Lagrangian approach to three-dimensional azimuthal magnetorotational instability with and without resistivity

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（抵抗有り・無しの３次元方位磁気回転不安定性に対するラグランジュ的方法）

区　　分：甲

論　文　内　容　の　要　旨

Magnetorotational instability (MRI) is the desired mechanism to trigger turbulence in the accretion disk, to excite the outward transportation of the angular momentum while the mass accretes center. We use Lagrangian approach to obtain Hain-Lüst equation which is a second order differential equation. The WKB approximation is taken based on this equation. It is a lengthy work to deduce the Hain-Lüst equation but avoids the ignorance of terms in the usual WKB treatment. The usual WKB treatment uses WKB approximation to a group of equations rather than a single one and likely to intuitively ignore important term for studying nonaxisymmetric disturbances.

In chapter 2 short-wavelength stability analysis is made of axisymmetric rotating flows of a perfectly conducting fluid (MHD), subjected to external azimuthal magnetic field B to nonaxisymmetric as well as axisymmetric perturbations. Our WKB approximation is based on the Hain-Lüst equation which is deduced here in a slightly different way from the deduction from the famous Friman-Rotenburg equation. When the magnetic field is sufficiently weak, the instability occurs for Rossby number Ro , which designates the shear of velocity, close to zero and the maximum growth rate close to the Oort A-value. As the magnetic field is increased, the flow becomes unstable to waves of very short axial wavelengths for the whole range of Ro when magnetic Rossby number, which designates the shear of magnetic field, $Rb > -3/4$, and to waves of very long axial wavelengths for a finite range of Ro when $Rb < -1/2$. For the both waves, the maximum growth rate increases, beyond the Oort A-value, without bound in proportion to the strength of the magnetic field.

In chapter 3 short-wavelength analysis is also considered for the azimuthal magnetorotational instability (AMRI), the instability of rotating flows of an electrically conducting fluid, subject to the azimuthal magnetic field, to three-dimensional disturbances. Different from chapter 2, the viscosity and electrical resistivity are considered. The extended Hain-Lüst equation for the radial Lagrangian displacement is obtained. We apply the WKB approximation to the extended Hain-Lüst equation, whereby we can retain all the terms, being otherwise liable to be missed for nonaxisymmetric disturbances. The electric resistivity is assumed much larger than viscosity and this case is called inductionless limit. For inductionless nonaxisymmetric AMRI, unlike the case of a perfectly conducting fluid, no instability occurs for sufficiently weak magnetic field. When the magnetic field is sufficiently strong, there are mainly two modes considered for instability dependent on *Rb*. For Keplerian flow, for arbitrary *Rb,* it is always unstable. The growth rate is found to increase without bound in proportion to the square of the strength of the magnetic field which is faster than AMRI for the ideal MHD as shown in chapter 2.

In chapter 4, the energy of the ideal MHD by means of the Lagrangian displacement is studied. Ideal MHD is a Hamiltonian system. Krein’s collision theorem states that when two real eigenvalue having disturbance wave energy of opposite sign (positive and negative) collides, the real eigenvalues would bifurcate to complex ones which have imaginary part and cause instability. Starting with a lengthy energy formula, we obtain two energy formulas expressible by the Lagrangian displacement and a induced variable which describes the magnetic field ’displacement’. By using the simplified formula, we calculate the energy of a rigid rotation flow subject to magnetic field made up of an azimuthal component with linear dependence on radius r and an axial constant component and then we find the bifurcation which is in perfect accordance with Krein's collision theorem.