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## Generalization Capability of Radial Basis Function Controller Using Random Search Method with Variable Search Length in Universal Learning Network

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**Abstract:** In this paper, generalization capability of a Radial Basis Function controller using RasVal in Universal Learning Network was studied. RasVal is an abbreviation of Random Search with Variable Search Length and it can search for a global minimum systematically and effectively in a single framework which is not a combination of different methods. In this paper, a new method to overcome the over-fitting problem in nonlinear control systems is proposed, where the weighting coefficients of control variables in the criterion function are increased in order to obtain the generalization capability of RasVal. From simulation results of a nonlinear crane system, it has been shown that the smaller the scale of the R.B.F. controller is, the smaller the weighting coefficients of the control variables could be.

**Keywords:** Universal learning network, Random search method, Nonlinear control, Neural network, Radial basis function, Generalization capability.

### 1. Introduction

Universal Learning Network (U.L.N.)<sup>1)</sup> is a new-type of network which can be used to model and control large-scale complicated systems such as economic, social and living phenomena as well as industrial plants. Universal Learning Network consists of nonlinearly operated nodes and multi-branches that may have arbitrary time delays between the nodes. A new control method has been already presented for nonlinear systems using Universal Learning Network with radial basis function(R.B.F.) and it has been compared to the commonly used control method using neural networks. In the above system using Universal Learning Network, as learning algorithm of parameters in the controller was based on the gradient method, the problem of falling into a local minimum that leads to low efficiency of learning could not be solved. To overcome this problem, a new learning algorithm that can find a global minimum was presented and it was applied to build the optimal controller of a nonlinear control system. The proposed learning algorithm is called RasVal<sup>2)</sup> which is an abbreviation of Random Search with Variable Search Length and it can search for a global minimum systematically and effectively in a single framework which is not a combination of different methods. RasVal is a kind

of random search based on the probability density function of searching, which can be modified using informations on the results of the past searching in order to execute the intensified and diversified searching. The features of RasVal are as follows.

(1) it does not require differential calculations as the gradient method, therefore, it takes a shorter calculation time than the gradient method.

(2) random search with the intensification and diversification is carried out in order to solve the local minimum problem.

By applying RasVal to a nonlinear crane control system, it has been proved that a new learning algorithm is superior in performance to the back propagation learning algorithm<sup>3)</sup>.

But, in the simulations to study the generalization capability of RasVal, it was found that too much learning causes the over-fitting problem, that is, the control system becomes instable at the different condition from that of learning. In this paper, a new method to overcome the over-fitting problem in the nonlinear control systems is proposed, where the weighting coefficients of control variables in the criterion function are increased in order to obtain the generalization capability of RasVal. From simulation results of a nonlinear crane system, it has been shown that the smaller the scale of the R.B.F. controller is, the smaller the weighting coefficients of the control variables could be.

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## 2. Universal Learning Network with Radial Basis Function

In this section, Universal Learning Network with Radial Basis Function is summarized, which is used to model the system to be controlled and its controller.

Universal Learning Network (U.L.N.) is a new-type of network, it consists of nonlinearly operated nodes and branches that may have arbitrary time delays including zero or minus ones. Structure of U.L.N. is shown in Fig.1.

Basic equation of U.L.N. with Multi Branches is represented by Eq.(1):

$$h_j(t) = O_j(\{h_i(t - D_{ij}(p)) | i \in JF(j), p \in B(i, j)\}, \{r_n(t) | n \in N(j)\}, \{\lambda_m(t) | m \in M(j)\}) \quad (1)$$

$$j \in J, \quad t \in T$$

where,

$h_j(t)$ : output value of node  $j$  at time  $t$ ;

$\lambda_m(t)$ : value of  $m$ th parameter at time  $t$ ;

$r_n(t)$ : value of  $n$ th external input variable at time  $t$ ;

$O_j$ : nonlinear function of node  $j$ ;

$D_{ij}(p)$ : time delay of  $p$ th branch from node  $i$  to node  $j$ ;

$JF(j)$ : set of node numbers whose outputs are connected to node  $j$ ;

$JB(j)$ : set of node numbers whose inputs are connected from node  $j$ ;

$B(i, j)$ : set of branches from node  $i$  to node  $j$ ;

$N(j)$ : set of external input variable numbers that are connected to node  $j$ ;

$N$ : set of external input variable numbers;

$M(j)$ : set of parameter numbers, with respect to these parameters, output of node  $j$  can be partially differentiable;

$M$ : set of parameter numbers;

$J$ : set of node numbers;

$T$ : set of sampling times;

Let a criterion function be written as Eq.(2):

$$E = E(\{h_r(s)\}, \{\lambda_m(s)\}) \quad (2)$$

$$r \in J_0, \quad m \in M_0, \quad s \in T_0$$

Hwhere

$J_0$ : set of node numbers related to evaluation;

$M_0$ : set of parameter numbers related to evaluation;

$T_0$ : set of sampling times related to evaluation .

Therefore, U.L.N. forms a superset of all kinds of neural network paradigms with supervised learning

capability.

The important features of U.L.N. are that functions of the nodes can take any nonlinear functions and the nodes can be connected to each other arbitrarily. So the structure of U.L.N. is a general one in the sense that U.L.N. with sigmoid functions and one sampling time delays corresponds to the recurrent neural network.

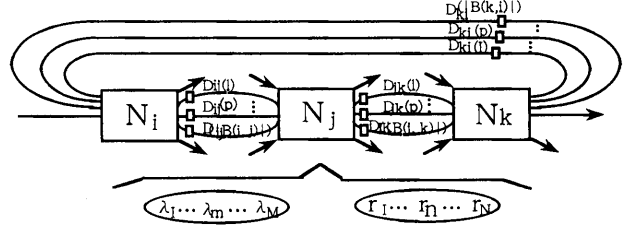


Fig.1 Structure of U.L.N. with Multi Branches

U.L.N. with R.B.F. can be expressed as follows.

$$h_j(t) = \sum_{m \in L(j)} f_{jm}(x_{jm}) + b_j \quad (3)$$

$$f_{jm}(x_{jm}) = k_{jm} \exp(x_{jm}) \quad (4)$$

$$x_{jm} = -\frac{1}{2} \sum_{i \in JF(j)} \sum_{p \in B(i,j)} \left( \frac{h_i(t - D_{ij}(p)) - h_{jm}^i(p)}{\sigma_{jm}^i(p)} \right)^2 \quad (5)$$

where,

$L(j)$ : set of the numbers of R.B.F. functions of node  $j$ ;

$k_{jm}, h_{jm}^i(p), \sigma_{jm}^i(p), b_j$ : parameters for node  $j$ .

## 3. Random Search Method with Variable Search Length (RasVal)

In this section, RasVal is summarized, which is used for learning of the parameters in the controller of the system.

RasVal is a kind of random search method based on the probability density functions of searching, which can be modified using informations on success or failure of the past searching.

The features of RasVal are as follows. (1)it does not require differential calculations as the gradient method, therefore, it takes a shorter calculation time than the gradient method, and (2)random search with the intensification and diversification can lead to the solution of the local minimum problem.

Calculation procedure of RasVal is as follows.

$$\text{if } E(\lambda + x) < E(\lambda) \implies \lambda \leftarrow \lambda + x; \quad (6)$$

(searching is success)

$$\text{if } E(\lambda + x) \geq E(\lambda) \implies \lambda \leftarrow \lambda. \quad (7)$$

(searching is failure)

where,  $E$  : criterion function;

$\lambda = [\lambda_1 \dots \lambda_m \dots \lambda_{|M|}]^T$  : parameter vector;

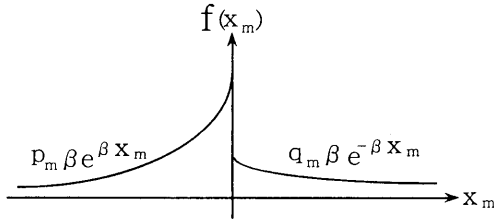
$x = [x_1 \dots x_m \dots x_{|M|}]^T$  : parameter search vector.

The probability density function  $f(x_m)$  of searching  $x_m$  (see **Fig.2**) is represented as Eq.(8),(9) and (10):

$$f(x_m) = p_m \beta e^{\beta x_m}, x_m \leq 0; \quad (8)$$

$$f(x_m) = q_m \beta e^{-\beta x_m}, x_m > 0; \quad (9)$$

$$p_m + q_m = 1.0 \quad (10)$$



**Fig.2** Probability Density Function of Searching  $x_m$

Therefore  $x_m$  can be calculated as follows:

$$\text{if } 0 \leq z \leq p_m \implies x_m = \frac{1}{\beta} \ln\left(\frac{z}{p_m}\right) \quad (11)$$

$$\text{if } p_m < z \leq 1.0 \implies x_m = -\frac{1}{\beta} \ln\left(\frac{1-z}{q_m}\right) \quad (12)$$

where,  $z$  : random numbers in  $[0,1]$ .

Parameters  $\beta, p_m, q_m$  of  $f(x_m)$  which are related to searching range and direction are modified based on the informations of success or failure of the past searching as follows.

$$\beta = \bar{\beta} e^{-\eta n} + \underline{\beta} \quad (13)$$

In case of negative direction searching :

$$p_m \leftarrow \alpha p_m + (1 - \alpha) \cdot SF \quad (14)$$

In case of positive direction searching :

$$q_m \leftarrow \alpha q_m + (1 - \alpha) \cdot SF \quad (15)$$

In case of failure,

$$n \leftarrow n + 1 \quad (16)$$

In case of success,

$$n \leftarrow n, \quad n = 0; \quad (17)$$

$$n \leftarrow n - 1, \quad 0 < n \leq n_0 \quad (18)$$

$$n \leftarrow n_0, \quad n > n_0 \quad (19)$$

where,

$SF = 1.0$ , in case of success;

$SF = 0.0$ , in case of failure;

$\alpha$  : exponential filter coefficient;

$\bar{\beta} + \underline{\beta}$  : upper limit of  $\beta$ ;

$\underline{\beta}$  : lower limit of  $\beta$ ;

$\eta$  : coefficient.

From Eq.(6) ~ (19), the intensification and diversification of the search can be realized such that when there is quite a possibility of finding good solutions around the current one, the intensified search for the vicinity of the current solution is carried out; on the other hand, when there is no possibility of finding good solutions, the diversified search is executed in order to find good solutions in the region far from the current solution.

#### 4. Generalization Capability of Non-linear Crane Control Systems Using RasVal

Generally, the words “generalization capability” means the ability of assurance of that the system works well even in the different environments from that at learning stage. It is commonly said that the generalization capability will be improved by learning a great number of cases with different environments and also by reducing the scale of networks for learning. Recently, some papers on the enhancement of the generalization capability have been reported by using the second order derivatives in Universal Learning Network<sup>4</sup>). These methods are based on the idea that a criterion function relating to the improvement of system robustness is added to the usual criterion function in order to enhance the generalization capability.

In this paper, a new method for the enhancement of the generalization capability in the nonlinear control systems is presented, where control signals to a plant to be controlled are suppressed by increasing the weighting coefficients related to the control signals in the criterion function. In the next section, simulations of a nonlinear crane system are carried out in order to study the above new method.

### 5. Simulation

#### 5.1 Nonlinear Crane Control System

In order to investigate the performance of the generalization capability of RasVal learning for nonlinear control systems with R.B.F. controller, a nonlinear crane control system was studied(**Fig.3**). The

aim of control is to bring the trolley to the target position, and to winch the load to the target height at the same time by minimizing the criterion function.

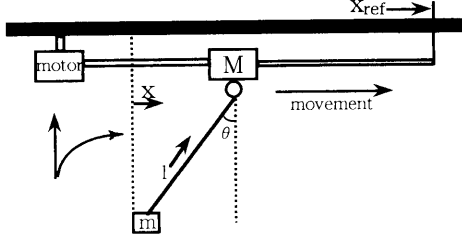


Fig.3 Nonlinear Crane System

The equations of the crane system are represented in the followings:

$$\ddot{x} = \frac{mg}{M}\theta - \frac{D+G}{M}\dot{x} + \frac{G}{M}u_d \quad (20)$$

$$\ddot{\theta} = \frac{M+m}{lM}g\theta - \frac{D+G}{lM}\dot{\theta} + \frac{G}{lM}u_d \quad (21)$$

$$\ddot{l} = \frac{c+G_m}{m}\dot{l} + \frac{G_m}{m}u_m \quad (22)$$

where,  $M$ : mass of the trolley;  $m$ : mass of the load;  $l$ : height of the load from the initial position;  $\theta$ : angle of the load;  $x$ : location of the trolley;  $C, D$ : coefficients of the friction.  $u_d, u_m$  are input voltage control values from the controller to the crane system.

Assuming the following notations,

$$\begin{aligned} h_1(t) &= x(t) & h_2(t) &= \dot{x}(t) \\ h_3(t) &= \theta(t) & h_4(t) &= \dot{\theta}(t) \\ h_5(t) &= l(t) & h_6(t) &= \dot{l}(t) \end{aligned}$$

then, equations can be expressed in the discrete forms.

$$h_1(t) = a_{11}h_1(t-1) + a_{21}h_2(t-1) \quad (23)$$

$$h_2(t) = a_{22}h_2(t-1) + a_{32}h_3(t-1) + b_1u_d(t) \quad (24)$$

$$h_3(t) = a_{33}h_3(t-1) + a_{43}h_4(t-1) \quad (25)$$

$$h_4(t) = a_{24}\frac{h_2(t-1)}{h_5(t-1)} + a_{34}\frac{h_3(t-1)}{h_5(t-1)} + a_{44}h_4(t-1) + \frac{b_1}{h_5(t-1)}u_d(t) \quad (26)$$

$$h_5(t) = a_{55}h_5(t-1) + a_{65}h_6(t-1) \quad (27)$$

$$h_6(t) = a_{66}h_6(t-1) + b_2u_m(t) \quad (28)$$

The structure of the nonlinear crane control system is shown in Fig.4. The controller is constructed by the radial basis function network. The arbitrary time delay is assumed to be 1.0 sampling time. The nonlinear crane control system has two parts. The

upper part is a crane system which has 6 nodes (real line frame); the lower part is a controller (dotted line frame). The U.L.N. with R.B.F. controller has two R.B.F. controllers, each of the R.B.F. controllers has a three layered structure. In the controller, the left controller has two inputs ( $x, \theta$ ), the right controller has two inputs ( $l, \dot{l}$ ).

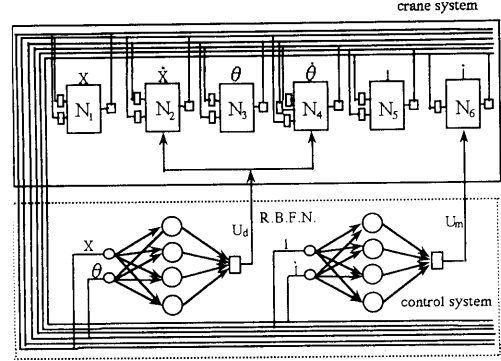


Fig.4 Structure of Nonlinear Crane Control System with R.B.F. Controller

## 5.2 Simulation Results

Simulations were carried out to study the generalization capability of the proposed method. In the simulations, control time is 40 seconds. And the criterion function for RasVal learning can be expressed as follows.

$$\begin{aligned} E &= \frac{1}{2} \sum_{t=0}^T [Q_1(l_{ref} - l(t))^2 \\ &\quad + Q_2(x_{ref} - x(t))^2 + Q_3\theta^2(t) + Q_4\dot{\theta}^2(t) \\ &\quad + Q_5u_m^2(t) + Q_6u_d^2(t)] \\ &\quad + \frac{1}{2} (Q_7\dot{x}^2(t_f) + Q_8\dot{l}^2(t_f)) \end{aligned} \quad (29)$$

where,  $l_{ref}, x_{ref}$ : reference value of  $l, x$ ;  $t_f$ : final sampling time;  $Q_i$ : coefficient of the criterion function.

Simulation conditions were shown in Table 1.

The generalization capability was investigated as follows. While learning of parameters was carried out so as to bring  $x$  from 0.0m to 0.2m and  $l$  from 2.0m to 1.7m in the 40 seconds, kinetic dynamics for investigating the generalization capability was calculated by changing the reference input from  $x = 0.0m$  to 0.2m, from  $l = 2.0m$  to 1.7m in the first 20 seconds; and from  $x = 0.2m$  to 0.4m, from  $l = 1.7m$  to 2.0m in the last 20 seconds. By changing the initial parameters of the R.B.F. controller randomly, simulations were carried out 5 times. Fig.5 show the average learning curves and  $l(t), x(t), \theta(t)$

of the nonlinear crane system which were obtained for the study of the generalization capability, on the condition that  $L(j)=4$  (four R.B.F. functions),  $Q_5 = Q_6 = 0.001$  and learning is carried out 20000 times and 80000 times respectively.

From **Fig.6** it is shown that when the learning was continued until 80000 times, dynamics of the system becomes instable because of the over-fitting of the learning. Curved surfaces of control signals  $u_d, u_m$  which are the function of  $(x, \theta)$  and  $(l, \dot{l})$  respectively are shown in **Fig.7** and **Fig.8**. It is understood that curved surfaces of **Fig.8** obtained when the learning is carried out too much can not calculate the appropriate control signal  $u_m$  around  $l_{ref} - l = 0.3$ . Therefore, suppressing of the control signals was tried to make by increasing the weighting coefficients  $Q_5, Q_6$  related to  $u_m, u_d$  in the criterion function.

**Fig.9 ~ Fig.12** show the curved surfaces of control signals  $u_d, u_m$  obtained by the condition that four R.B.F. functions,  $Q_5 = Q_6 = 1.0$ ; three R.B.F. functions,  $Q_5 = Q_6 = 0.4$ ; two R.B.F. functions,  $Q_5 = Q_6 = 0.3$ ; and one R.B.F. function,  $Q_5 = Q_6 = 1.0$  are used and learning is carried out 80000 times.  $Q_5, Q_6$  in **Fig.9 ~ Fig.11** are the lowest value, where stable dynamics is obtained even when the reference inputs  $x_{ref}, l_{ref}$  are changed in the middle of the control.

From **Fig.9 ~ Fig.12** and **Table 2**, it is known that the larger the number of R.B.F. functions is, the more needed the suppression of control signals are, and it is also known that sufficient generalization capability can not be obtained by using one R.B.F. function.

### 6. Conclusion

In this paper, the generalization capability of R.B.F. controller using RasVal in Universal Learning Network was studied. From simulations of a nonlinear control system, it has been proved that the generalization capability is enhanced by suppressing the control signals, and a great numbers of R.B.F. functions in the controller of the system deteriorate the generalization capability.

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**Table 1.** Simulation Conditions

mass of the trolley M	40.0 kg
mass of the load m	2.0 kg
coefficients of the friction D	300.0 kg/sec
number of sampling times	2000
learning times	20000 ~ 80000times
control time	40 s
random number range for parameter variables	[ 0, 1 ]
coefficient of criterion function	Q1=1.0 Q2=1.0 Q3=1.0 Q4=1.0 Q5=0.001~1.0 Q6=0.001~1.0 Q7=1.0 Q8=1.0
RasVal	
exponential filter coefficient $\alpha$	0.45
upper limit of $\underline{a} + \bar{b}$	500
lower limit of $\underline{a}$	200
coefficient $\eta$	0.0001

**Table 2.** Relation between  $L(j)$  and the lowest bound of  $Q_5$  and  $Q_6$

$L(j)$	The lowest coefficient to assure the generalization capability
1	—————
2	$Q_5=Q_6 \geq 0.3$
3	$Q_5=Q_6 \geq 0.4$
4	$Q_5=Q_6 \geq 1.0$

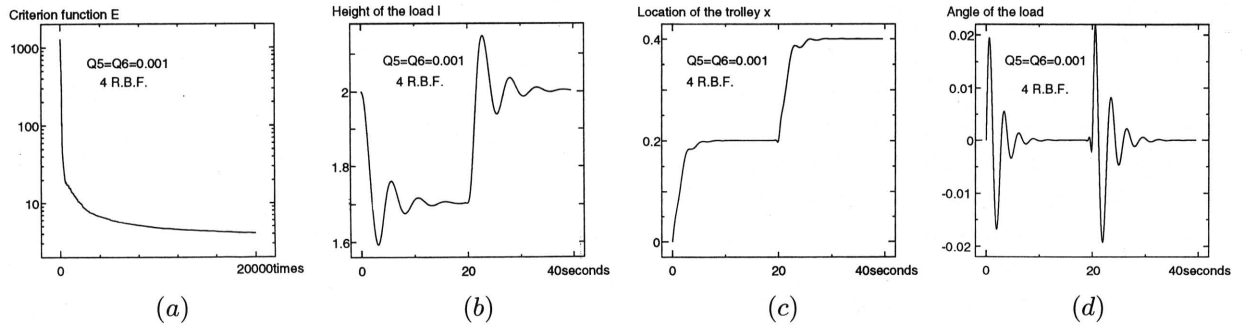


Fig.5 Learning Curves and Kinetic Dynamics for Studying Generalization Capability (Learning Times: 20000)

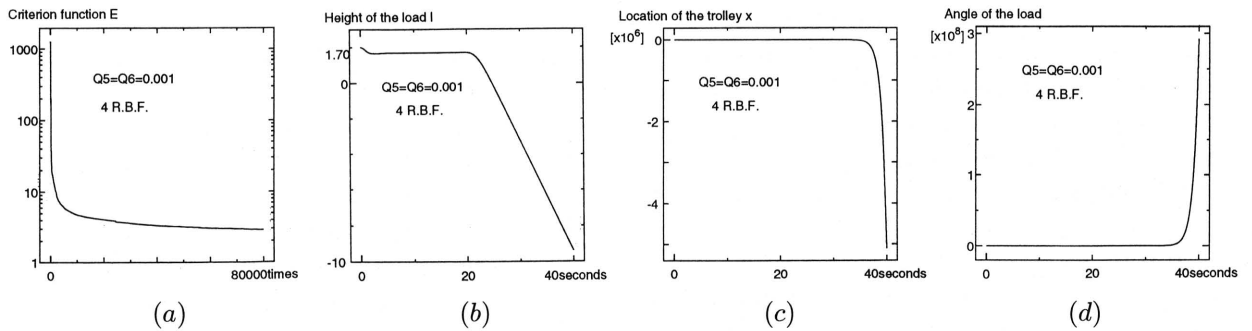


Fig.6 Learning Curves and Kinetic Dynamics for Studying Generalization Capability (Learning Times: 80000)

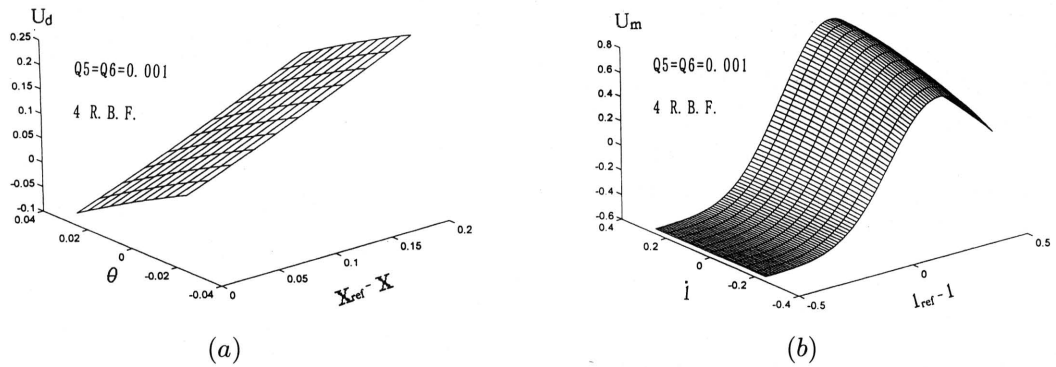


Fig.7 Curved Surfaces of Control Signals  $U_d$  and  $U_m$  (Learning Times: 20000)

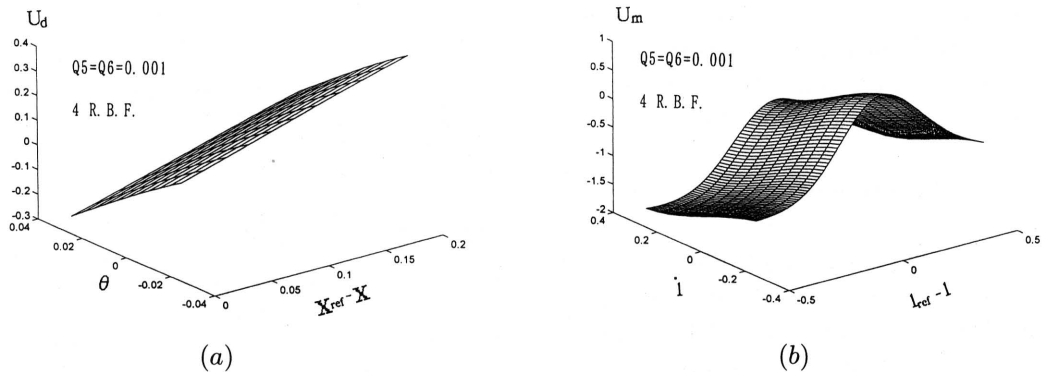
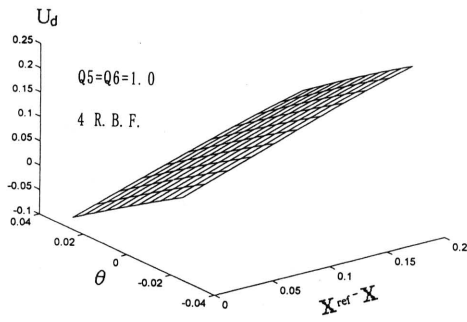
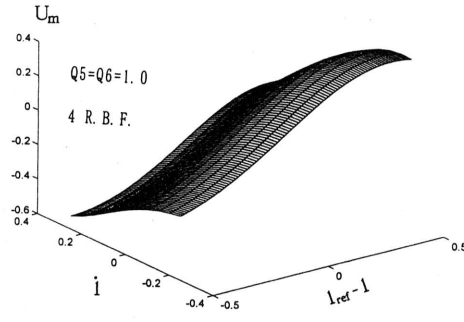


Fig.8 Curved Surfaces of Control Signals  $U_d$  and  $U_m$  (Learning Times: 80000)

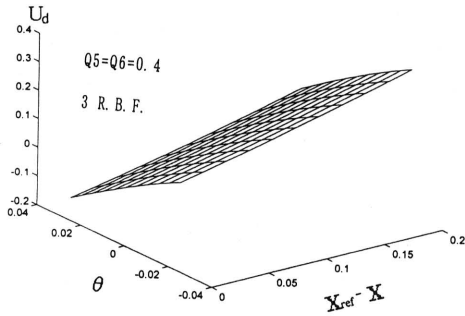


(a)

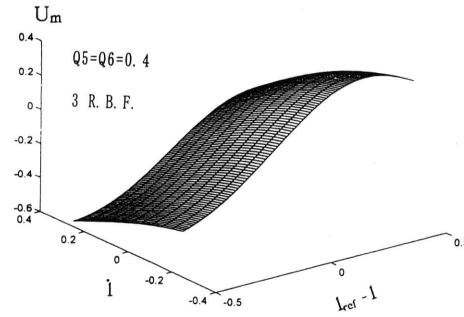


(b)

**Fig.9** Curved Surfaces of Control Signals  $U_d$  and  $U_m$  (Four R.B.F. Nodes)

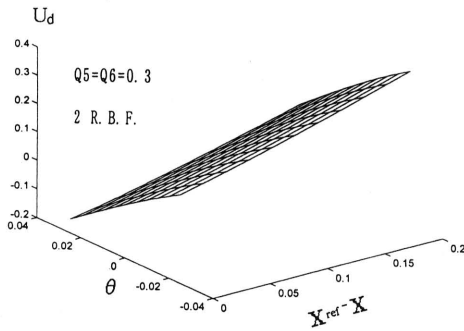


(a)

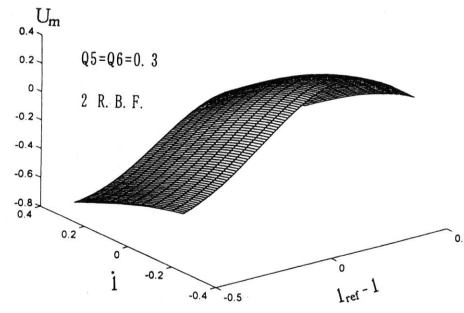


(b)

**Fig.10** Curved Surfaces of Control Signals  $U_d$  and  $U_m$  (Three R.B.F. Nodes)

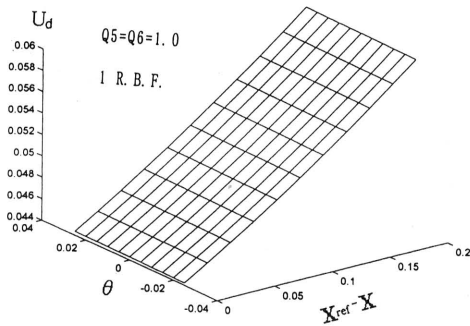


(a)

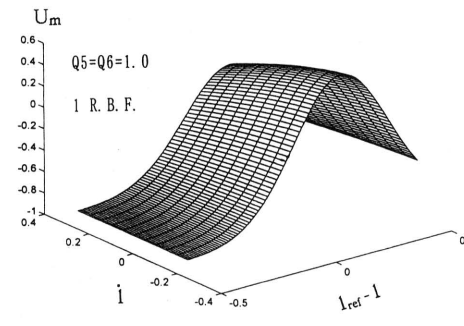


(b)

**Fig.11** Curved Surfaces of Control Signals  $U_d$  and  $U_m$  (Two R.B.F. Nodes)



(a)



(b)

**Fig.12** Curved Surfaces of Control Signals  $U_d$  and  $U_m$  (One R.B.F. Node)