A Realizability Condition for Bandpass Ladder Networks Composed of Two Kinds of LC Resonant Sections

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A Realizability Condition for Bandpass Ladder Networks Composed of Two Kinds of LC Resonant Sections

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Abstract: This paper investigates some properties of bandpass ladder networks composed only of two kinds of basic LC resonant sections and gives the necessary and sufficient condition for the realization. This condition can be regarded as a direct generalization of the famous Fujisawa's necessary and sufficient condition for LC low-pass ladder networks. Examples are shown to illustrate the synthesis process.

Keywords: Realizability, LC ladder network, Dielectric filter, Fujisawa's condition

1. Introduction

Typical bandpass filters used in portable telephone handsets are monoblock dielectric filters. For these filters Lu et al.[1),2)] recently proposed a new equivalent circuit which consists only of two basic kinds of resonant circuits, each of them being composed of a lumped-capacitor and a distributed-element.

Though these circuits can be regarded as two-variable networks, the realizability conditions for the above type of networks have not been shown in the synthesis theory of two-variable networks.

The synthesis problem of the above mixed lumped- and distributed-networks can be reduced to that of a kind of tansformerless LC ladder networks. Although the realization of transformerless ladder networks has extensively studied in particular in 1960's and 1970's, the realization theorem was not given for the above kinds of networks. Closely related results were however given as the famous Fujisawa's necessary and sufficient condition for mid-series or mid-shunt low-pass ladders[3)], Watanabe's sufficient condition for bandpass ladder networks[4)], Ozaki's sufficient condition for series-paralell RC three-terminal networks[5)], and Nishi's sufficient conditions for some kinds of RC 3-terminal networks[6),7)].

This paper gives the necessary and sufficient condition for the realization of bandpass ladders composed only of two basic LC resonant sections. This result can be considered as a direct extension of Fujisawa's Theorem[3)] for low-pass ladders.

It is expected that the new criteria can be utilized to design some practical bandpass filters with finite attenuation poles and can also be applied to the design of mixed structures composed of both lumped and distributed elements.

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2. Main Theorem

2.1 Notations and definitions

The bandpass ladder network we deal with is an LC 2-port composed of two kinds of basic LC resonant sections, that is, Type-A sections and Type-B sections shown in Figs.1(a) and 1(b), respectively. These two sections are alternately connected in cascade so that the right-most section is a Type-A section, while the left-most section is either a Type-A or Type-B section. Let the number of Type-B sections be m. Then the number of Type-A sections is m or $m+1$. The ladder 2-port we consider is thus shown in Fig. 2 (surrounded by a dotted line) and is denoted by N.

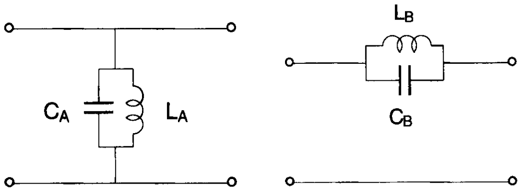
We assume that the resonant angular frequencies of all Type-A sections are identical and that they are denoted by ω_A. This assumption is reasonable because a filter for a portable telephone has a very narrow bandwidth and ω_A roughly corresponds to the center frequency of this filter.

On the other hand the angular frequencies of Type-B sections are denoted by $\omega_{B_i}, (i = 1, 2, \ldots, m)$. Let $m_l (\geq 0)$ (resp. $m_h (\geq 0)$) be the number of ω_{B_i} which are smaller (resp. larger) than ω_A. Of course $m = m_l + m_h$ holds. The frequencies ω_{B_i} correspond to the finite attenuation poles (abbreviated as FAP's) of the bandpass ladder filter.

The driving-point admittance function $Y(s)$ of the 2-port N terminated with a conductance g (as illustrated in Fig.2) can be written as

$$Y(s) = \frac{N(s)}{M(s)} = \frac{N_e(s) + sN_o(s)}{s^2 M_e(s) + sM_o(s)} \tag{1}$$

where the polynomials $N(s)$ and $M(s)$ are relatively prime, i.e., $(N, M) = 1$; $N_e(s)$ and $s^2 M_e(s)$ (resp. $sN_o(s)$ and $sM_o(s)$) are respectively the even (resp. odd) parts of the numerator and the denominator.



(a) A Type-A section (b) A Type-B section

Fig.1 Basic resonant sections

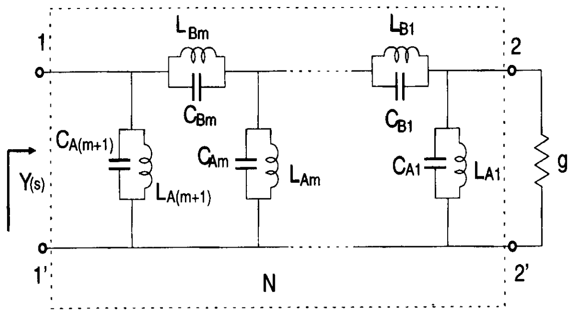


Fig.2 A bandpass ladder 2-port N terminated with a conductance g

2.2 Some properties of $Y(s)$ and y_{1f}

The following lemmas concerning $Y(s)$ (Eq.1) can be readily derived.

Lemma 1.

Let $n = deg[Y(s)] (\geq 1)$ be the degree of $Y(s)$ in Eq.(1). Of course $n = 2m$ holds. Then we have:

1)

$$\left.\begin{array}{ll} deg[N_e] = n & deg[N_o] = n-1 \\ deg[M_e] = n-2 & deg[M_o] = n-1 \end{array}\right\} \quad (2)$$

2)

$$\left.\begin{array}{l} (N_e, N_o) = 1 \\ or \\ (N_e, N_o) = s^2 + \omega_{B i_0}^2 \end{array}\right\} \quad (3)$$

where $\omega_{Bi_0} (1 \geq i_0 \geq m)$ is one of the finite attenuation poles.

The impedance parameters z_{ij} and the admittance parameters y_{ij} of the LC two-port in Fig. 2 can be easily derived from the input admittance $Y(s)$ as follows:

Lemma 2

Impedance parameters are given as follows.

$$\left.\begin{array}{rl} z_{11} = \dfrac{sM_o}{N_e}, & z_{22} = \dfrac{sN_o}{gN_e} \\ \pm z_{12} = & \dfrac{s\sqrt{M_o N_o - M_e N_e}}{\sqrt{g} N_e} \\ = & \dfrac{\sqrt{K_{12}}\, s \prod_{i=1}^{m} (s^2 + \omega_{Bi}^2)}{\sqrt{g} N_e} \end{array}\right\} \quad (4)$$

where K_{12} is a positive constant.

Admittance parameters are also given as follows.

$$\left.\begin{array}{rl} y_{11} = \dfrac{N_o}{sM_e}, & y_{22} = \dfrac{gM_o}{sM_e} \\ \pm y_{12} = & \sqrt{g} \dfrac{\sqrt{M_o N_o - M_e N_e}}{sM_e} \\ = & \sqrt{g} \dfrac{\sqrt{K_{12}} \prod_{i=1}^{m} (s^2 + \omega_{Bi}^2)}{sM_e} \end{array}\right\} \quad (5)$$

Let $y_{1f} = z_{11}^{-1}$ be the input admittance of N with output-port open-circuited. We easily see from the configuration of N that y_{1f} has a zero at $s = j\omega_A$. Therefore we have

$$N_e(s) = K(s^2 + \omega_A^2) \prod_{1}^{m} (s^2 + b_i^2), \quad (6)$$

where K is a positive constant. We assume without loss of generality that

$$0 < b_1 < b_2 < \ldots < b_m. \quad (7)$$

Then we can write y_{1f} as

$$y_{1f}(s) = \frac{N_e(s)}{sM_o(s)} = \frac{K(s^2 + \omega_A^2) \prod_1^m (s^2 + b_i^2)}{s \prod_1^m (s^2 + \alpha_i^2)} \quad (8)$$

where we assume:

$$0 < \alpha_1 < \alpha_2 < \cdots < \alpha_m$$

It is obvious that y_{1f} is a susceptance (reactance) function.

Lemma 3

Let us denote $\hat{y}_{1f}$ as the augmented y_{1f} after connecting a Type-A section to N. Then we have

$$\hat{y}_{1f}(s) = y_{1f}(s) + \frac{C_A(s^2 + \omega_A^2)}{s} = \frac{\left[\begin{array}{l} (s^2 + \omega_A^2) \\ \times \{K \prod_1^m (s^2 + b_i^2) + C_A \prod_1^m (s^2 + \alpha_i^2)\} \end{array}\right]}{s \prod_1^m (s^2 + \alpha_i^2)} \quad (9)$$

where C_A is a positive constant and $\hat{y}_{1f}$ is in the same form as Eq.(8).

Assume that for some $k(0 \leq k \leq m)$

$$b_k < \omega_A < b_{k+1}. \tag{10}$$

If a polynomial $P(x)$ is expressed as

$$P(x) = K\prod_{1}^{m}(b_i^2 - x^2) + C_A\prod_{1}^{m}(\alpha_i^2 - x^2) = (K + C_A)\prod_{1}^{m}(\hat{b}_i^2 - x^2) \tag{11}$$

Then $P(x)$ has m zeros, each of which locates in one of the m open intervals. i.e., $\hat{b}_i \in (b_i, \alpha_i)$ for $1 \leq i \leq k$ and $\hat{b}_i \in (\alpha_i, b_i)$ for $k+1 \leq i \leq m$.

2.3 Necessary and sufficient conditions

Theorem 1. *The function $Y(s)$ in Eq.(1) represents the driving-point admittance of the aforementioned bandpass ladder network if and only if*

1) $Y(s)$ is a positive real fucntion.

2) $Y(s)$ has a pole at the origin ($s = 0$) and at infinity ($s = \infty$) respectively.

3). The even part of $Y(s)$ can be written as:

$$Ev[Y(s)] = \frac{M_eN_e - M_oN_o}{s^2M_e^2 - M_o^2} = \frac{K_e\prod_{i=1}^{m}(s^2 + \omega_{Bi}^2)^2}{M_o^2 - s^2M_e^2} \tag{12}$$

where K_e is a positive constant, and ω_{B_i} ($i = 1, 2, \ldots, m$) are finite zeros of $Ev[Y(s)]$, and m satisfies the following relationship: $m = m_l + m_h = [n/2] - 1$ where $n = deg[Y(s)]$.

4) Let ω_{B_i} and the $m+1$ pairs of zeros of $N_e(s)$ be arranged respectively in the order of increasing magnitude as

$$0 < \omega_{B1} \leq \ldots, \leq \omega_{Bm_l} < \omega_A < \omega_{Bm_l+1} \leq \cdots \leq \omega_{Bm} < \infty \tag{13}$$

$$b_1 < \ldots < b_{m_l} < \omega_A < b_{m_l+1} < \ldots < b_m \tag{14}$$

Then for every $i \leq m_l$ and every $j \leq m_h$ there are:

$$\left.\begin{array}{ll} \omega_{B_i} \leq b_i < \omega_A & 1 \leq i \leq m_l \\ \text{and} & \\ \omega_A < b_{m_l+j} \leq \omega_{B_{m_l+j}} & 1 \leq j \leq m_h \end{array}\right\} \tag{15}$$

Comment 1 In Conditions 3 and 4 of Theorem 1 m_l may possibly be 0 or m. Thus Theorem 1 holds even in the following simple cases:

1) ω_A is larger than all ω_{B_m}; i.e. $\omega_{B_i} < \omega_A$, which indicates that $m_l = m$.

2) ω_A is smaller than ω_{B_j}; i.e. $\omega_A < \omega_{B_1}$, which indicates that $m_l = 0$.

Furthermore, in the second special case, if $\omega_A = 0$, then $Y(s)$ has no pole at origin, which means that condition 2 turns into "$Y(s)$ has a pole at infinity". In this case Theorem 1 is equivalent to Fujisawa's First Theorem for low-pass ladders[3].

Comment 2 Condition 4 of Theorem 1 is an extended version of Fujisawa's condition[3] and is in the same form as Watanabe's sufficient condition[4]. However, Condition 4 is both necessary and sufficient for the proposed circuit configuration (Fig.2(a)).

Proof of the above theorem is obtained by using Lemmas 2 and 3. It is however considerably lengthy and involved, but are resemble to those of Fujisawa's theorem and Watanabe's one. So we omit it.

3. Synthesis Procedure

The synthesis procedure is an iterative process of the following two steps (illustrated in Figs. 3(a) and 3(b)).

Step I: If $Y(s)$ has no zero at any FAPs ($s = j\omega_{B_i}$), then remove a shunt arm (Type-A section) so that the residual function $Y_n(s)$ has a zero at one of the FAPs.

Step II. If $Y(s)$ has a zero at one of the FAPs ($s = j\omega_{B_i}$), then extract a Type-B section to realize the FAP.

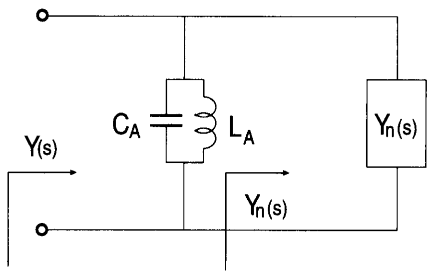


Fig.3(a) Step I: Removal of a Type-A section from $Y(s)$

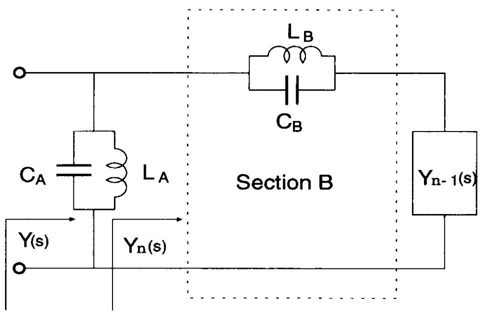


Fig.3(b) Step II: Removal of a Type-B section from a reminder admittance function $Y_n(s)$

4. Examples

In this section we demonstrate the detail synthesis process described above using two simple examples.

Example 1. Let the $Y(s)$ of a bandpass ladder be

$$Y(s) = \frac{\begin{bmatrix} (s^2+1)(8s^4+71s^2+135) \\ +s(5s^4+40s^2+59) \end{bmatrix}}{s(5s^4+35s^2+54)+s^2(3s^2+14)} \quad (16)$$

The even part of $Y(s)$ is

$$Ev[Y(s)] = \frac{(s^2+4)^2(s^2+9)^2}{D} \quad (17)$$

where

$$D = (5s^4+35s^2+54)^2 - s^2(3s^2+14)^2$$

It is obvious that $\omega_{B1} = 2$, $\omega_{B2} = 3$ and both are larger than $\omega_A (= 1)$ so in this case $m_l = 0, m_h = 2$.

It can be easily examined that $Y(s)$ satisfies the four conditions of Theorem 1 and that $Y(s)$ owns no zero at either $\omega_{B1} = 2$ or $\omega_{B2} = 3$. So we have to apply step I and extract a Type-A section first. Then by carrying out step II and step I iteratively we can synthesize it into a bandpass ladder as shown in Fig.4.

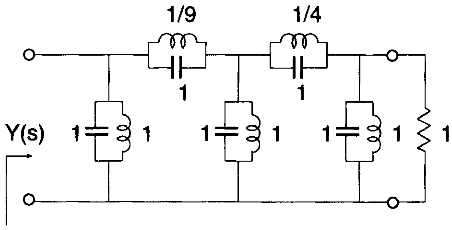


Fig.4 The final bandpass ladder of Example 1

Example 2. Here we deal the case that ω_{Bi} locates both below and above ω_A. Let the $Y(s)$ of a bandpass ladder be

$$\frac{\begin{bmatrix} (s^2+1)(8s^4+27.25s^2+12.5) \\ +s(5s^4+17.5s^2+10.25) \end{bmatrix}}{s(5s^4+13.75s^2+6.5)+s^2(3s^2+5.25)} \quad (19)$$

Then the even part of $Y(s)$ is

$$Ev[Y(s)] = \frac{(s^2+\frac{1}{4})^2(s^2+4)^2}{(5s^4+13.75s^2+6.5)^2 - s^2(3s^2+5.25)^2} \quad (20)$$

It is obvious that $\omega_{B1} = 1/2$, $\omega_{B2} = 2$, with the former being smaller and the latter being bigger than $\omega_A (= 1)$. So this is the case $m_l = 1, m_h = 1$. It can be easily examined that $Y(s)$ satisfies the four conditions of Theorem 1 and that $Y(s)$ owns no zero at either of the two ω_{Bi}. So we should begin by extracting a Type-A section first. Then by carrying out step II and step I appropriately, we can synthesize it into a bandpass ladder. The final ladder configuration is shown in Fig.5.

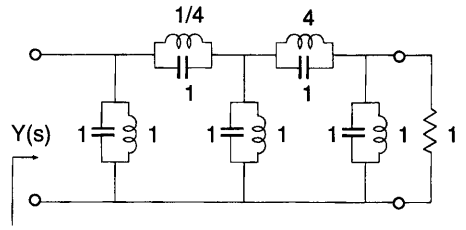


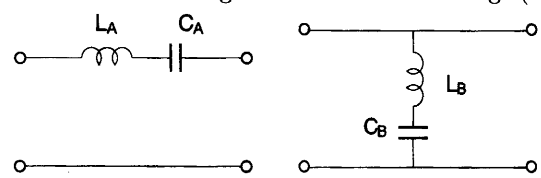
Fig.5 The final bandpass ladder of example 2

The values of each element in Fig.5 are shown below:

$$\left.\begin{array}{l} C_A(3) = 1 \;\; L_A(3) = 1 \\ C_B(2) = 1 \;\; L_B(2) = 1/4 \\ C_A(2) = 1 \;\; L_A(2) = 1 \\ C_B(1) = 1 \;\; L_B(1) = 4 \\ C_A(1) = 1 \;\; L_A(1) = 1 \\ g = 1 \end{array}\right\} \quad (21)$$

5. The Dual Form of Theorem 1

Theorem 1 is expressed in the form of driving-point admittance function of the bandpass ladders. However, when it is expressed in the form of driving-point impedance function, we can get a new necessary and sufficient condition for realizing a bandpass ladder that is composed of two kinds of series resonant section (series resonant Type-A section and B, as depicted in Figs.6(a) and (b)). This new theorem is the dual form of Theorem 1 and is named as Theorem 2. The overall ladder network as shown in Fig.7 is the dual form of Fig.2(a).



(a) Basic series resonant section (b) Basic series resonant section

Fig.6 Basic series resonant section

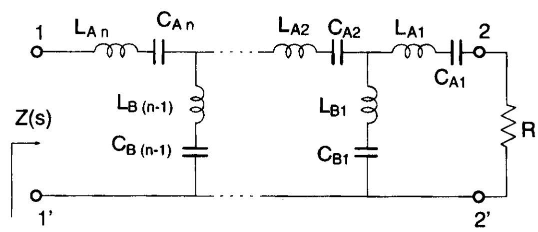


Fig.7 The dual form of Fig.2(a) in series sections

Theorem 2. *Write*

$$Z(s) = \frac{N(s)}{M(s)} = \frac{N_e(s)+N_o(s)}{M_e(s)+M_o(s)} = \frac{N_1(s)+sN_2(s)}{s(M_1(s)+sM_2(s))} \quad (22)$$

where the polynomials $N(s)$ and $M(s)$ are relatively *prime; i.e.,* $(N, M) = 1$; $N_e(s)$ *and* $M_e(s)$, $N_o(s)$ *and* $M_o(s)$ *are the even and the odd parts of the numerator and the denominator respectively, and* $M_o(s) = sM_1(s)$ *and* $M_e(s) = s^2M_2(s)$, $N_e(s) = N_1(s)$, $N_o(s) = sN_2(s)$. *(*$N_1(s), N_2(s), M_1(s)$ *and* $M_2(s)$ *are even polynomials.) Then* $Z(s)$ *represents the driving-point impedance of the aforementioned bandpass ladder, if and only if*

1). $Z(s)$ *is positive real.*

2). $Z(s)$ *has a pole at the origin (*$s = j0$*) and at infinity (*$s = j\infty$*), respectively.*

3). The even part of $Z(s)$ *can be written as:*

$$Ev[Z(s)] = \frac{N_eM_e - N_oM_o}{M_e^2 - M_o^2} = \frac{K_e\prod_{i=1}^{m}(s^2+\omega_{Bi}^2)^2}{M_1^2 - s^2M_2^2} \tag{23}$$

where K_e *is a positive constant,* ω_{Bi}, $i = 1, 2, \ldots, m_l, m_l+1, \ldots, m$ *are the finite real frequency zeros of* $Ev[Z(s)]$ *(defined as FAPs). Let* m *be the total number of the FAPs, then* $m = integer[n/2] - 1$ *and here* $n = deg[Z(s)]$ *is the order of* $Z(s)$; m_l *and* m_h *are respectively the number of zeros below and above* ω_A. *The FAPs can be arranged according to the magnitude in the increasing sequence as follows*

$$\begin{aligned} &0 < \omega_{B1} \le \ldots \le \omega_{B_{m_l}} < \omega_A \\ &< \omega_{B_{m_l+1}} \le \ldots \le \omega_{B_m} < \infty \end{aligned} \tag{24}$$

4). The even polynomial $N_e(s)$ *possesses* $m+1$ (= $m_l + m_h + 1$) *positive real frequency zeros* $b_1, \ldots, b_{m_l-1}, b_{m_l}, \omega_A, b_{m_l+1}, b_{m_l+2}, \ldots, b_m$. *And for every* $i \le m_l$ *and every* $j \le m_h$ *there are:*

$$\left.\begin{aligned} &\omega_{B_i} \le b_i < \omega_A && 1 \le i \le m_l \\ &\text{and} && \\ &\omega_A < b_{m_l+j} \le \omega_{B_{m_l+j}} && 1 \le j \le m_h \end{aligned}\right\} \tag{25}$$

6. Conclusion

The necessary and sufficient realizability theorems for bandpass ladders composed of two kinds of resonant sections have been provided in this paper. The conditions can be regarded as direct extensions of Fujisawa's necessary and sufficient condition for low-pass ladder networks[3]. Examples were provided to give concrete form to theoretical developments. It is expected that the new theorems can be extended into more general cases.

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