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Instability of plane Poiseuille flow in viscous compressible gas

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Abstract

Instability of plane Poiseuille flow in viscous compressible gas is investigated. A condition for the Reynolds and Mach numbers is given in order for plane Poiseuille flow to be unstable. It turns out that plane Poiseuille flow is unstable for Reynolds numbers much less than the critical Reynolds number for the incompressible flow when the Mach number is suitably large. It is proved by the analytic perturbation theory that the linearized operator around plane Poiseuille flow has eigenvalues with positive real part when the instability condition for the Reynolds and Mach numbers is satisfied.

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1 Introduction

This paper is concerned with the stability of plane Poiseuille flow of the compressible Navier-Stokes equation. We consider the following system of equations

$$\partial_{\tilde{t}}\tilde{\rho} + \mathrm{div}\,(\tilde{\rho}\tilde{v}) = 0, \tag{1.1}$$

$$\tilde{\rho}(\partial_{\tilde{t}}\tilde{v} + \tilde{v}\cdot\nabla\tilde{v}) - \mu\Delta\tilde{v} - (\mu+\mu')\nabla\mathrm{div}\,\tilde{v} + \nabla\tilde{P}(\tilde{\rho}) = \tilde{\rho}\tilde{\boldsymbol{g}} \tag{1.2}$$

in a 3-dimensional infinite layer $\Omega_\ell = \mathbf{R}^2 \times (0,\ell)$:

$$\Omega_\ell = \{\tilde{x} = (\tilde{x}', \tilde{x}_3) \,:\, \tilde{x}' = (\tilde{x}_1, \tilde{x}_2) \in \mathbf{R}^2,\, 0 < \tilde{x}_3 < \ell\}.$$

Here $\tilde{\rho} = \tilde{\rho}(\tilde{x}, \tilde{t})$ and $\tilde{v} = {}^\top(\tilde{v}^1(\tilde{x}, \tilde{t}), \tilde{v}^2(\tilde{x}, \tilde{t}), \tilde{v}^3(\tilde{x}, \tilde{t}))$ denote the density and velocity at time $\tilde{t} \geq 0$ and position $\tilde{x} \in \Omega_\ell$, respectively; $\tilde{P} = \tilde{P}(\tilde{\rho})$ is the pressure that is assumed to be a smooth function of $\tilde{\rho}$ satisfying

$$\tilde{P}'(\rho_*) > 0$$

for a given constant $\rho_* > 0$; μ and μ' are the viscosity and the second viscosity coefficients, respectively, that are assumed to be constants and satisfy

$$\mu > 0, \ \ \frac{2}{3}\mu + \mu' \geq 0;$$

div , ∇ and Δ denote the usual divergence, gradient and Laplacian with respect to $\tilde{x}$; and $\tilde{\boldsymbol{g}}$ is a given external force. Here and in what follows ${}^\top\cdot$ stands for the transposition.

We assume that the external force $\tilde{\boldsymbol{g}}$ takes the form

$$\tilde{\boldsymbol{g}} = g\boldsymbol{e}_1,$$

where g is a positive constant and $\boldsymbol{e}_1 = {}^\top(1, 0, 0) \in \mathbf{R}^3$.

The system (1.1)–(1.2) is considered under the boundary condition

$$\tilde{v}|_{\tilde{x}_3=0,\ell} = 0. \tag{1.3}$$

It is easily seen that (1.1)–(1.3) has a stationary solution $\tilde{u}_s = {}^\top(\tilde{\phi}_s, \tilde{v}_s)$ satisfying

$$\tilde{\phi}_s = \rho_*, \quad \tilde{v}_s = \frac{\rho_* g}{2\mu}\tilde{x}_3(\ell - \tilde{x}_3)\boldsymbol{e}_1,$$

that is the so-called plane Poiseuille flow.

The aim of this paper is to give a condition for the Reynolds and Mach numbers in order for plane Poiseuille flow to be unstable.

The function $\tilde{u}_s$ is also a stationary solution of the incompressible Navier-Stokes equation

$$\mathrm{div}\,\tilde{v} = 0, \tag{1.4}$$

$$\rho_*(\partial_{\tilde{t}}\tilde{v} + \tilde{v} \cdot \nabla\tilde{v}) - \mu\Delta\tilde{v} + \nabla\tilde{p} = 0, \tag{1.5}$$

$$\tilde{v}|_{\tilde{x}_3=0,\ell} = 0 \tag{1.6}$$

with $\tilde{p} = \rho_* g\tilde{x}_1$.

It is well known that stationary parallel flow of the incompressible Navier-Stokes equation is in general stable under arbitrary size of initial perturbations in L^2 if the Reynolds number R is sufficiently small. Furthermore, plane Poiseuille flow is stable under sufficiently small initial perturbations if $\mathrm{R} < \mathrm{R}_c$ for a critical number $\mathrm{R}_c \sim 5772$, and unstable if $\mathrm{R} > \mathrm{R}_c$.

In the case of the compressible Navier-Stokes equation, Iooss-Padula [1] investigated the linearized stability of stationary parallel flow in a cylindrical domain under perturbations that are periodic in the unbounded direction of the domain. It was shown in [1] that stationary parallel flow is linearly stable for suitably small Reynolds number. In [2] (cf., [3]), nonlinear stability of parallel flow in the infinite layer Ω_ℓ was studied; and it was proved that parallel flow is asymptotically stable under perturbations sufficiently small in some Sobolev space over Ω_ℓ if the Reynolds and Mach numbers are sufficiently small. In this paper we will show that plane Poiseuille flow of (1.1)–(1.2) is linearly unstable if $(3/Re+1/Re')/Re \leq 30(1/280-1/Ma^2)/Ma^2$, provided that $Ma^2 > 280$. Here Re, Re' and Ma are the numbers given by $Re = 16\mathrm{R}$, $Re' = 16\mathrm{R}'$ and $Ma = \mathrm{M}/8$ with the Reynolds number R, second Reynolds number R' and Mach number M defined by

$$\mathrm{R} = \frac{\rho_* \ell V_0}{2\mu}, \quad \mathrm{R}' = \frac{\rho_* \ell V_0}{2\mu'}, \quad \mathrm{M} = \frac{\sqrt{P'(\rho_*)}}{V_0}.$$

Here V_0 is the maximum velocity $\frac{\rho_* g \ell^2}{8\mu}$ of plane Poiseuille flow. In particular, this result shows that there appears an instability even when $\mathrm{R} \ll \mathrm{R}_c$ in the case of compressible flows.

To prove our result, we consider the spectrum of the linearized operator under periodic boundary condition in $x' = (x_1, x_2)$ to find eigenvalues with positive real part. As in the case of cylindrical domain analyzed in [1], the linearized operator generates a C_0-semigroup on $L^2_{per}(\mathcal{P}_{\alpha_1,\alpha_2} \times (0,1))$. Here $\mathcal{P}_{\alpha_1,\alpha_2}$ denotes the basic period cell $[-\frac{\pi}{\alpha_1}, \frac{\pi}{\alpha_1}) \times [-\frac{\pi}{\alpha_2}, \frac{\pi}{\alpha_2})$ with $\alpha_1, \alpha_2 > 0$. We will investigate the spectrum of the linearized operator on $L^2_{per}(\mathcal{P}_{\alpha_1,\alpha_2} \times (0,1))$ for sufficiently small α_1 and α_2 by using the analytic perturbation theory to obtain our instability criterion mentioned above.

This paper is organized as follows. In section 2 we deduce a non-dimensional form of system (1.1)–(1.2) and rewrite it into the system of equations for perturbations. We also introduce notations used in this paper. In section 3 we state the main result of this paper precisely. Sections 4–6 are devoted to the proof of the main result. In section 4 we consider the Fourier series expansion in $x' = (x_1, x_2) \in \mathcal{P}_{\alpha_1,\alpha_2}$ and reduce the spectral analysis of the linearized operator to the one for the Fourier coefficients that are functions of x_3. Section 5 is devoted to the study of the spectrum of the zero frequency part of the linearized operator. In section 6 we investigate the spectrum of the low frequency part of the linearized operator by the analytic perturbation theory and complete the proof of our instability result.

2 Preliminaries

In this section we first deduce a non-dimensional form of system (1.1)–(1.2) and then give the system of equations for perturbations. In the end of this section we introduce function spaces used in this paper.

We introduce the following non-dimensional variables:

$$\tilde{x} = \ell x, \ \tilde{t} = \frac{\ell}{V} t, \ \tilde{v} = V v, \ \tilde{\rho} = \rho_* \rho, \ \tilde{P} = \rho_* V^2 P$$

with

$$V = \frac{\rho_* g \ell^2}{\mu}.$$

Under this transformation, Ω_ℓ is transformed into $\Omega = \Omega_1$:

$$\Omega = \{x = (x', x_3) \,:\, x' = (x_1, x_2) \in \mathbf{R}^2,\ 0 < x_3 < 1\}.$$

Using the relations $\partial_{\tilde{x}} = \frac{1}{\ell}\partial_x$, $\partial_{\tilde{t}} = \frac{V}{\ell}\partial_t$, we see that (1.1) and (1.2) are transformed into

$$\partial_t \rho + \mathrm{div}\,(\rho v) = 0, \tag{2.1}$$

$$\rho(\partial_t v + v \cdot \nabla v) - \nu \Delta v - (\nu + \nu')\nabla \mathrm{div}\, v + \nabla P(\rho) = \nu \rho \boldsymbol{e}_1. \tag{2.2}$$

Here, div , ∇ and Δ denote the divergence, gradient and Laplacian with respect to x; and ν and ν' are the non-dimensional parameters given by

$$\nu = \frac{\mu}{\rho_* \ell V}, \quad \nu' = \frac{\mu'}{\rho_* \ell V}.$$

To derive (2.2) we have used the relation $\frac{\ell g}{V^2} = \nu$. The assumption $\tilde{P}'(\rho_*) > 0$ is restated as

$$P'(1) > 0.$$

We next introduce plane Poiseuille flow. Let us consider the stationary problem

$$\mathrm{div}\,(\rho v) = 0, \tag{2.3}$$

$$\rho v \cdot \nabla v - \nu \Delta v - (\nu + \nu')\nabla \mathrm{div}\, v + \nabla P(\rho) = \nu \rho \boldsymbol{e}_1 \tag{2.4}$$

in Ω under the boundary condition

$$v|_{x_3 = 0,1} = 0. \tag{2.5}$$

Proposition 2.1. *Problem* (2.3)–(2.5) *has a stationary solution* (*plane Poiseuille flow*) $u_s = {}^\top(\rho_s, v_s)$, *where*

$$\rho_s = 1, \quad v_s = {}^\top(v_s^1(x_3), 0, 0), \quad v_s^1(x_3) = \frac{1}{2}(-x_3^2 + x_3).$$

Proof. Set $\rho = 1$ and $v = {}^\top(v^1(x_3), 0, 0)$ in (2.3) and (2.4). Then, since

$$\mathrm{div}\, v = \partial_{x_1} v^1(x_3) = 0, \quad v \cdot \nabla v^1 = v^1 \partial_{x_1} v^1(x_3) = 0,$$

together with (2.5), we have $-\partial_{x_3}^2 v^1 = 1$ and $v^1|_{x_3=0,1} = 0$, from which we obtain $v^1(x_3) = \frac{1}{2}(-x_3^2 + x_3)$. This completes the proof. □

We next derive the system of equations for perturbations. We substitute $u(t) = {}^\top(\phi(t), w(t)) \equiv {}^\top(\gamma^2(\rho(t) - \rho_s), v(t) - v_s)$ into (2.1) and (2.2), where γ is the non-dimensional number given by

$$\gamma = \sqrt{P'(1)} = \frac{\sqrt{\tilde{P}'(\rho_*)}}{V}.$$

Noting that $\rho_s = 1$, $v_s = {}^\top(v_s^1(x_3), 0, 0)$ and $-\Delta v_s = \boldsymbol{e}_1$, we obtain the following system of equations

$$\partial_t \phi + v_s^1 \partial_{x_1} \phi + \gamma^2 \mathrm{div}\, w = f^0, \tag{2.6}$$

$$\partial_t w - \nu \Delta w - \tilde{\nu} \nabla \mathrm{div}\, w + \nabla \phi - \frac{\nu}{\gamma^2} \phi \boldsymbol{e}_1 + v_s^1 \partial_{x_1} w + (\partial_{x_3} v_s^1) w^3 \boldsymbol{e}_1 = \boldsymbol{f}. \tag{2.7}$$

Here $\boldsymbol{e}_1 = {}^\top(1, 0, 0) \in \mathbf{R}^3$; and f^0 and $\boldsymbol{f} = {}^\top(\boldsymbol{f}', f^3)$ with $\boldsymbol{f}' = {}^\top(f^1, f^2)$ denote the nonlinearities:

$$f^0 = -\mathrm{div}\,(\phi w),$$

$$\begin{aligned} \boldsymbol{f} \;\; = \;\; & -w \cdot \nabla w - \tfrac{\phi}{\gamma^2+\phi} \Big(\nu \Delta w + \tfrac{\nu}{\gamma^2} \phi \boldsymbol{e}_1 + \tilde{\nu} \nabla \mathrm{div}\, w \Big) \\ & + \tfrac{\phi}{\gamma^4} \nabla \left(P^{(1)}(\gamma^{-2}\phi)\phi \right) + \tfrac{\phi^2}{\gamma^2(\gamma^2+\phi)} \nabla \left(P(1+\gamma^{-2}\phi) \right) + \tfrac{1}{\gamma^4} \nabla \left(P^{(2)}(\gamma^{-2}\phi)\phi^2 \right), \end{aligned}$$

where

$$P^{(1)}(\gamma^{-2}\phi) = \int_0^1 P'(1+\theta\gamma^{-2}\phi)\, d\theta$$

and

$$P^{(2)}(\gamma^{-2}\phi) = \int_0^1 (1-\theta) P''(1+\theta\gamma^{-2}\phi)\, d\theta.$$

We consider (2.6)–(2.7) under the boundary conditions

$$w|_{x_3=0,1} = 0, \quad \phi,\ w\text{: } \tfrac{2\pi}{\alpha_j}\text{-periodic in } x_j\ (j = 1, 2), \tag{2.8}$$

and the initial condition

$$u|_{t=0} = u_0 = {}^\top(\phi_0, w_0). \tag{2.9}$$

Here α_1 and α_2 are given positive numbers.

We are interested in the instability of plane Poiseuille flow. We will thus consider the linearized problem for problem (2.6)–(2.9), i.e., with $f^0 = 0$ and $\boldsymbol{f} = \mathbf{0}$.

In the remaining of this section we introduce some notations used in this paper. For given α_1, $\alpha_2 > 0$, we denote the basic period cell by

$$\mathcal{P}_{\alpha_1,\alpha_2} = \left[-\tfrac{\pi}{\alpha_1}, \tfrac{\pi}{\alpha_1}\right) \times \left[-\tfrac{\pi}{\alpha_2}, \tfrac{\pi}{\alpha_2}\right).$$

We set

$$\Omega_{\alpha_1,\alpha_2} = \mathcal{P}_{\alpha_1,\alpha_2} \times (0, 1).$$

We denote by $C^\infty_{0,per}(\Omega_{\alpha_1,\alpha_2})$ the space of restrictions of functions in $C^\infty(\Omega)$ which are $\mathcal{P}_{\alpha_1,\alpha_2}$-periodic in $x' = (x_1, x_2)$ and vanish near $x_3 = 0, 1$. We set

$$L^2_{per}(\Omega_{\alpha_1,\alpha_2}) = \text{the } L^2(\Omega_{\alpha_1,\alpha_2})\text{-closure of } C^\infty_{0,per}(\Omega_{\alpha_1,\alpha_2}),$$

$$H^1_{0,per}(\Omega_{\alpha_1,\alpha_2}) = \text{the } H^1(\Omega_{\alpha_1,\alpha_2})\text{-closure of } C^\infty_{0,per}(\Omega_{\alpha_1,\alpha_2}).$$

We note that if $f \in H^1_{0,per}(\Omega_{\alpha_1,\alpha_2})$, then $f|_{x_j=-\pi/\alpha_j} = f|_{x_j=\pi/\alpha_j}$ $(j = 1, 2)$ and $f|_{x_3=0,1} = 0$.

For simplicity the set of all vector fields whose components are in $L^2_{per}(\Omega_{\alpha_1,\alpha_2})$ (resp. $H^1_{0,per}(\Omega_{\alpha_1,\alpha_2})$) is also denoted by $L^2_{per}(\Omega_{\alpha_1,\alpha_2})$ (resp. $H^1_{0,per}(\Omega_{\alpha_1,\alpha_2})$) if no confusion will occur.

We also use notation $L^2_{per}(\Omega_{\alpha_1,\alpha_2})$ for the set of all $u = {}^\top(\phi, w)$ with $\phi \in L^2_{per}(\Omega_{\alpha_1,\alpha_2})$ and $w = {}^\top(w^1, w^2, w^3) \in L^2_{per}(\Omega_{\alpha_1,\alpha_2})$ if no confusion will occur. The inner product of $u_j = {}^\top(\phi_j, w_j) \in L^2_{per}(\Omega_{\alpha_1,\alpha_2})$ $(j = 1, 2)$ is defined by

$$(u_1, u_2) = \frac{1}{\gamma^2} \int_{\Omega_{\alpha_1,\alpha_2}} \phi_1(x)\overline{\phi_2(x)}\, dx + \int_{\Omega_{\alpha_1,\alpha_2}} w_1(x) \cdot \overline{w_2(x)}\, dx,$$

where $\overline{z}$ denotes the complex conjugate of z.

We denote by $L^2(0, 1)$ the usual L^2 space on $(0, 1)$ with norm $|\cdot|_{L^2}$, and, likewise, by $H^k(0, 1)$ the kth order L^2-Sobolev space on $(0, 1)$ with norm $|\cdot|_{H^k}$. The H^1-closure of $C_0^\infty(0, 1)$ is denoted by $H^1_0(0, 1)$. As in the case of functions on $\Omega_{\alpha_1,\alpha_2}$, function spaces of vector fields $w = {}^\top(w^1, w^2, w^3)$ and, also, those of $u = {}^\top(\phi, w)$, are simply denoted by $L^2(0, 1)$, $H^1_0(0, 1)$, and so on, if no confusion will occur. We define an inner product $\langle u_1, u_2\rangle$ of $u_j = {}^\top(\phi_j, w_j) \in L^2(0, 1)$ $(j = 1, 2)$, by

$$\langle u_1, u_2\rangle = \frac{1}{\gamma^2} \int_0^1 \phi_1(x_3)\overline{\phi_2(x_3)}\, dx_3 + \int_0^1 w_1(x_3) \cdot \overline{w_2(x_3)}\, dx_3.$$

The mean value of a function $\phi(x_3)$ over $(0, 1)$ is denoted by $\langle\phi\rangle$:

$$\langle\phi\rangle = \int_0^1 \phi(x_3)\, dx_3.$$

The set of all $\phi \in L^2(0, 1)$ with $\langle\phi\rangle = 0$ is denoted by $L^2_*(0, 1)$, i.e.,

$$L^2_*(0, 1) = \{\phi \in L^2(0, 1) : \langle\phi\rangle = 0\}.$$

We define 4×4 diagonal matrices Q_0 and $\tilde{Q}$ by

$$Q_0 = \mathrm{diag}\,(1, 0, 0, 0), \quad \tilde{Q} = \mathrm{diag}\,(0, 1, 1, 1).$$

Note that

$$Q_0u = {}^\top(\phi, 0), \quad \tilde{Q}u = {}^\top(0, w) \ \text{ for } \ u = {}^\top(\phi, w).$$

We denote the resolvent set of a closed operator A by $\rho(A)$ and the spectrum of A by $\sigma(A)$. The kernel and the range of A are denoted by $\mathrm{Ker}\, A$ and $R(A)$, respectively.

3 Main result

In this section we state our main result of this paper.

The linearized problem is written as

$$\partial_t\phi + v_s^1\partial_{x_1}\phi + \gamma^2\mathrm{div}\, w = 0, \tag{3.1}$$

$$\partial_t w - \nu\Delta w - \tilde{\nu}\nabla \mathrm{div}\, w + \nabla\phi - \frac{\nu}{\gamma^2}\phi \boldsymbol{e}_1 + v_s^1 \partial_{x_1} w + (\partial_{x_3} v_s^1) w^3 \boldsymbol{e}_1 = 0, \tag{3.2}$$

$$w|_{x_3=0,1} = 0, \quad \phi, w:\ \mathcal{P}_{\alpha_1,\alpha_2}\text{-periodic in } x', \tag{3.3}$$

$$u|_{t=0} = u_0 = {}^\top(\phi_0, w_0). \tag{3.4}$$

We define the operator L on $L^2_{per}(\Omega_{\alpha_1,\alpha_2})$ by

$$D(L) = \left\{ u = {}^\top(\phi, w) \in L^2_{per}(\Omega_{\alpha_1,\alpha_2}) :\ w \in H^1_{0,per}(\Omega_{\alpha_1,\alpha_2}),\ Lu \in L^2_{per}(\Omega_{\alpha_1,\alpha_2}) \right\},$$

$$L = \begin{pmatrix} v_s^1 \partial_{x_1} & \gamma^2 \mathrm{div} \\ \nabla & -\nu\Delta - \tilde{\nu}\nabla \mathrm{div} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -\frac{\nu}{\gamma^2} & v_s^1 \partial_{x_1} + (\partial_{x_3} v_s^1) \boldsymbol{e}_1 {}^\top \boldsymbol{e}_3 \end{pmatrix}.$$

As in [1] one can see that $-L$ generates a C_0-semigroup in $L^2_{per}(\Omega_{\alpha_1,\alpha_2})$.

We now state our main result of this paper. For $\alpha' = (\alpha_1, \alpha_2)$ with $\alpha_1, \alpha_2 > 0$ and $(m_1, m_2) \in \mathbf{Z}^2$, we introduce the notations $|\alpha'|$ and α'_{m_1,m_2} by

$$|\alpha'| = (\alpha_1^2 + \alpha_2^2)^{\frac{1}{2}} \text{ and } \alpha'_{m_1,m_2} = (\alpha_1 m_1, \alpha_2 m_2).$$

Theorem 3.1. *There exist constants $r_0 > 0$ and $\eta_0 > 0$ such that if $|\alpha'| \le r_0$, then*

$$\sigma(-L) \cap \left\{ \lambda \in \mathbf{C} : |\lambda| \le \eta_0 \right\} = \{ \lambda_{m_1,m_2} :\ |m_1| = 0, 1, \cdots, k_1, |m_2| = 0, 1, \cdots, k_2 \}$$

for some $k_1, k_2 \in \mathbf{N}$, where λ_{m_1,m_2} are eigenvalues of $-L$ that satisfies

$$\lambda_{m_1,m_2} = -\frac{i}{6}(\alpha_1 m_1) + \kappa_0 (\alpha_1 m_1)^2 - \frac{\gamma^2}{12\nu}(\alpha_2 m_2)^2 + O(|\alpha'_{m_1,m_2}|^3)$$

as $|\alpha'_{m_1,m_2}| \to 0$. Here κ_0 is the number given by

$$\kappa_0 = \frac{1}{12\nu}\left(\frac{1}{280} - \gamma^2 - \frac{\nu^2}{15\gamma^2} - \frac{\nu\tilde{\nu}}{30\gamma^2} \right).$$

As a consequence, if $\gamma^2 < \frac{1}{280}$ and $2\nu^2 + \nu\tilde{\nu} \le 30\gamma^2 \left(\frac{1}{280} - \gamma^2 \right)$, then $\kappa_0 > 0$ and plane Poiseuille flow $u_s = {}^\top(\phi_s, v_s)$ is linearly unstable.

Remark 3.2. Let, for example, $\gamma = 0.05$, $\nu = 1/173$ and $\nu' = -2\nu/3$. Then $\kappa_0 > 0$ and thus plane Poiseuille flow is unstable. In this case, the Reynolds number $\mathrm{R} = 1/(16\nu) \sim 10.81$ and the Mach number $\mathrm{M} = 8/\gamma = 160$.

We will prove Theorem 3.1 in the subsequent sections. In section 4 we consider the Fourier series expansion in $x' = (x_1, x_2) \in \mathcal{P}_{\alpha_1,\alpha_2}$ and reduce the problem to the ones for the Fourier coefficients $\hat{u}(\alpha'_{m_1,m_2}, x_3)$. In section 5 we investigate the spectrum for the case $\alpha'_{m_1,m_2} = 0$. In section 6 we complete the proof of Theorem 3.1 by applying the analytic perturbation theory for small α'_{m_1,m_2} based on the analysis in section 5.

4 Fourier series expansion in $x' \in \mathcal{P}_{\alpha_1,\alpha_2}$

In this section we consider the Fourier series expansion in $x' = (x_1, x_2) \in \mathcal{P}_{\alpha_1,\alpha_2}$ and reduce the problem to the ones for the Fourier coefficients $\hat{u}(\alpha'_{m_1,m_2})$.

To investigate the spectrum of $-L$, we consider the Fourier series expansion of (3.1)–(3.4) in $x' \in \mathcal{P}_{\alpha_1,\alpha_2}$:

$$\partial_t \hat{\phi} + i\xi_1 v_s^1 \hat{\phi} + i\gamma^2 \xi' \cdot \hat{w}' + \gamma^2 \partial_{x_3} \hat{w}^3 = 0, \tag{4.1}$$

$$\begin{aligned} \partial_t \hat{w}' + \nu(|\xi'|^2 - \partial_{x_n}^2)\hat{w}' - i\tilde{\nu}\xi'(i\xi' \cdot \hat{w}' + \partial_{x_3}\hat{w}^3) + i\xi'\hat{\phi} \\ -\tfrac{\nu}{\gamma^2}\hat{\phi}\boldsymbol{e}'_1 + i\xi_1 v_s^1 \hat{w}' + (\partial_{x_3} v_s^1)\hat{w}^3 \boldsymbol{e}'_1 = 0, \end{aligned} \tag{4.2}$$

$$\partial_t \hat{w}^3 + \nu(|\xi'|^2 - \partial_{x_n}^2)\hat{w}^3 - \tilde{\nu}\partial_{x_3}(i\xi' \cdot \hat{w}' + \partial_{x_3}\hat{w}^3) + \partial_{x_3}\hat{\phi} + i\xi_1 v_s^1 \hat{w}^3 = 0, \tag{4.3}$$

$$\hat{w}|_{x_n=0,1} = 0, \tag{4.4}$$

$$\hat{u}|_{t=0} = \hat{u}_0 = {}^\top(\hat{\phi}_0, \hat{w}_0). \tag{4.5}$$

Here and in what follows we simply write $\alpha'_{m_1,m_2} = (\alpha_1 m_1, \alpha_2 m_2)$ ($(m_1, m_2) \in \mathbf{Z}^2$) as $\xi' = (\xi_1, \xi_2)$; $\hat{\phi} = \hat{\phi}(\xi', x_3, t)$ and $\hat{w}^j = \hat{w}^j(\xi', x_3, t)$ ($j = 1, 2, 3$) are the Fourier coefficients of $\phi = \phi(x', x_3, t)$ and $w^j = w^j(x', x_3, t)$ ($j = 1, 2, 3$) with respect to $x' = (x_1, x_2) \in \mathcal{P}_{\alpha_1,\alpha_2}$, respectively, with $w' = {}^\top(w^1, w^2)$; and $\boldsymbol{e}'_1 = {}^\top(1, 0) \in \mathbf{R}^2$.

We thus arrive at the following problem

$$\partial_t u + \hat{L}_{\xi'} u = 0, \quad u|_{t=0} = u_0 \tag{4.6}$$

with a parameter $\xi' = (\xi_1, \xi_2) \in \mathbf{R}^2$, where $\hat{L}_{\xi'}$ is the operator on $L^2(0, 1)$ of the form

$$\hat{L}_{\xi'} = \hat{A}_{\xi'} + \hat{B}_{\xi'} + \hat{C}_0$$

with domain

$$D(\hat{L}_{\xi'}) = \{u = {}^\top(\phi, w) \in L^2(0, 1) : w \in H_0^1(0, 1), \hat{L}_{\xi'} u \in L^2(0, 1)\}.$$

Here

$$\hat{A}_{\xi'} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \nu(|\xi'|^2 - \partial_{x_3}^2)I_2 + \tilde{\nu}\xi'^\top\xi' & -i\tilde{\nu}\xi'\partial_{x_3} \\ 0 & -i\tilde{\nu}^\top\xi'\partial_{x_3} & \nu(|\xi'|^2 - \partial_{x_3}^2) - \tilde{\nu}\partial_{x_3}^2 \end{pmatrix},$$

where I_2 denotes the 2×2 identity matrix,

$$\hat{B}_{\xi'} = \begin{pmatrix} i\xi_1 v_s^1 & i\gamma^2\, {}^\top\xi' & \gamma^2\partial_{x_3} \\ i\xi' & i\xi_1 v_s^1 I_2 & 0 \\ \partial_{x_3} & 0 & i\xi_1 v_s^1 \end{pmatrix},$$

and

$$\hat{C}_0 = \begin{pmatrix} 0 & 0 & 0 \\ -\frac{\nu}{\gamma^2}\boldsymbol{e}_1' & 0 & (\partial_{x_3} v_s^1)\boldsymbol{e}_1' \\ 0 & 0 & 0 \end{pmatrix}.$$

Note that $D(\hat{L}_{\xi'}) = D(\hat{L}_0)$ for all $\xi' \in \mathbf{R}^2$.

5 Spectrum of $-\hat{L}_0$

To prove Theorem 3.1, we first consider the spectrum of $-\hat{L}_0$, i.e., $-\hat{L}_{\xi'}$ with $\xi' = 0$:

$$\hat{L}_0 = \begin{pmatrix} 0 & 0 & \gamma^2 \partial_{x_3} \\ -\frac{\nu}{\gamma^2}\boldsymbol{e}_1' & -\nu\partial_{x_3}^2 I_2 & (\partial_{x_3} v_s^1)\boldsymbol{e}_1' \\ \partial_{x_3} & 0 & -(\nu+\tilde{\nu})\partial_{x_3}^2 \end{pmatrix}.$$

Let us introduce the adjoint operator $\hat{L}_{\xi'}^*$ of $\hat{L}_{\xi'}$ with respect to the inner product $\langle\cdot,\cdot\rangle$:

$$\hat{L}_{\xi'}^* = \hat{A}_{\xi'} - \hat{B}_{\xi'} + \hat{C}_0^*,$$

where

$$\hat{C}_0^* = \begin{pmatrix} 0 & -\nu\,{}^\top\boldsymbol{e}_1' & 0 \\ 0 & 0 & 0 \\ 0 & (\partial_{x_3} v_s^1)\,{}^\top\boldsymbol{e}_1' & 0 \end{pmatrix}.$$

We consider $\hat{L}_{\xi'}^*$ as an operator on $L^2(0,1)$ with domain

$$D(\hat{L}_{\xi'}^*) = \{u = {}^\top(\phi, w) \in L^2(0,1) : w \in H_0^1(0,1), \hat{L}_{\xi'}^* u \in L^2(0,1)\}.$$

Note that

$$\hat{L}_0^* = \begin{pmatrix} 0 & -\nu\,{}^\top\boldsymbol{e}_1' & -\gamma^2\partial_{x_3} \\ 0 & -\nu\partial_{x_3}^2 I_2 & 0 \\ -\partial_{x_3} & (\partial_{x_3} v_s^1)\,{}^\top\boldsymbol{e}_1' & -(\nu+\tilde{\nu})\partial_{x_3}^2 \end{pmatrix}.$$

In this paper we only consider the spectrum near the origin since we focus on the instability of plane Poiseuille flow.

Lemma 5.1. *The following assertions hold true.*

(i) *There is a positive number $\eta_1 = \eta_1(\nu, \tilde{\nu}, \gamma)$ such that $\{\lambda \in \mathbf{C} : |\lambda| < \eta_1\} \backslash \{0\} \subset \rho(-\hat{L}_0)$. Furthermore, the following estimate holds uniformly for $\lambda \in \{\lambda \in \mathbf{C} : |\lambda| \leq \eta_1/2\} \setminus \{0\}$:*

$$\left|(\lambda + \hat{L}_0)^{-1} f\right|_{L^2} + \left|\partial_{x_3}\tilde{Q}(\lambda + \hat{L}_0)^{-1} f\right|_2 \leq \frac{C}{|\lambda|}|f|_{L^2}.$$

The same assertion holds with $\hat{L}_0$ *replaced by* $\hat{L}_0^*$.

(ii) $\lambda = 0$ *is a simple eigenvalue of* $-\hat{L}_0$, *i.e.,* $R(\hat{L}_0)$ *is closed and*

$$L^2(0,1) = \mathrm{Ker}\,\hat{L}_0 \oplus R(\hat{L}_0) \quad \textit{with} \ \dim \mathrm{Ker}\,\hat{L}_0 = 1.$$

The same assertion holds with $\hat{L}_0$ *replaced by* $\hat{L}_0^*$.

(iii) *The eigenspaces for* $\lambda = 0$ *of* $\hat{L}_0$ *and* $\hat{L}_0^*$ *are spanned by* $u^{(0)}$ *and* $u^{(0)*}$ *respectively, where*

$$u^{(0)} = {}^\top(\phi^{(0)}, w^{(0)}), \quad w^{(0)} = {}^\top(w^{(0),1}, 0, 0)$$

and

$$u^{(0)*} = {}^\top(\phi^{(0)*}, w^{(0)*}), \quad w^{(0)*} = {}^\top(0, 0, 0)$$

with

$$\phi^{(0)}(x_3) = 1, \quad w^{(0),1}(x_3) = \frac{1}{2\gamma^2}(-x_3^2 + x_3), \quad \phi^{(0)*}(x_3) = \gamma^2.$$

(iv) *The eigenprojections* $\hat{\varPi}^{(0)}$ *and* $\hat{\varPi}^{(0)*}$ *for* $\lambda = 0$ *of* $-\hat{L}_0$ *and* $-\hat{L}_0^*$ *are given by*

$$\hat{\varPi}^{(0)} u = \langle u, u^{(0)*} \rangle u^{(0)} = \langle \phi \rangle u^{(0)},$$

and

$$\hat{\varPi}^{(0)*} u = \langle u, u^{(0)} \rangle u^{(0)*}$$

for $u = {}^\top(\phi, w)$, *respectively. In particular, it holds that*

$$u = {}^\top(\phi, w) \in R(I - \hat{\varPi}^{(0)}) \quad \textit{if and only if} \ \ \langle \phi \rangle = \langle u, u^{(0)*} \rangle = 0.$$

To prove Lemma 5.1, we introduce some operators. We define 2×2 matrix operators $\tilde{L}_0$, $\tilde{L}_0^*$ on $L^2(0,1)^2 = L^2(0,1) \times L^2(0,1)$, and A, $\tilde{C}_0$, $\tilde{C}_0^*$ on $L^2(0,1)^2$ by

$$\tilde{L}_0 = \begin{pmatrix} 0 & \gamma^2 \partial_{x_3} \\ \partial_{x_3} & -(\nu + \tilde{\nu})\partial_{x_3}^2 \end{pmatrix},$$

$$\tilde{L}_0^* = \begin{pmatrix} 0 & -\gamma^2 \partial_{x_3} \\ -\partial_{x_3} & -(\nu + \tilde{\nu})\partial_{x_3}^2 \end{pmatrix}$$

with domain

$$D(\tilde{L}_0) = \{\tilde{u} = {}^\top(\phi, w^3) \in L^2(0,1)^2 : w \in H_0^1(0,1), \tilde{L}_0 u \in L^2(0,1)^2\},$$

$$D(\tilde{L}_0^*) = \{u = {}^\top(\phi, w^3) \in L^2(0,1)^2 : w \in H_0^1(0,1), \tilde{L}_0^* u \in L^2(0,1)^2\},$$

and

$$A = \begin{pmatrix} -\nu \partial_{x_3}^2 & 0 \\ 0 & -\nu \partial_{x_3}^2 \end{pmatrix}$$

with domain $D(A) = [H^2(0,1) \cap H_0^1(0,1)]^2$, and

$$\tilde{C}_0 = \begin{pmatrix} -\frac{\nu}{\gamma^2} & (\partial_{x_3} v_s^1) \\ 0 & 0 \end{pmatrix},$$

$$\tilde{C}_0^* = \begin{pmatrix} -\nu & 0 \\ (\partial_{x_3} v_s^1) & 0 \end{pmatrix}$$

with domain $D(\tilde{C}_0) = D(\tilde{C}_0^*) = L^2(0,1)^2$.

Lemma 5.2. (i) *It holds that*

$$\sigma(-\tilde{L}_0) \cap \{\lambda \in \mathbf{C} : |\lambda| < \tilde{\eta}_0\} = \{0\}, \quad \{\lambda \in \mathbf{C} : |\lambda| < \nu\pi\} \subset \rho(-A),$$

for some constant $\tilde{\eta}_0 = \tilde{\eta}_0(\nu, \tilde{\nu}, \gamma^2) > 0$. *Furthermore,*

$$L^2(0,1)^2 = \mathrm{Ker}\, \tilde{L}_0 \oplus R(\tilde{L}_0)$$

with

$$\mathrm{Ker}\, \tilde{L}_0 = \mathrm{span}\, \{\tilde{u}^{(0)}\}, \quad \tilde{u}^{(0)} = {}^\top(1,0),$$
$$R(\tilde{L}_0) = L_*^2(0,1) \times L^2(0,1).$$

In particular, 0 *is a simple eigenvalue of* $-\tilde{L}_0$ *with eigenprojection* $\tilde{\Pi}_0$ *given by* $\tilde{\Pi}_0 \tilde{u} = \langle \phi \rangle \tilde{u}^{(0)}$ $(\tilde{u} = {}^\top(\phi, w^3))$.

(ii) *There hold the estimates*

$$\left|(\lambda + \tilde{L}_0)^{-1} g\right|_{L^2} + \left|\partial_{x_3} \tilde{Q}_2 (\lambda + \tilde{L}_0)^{-1} g\right|_{L^2} \le C\Big(\frac{1}{|\lambda|} + \frac{1}{\tilde{\eta}_0 - |\lambda|}\Big)|g|_{L^2}$$

uniformly for $\lambda \in \{\lambda \in \mathbf{C} : |\lambda| < \tilde{\eta}_0\} \setminus \{0\}$ *and* $g = {}^\top(f^0, f^3) \in L^2(0,1)^2$, *and*

$$\left|\partial_{x_3}^l (\lambda + A)^{-1} f'\right|_{L^2} \le \frac{1}{\nu\pi^2 - |\lambda|}|f'|_{L^2}, \quad l = 0, 1.$$

uniformly for $\lambda \in \{\lambda \in \mathbf{C} : |\lambda| < \nu\pi^2\}$ *and* $f' \in L^2(0,1)^2$. *Here* $\tilde{Q}_2 = \mathrm{diag}\,(0,1)$.

(iii) *The assertions in* (i) *and* (ii) *also hold with* $\tilde{L}_0$ *replaced by* $\tilde{L}_0^*$.

Proof. The assertions for A is well-known, and so we here omit the proof for A.

As for $\tilde{L}_0$, let us consider the problem to find $\tilde{u}$ satisfying

$$\tilde{L}_0 \tilde{u} = g, \quad \tilde{u} = {}^\top(\phi, w^3) \in D(\tilde{L}_0) \tag{5.1}$$

for a given $g = {}^\top(f^0, f^3) \in L^2(0,1)^2$.

To solve this problem, we expand ϕ and w^3 into the Fourier cosine and sine series respectively:

$$\phi = \sum_{n=0}^{\infty} \phi_n \cos n\pi x_3, \quad w^3 = \sum_{n=1}^{\infty} w_n^3 \sin n\pi x_3,$$

and likewise,

$$f^0 = \sum_{n=0}^{\infty} f_n^0 \cos n\pi x_3, \quad f^3 = \sum_{n=1}^{\infty} f_n^3 \sin n\pi x_3.$$

It then follows from (5.1) that

$$f_0^0 = 0,$$

and, for $n \geq 1$,

$$w_n^3 = \frac{1}{\gamma^2} \frac{1}{n\pi} f_n^0,$$

$$\phi_n = \frac{\nu + \tilde{\nu}}{\gamma^2} f_n^0 - \frac{1}{n\pi} f_n^3.$$

We thus see that problem (5.1) is uniquely solvable if and only if $\langle f^0 \rangle = f_0^0 = 0$ and $\langle \phi \rangle = \phi^0 = 0$; and in this case, the unique solution is given by

$$\phi = \frac{\nu + \tilde{\nu}}{\gamma^2} f^0 + F^3, \quad w^3 = \frac{1}{\gamma^2} \int_0^{x_3} f^0(y)\, dy, \tag{5.2}$$

where $F^3 = -\sum_{n=1}^{\infty} \frac{1}{n\pi} f_n^3 \cos n\pi x_3$. Furthermore, it holds that

$$|\tilde{u}|_{L^2} \leq \tilde{\eta}_0^{-1} |g|_{L^2}, \quad |\partial_{x_3} w^3|_{L^2} \leq \frac{1}{\gamma^2} |f^0|_{L^2}, \tag{5.3}$$

where $\tilde{\eta}_0 = \Big[\max\{\frac{\nu+\tilde{\nu}}{\gamma^2}, \frac{1}{\gamma^2 \pi}, \frac{1}{\pi}\}\Big]^{-1}$. Therefore, we see that $R(\tilde{L}_0) = L^2_*(0,1) \times L^2(0,1)$. Moreover, we find that $\operatorname{Ker} \tilde{L}_0 = \operatorname{span}\{\tilde{u}^{(0)}\}$ with $\tilde{u}^{(0)} = {}^\top(1,0)$ and $L^2(0,1)^2 = \operatorname{Ker} \tilde{L}_0 \oplus R(\tilde{L}_0)$. It then follows that 0 is a simple eigenvalue of $-\tilde{L}_0$ and the eigenspace for 0 is spanned by $\tilde{u}^{(0)}$. The eigenprojection $\tilde{\Pi}_0$ for 0 is given by $\tilde{\Pi}_0 \tilde{u} = \langle \phi \rangle \tilde{u}^{(0)}$ for $\tilde{u} = {}^\top(\phi, w^3) \in L^2(0,1)$.

We decompose $L^2(0,1)^2$ as $L^2(0,1)^2 = X_0 \oplus X_1$, where $X_0 = \tilde{\Pi}_0 L^2(0,1)^2 = \operatorname{Ker} \tilde{L}_0$ and $X_1 = \tilde{\Pi}_1 L^2(0,1)^2 = R(\tilde{L}_0)$ with $\tilde{\Pi}_1 = I - \tilde{\Pi}_0$.

Let us consider the resolvent problem

$$\lambda \tilde{u} + \tilde{L}_0 \tilde{u} = g, \tilde{u} \in D(\tilde{L}_0). \tag{5.4}$$

This is equivalent to

$$\lambda \tilde{u} = g, \quad \tilde{u} \in X_0 \tag{5.5}$$

for $g \in X_0$ and

$$\lambda \tilde{u} + \tilde{L}_0 \tilde{u} = g, \quad \tilde{u} \in X_1 \cap D(\tilde{L}_0) \tag{5.6}$$

for $g \in X_1$.

We see from (5.3) that if $|\lambda| < \tilde{\eta}_0$, then $\lambda \in \rho(-\tilde{L}_0|_{X_1})$ and

$$|((\lambda + \tilde{L}_0)|_{X_1})^{-1} g|_{L^2} \leq \frac{1}{\tilde{\eta}_0 - |\lambda|} |g|_{L^2},$$

$$|\partial_{x_3} \tilde{Q}_2 ((\lambda + \tilde{L}_0)|_{X_1})^{-1} g|_{L^2} \leq \frac{1}{\gamma^2} \frac{\tilde{\eta}_0}{\tilde{\eta}_0 - |\lambda|} |g|_{L^2}$$

for $|\lambda| < \tilde{\eta}_0$ and $g \in X_1$. On the other hand, it follows from (5.5) that if $\lambda \neq 0$, then

$$\tilde{\Pi}_0 \tilde{u} = \frac{1}{\lambda} \tilde{\Pi}_0 g.$$

As a result, we see that

$$(\lambda + \tilde{L}_0)^{-1} g = \frac{1}{\lambda} \tilde{\Pi}_0 g + ((\lambda + \tilde{L}_0)|_{X_1})^{-1} \tilde{\Pi}_1 g$$

for $\lambda \neq 0$ with $|\lambda| < \tilde{\eta}_0$ and $g \in L^2(0,1)^2$. The desired estimate for $\tilde{L}_0$ now follows. The assertion for $\tilde{L}_0^*$ can be shown in a similar manner. This completes the proof. □

We now give a proof of Lemma 5.1.

Proof of Lemma 5.1. For $u = {}^\top(\phi, w)$, $w = {}^\top(w^1, w^2, w^3)$, we write

$$\tilde{u} = {}^\top(\phi, w^3), \quad w' = {}^\top(w^1, w^2).$$

Then the resolvent problem

$$(\lambda + \hat{L}_0) u = f \tag{5.7}$$

is written as

$$(\lambda + \tilde{L}_0)\tilde{u} = g, \tag{5.8}$$

$$(\lambda + A) w' = f' - \tilde{C}_0 \tilde{u}. \tag{5.9}$$

Here $f = {}^\top(f^0, f^1, f^2, f^3)$, $g = {}^\top(f^0, f^3)$ and $f' = {}^\top(f^1, f^2)$.

Set $\eta_1 = \min\{\tilde{\eta}_0, \nu\pi^2\}$. Since $|\tilde{C}_0 \tilde{u}|_{L^2} \le C|\tilde{u}|_{L^2}$, we see from Lemma 5.2 that $\{\lambda \in \mathbf{C} : |\lambda| < \eta_1\} \backslash \{0\} \subset \rho(-\hat{L}_0)$. Furthermore, if $\lambda \in \{\lambda \in \mathbf{C} : |\lambda| < \eta_1\} \backslash \{0\}$, then the $\tilde{u}$- and w'-components of $u = (\lambda + \hat{L}_0)^{-1} f$ are given by $\tilde{u} = {}^\top(\phi, w^3) = (\lambda + \tilde{L}_0)^{-1} g$ and $w' = {}^\top(w^1, w^2) = (\lambda + A)^{-1}[f' - \tilde{C}_0 \tilde{u}]$, respectively, and it holds that

$$|\tilde{u}|_{L^2} + \left|\partial_{x_3} w^3\right|_{L^2} \le C\Big(\frac{1}{|\lambda|} + \frac{1}{\tilde{\eta}_0 - |\lambda|}\Big)|g|_{L^2}$$

and

$$\left|\partial_{x_3}^l w'\right|_{L^2} \le \frac{C}{(\nu\pi^2 - |\lambda|)} \left(|f'|_{L^2} + \Big(\frac{1}{|\lambda|} + \frac{1}{\tilde{\eta}_0 - |\lambda|}\Big)|g|_{L^2} \right), \quad l = 0, 1.$$

We thus find that if $|\lambda| \le \frac{\eta_1}{2}$, then

$$\left|(\lambda + \hat{L}_0)^{-1} f\right|_{L^2} + \left|\partial_{x_3} \tilde{Q} (\lambda + \hat{L}_0)^{-1} f\right|_{L^2} \le \frac{C}{|\lambda|} |f|_{L^2}.$$

This proves (i) for $\hat{L}_0$.

We next prove assertions (ii)–(iv) for $\hat{L}_0$. We first prove $\operatorname{Ker} \hat{L}_0 = \operatorname{span}\{u^0\}$. Let $\hat{L}_0 u = 0$. Then $\tilde{L}_0 \tilde{u} = 0$ and $A w' = -\tilde{C}_0 \tilde{u}$. It follows from Lemma 5.2 that $u = \alpha u^{(0)}$ (α: constant) and, hence, $\operatorname{Ker} \hat{L}_0 = \operatorname{span}\{u^0\}$.

Let us show that $R(\hat{L}_0)$ is closed and $L^2(0,1) = \operatorname{Ker}\hat{L}_0 \oplus R(\hat{L}_0)$. Set $\mathit{\Pi}^{(0)}u = \langle u, u^{(0)*}\rangle u^{(0)} = \langle \phi \rangle u^{(0)}$ for $u = {}^\top(\phi, w) \in L^2(0,1)$. It then follows that $\mathit{\Pi}^{(0)}$ is a projection onto $\operatorname{Ker}\hat{L}_0$ with property $\mathit{\Pi}^{(0)}\hat{L}_0 \subset \hat{L}_0\mathit{\Pi}^{(0)}$. Furthermore, it holds that $f = {}^\top(f^0, f', f^3) \in R(I - \mathit{\Pi}^{(0)})$ if and only if $\langle f, u^{(0)*}\rangle = \langle f^0 \rangle = 0$.

Let $\langle f, u^{(0)*}\rangle = \langle f^0 \rangle = 0$. Then (5.7) with $\lambda = 0$ is written in the form of (5.8)–(5.9) with $\lambda = 0$ and $g = {}^\top(f^0, f^3) \in L^2_*(0,1) \times L^2(0,1)$. It then follows from Lemma 5.2 that (5.8) with $\lambda = 0$ has a unique solution $\tilde{u} \in D(\tilde{L}_0)$ and, hence, (5.9) with $\lambda = 0$ has a unique solution $w' \in D(A)$. As a result, (5.7) with $\lambda = 0$ has a unique solution $u = {}^\top(\phi, w', w^3)$ that is given by

$$\tilde{u} = {}^\top(\phi, w^3) = (\tilde{L}_0|_{X_1})^{-1}g, \quad g = {}^\top(f^0, f^3),$$

$$w' = A^{-1}[f' - \tilde{C}_0\tilde{u}].$$

We thus find that $R(I - \mathit{\Pi}^{(0)}) \subset R(\hat{L}_0)$. On the other hand, if $f = {}^\top(f^0, f', f^3) \in R(\hat{L}_0)$, then it is easy to see that $\langle f, u^{(0)*}\rangle = \langle f^0 \rangle = 0$, and, hence, $f \in R(I - \mathit{\Pi}^{(0)})$. We thus find that $R(I - \mathit{\Pi}^{(0)}) = R(\hat{L}_0)$. Consequently, we see that $R(\hat{L}_0)$ is closed and $L^2(0,1) = \operatorname{Ker}\hat{L}_0 \oplus R(\hat{L}_0)$. This proves (ii)–(iv) for $\hat{L}_0$.

The assertions for $\hat{L}_0^*$ can be obtained similarly and we omit the details. This completes the proof. □

6 Perturbation argument

In this section we investigate $\sigma(-\hat{L}_{\xi'}) \cap \{|\lambda| \le \eta_1/2\}$ for $|\xi'| \ll 1$.

Let $\hat{L}_{\xi'}$ be denoted by

$$\hat{L}_\xi = \hat{L}_0 + \sum_{j=1}^{2} \xi_j \hat{L}_j^{(1)} + \sum_{j,k=1}^{2} \xi_j\xi_k \hat{L}_{jk}^{(2)},$$

where

$$\hat{L}_0 = \begin{pmatrix} 0 & 0 & \gamma^2\partial_{x_3} \\ -\frac{\nu}{\gamma^2}\boldsymbol{e}_1' & -\nu\partial_{x_3}^2 I_2 & (\partial_{x_3}v_s^1)\boldsymbol{e}_1' \\ \partial_{x_3} & 0 & -(\nu+\tilde{\nu})\partial_{x_3}^2 \end{pmatrix},$$

$$\hat{L}_j^{(1)} = \begin{pmatrix} i\delta_{1j}v_s^1 & i\gamma^{2\top}\boldsymbol{e}_j' & 0 \\ i\boldsymbol{e}_j' & iv_s^1\delta_{1j}I_2 & -i\tilde{\nu}\boldsymbol{e}_j'\partial_{x_3} \\ 0 & -i\tilde{\nu}^\top\boldsymbol{e}_j'\partial_{x_3} & iv_s^1\delta_{1j} \end{pmatrix}, \quad j = 1, 2,$$

$$\hat{L}_{jk}^{(2)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \nu\delta_{jk}I_2 + \tilde{\nu}\boldsymbol{e}_j'{}^\top\boldsymbol{e}_k' & 0 \\ 0 & 0 & \nu\delta_{jk} \end{pmatrix}, \quad j, k = 1, 2.$$

Here $\boldsymbol{e}'_1 = {}^\top(1,0)$ and $\boldsymbol{e}'_2 = {}^\top(0,1)$.

We will apply the analytic perturbation theory to prove Theorem 3.1. To do so, we prepare the following estimates.

Lemma 6.1. *There hold the following estimates uniformly for λ with $|\lambda| = \frac{\eta_1}{2}$ and $f \in L^2(0,1)$:*

$$\left| \hat{L}_j^{(1)}(\lambda + \hat{L}_0)^{-1} f \right|_{L^2} \le C|f|_{L^2}, \quad j = 1,2,$$

$$\left| \hat{L}_{jk}^{(2)}(\lambda + \hat{L}_0)^{-1} f \right|_{L^2} \le C|f|_{L^2}, \quad j,k = 1,2.$$

Proof. Let λ satisfy $|\lambda| = \frac{\eta_1}{2}$. It then follows from Lemma 5.1 that

$$\left| \hat{L}_j^{(1)}(\lambda + \hat{L}_0)^{-1} f \right|_{L^2} \le C \left| (\lambda + \hat{L}_0)^{-1} f \right|_{L^2 \times H^1} \le C|f|_{L^2}$$

and

$$\left| \hat{L}_{jk}^{(2)}(\lambda + \hat{L}_0)^{-1} f \right|_{L^2} \le C \left| (\lambda + \hat{L}_0)^{-1} f \right|_{L^2} \le C|f|_{L^2}.$$

This completes the proof. □

Theorem 3.1 follows from the following result on the spectrum of $-\hat{L}_{\xi'}$.

Theorem 6.2. *There exists a positive number $r_1 = r_1(\nu, \tilde{\nu}, \gamma)$ such that if $|\xi'| \le r_1$, then it holds*

$$\sigma(-\hat{L}_{\xi'}) \cap \left\{ \lambda \in \mathbf{C} : |\lambda| \le \frac{\eta_0}{2} \right\} = \{\lambda_{\xi'}\},$$

where $\lambda_{\xi'}$ is a simple eigenvalue of $-\hat{L}_{\xi'}$ and it satisfies

$$\lambda_{\xi'} = -\frac{i}{6}\xi_1 + \kappa_0 \xi_1^2 - \frac{\gamma^2}{12\nu}\xi_2^2 + O(|\xi'|^3)$$

as $\xi' \to 0$. Here

$$\kappa_0 = \frac{1}{12\nu}\left(\frac{1}{280} - \gamma^2 - \frac{\nu^2}{15\gamma^2} - \frac{\nu\tilde{\nu}}{30\gamma^2} \right).$$

Therefore, if $\gamma^2 < \frac{1}{280}$ and $2\nu^2 + \nu\tilde{\nu} \le 30\gamma^2\left(\frac{1}{280} - \gamma^2\right)$, then $\kappa_0 > 0$.

Proof of Theorem 6.2. Based on Lemma 5.1 and Lemma 6.1 we can apply the analytic perturbation theory (see, e.g., [4, Chap VII], [5, Chap. XII]) to see that if $|\xi'| \ll 1$, then

$$\sigma(-\hat{L}_{\xi'}) \cap \left\{ \lambda \in \mathbf{C} : \ |\lambda| \le \frac{\eta_1}{2} \right\} = \{\lambda_{\xi'}\},$$

where $\lambda_{\xi'}$ is a simple eigenvalue. Furthermore, $\lambda_{\xi'}$ is given by

$$\lambda_{\xi'} = \lambda_0 + \sum_{j=1}^{2} \xi_j \lambda_j^{(1)} + \sum_{j,k=1}^{2} \xi_j \xi_k \lambda_{jk}^{(2)} + O(|\xi'|^3).$$

Here $\lambda_{jk}^{(2)} = \lambda_{kj}^{(2)}$,

$$\lambda_0 = 0,$$
$$\lambda_j^{(1)} = -\langle \hat{L}_j^{(1)} u^{(0)}, u^{(0)*} \rangle,$$
$$\lambda_{jk}^{(2)} = -\langle \frac{1}{2}(\hat{L}_{jk}^{(2)} + \hat{L}_{kj}^{(2)}) u^{(0)}, u^{(0)*} \rangle + \langle \frac{1}{2}(\hat{L}_j^{(1)} \hat{S} \hat{L}_k^{(1)} + \hat{L}_k^{(1)} \hat{S} \hat{L}_j^{(1)}) u^{(0)}, u^{(0)*} \rangle,$$

where $\hat{S} = [(I - \mathit{\Pi}^{(0)}) \hat{L}_0 (I - \mathit{\Pi}^{(0)})]^{-1}$.

The proof of Theorem 6.2 will be completed if we compute $\lambda_j^{(1)}$ and $\lambda_{jk}^{(2)}$. We will compute them in the following propositions.

Proposition 6.3. $\lambda_j^{(1)} = -\frac{i}{6}\delta_{1j}$, $j = 1, 2$.

Proof. We have

$$\hat{L}_1^{(1)} u^{(0)} = i \begin{pmatrix} v_s^1 + \gamma^2 w^{(0),1} \\ (1 + v_s^1 w^{(0),1}) \boldsymbol{e}_1' \\ -\tilde{\nu} \partial_{x_3} w^{(0),1} \end{pmatrix} = i \begin{pmatrix} -x_3^2 + x_3 \\ 1 + \frac{1}{4\gamma^2}(-x_3^2 + x_3)^2 \\ 0 \\ -\frac{\tilde{\nu}}{2\gamma^2}(-2x_3 + 1) \end{pmatrix}, \tag{6.1}$$

$$\hat{L}_2^{(1)} u^{(0)} = i \begin{pmatrix} 0 \\ \boldsymbol{e}_2' \\ 0 \end{pmatrix} = i \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}. \tag{6.2}$$

It then follows that

$$\lambda_1^{(1)} = -\langle \hat{L}_1^{(1)} u^{(0)}, u^{(0)*} \rangle = -i \langle v_s^1 + \gamma^2 w^{(0),1} \rangle = -i \int_0^1 (-y^2 + y)\, dy = -\frac{i}{6}$$

and $\lambda_2^{(1)} = 0$. This completes the proof. □

Proposition 6.4. $\lambda_{22}^{(2)} = -\frac{\gamma^2}{12\nu}$.

Proof. Since $\hat{L}_{jk}^{(2)} u^{(0)} = {}^\top(0, *, *, *)$, we have

$$\langle \hat{L}_{jk}^{(2)} u^{(0)}, u^{(0)*} \rangle = 0 \quad \text{for} \quad j, k = 1, 2. \tag{6.3}$$

Let us compute $\hat{L}_j^{(1)} \hat{S} \hat{L}_2^{(1)} u^{(0)}$. We see from (6.2) that $\langle \hat{L}_2^{(1)} u^{(0)}, u^{(0)*} \rangle = 0$. Therefore, $\hat{L}_2^{(1)} u^{(0)} \in R(I - \hat{\mathit{\Pi}}^{(0)})$, and so, $\hat{S} \hat{L}_2^{(1)} u^{(0)}$ is a unique solution $u = {}^\top(\phi, w)$ of

$$\hat{L}_0 u = \hat{L}_2^{(1)} u^{(0)}, \quad \langle \phi \rangle = 0.$$

By (6.2), we see that the solution u of this problem is given by $\phi = w^1 = w^3 = 0$ and w^2 that satisfies $-\nu\partial_{x_3}^2 w^2 = i$ and $w^2|_{x_3=0,1} = 0$. We thus obtain

$$\hat{S}\hat{L}_2^{(1)}u^{(0)} = {}^\top(0, 0, \frac{i}{2\nu}(-x_3^2 + x_3), 0).$$

This implies that

$$\hat{L}_1^{(1)}\hat{S}\hat{L}_2^{(1)}u^{(0)} = 0 \tag{6.4}$$

and

$$\hat{L}_2^{(1)}\hat{S}\hat{L}_2^{(1)}u^{(0)} = {}^\top(-\frac{\gamma^2}{2\nu}(-x_3^2 + x_3), *, *, *). \tag{6.5}$$

It then follows from (6.3) and (6.5) that

$$\lambda_{22}^{(2)} = -\frac{\gamma^2}{2\nu}\langle -x_3^2 + x_3\rangle = -\frac{\gamma^2}{12\nu}.$$

This completes the proof. □

To obtain $\lambda_{j1}^{(2)} (= \lambda_{1j}^{(2)})$, we compute $\hat{S}\hat{L}_1^{(1)}u^{(0)}$.

Proposition 6.5. *$\hat{S}\hat{L}_1^{(1)}u^{(0)}$ is given by*

$$\hat{S}\hat{L}_1^{(1)}u^{(0)} = u_1^{(1)},$$

where $u_1^{(1)} = {}^\top(\phi_1^{(1)}, w_1^{(1),1}, w_1^{(1),2}, w_1^{(1),3})$ with

$$\begin{aligned}
\phi_1^{(1)}(x_3) &= i\left(\tfrac{\nu}{\gamma^2} + \tfrac{\tilde{\nu}}{2\gamma^2}\right)\left(-x_3^2 + x_3 - \tfrac{1}{6}\right), \\
w_1^{(1),1}(x_3) &= i\left(\tfrac{\nu}{\gamma^4} + \tfrac{\tilde{\nu}}{2\gamma^4}\right)\left(\tfrac{1}{12}x_3^4 - \tfrac{1}{6}x_3^3 + \tfrac{1}{12}x_3^2\right) \\
&\quad + \tfrac{i}{12\nu\gamma^2}\left(\tfrac{1}{30}x_3^6 - \tfrac{1}{10}x_3^5 + \tfrac{1}{12}x_3^4 - \tfrac{1}{60}x_3\right) + \tfrac{i}{2\nu}(-x_3^2 + x_3), \\
w_1^{(1),2}(x_3) &= 0, \\
w_1^{(1),3}(x_3) &= \tfrac{i}{\gamma^2}\left(-\tfrac{1}{3}x_3^3 + \tfrac{1}{2}x_3^2 - \tfrac{1}{6}x_3\right).
\end{aligned}$$

Proof. We set $f = {}^\top(f^0, f^1, f^2, f^3) = (I - \hat{\Pi}^{(0)})\hat{L}_1^{(1)}u^{(0)}$. Then $\hat{S}\hat{L}_1^{(1)}u^{(0)}$ is a unique solution u of

$$\hat{L}_0 u = f, \quad \langle\phi\rangle = 0,$$

namely, $u = {}^\top(\phi, w^1, w^2, w^3)$ is a solution of

$$\gamma^2\partial_{x_3}w^3 = f^0, \tag{6.6}$$

$$-\frac{\nu}{\gamma^2}\phi - \nu\partial_{x_3}^2 w^1 + (\partial_{x_3}v_s^1)w^3 = f^1, \tag{6.7}$$

$$-\nu\partial_{x_3}^2 w^2 = f^2, \tag{6.8}$$

$$\partial_{x_3}\phi - (\nu + \tilde{\nu})\partial_{x_3}^2 w^3 = f^3, \tag{6.9}$$

$$w|_{x_3=0,1} = 0, \tag{6.10}$$

$$\langle \phi \rangle = 0. \tag{6.11}$$

To solve (6.6)–(6.11), let us first compute f. Since

$$\hat{\Pi}^{(0)}\hat{L}_1^{(1)}u^{(0)} = \langle \hat{L}_1^{(1)}u^{(0)}, u^{(0)*}\rangle u^{(0)} = -\lambda_1^{(1)}u^{(0)} = \frac{i}{6}u^{(0)},$$

we have

$$\begin{aligned} f &= \hat{L}_1^{(1)}u^{(0)} - \tfrac{i}{6}u^{(0)} \\ &= i\begin{pmatrix} v_s^1 + \gamma^2 w^{(0),1} - \frac{1}{6}\phi^{(0)} \\ 1 + v_s^1 w^{(0),1} - \frac{1}{6}w^{(0),1} \\ 0 \\ -\tilde{\nu}\partial_{x_3}w^{(0),1} \end{pmatrix} \\ &= i\begin{pmatrix} -x_3^2 + x_3 - \frac{1}{6} \\ \frac{1}{12\gamma^2}(3x_3^4 - 6x_3^3 + 4x_3^2 - x_3) + 1 \\ 0 \\ \frac{\tilde{\nu}}{2\gamma^2}(2x_3 - 1) \end{pmatrix}. \end{aligned} \tag{6.12}$$

<u>Computation of w^2</u>: It follows from (6.8), (6.10) and (6.12) that $w^2 = 0$.

<u>Computation of w^3</u>: Integrating (6.6), we have

$$w^3(x_3) = \int_0^{x_3} f^0(y)\,dy = \frac{i}{\gamma^2}\left(-\frac{1}{3}x_3^3 + \frac{1}{2}x_3^2 - \frac{1}{6}x_3\right). \tag{6.13}$$

Note that this w^3 also satisfies (6.10) since $\langle f^0\rangle = 0$.

<u>Computation of ϕ</u>: We see from (6.9)–(6.11), (6.12) and (6.13) that

$$\phi(x_3) = i\left(\frac{\nu}{\gamma^2} + \frac{\tilde{\nu}}{2\gamma^2}\right)\left(-x_3^2 + x_3 - \frac{1}{6}\right). \tag{6.14}$$

<u>Computation of w^1</u>: From (6.7), we have

$$\partial_{x_3}^2 w^1 = -\frac{1}{\gamma^2}\phi + \frac{1}{\nu}(\partial_{x_3}v_s^1)w^3 - \frac{1}{\nu}f^1,$$

which, together with (6.12)–(6.14), gives

$$\begin{aligned} w^1(x_3) &= c_0 + c_1x_3 - i\left(\tfrac{\nu}{\gamma^4} + \tfrac{\tilde{\nu}}{2\gamma^4}\right)\left(-\tfrac{1}{12}x_3^4 + \tfrac{1}{6}x_3^3 - \tfrac{1}{12}x_3^2\right) \\ &\quad + \tfrac{i}{12\nu\gamma^2}\left(\tfrac{1}{30}x_3^6 - \tfrac{1}{10}x_3^5 + \tfrac{1}{12}x_3^4\right) - \tfrac{i}{2\nu}x_3^2. \end{aligned}$$

Here c_0 and c_1 are some constants. Since $w^1(0) = w^1(1) = 0$, we see that $c_0 = 0$ and $c_1 = -\frac{i}{12\nu\gamma^2} \cdot \frac{1}{60} + \frac{i}{2\nu}$. We thus find that

$$\begin{aligned} w^1(x_3) &= -i\left(\tfrac{\nu}{\gamma^4} + \tfrac{\tilde{\nu}}{2\gamma^4}\right)\left(-\tfrac{1}{12}x_3^4 + \tfrac{1}{6}x_3^3 - \tfrac{1}{12}x_3^2\right) \\ &\quad + \tfrac{i}{12\nu\gamma^2}\left(\tfrac{1}{30}x_3^6 - \tfrac{1}{10}x_3^5 + \tfrac{1}{12}x_3^4 - \tfrac{1}{60}x_3\right) + \tfrac{i}{2\nu}(-x_3^2 + x_3). \end{aligned} \tag{6.15}$$

This completes the proof. □

Proposition 6.6. $\lambda_{j1}^{(2)} = \lambda_{1j}^{(2)} = \kappa_0\delta_{j1}$, $j = 1, 2$. *Here* κ_0 *is given by*

$$\kappa_0 = \frac{1}{12\nu}\left(\frac{1}{280} - \gamma^2 - \frac{\nu^2}{15\gamma^2} - \frac{\nu\tilde{\nu}}{30\gamma^2}\right).$$

Proof. Since $w_1^{(1),2} = 0$ by Proposition 6.5, we have $\hat{L}_2^{(1)}\hat{S}\hat{L}_1^{(1)}u^{(0)} = {}^\top(0, *, *, *)$. It then follows that $\langle \hat{L}_2^{(1)}\hat{S}\hat{L}_1^{(1)}u^{(0)}, u^{(0)*}\rangle = 0$. This, together with (6.3) and (6.4), implies $\lambda_{21}^{(2)} = \lambda_{12}^{(2)} = 0$.

We next compute $\lambda_{11}^{(2)}$. Since

$$\begin{aligned} v_s^1\phi_1^{(1)} + \gamma^2 w_1^{(1),1} &= \tfrac{i}{12}\left(\tfrac{\nu}{\gamma^2} + \tfrac{\tilde{\nu}}{2\gamma^2}\right)(7x_3^4 - 14x_3^3 + 8x_3^2 - x_3) \\ &\quad + \tfrac{i}{12\nu}\left(\tfrac{1}{30}x_3^6 - \tfrac{1}{10}x_3^5 + \tfrac{1}{12}x_3^4 - \tfrac{1}{60}x_3\right) + \tfrac{i\gamma^2}{2\nu}(-x_3^2 + x_3), \end{aligned}$$

we have

$$\begin{aligned} \lambda_{11}^{(2)} &= \langle \hat{L}_1^{(1)}\hat{S}\hat{L}_1^{(1)}u^{(0)}, u^{(0)*}\rangle \\ &= i\langle v_s^1\phi_1^{(1)} + \gamma^2 w_1^{(1),1}\rangle \\ &= -\tfrac{1}{12}\left(\tfrac{\nu}{\gamma^2} + \tfrac{\tilde{\nu}}{2\gamma^2}\right)\cdot\tfrac{1}{15} + \tfrac{1}{12\nu}\cdot\tfrac{1}{280} - \tfrac{\gamma^2}{2\nu}\cdot\tfrac{1}{6} = \kappa_0. \end{aligned}$$

This completes the proof. □

The proof of Theorem 6.2 is now completed in view of Propositions 6.3, 6.4 and 6.6. □

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