2) In the case of $U = (-\infty,a]$ and $V = (a,\infty)$: it is trivial that $U\cup V = \mathbb{R}$ and $U\cap V = \emptyset$. Therefore we have $\mathbb{R} \notin \mathcal{C}_0$, the absurdity.

(3-3) In the case of $U = (-\infty,a)$ and $V = [a,\infty)$: it is symmetric to (3-2).

(3-4) In the case of $U = (-\infty,a)$ and $V = (a,\infty)$:
If U is closed with respect to $\mathcal{O}$, then it is clopen and hence $\mathbb{R} \notin \mathcal{C}_0$, the absurdity. Otherwise $V^c = (-\infty,a] = U\cup\{a\}$ is the smallest closed subset containing U, namely the closure of U, and hence $(-\infty,a] \in \mathcal{C}_0$ by $\bullet_3$ (for $U \in \mathcal{C}_0$), the absurdity.
(4) In the case of $(a,\infty)\cap U \neq \emptyset$ and $(-\infty,a)\cap V \neq \emptyset$: it is symmetric to (3).
This completes the proof. □

5. Conclusion

The paper pointed out that the set of all connected subsets of a topological space forms a set of collisions, and gave a counter example of sets of collisions which doesn't originate with the set of all connected subsets of any topological space.

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