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A REMARK ON THE GLOBAL EXISTENCE OF A THIRD ORDER DISPERSIVE FLOW INTO LOCALLY HERMITIAN SYMMETRIC SPACES

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ABSTRACT. We prove global existence of solutions to the initial value problem for a third order dispersive flow into compact locally Hermitian symmetric spaces. The equation under consideration generalizes two-sphere-valued completely integrable systems which model the motion of vortex filament. Unlike one-dimensional Schrödinger maps, our third order equation is not completely integrable under the curvature condition on the target manifold in general. The idea of our proof is to exploit two conservation laws and an energy which is not necessarily preserved in time but does not blow up in finite time.

1. INTRODUCTION

Let (N, J, g) be a compact almost Hermitian manifold with an almost complex structure J and a Hermitian metric g . Let ∇ be the Levi-Civita connection with respect to g . Consider the initial value problem (IVP) for a third order dispersive partial differential equation of the form

$$u_t = a \nabla_x^2 u_x + J_u \nabla_x u_x + b g(u_x, u_x) u_x \quad \text{in } \mathbb{R} \times X, \quad (1)$$

$$u(0, x) = u_0(x) \quad \text{in } X, \quad (2)$$

where u is an unknown mapping of $\mathbb{R} \times X$ to N , $(t, x) \in \mathbb{R} \times X$, X denotes \mathbb{R} or $\mathbb{T}(= \mathbb{R}/\mathbb{Z})$, $u_t = du(\partial/\partial t)$, $u_x = du(\partial/\partial x)$, du is the differential of the mapping u , u_0 is a given initial curve on N , and $a, b \in \mathbb{R}$ are constant. $u(t)$ is a curve on N for fixed $t \in \mathbb{R}$, and u describes the motion of a curve subject to (1). ∇_x is the covariant derivative induced from ∇ in the direction x along the mapping u , and J_u denotes the almost complex structure at $u \in N$.

The equation (1) geometrically generalizes two-sphere-valued completely integrable systems which model the motion of vortex filament. In [4], Da Rios first formulated the motion of vortex filament as

$$\vec{u}_t = \vec{u} \times \vec{u}_{xx}, \quad (3)$$

where $\vec{u} = (u^1, u^2, u^3)$ is an \mathbb{S}^2 -valued function of (t, x) , \mathbb{S}^2 is a unit sphere in \mathbb{R}^3 with a center at the origin, and \times is the exterior product in \mathbb{R}^3 . The physical meanings of \vec{u} and x are the tangent vector and the signed arc length of vortex filament respectively. When $a, b = 0$, (1) generalizes (3) and solutions to (1) are called one-dimensional Schrödinger maps. In [6], Fukumoto and Miyazaki proposed a modified model equation of vortex filament

$$\vec{u}_t = \vec{u} \times \vec{u}_{xx} + a \left[\vec{u}_{xxx} + \frac{3}{2} \{ \vec{u}_x \times (\vec{u} \times \vec{u}_x) \}_x \right]. \quad (4)$$

When $b = a/2$, (1) generalizes (4). We call solutions to (1) dispersive flows.

In recent ten years, the generalized form (1) has been studied in order to understand the relation between the structure of (1) as a partial differential equation and the geometric setting for N . In this article, having same motivation in mind, we are concerned with the existence (and the uniqueness) of solutions to the IVP for (1)-(2).

For \mathbb{S}^2 -valued physical models such as (3) and (4), time-local and global existence theorem is well studied. More precisely, Sulem, Sulem and Bardos proved time-local and global existence of a

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unique solution to the IVP for (3) in [19]. Nishiyama and Tani showed time-local and global existence theorem for (4) in [14] and [20]. In their results, some conservation laws of the equation played the crucial parts.

Restricting to the case of Kähler manifolds as N , short-time existence results for (1)-(2) have already been well established. Roughly speaking, the Kähler condition $\nabla J \equiv 0$ ensures that the equation behaves as symmetric hyperbolic systems and hence the mix of the classical energy method and geometric analysis works to their proof. When $a, b = 0$, Koiso showed the short-time existence of a unique solution in the class $H^{m+1}(\mathbb{T}; N)$ for any integer $m \geq 1$. See [8] (see [18] if $X = \mathbb{R}$). His work was pioneering in the sense that the L^2 -based bundle-valued Sobolev space $H^m(X; TN)$ for u_x was revealed to be suitable to understand the structure of the equation for the first time. After that, short-time existence results for higher-dimensional Schrödinger maps were established. See, [5], [12] and references therein. When $a \neq 0, b \in \mathbb{R}$, the author showed the short-time existence of a unique solution in the class $H^{m+1}(X; N)$ for any integer $m \geq 2$ (see [15]).

If $\nabla J \neq 0$, a loss of one derivative occurs in the equation and the classical energy method does not work well. However, very recently, Chihara succeeded to prove short-time existence theorem for higher-dimensional Schrödinger maps without assuming the Kähler condition in [2]. Also for the third order equation (1), he and the author showed short-time existence theorem when $a \neq 0$ and $b \in \mathbb{R}$ without assuming the Kähler condition. See [3] and [17]. The idea of their proof is to construct a gauge transformation on the pull-back bundle $u^{-1}TN$ to eliminate the seemingly bad first order derivative loss. These results require more regularity $m \geq 4$ for the class of the solution.

On the other hands, global existence results for (1)-(2) have been studied by adding some more conditions on N . When $a, b = 0$ and $X = \mathbb{T}$, Koiso proved that the solution exists globally in time if the Kähler manifold N is the locally Hermitian symmetric space ($\nabla R \equiv 0$) by finding a conservation law in [8]. Pang, Wang and Wang obtained the same results when $a, b = 0$ and $X = \mathbb{R}$ in [18]. Being inspired with Hasimoto's pioneering work in [7], Chang, Shatah and Uhlenbeck constructed a good moving frame along the map and rigorously reduced the equation for the one-dimensional Schrödinger map to a simple form of a complex-valued nonlinear Schrödinger equation to discuss the global existence of the Schrödinger map into Riemann surfaces. Though their argument is restricted only to the case where $X = \mathbb{R}$ and the map is assumed to have a fixed point on N as $x \rightarrow -\infty$, this reduction gives us understandings on an essential structure of one-dimensional Schrödinger maps. (see [1]). For the case $a \neq 0$, the author proved the global existence theorem by assuming that N is the compact Riemann surface with constant Gaussian curvature K and $b = aK/2$ in [15]. Under the condition, (1) behaves as completely integrable systems and some conservation laws of the equation work in the proof. However, without such assumption, (1) cannot be expected to be completely integrable in general, even if $\nabla R \equiv 0$ is assumed as in the case $a, b = 0$.

The aim of this article is to establish a global existence theorem for (1)-(2) under the condition $\nabla R \equiv 0$ also when $a \neq 0$, without the previous assumption in [15]. The main theorem is the following:

Theorem 1. *Let (N, J, g) be a compact locally Hermitian symmetric space, $a \neq 0, b \in \mathbb{R}$, and let m be a positive integer satisfying $m \geq 2$. Then, for any $u_0 \in H^{m+1}(X; N)$, the initial value problem (1)-(2) admits a unique solution $u \in C(\mathbb{R}; H^{m+1}(X; N))$.*

Theorem 1 gives not only an extension of the previous result by the author in [15] for the case $a \neq 0$ but also an analogue of the result by Koiso in [8] for the case $a, b = 0$.

To prove the theorem, we apply two conservation laws and an energy quantity for this equation. More precisely, we use the following integral quantities of the form

$$\begin{aligned} E_1(u) &= a \|\nabla_x u_x\|_{L^2}^2 - \frac{b}{2} \int_X (g(u_x, u_x))^2 dx - \int_X g(u_x, J \nabla_x u_x) dx, \\ E_2(u) &= 3a \|\nabla_x^2 u_x\|_{L^2}^2 - 10b \int_X (g(u_x, \nabla_x u_x))^2 dx \end{aligned} \quad (5)$$

$$-5b \int_X g(u_x, u_x) g(\nabla_x u_x, \nabla_x u_x) dx + 2a \int_X g(R(u_x, \nabla_x u_x) u_x, \nabla_x u_x) dx. \quad (6)$$

While $\|u_x(t)\|_{L^2}^2$ and $E_1(u(t))$ are preserved in time, $E_2(u(t))$ is not necessarily preserved in time. However, the a priori estimate itself for $\|\nabla_x^2 u_x(t)\|_{L^2}^2$ can be obtained by careful computation. They imply a bound for $u_x(t)$ in $H^2(X; TN)$, which, in view of the local existence result, prevents the formation of a finite-time singularity.

The idea of finding such quantities comes from [10] and [16]. To explain this, assume that N is a compact Riemann surface with constant Gaussian curvature K and $X = \mathbb{R}$. From [16, Theorem 1], the equation (1) for $u(t, x) : \mathbb{R}_t \times \mathbb{R}_x \rightarrow N$ which has a fixed point on N as $x \rightarrow -\infty$ can be reduced to a third order dispersive equation with constant coefficient of the form

$$q_t - a q_{xxx} - \sqrt{-1} q_{xx} = \left(\frac{a}{2}K + 2b\right) |q|^2 q_x - \left(\frac{a}{2}K - b\right) q^2 \bar{q}_x + \frac{\sqrt{-1}}{2} K |q|^2 q \quad (7)$$

for complex-valued function $q(t, x) : \mathbb{R}_t \times \mathbb{R}_x \rightarrow \mathbb{C}$. This reduction is obtained via the relation

$$u_x = q_1 e + q_2 J e, \quad q = q_1 + \sqrt{-1} q_2, \quad \nabla_x e = 0, \quad (8)$$

where $\{e, J e\}$ is the moving frame along u introduced by Chang, Shatah and Uhlenbeck in [1]. On the other hands, the global existence theorem for the equation of the form

$$q_t + A q_{xxx} - \sqrt{-1} B q_{xx} = -\sqrt{-1} \alpha |q|^2 q + \beta |q|_x^2 q + \gamma |q|^2 q_x \quad (9)$$

was established in the class $H^2(X; \mathbb{C})$ by Laurey in [10], where $A, B, \alpha, \beta, \gamma \in \mathbb{R}$ and $A \neq 0, \beta \neq 0$. The key idea of her proof was to exposit nice quantities of the form

$$-3A\beta \|q_x\|_2^2 - \beta \left(\beta + \frac{\gamma}{2}\right) \|q\|_4^2 + \sqrt{-1} (B(2\beta + \gamma) - 3A\alpha) \int_X q \bar{q}_x dx, \quad (10)$$

$$3A \|q_{xx}\|_2^2 + (6\beta + 4\gamma) \int_X |q|^2 |q_x|^2 dx + (4\beta + \gamma) \operatorname{Re} \int_X q^2 \bar{q}_x^2 dx, \quad (11)$$

and $\|q\|_2^2$, where $\|\cdot\|_p$ is the standard L^p -norm for complex-valued function on X . See (5.8), (5.12) and (5.4) respectively in [10]. If we set

$$A = -a, \quad B = 1, \quad \alpha = -K/2, \quad \beta = b - aK/2, \quad \gamma = b + aK \quad (12)$$

and take (10) / 3β , (11) $\times -1$, we get

$$a \|q_x\|_2^2 - \frac{b}{2} \|q\|_4^2 + \sqrt{-1} \int_X q \bar{q}_x dx, \quad (13)$$

$$3a \|q_{xx}\|_2^2 - (aK + 10b) \int_X |q|^2 |q_x|^2 dx - (-aK + 5b) \operatorname{Re} \int_X q^2 \bar{q}_x^2 dx. \quad (14)$$

In fact, via the relation (8), these quantities (13), (14) and $\|q\|_2^2$ are reformulated as $E_1(u)$, $E_2(u)$ and $\|u_x\|_{L^2}^2$ respectively. These quantities make sense and work effectively to prove Theorem 1 also when $X = \mathbb{T}$ or when the solution has no fixed point as $x \rightarrow -\infty$, as far as the Kähler manifold N satisfies the condition $\nabla R \equiv 0$. Therefore, we can say that $\nabla J \equiv \nabla R \equiv 0$ is the assumption for the original equation (1) to behave essentially as a third order complex-valued nonlinear dispersive equation with constant coefficients, whose global existence result is well known. The proof of Theorem 1 itself will be given in the next section.

Remark 2. It seems to be reasonable to state the difference between our result and previous ones through the nonlinear structure of the equation (7). It is known that the equation (7) is not necessarily completely integrable when $a \neq 0$ and $b \in \mathbb{R}$, which is unlike the case for $a, b = 0$. See, e.g., [6], [11], [21]. However, if $a \neq 0$ and $b = aK/2$, the equation (7) is so-called the Hirota equation which is completely integrable. This is strongly related to the fact that there exists a conservation law to control $\nabla_x^2 u_x(t)$ if N is a Riemann surface with constant curvature K and $b = aK/2$. See [15, Lemma 6.1].

2. PROOF OF THE TIME-GLOBAL EXISTENCE THEOREM

First, we recall basic notation and facts to get estimation. We make use of basic techniques of geometric analysis of nonlinear problems. See [13] for instance. For $u : X \rightarrow N$, $\Gamma(u^{-1}TN)$ denotes the set of the section of $u^{-1}TN$, and $\|\cdot\|_{L^2}$ is a norm of $L^2(X; TN)$ defined by

$$\|V\|_{L^2}^2 = \int_X g(V, V)dx \quad \text{for } V \in \Gamma(u^{-1}TN).$$

For positive integer k , $H^{k+1}(X; N)$ denotes the set of all continuous mappings $u : X \rightarrow N$ satisfying $u_x \in H^k(X; TN)$, that is,

$$\|u_x\|_{H^k(X; TN)}^2 = \sum_{l=0}^k \|\nabla_x^l u_x\|_{L^2}^2 = \sum_{l=0}^k \int_X g_{u(x)}(\nabla_x^l u_x(x), \nabla_x^l u_x(x))dx < +\infty.$$

The main tools of the computation below are

$$\int_X g(\nabla_x V, W)dx = - \int_X g(V, \nabla_x W)dx, \quad (15)$$

$$\nabla_x u_t = \nabla_t u_x, \quad (16)$$

$$\nabla_x^{k+1} u_t = \nabla_t \nabla_x^k u_x + \sum_{l=0}^{k-1} \nabla_x^l \left[R(u_x, u_t) \nabla_x^{k-(l+1)} u_x \right], \quad k \in \mathbb{N}, \quad (17)$$

$$R(V, W) = -R(W, V), \quad \text{in particular } R(V, V) = 0, \quad (18)$$

$$g(R(V_1, V_2)V_3, V_4) = g(R(V_3, V_4)V_1, V_2) \quad (19)$$

for $V, W, V_j \in \Gamma(u^{-1}TN)$, $j = 1, 2, 3, 4$, where R is the Riemannian curvature tensor on N . In addition, the notation like C or $C(\cdot, \dots, \cdot)$ will be sometimes used to denote a positive constant depending on certain parameters, such as a, b , geometric properties of N , et al.

We start the proof of Theorem 1 from a short time existence result. Since the locally Hermitian symmetric space is the Kähler manifold, short-time existence is ensured by the following:

Theorem 3 (Theorem 1.1 in [15] and Theorem 1.2 in [17]). *Let (N, J, g) be a compact Kähler manifold and let $a \neq 0$ and $b \in \mathbb{R}$. Then for any $u_0 \in H^{m+1}(X; N)$ with an integer $m \geq 2$, there exists a constant $T > 0$ depending only on a, b, N and $\|u_{0x}\|_{H^2}$ such that the initial value problem (1)-(2) possesses a unique solution $u \in C([-T, T]; H^{m+1}(X; N))$.*

Let T be the largest number such that a solution $u(t, x)$ with the initial data $u_0 \in H^{m+1}$ exists on the interval $0 \leq t < T$. If $\|u_x(t)\|_{H^m}$ is uniformly bounded on $[0, T)$, then we can extend the solution beyond T , which implies that the maximal existence time is infinite. Therefore, it suffices to show the following.

Proposition 4. *Let $u(t, x)$ be a solution of (1) with initial data $u_0 \in H^{m+1}(X; N)$ on $[0, T)$, where T is positive and finite number. Then $\|u_x(t)\|_{H^m}$ is uniformly bounded on $[0, T)$.*

Proof of Proposition 4. We show the proof only for the case $X = \mathbb{T}$, since the argument for the case $X = \mathbb{R}$ is essentially parallel to the case $X = \mathbb{T}$. We sometimes use Sobolev's inequality of the form

$$\|V\|_{L^\infty}^2 \leq C\|V\|_{L^2}(\|V\|_{L^2} + \|\nabla_x V\|_{L^2}) \quad (20)$$

for $V \in \Gamma(u^{-1}TN)$ below with no mention. See, e.g., [9, Lemma 1. 3. and 1. 4.] for the proof.

Now, we establish two conservation laws and a semi-conservation law on $[0, T)$ of the form

$$\frac{d}{dt} \|u_x(t)\|_{L^2}^2 = 0, \quad (21)$$

$$\frac{d}{dt} E_1(u(t)) = 0, \quad (22)$$

$$\frac{d}{dt}E_2(u(t)) = F(u(t)) \quad (23)$$

for the solution $u(t, x)$, where

$$|F(u(t))| \leq C(a, b, N, \|u_x(t)\|_{H^1})(1 + \|\nabla_x^2 u_x(t)\|_{L^2}^2). \quad (24)$$

Proposition 4 is proved by (21)-(24) in the following manner: If (21) is true, then $\|u_x(t)\|_{L^2} = \|u_{0x}\|$ holds for $t \in [0, T]$. In addition, if (22) is true, by integrating (22) in t and by using the inequality (20), we have

$$\begin{aligned} a \|\nabla_x u_x\|_{L^2}^2 &= \frac{b}{2} \int_X (g(u_x, u_x))^2 dx + \int_X g(u_x, J\nabla_x u_x) dx + E_1(u_0) \\ &\leq C_1(a, b, \|u_{0x}\|_{H^1}) + C_2(b, \|u_{0x}\|_{L^2})(1 + \|\nabla_x u_x\|_{L^2}). \end{aligned}$$

It means that $\|u_x(t)\|_{H^1}$ is uniformly bounded by some constant $C(a, b, \|u_{0x}\|_{H^1})$ on $[0, T]$. Thus if (23) and (24) are also true, after integrating (23) in t , we get

$$\begin{aligned} a \|\nabla_x^2 u_x(t)\|_{L^2}^2 &= 10b \int_X (g(u_x, \nabla_x u_x))^2(t) dx + 5b \int_X g(u_x, u_x) g(\nabla_x u_x, \nabla_x u_x)(t) dx \\ &\quad - 2a \int_X g(R(u_x, \nabla_x u_x)u_x, \nabla_x u_x)(t) dx + E_2(u_0) + \int_0^t F(u(\tau)) d\tau \\ &\leq C_1(a, b, N, \|u_{0x}\|_{H^2}) + C_2(a, b, N, \|u_{0x}\|_{H^1}) \int_0^t (1 + \|\nabla_x^2 u_x(\tau)\|_{L^2}^2) d\tau. \end{aligned}$$

Therefore, Gronwall's lemma implies that $\|\nabla_x^2 u_x(t)\|_{L^2}$ is uniformly bounded on $[0, T]$ and thus $\|u_x(t)\|_{H^2}$ is uniformly bounded on $[0, T]$. Finally, the desired H^m -uniform estimate is obtained by using the estimate

$$\frac{d}{dt} \|u_x(t)\|_{H^k}^2 \leq C(a, b, N) P(\|u_x(t)\|_{H^{k-1}}) \|u_x(t)\|_{H^k}^2 \quad (25)$$

inductively for $3 \leq k \leq m$, where $P(\cdot)$ is some polynomial function on \mathbb{R} . The estimate (25) has already been shown in [15, Lemma 4.1] to prove the short-time existence theorem.

From now on, we check (22)-(24). (First conservation law (21) is obvious, so we omit the computation.) We often use (15)-(20) with no mention below.

To obtain (22), we first deduce

$$\begin{aligned} \frac{d}{dt} [a \|\nabla_x u_x\|_{L^2}^2] &= 2a \int_X g(\nabla_x u_x, \nabla_t \nabla_x u_x) dx \\ &= 2a \int_X g(\nabla_x u_x, \nabla_x^2 u_t) dx + 2a \int_X g(\nabla_x u_x, R(u_t, u_x)u_x) dx \\ &= 2a \int_X g(\nabla_x^3 u_x, u_t) dx - 2a \int_X g(R(u_x, \nabla_x u_x)u_x, u_t) dx. \end{aligned} \quad (26)$$

Since $u(t, x)$ solves the equation (1), we have

$$\begin{aligned} 2a \int_X g(\nabla_x^3 u_x, u_t) dx &= 2a \int_X g(\nabla_x^3 u_x, a \nabla_x^2 u_x) dx \\ &\quad + 2a \int_X g(\nabla_x^3 u_x, J\nabla_x u_x) dx \\ &\quad + 2a \int_X g(\nabla_x^3 u_x, b g(u_x, u_x)u_x) dx \\ &= 2ab \int_X g(\nabla_x^3 u_x, g(u_x, u_x)u_x) dx \end{aligned}$$

$$\begin{aligned}
&= -4ab \int_X g(\nabla_x u_x, u_x) g(u_x, \nabla_x^2 u_x) dx \\
&\quad - 2ab \int_X g(u_x, u_x) g(\nabla_x u_x, \nabla_x^2 u_x) dx \\
&= 6ab \int_X g(\nabla_x u_x, u_x) g(\nabla_x u_x, \nabla_x u_x) dx, \tag{27}
\end{aligned}$$

$$\begin{aligned}
-2a \int_X g(R(u_x, \nabla_x u_x) u_x, u_t) dx &= -2a \int_X g(R(u_x, \nabla_x u_x) u_x, a \nabla_x^2 u_x) dx \\
&\quad - 2a \int_X g(R(u_x, \nabla_x u_x) u_x, J \nabla_x u_x) dx \\
&\quad - 2a \int_X g(R(u_x, \nabla_x u_x) u_x, b g(u_x, u_x) u_x) dx \\
&= -2a^2 \int_X g(R(u_x, \nabla_x u_x) u_x, \nabla_x^2 u_x) dx \\
&\quad - 2a \int_X g(R(u_x, \nabla_x u_x) u_x, J \nabla_x u_x) dx \\
&= a^2 \int_X g((\nabla R)(u_x)(u_x, \nabla_x u_x) u_x, \nabla_x u_x) dx \\
&\quad - 2a \int_X g(R(u_x, \nabla_x u_x) u_x, J \nabla_x u_x) dx. \tag{28}
\end{aligned}$$

Remark that the second equality of (28) follows from (18) and the final equality of (28) follows from (15) and (19). Substituting (27) and (28) into (26), we obtain

$$\begin{aligned}
\frac{d}{dt} [a \|\nabla_x u_x\|_{L^2}^2] &= 6ab \int_X g(\nabla_x u_x, u_x) g(\nabla_x u_x, \nabla_x u_x) dx \\
&\quad + a^2 \int_X g((\nabla R)(u_x)(u_x, \nabla_x u_x) u_x, \nabla_x u_x) dx \\
&\quad - 2a \int_X g(R(u_x, \nabla_x u_x) u_x, J \nabla_x u_x) dx. \tag{29}
\end{aligned}$$

In the same way, we deduce

$$\begin{aligned}
&\frac{d}{dt} \left[-\frac{b}{2} \int_X (g(u_x, u_x))^2 dx \right] \\
&= -2b \int_X g(u_x, u_x) g(u_x, \nabla_t u_x) dx \\
&= -2b \int_X g(u_x, u_x) g(u_x, \nabla_x u_t) dx \\
&= 4b \int_X g(\nabla_x u_x, u_x) g(u_x, u_t) dx + 2b \int_X g(u_x, u_x) g(\nabla_x u_x, u_t) dx \\
&= 4b \int_X g(\nabla_x u_x, u_x) g(u_x, a \nabla_x^2 u_x) dx \\
&\quad + 4b \int_X g(\nabla_x u_x, u_x) g(u_x, J \nabla_x u_x) dx \\
&\quad + 4b \int_X g(\nabla_x u_x, u_x) g(u_x, b g(u_x, u_x) u_x) dx \\
&\quad + 2b \int_X g(u_x, u_x) g(\nabla_x u_x, a \nabla_x^2 u_x) dx
\end{aligned}$$

$$\begin{aligned}
& + 2b \int_X g(u_x, u_x) g(\nabla_x u_x, J \nabla_x u_x) dx \\
& + 2b \int_X g(u_x, u_x) g(\nabla_x u_x, b g(u_x, u_x) u_x) dx \\
& = 4ab \int_X g(\nabla_x u_x, u_x) g(u_x, \nabla_x^2 u_x) dx \\
& + 4b \int_X g(\nabla_x u_x, u_x) g(u_x, J \nabla_x u_x) dx \\
& + 2ab \int_X g(u_x, u_x) g(\nabla_x u_x, \nabla_x^2 u_x) dx \\
& = -6ab \int_X g(\nabla_x u_x, u_x) g(\nabla_x u_x, \nabla_x u_x) dx \\
& + 4b \int_X g(\nabla_x u_x, u_x) g(u_x, J \nabla_x u_x) dx.
\end{aligned} \tag{30}$$

Note that the final equality of (30) comes from

$$\int_X (g(u_x, u_x))^2 g(u_x, \nabla_x u_x) dx = \frac{1}{6} \int_X \left[(g(u_x, u_x))^3 \right]_x dx = 0.$$

Furthermore we deduce

$$\begin{aligned}
& \frac{d}{dt} \left[- \int_X g(u_x, J \nabla_x u_x) dx \right] \\
& = - \int_X g(\nabla_t u_x, J \nabla_x u_x) dx - \int_X g(u_x, J \nabla_t \nabla_x u_x) dx \\
& = - \int_X g(\nabla_x u_t, J \nabla_x u_x) dx - \int_X g(u_x, J \nabla_x^2 u_t + J R(u_t, u_x) u_x) dx \\
& = 2 \int_X g(u_t, J \nabla_x^2 u_x) dx - \int_X g(R(u_x, J u_x) u_x, u_t) dx \\
& = 2 \int_X g(b g(u_x, u_x) u_x, J \nabla_x^2 u_x) dx \\
& \quad - \int_X g(R(u_x, J u_x) u_x, a \nabla_x^2 u_x) dx \\
& \quad - \int_X g(R(u_x, J u_x) u_x, J \nabla_x u_x) dx.
\end{aligned} \tag{31}$$

Here, for each term of right hand side of the above, a simple computation shows

$$\begin{aligned}
& 2 \int_X g(b g(u_x, u_x) u_x, J \nabla_x^2 u_x) dx \\
& \quad = -4b \int_X g(\nabla_x u_x, u_x) g(u_x, J \nabla_x u_x) dx, \\
& \quad - \int_X g(R(u_x, J u_x) u_x, a \nabla_x^2 u_x) dx \\
& \quad = a \int_X g((\nabla R)(u_x)(u_x, J u_x) u_x, \nabla_x u_x) dx \\
& \quad + a \int_X g(R(\nabla_x u_x, J u_x) u_x, \nabla_x u_x) dx \\
& \quad + a \int_X g(R(u_x, J \nabla_x u_x) u_x, \nabla_x u_x) dx
\end{aligned} \tag{32}$$

$$\begin{aligned}
&= a \int_X g((\nabla R)(u_x)(u_x, Ju_x)u_x, \nabla_x u_x) dx \\
&\quad + 2a \int_X g(R(u_x, \nabla_x u_x)u_x, J\nabla_x u_x) dx, \\
&- \int_X g(R(u_x, Ju_x)u_x, J\nabla_x u_x) dx \\
&= -\frac{3}{4} \int_X g(R(u_x, Ju_x)u_x, J\nabla_x u_x) dx \\
&\quad + \frac{1}{4} \int_X g((\nabla R)(u_x)(u_x, Ju_x)u_x, Ju_x) dx \\
&\quad + \frac{1}{4} \int_X g(R(\nabla_x u_x, Ju_x)u_x, Ju_x) dx \\
&\quad + \frac{1}{4} \int_X g(R(u_x, J\nabla_x u_x)u_x, Ju_x) dx \\
&\quad + \frac{1}{4} \int_X g(R(u_x, Ju_x)\nabla_x u_x, Ju_x) dx \\
&= \frac{1}{4} \int_X g((\nabla R)(u_x)(u_x, Ju_x)u_x, Ju_x) dx.
\end{aligned} \tag{33}$$

$$\begin{aligned}
&= \frac{1}{4} \int_X g((\nabla R)(u_x)(u_x, Ju_x)u_x, Ju_x) dx.
\end{aligned} \tag{34}$$

Substituting (32)-(34) into (31), we obtain

$$\begin{aligned}
&\frac{d}{dt} \left[- \int_X g(u_x, J\nabla_x u_x) dx \right] \\
&= -4b \int_X g(\nabla_x u_x, u_x) g(u_x, J\nabla_x u_x) dx \\
&\quad + a \int_X g((\nabla R)(u_x)(u_x, Ju_x)u_x, \nabla_x u_x) dx \\
&\quad + 2a \int_X g(R(u_x, \nabla_x u_x)u_x, J\nabla_x u_x) dx \\
&\quad + \frac{1}{4} \int_X g((\nabla R)(u_x)(u_x, Ju_x)u_x, Ju_x) dx.
\end{aligned} \tag{35}$$

Consequently, by adding (29), (30) and (35), we obtain

$$\begin{aligned}
&\frac{d}{dt} \left[a \|\nabla_x u_x\|_{L^2}^2 - \frac{b}{2} \int_X (g(u_x, u_x))^2 dx - \int_X g(u_x, J\nabla_x u_x) dx \right] \\
&= a^2 \int_X g((\nabla R)(u_x)(u_x, \nabla_x u_x)u_x, \nabla_x u_x) dx \\
&\quad + a \int_X g((\nabla R)(u_x)(u_x, Ju_x)u_x, \nabla_x u_x) dx \\
&\quad + \frac{1}{4} \int_X g((\nabla R)(u_x)(u_x, Ju_x)u_x, Ju_x) dx,
\end{aligned}$$

and the right hand side of the above vanishes due to the assumption $\nabla R \equiv 0$. Thus we obtain the conservation law (22).

We next show (23). A simple computation gives

$$\begin{aligned}
&\frac{d}{dt} [3a \|\nabla_x^2 u_x\|_{L^2}^2] \\
&= 6a \int_X g(\nabla_t \nabla_x^2 u_x, \nabla_x^2 u_x) dx
\end{aligned}$$

$$\begin{aligned}
&= 6a \int_X g(\nabla_x^3 u_t + \nabla_x [R(u_t, u_x)u_x] + R(u_t, u_x)\nabla_x u_x, \nabla_x^2 u_x) dx \\
&= -6a \int_X g(\nabla_x^5 u_x, u_t) dx \\
&\quad + 6a \int_X g(R(u_x, \nabla_x^3 u_x)u_x, u_t) dx \\
&\quad - 6a \int_X g(R(\nabla_x u_x, \nabla_x^2 u_x)u_x, u_t) dx \\
&= -6a \int_X g(\nabla_x^5 u_x, b g(u_x, u_x)u_x) dx \\
&\quad + 6a \int_X g(R(u_x, \nabla_x^3 u_x)u_x, a \nabla_x^2 u_x) dx \\
&\quad + 6a \int_X g(R(u_x, \nabla_x^3 u_x)u_x, J \nabla_x u_x) dx \\
&\quad - 6a \int_X g(R(\nabla_x u_x, \nabla_x^2 u_x)u_x, a \nabla_x^2 u_x) dx \\
&\quad - 6a \int_X g(R(\nabla_x u_x, \nabla_x^2 u_x)u_x, J \nabla_x u_x) dx.
\end{aligned} \tag{36}$$

Here, the integration by parts and the property of the Riemannian curvature tensor yield

$$\begin{aligned}
\int_X g(\nabla_x^5 u_x, g(u_x, u_x)u_x) dx &= -10 \int_X g(\nabla_x^2 u_x, \nabla_x u_x) g(\nabla_x^2 u_x, u_x) dx \\
&\quad - 5 \int_X g(\nabla_x^2 u_x, \nabla_x^2 u_x) g(\nabla_x u_x, u_x) dx,
\end{aligned} \tag{37}$$

$$\begin{aligned}
\int_X g(R(u_x, \nabla_x^3 u_x)u_x, \nabla_x^2 u_x) dx &= - \int_X g(R(\nabla_x u_x, \nabla_x^2 u_x)u_x, \nabla_x^2 u_x) dx \\
&\quad - \frac{1}{2} \int_X g((\nabla R)(u_x)(u_x, \nabla_x^2 u_x)u_x, \nabla_x^2 u_x) dx,
\end{aligned} \tag{38}$$

$$\begin{aligned}
\int_X g(R(u_x, \nabla_x^3 u_x)u_x, J \nabla_x u_x) dx &= - \int_X g((\nabla R)(u_x)(u_x, \nabla_x^2 u_x)u_x, J \nabla_x u_x) dx \\
&\quad - \int_X g(R(\nabla_x u_x, \nabla_x^2 u_x)u_x, J \nabla_x u_x) dx \\
&\quad - \int_X g(R(u_x, \nabla_x^2 u_x)\nabla_x u_x, J \nabla_x u_x) dx \\
&\quad - \int_X g(R(u_x, \nabla_x^2 u_x)u_x, J \nabla_x^2 u_x) dx.
\end{aligned} \tag{39}$$

By substituting (37)-(39) into (36), we obtain

$$\begin{aligned}
\frac{d}{dt} [3a \|\nabla_x^2 u_x\|_{L^2}^2] &= -12a^2 \int_X g(R(\nabla_x u_x, \nabla_x^2 u_x)u_x, \nabla_x^2 u_x) dx \\
&\quad + 60ab \int_X g(\nabla_x^2 u_x, \nabla_x u_x) g(\nabla_x^2 u_x, u_x) dx \\
&\quad + 30ab \int_X g(\nabla_x^2 u_x, \nabla_x^2 u_x) g(\nabla_x u_x, u_x) dx \\
&\quad + F_0,
\end{aligned} \tag{40}$$

where

$$\begin{aligned}
F_0 = & -12a \int_X g(R(\nabla_x u_x, \nabla_x^2 u_x)u_x, J\nabla_x u_x)dx \\
& - 6a \int_X g(R(u_x, \nabla_x^2 u_x)\nabla_x u_x, J\nabla_x u_x)dx \\
& - 6a \int_X g(R(u_x, \nabla_x^2 u_x)u_x, J\nabla_x^2 u_x)dx \\
& - 3a^2 \int_X g((\nabla R)(u_x)(u_x, \nabla_x^2 u_x)u_x, \nabla_x^2 u_x)dx \\
& - 6a \int_X g((\nabla R)(u_x)(u_x, \nabla_x^2 u_x)u_x, J\nabla_x u_x)dx.
\end{aligned} \tag{41}$$

Here, F_0 has the same estimate as (24). To get the estimate, note that (20) implies

$$\|\nabla_x u_x(t)\|_{L^\infty} \leq C(\|u_x(t)\|_{H^1}) (1 + \|\nabla_x u_x(t)\|_{L^2}^2)^{1/2}. \tag{42}$$

Then, it is easy to get

$$\begin{aligned}
|F_0| \leq & C(a, N) \{ \|u_x\|_{L^\infty} \|\nabla_x u_x\|_{L^2} \|\nabla_x u_x\|_{L^\infty} \|\nabla_x^2 u_x\|_{L^2} \\
& + (\|u_x\|_{L^\infty}^2 + \|u_x\|_{L^\infty}^3) \|\nabla_x^2 u_x\|_{L^2}^2 \\
& + \|u_x\|_{L^\infty}^3 \|\nabla_x u_x\|_{L^2} \|\nabla_x^2 u_x\|_{L^2} \} \\
\leq & C(a, N, \|u_x\|_{H^1}) (1 + \|\nabla_x^2 u_x\|_{L^2}^2).
\end{aligned} \tag{43}$$

To cancel the terms with higher order derivatives in the right hand side of (40) except for F_0 , we apply the rest part of the energy E_2 . To neglect the effect of the lower order terms such as F_0 , we use the notation $f \equiv 0$ for any function $f(t)$ on $[0, T)$ if

$$|f(t)| \leq C(a, b, N, \|u_x(t)\|_{H^1}) (1 + \|\nabla_x^2 u_x(t)\|_{L^2}^2). \tag{44}$$

As we can see also from (41)-(43), the integral where the sum of the order of the covariant derivative operator is less than five can be estimated as (44). In other words, we have only to pay attention to the integral where the sum of the order of the covariant derivative is five.

Having them in mind, we first deduce

$$\begin{aligned}
& \frac{d}{dt} \left[-10b \int_X (g(u_x, \nabla_x u_x))^2 dx \right] \\
& = -20b \int_X g(u_x, \nabla_x u_x) g(u_x, \nabla_t \nabla_x u_x) dx \\
& \quad - 20b \int_X g(u_x, \nabla_x u_x) g(\nabla_t u_x, \nabla_x u_x) dx \\
& = -20b \int_X g(u_x, \nabla_x u_x) g(u_x, \nabla_x^2 u_t) dx \\
& \quad - 20b \int_X g(u_x, \nabla_x u_x) g(u_x, R(u_t, u_x)u_x) dx \\
& \quad - 20b \int_X g(u_x, \nabla_x u_x) g(\nabla_x u_t, \nabla_x u_x) dx \\
& = 20b \int_X [g(u_x, \nabla_x u_x)]_x g(u_x, \nabla_x u_t) dx \\
& = -20b \int_X g(u_x, \nabla_x^3 u_x) g(u_x, u_t) dx
\end{aligned}$$

$$\begin{aligned}
& -60b \int_X g(\nabla_x^2 u_x, \nabla_x u_x) g(u_x, u_t) dx \\
& -20b \int_X g(\nabla_x u_x, \nabla_x u_x) g(\nabla_x u_x, u_t) dx \\
& -20b \int_X g(u_x, \nabla_x^2 u_x) g(\nabla_x u_x, u_t) dx \\
& \equiv -20b \int_X g(u_x, \nabla_x^3 u_x) g(u_x, a \nabla_x^2 u_x) dx \\
& -60b \int_X g(\nabla_x^2 u_x, \nabla_x u_x) g(u_x, a \nabla_x^2 u_x) dx \\
& -20b \int_X g(\nabla_x u_x, \nabla_x u_x) g(\nabla_x u_x, a \nabla_x^2 u_x) dx \\
& -20b \int_X g(u_x, \nabla_x^2 u_x) g(\nabla_x u_x, a \nabla_x^2 u_x) dx \\
& = -60ab \int_X g(\nabla_x u_x, \nabla_x^2 u_x) g(u_x, \nabla_x^2 u_x) dx,
\end{aligned} \tag{45}$$

where the last equality follows from

$$\int_X g(u_x, \nabla_x^3 u_x) g(u_x, \nabla_x^2 u_x) dx = - \int_X g(\nabla_x u_x, \nabla_x^2 u_x) g(u_x, \nabla_x^2 u_x) dx, \tag{46}$$

$$\int_X g(\nabla_x u_x, \nabla_x u_x) g(\nabla_x u_x, \nabla_x^2 u_x) dx = \frac{1}{4} \int_X \left[(g(\nabla_x u_x, \nabla_x u_x))^2 \right]_x dx = 0. \tag{47}$$

Moreover, we deduce

$$\begin{aligned}
& \frac{d}{dt} \left[-5b \int_X g(u_x, u_x) g(\nabla_x u_x, \nabla_x u_x) dx \right] \\
& = -10b \int_X g(u_x, u_x) g(\nabla_x u_x, \nabla_t \nabla_x u_x) dx \\
& -10b \int_X g(\nabla_t u_x, u_x) g(\nabla_x u_x, \nabla_x u_x) dx \\
& = -10b \int_X g(u_x, u_x) g(\nabla_x u_x, \nabla_x^2 u_t) dx \\
& -10b \int_X g(u_x, u_x) g(\nabla_x u_x, R(u_t, u_x) u_x) dx \\
& -10b \int_X g(\nabla_x u_t, u_x) g(\nabla_x u_x, \nabla_x u_x) dx \\
& = -10b \int_X g(u_x, u_x) g(\nabla_x^3 u_x, u_t) dx \\
& -40b \int_X g(\nabla_x u_x, u_x) g(\nabla_x^2 u_x, u_t) dx \\
& -20b \int_X g(\nabla_x^2 u_x, u_x) g(\nabla_x u_x, u_t) dx \\
& +10b \int_X g(u_x, u_x) g(R(u_x, \nabla_x u_x) u_x, u_t) dx \\
& -20b \int_X g(\nabla_x u_x, \nabla_x u_x) g(\nabla_x u_x, u_t) dx
\end{aligned}$$

$$\begin{aligned}
& + 20b \int_X g(\nabla_x^2 u_x, \nabla_x u_x) g(u_x, u_t) dx \\
& + 10b \int_X g(\nabla_x u_x, \nabla_x u_x) g(\nabla_x u_x, u_t) dx \\
& \equiv - 10b \int_X g(u_x, u_x) g(\nabla_x^3 u_x, a \nabla_x^2 u_x) dx \\
& - 40b \int_X g(\nabla_x u_x, u_x) g(\nabla_x^2 u_x, a \nabla_x^2 u_x) dx \\
& - 20b \int_X g(\nabla_x^2 u_x, u_x) g(\nabla_x u_x, a \nabla_x^2 u_x) dx \\
& - 10b \int_X g(\nabla_x u_x, \nabla_x u_x) g(\nabla_x u_x, a \nabla_x^2 u_x) dx \\
& + 20b \int_X g(\nabla_x^2 u_x, \nabla_x u_x) g(u_x, a \nabla_x^2 u_x) dx \\
& = - 30ab \int_X g(\nabla_x u_x, u_x) g(\nabla_x^2 u_x, \nabla_x^2 u_x) dx.
\end{aligned} \tag{48}$$

Note that the last equality follows from (47) and

$$\int_X g(u_x, u_x) g(\nabla_x^3 u_x, \nabla_x^2 u_x) dx = - \int_X g(\nabla_x u_x, u_x) g(\nabla_x^2 u_x, \nabla_x^2 u_x) dx.$$

In the same way, we get

$$\begin{aligned}
& \frac{d}{dt} \left[2a \int_X g(R(u_x, \nabla_x u_x) u_x, \nabla_x u_x) dx \right] \\
& = 4a \int_X g(R(\nabla_t u_x, \nabla_x u_x) u_x, \nabla_x u_x) dx \\
& \quad + 4a \int_X g(R(u_x, \nabla_t \nabla_x u_x) u_x, \nabla_x u_x) dx \\
& = 4a \int_X g(R(\nabla_x u_t, \nabla_x u_x) u_x, \nabla_x u_x) dx \\
& \quad + 4a \int_X g(R(u_x, \nabla_x^2 u_t) u_x, \nabla_x u_x) dx \\
& \quad + 4a \int_X g(R(u_x, R(u_t, u_x) u_x) u_x, \nabla_x u_x) dx \\
& = - 4a \int_X g(R(u_x, \nabla_x u_t) u_x, \nabla_x^2 u_x) dx \\
& \quad - 8a \int_X g(R(\nabla_x u_x, \nabla_x u_t) u_x, \nabla_x u_x) dx \\
& \quad + 4a \int_X g(R(u_x, \nabla_x u_x) u_x, R(u_t, u_x) u_x) dx \\
& = 4a \int_X g(R(\nabla_x u_x, \nabla_x^2 u_x) u_x, u_t) dx \\
& \quad + 4a \int_X g(R(u_x, \nabla_x^3 u_x) u_x, u_t) dx \\
& \quad + 12a \int_X g(R(u_x, \nabla_x^2 u_x) \nabla_x u_x, u_t) dx
\end{aligned}$$

$$\begin{aligned}
& + 8a \int_X g(R(u_x, \nabla_x u_x) \nabla_x^2 u_x, u_t) dx \\
& - 4a \int_X g(R(u_x, R(u_x, \nabla_x u_x) u_x) u_x, u_t) dx \\
& \equiv 4a \int_X g(R(\nabla_x u_x, \nabla_x^2 u_x) u_x, a \nabla_x^2 u_x) dx \\
& + 4a \int_X g(R(u_x, \nabla_x^3 u_x) u_x, a \nabla_x^2 u_x) dx \\
& + 12a \int_X g(R(u_x, \nabla_x^2 u_x) \nabla_x u_x, a \nabla_x^2 u_x) dx \\
& + 8a \int_X g(R(u_x, \nabla_x u_x) \nabla_x^2 u_x, a \nabla_x^2 u_x) dx \tag{49}
\end{aligned}$$

$$\equiv 12a^2 \int_X g(R(u_x, \nabla_x^2 u_x) \nabla_x u_x, \nabla_x^2 u_x) dx. \tag{50}$$

Note that the last relation comes from the computation

$$\begin{aligned}
& \int_X g(R(u_x, \nabla_x^3 u_x) u_x, \nabla_x^2 u_x) dx \\
& = - \int_X g(R(\nabla_x u_x, \nabla_x^2 u_x) u_x, \nabla_x^2 u_x) dx \\
& \quad - \frac{1}{2} \int_X g((\nabla R)(u_x)(u_x, \nabla_x^2 u_x) u_x, \nabla_x^2 u_x) dx \\
& \equiv - \int_X g(R(\nabla_x u_x, \nabla_x^2 u_x) u_x, \nabla_x^2 u_x) dx,
\end{aligned}$$

and the fact that the last integral of the right hand side of (49) vanishes because of (18).

As a consequence, if we add (40), (45), (48) and (50), we obtain $(d/dt)E_2(u) \equiv F_0 \equiv 0$, which implies desired (23) and (24). Thus we complete the proof. \square

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