Spectural properties of the linearized semigroup of the compressible Navier-Stokes equation on a periodic layer

Kagei, Yoshiyuki
Faculty of Mathematics, Kyushu University

Makio, Naoki
Simplex Inc.

出版情報：MI Preprint Series. 2014-09, 2014-09. 九州大学大学院数理学研究院
バージョン：
権利関係：

MI Preprint Series

Mathematics for Industry
Kyushu University

Spectral properties of the linearized semigroup of the compressible Navier-Stokes equation on a periodic layer

Yoshiyuki Kagei
& Naoki Makio

MI 2014-9

(Received September 1, 2014)

Institute of Mathematics for Industry
Graduate School of Mathematics
Kyushu University
Fukuoka, JAPAN

Spectral properties of the linearized semigroup of the compressible Navier-Stokes equation on a periodic layer

Yoshiyuki Kagei[1] and Naoki Makio[2]

[1] Faculty of Mathematics,
Kyushu University,
Nishi-ku, Motooka 744,
Fukuoka 819-0395, Japan

[2] Simplex Inc.
Toranomon Hills Mori Tower,
1-23-1 Toranomon, Minato-ku,
Tokyo 105-6319, Japan

Abstract

The linearized problem for the compressible Navier-Stokes equation around a given constant state is considered in a periodic layer of $\mathbb{R}^n$ with $n \geq 2$, and spectral properties of the linearized semigroup is investigated. It is shown that the linearized operator generates a C_0-semigroup in L^2 over the periodic layer and the time-asymptotic leading part of the semigroup is given by a C_0-semigroup generated by an $n-1$ dimensional elliptic operator with constant coefficients that are determined by solutions of a Stokes system over the basic period domain.

1 Introduction

This paper is concerned with the initial boundary value problem for the following compressible Navier-Stokes equation in a periodic layer Ω:

$$\begin{cases} \partial_t \rho + \mathrm{div}\,(\rho u) = 0, \\ \rho(\partial_t v + v \cdot \nabla v) - \mu \Delta v - (\mu + \mu')\nabla \mathrm{div}\, v + \nabla(P(\rho)) = 0, \\ v|_{\partial\Omega} = 0, \\ (\rho, v)|_{t=0} = (\rho_0, v_0). \end{cases} \tag{1.1}$$

Here $\rho = \rho(x,t)$ and $v = {}^\top(v^1(x,t), \cdots, v^n(x,t))$ denote the unknown density and velocity, respectively, at time t and position x; Ω is a periodic layer defined by:

$$\Omega := \left\{ x = (x', x_n)\,;\, x' \in \mathbb{R}^{n-1},\, \omega_1(x') < x_n < \omega_2(x') \right\},$$

where ω_1 and ω_2 are nonconstant and smooth functions of x' satisfying the periodicity conditions $\omega_j(x' + \frac{2\pi}{\alpha_k}\boldsymbol{e}'_k) = \omega_j(x')$ $(j = 1, 2\ ;\ k = 1, \cdots, n-1)$ with constants $\alpha_k > 0$ and $\boldsymbol{e}'_k = {}^\top(0, \cdots, \overset{k}{1}, \cdots, 0) \in \mathbb{R}^{n-1}$; μ and μ' are the viscosity coefficients that are constants satisfying

$$\mu > 0,\ \frac{2}{n}\mu + \mu' \geq 0;$$

P is the pressure for which we assume that P is a smooth function of ρ that satisfies

$$P'(\rho_*) > 0$$

for a given positive constant ρ_*. Here and in what follows, ${}^\top\cdot$ stands for the transposition.

We are interested in the large time behavior of solutions to (1.1) around the constant equilibrium $u_s = {}^\top(\rho_*, 0)$. To establish a detailed asymptotic description of large time behavior, we here study spectral properties of the linearized semigroup for (1.1) around u_s as a first step of the analysis.

The system of equations for the perturbation is written as

$$\begin{cases} \partial_t\phi + \gamma \mathrm{div}\, w = f^0, \\ \partial_t w - \nu\Delta w - \tilde{\nu}\nabla\mathrm{div}\, w + \gamma\nabla\phi = f, \\ w|_{\partial\Omega} = 0, \\ u|_{t=0} = u_0 = (\phi_0, w_0). \end{cases} \tag{1.2}$$

Here $u = {}^\top(\phi, w)$ with $\phi = \frac{1}{\rho_*}(\rho - \rho_*)$ and $w = \frac{1}{\gamma}v$ denotes the (scaled) perturbation from $u_s = {}^\top(\rho_*, 0)$; ν, $\tilde{\nu}$ and γ are parameters given by

$$\nu = \frac{\mu}{\rho_*}, \qquad \tilde{\nu} = \frac{\mu + \mu'}{\rho_*}, \qquad \gamma = \sqrt{P'(\rho_*)};$$

and f^0 and f denote the nonlinearities

$$f^0 = -\gamma\mathrm{div}\,(\phi w),$$

$$f = \frac{\gamma\phi}{\rho_*(1+\phi)}\left\{\nu\Delta w + \tilde{\nu}\nabla\mathrm{div}\, w\right\} - \gamma^2 w\cdot\nabla w - \left\{\frac{1}{\rho_*(1+\phi)}\nabla(P(\rho_*(1+\phi))) - \frac{P'(\rho_*)}{\rho_*}\nabla\phi\right\}.$$

Large time behavior of solutions to the compressible Navier-Stokes equations has been extensively studied since the pioneering works by Matsumura-Nishida [16, 17, 18]. See, e.g., [5, 10, 11, 13, 14, 15, 20] and references therein. In [7, 8, 9], the stability of u_s was studied when the underlying domain is an n dimensional infinite layer:

$$\mathbb{R}^{n-1} \times (0, 1) = \{x = (x', x_n)\ ;\ x' = (x_1, \cdots, x_{n-1}) \in \mathbb{R}^{n-1},\ 0 < x_n < 1\}.$$

It was proved that u_s is stable under sufficiently small initial perturbations and the L^2 norm of the perturbation decays in the order $t^{-\frac{n-1}{4}}$ as $t \to \infty$. Furthermore, it was shown that the perturbation behaves like a solution of an $n-1$ dimensional heat equation. In this paper we extend the results on the asymptotic behavior of the linearized semigroup for (1.2) obtained in [7, 8] to the case of the periodic layer Ω. We will prove that the linearized semigroup behaves as $t \to \infty$ like a semigroup generated by an $n-1$ dimensional elliptic operator with constant coefficients. More precisely, we consider the linear problem

$$\partial_t u + Lu = 0, \quad u|_{t=0} = u_0, \tag{1.3}$$

where $u = {}^\top(\phi, w)$ is the unknown; $u_0 = {}^\top(\phi_0, w_0)$ is a given initial datum; and L is the operator of the form

$$L = \begin{pmatrix} 0 & \gamma \mathrm{div} \\ \gamma \nabla & -\nu\Delta - \tilde{\nu}\nabla \mathrm{div} \end{pmatrix}.$$

It is shown that $-L$ generates a contraction C_0-semigroup e^{-tL} on $L^2(\Omega)$ and e^{-tL} is decomposed as

$$e^{-tL} = e^{-tL}\Pi + e^{-tL}(I - \Pi).$$

Here Π is a bounded projection on $L^2(\Omega)$; and it holds that

$$\|e^{-tL}\Pi u_0\|_{L^2(\Omega)} \le C(1+t)^{-\frac{n-1}{4}}\|u_0\|_{L^1(\Omega)},$$

$$\|e^{-tL}(I-\Pi)u_0\|_{L^2(\Omega)} \le Ce^{-\beta t}\|u_0\|_{L^2(\Omega)}$$

and

$$\|e^{-tL}\Pi u_0 - [e^{-tH}\sigma_0]u^{(0)}\|_{L^2(\Omega)} \le Ct^{-\frac{n-1}{4}-\frac{1}{2}}\|u_0\|_{L^1(\Omega)}, \tag{1.4}$$

where β is a positive constant; and e^{-tH} is a C_0-semigroup in $L^2(\mathbb{R}^{n-1})$ generated by the operator $-H$:

$$H\sigma = -\frac{\gamma^2}{\nu}\sum_{i,j=1}^{n-1} a_{ij}\partial_{x_i}\partial_{x_j}\sigma.$$

Here (a_{ij}) is a positive definite symmetric matrix with constant components; and σ_0 and $u^{(0)}$ are given by

$$\sigma_0 = \frac{|Q|}{|\Omega_{per}|}\int_{\omega_1(x')}^{\omega_2(x')} \phi_0(x', x_n)dx_n, \quad u^{(0)} = {}^\top(1, 0),$$

where Ω_{per} is the basic period domain given by

$$\Omega_{per} = \{x = (x', x_n)\,;\, x' \in Q,\, \omega_1(x') < x_n < \omega_2(x')\}$$

with the basic period cell $Q = \prod_{j=1}^{n-1}[-\frac{\pi}{\alpha_j}, \frac{\pi}{\alpha_j})$. Here and in what follows, for a bounded domain D, $|D|$ denotes the volume of D. We note that the matrix (a_{ij}) is given by

$$a_{ij} = \frac{1}{|\Omega_{per}|}(\nabla w^{(i)}, \nabla w^{(j)})_{L^2(\Omega_{per})},$$

where $w^{(k)}$ ($k = 1, \cdots, n-1$) are functions Q-periodic in x' satisfying the following Stokes system:

$$\begin{cases} \operatorname{div} w^{(k)} = 0, \\ -\Delta w^{(k)} + \nabla \phi^{(k)} = \boldsymbol{e}_k, \\ w^{(k)}|_{x_n = \omega_1(x'), \omega_2(x')} = 0 \end{cases}$$

for some $\phi^{(k)} = \phi^{(k)}(x', x_n)$ being Q-periodic in x'. Here $\boldsymbol{e}_k = {}^\top(0, \cdots, \overset{k}{1}, \cdots, 0) \in \mathbb{R}^n$.

We will prove our results as follows. In the case of infinite layers analyzed in [7, 8, 9], the spectral properties of the linearized semigroup were investigated by using the Fourier transform in $x' \in \mathbb{R}^{n-1}$. In the case of the periodic layer Ω, the Fourier transform does not work well any longer, instead, we employ the Bloch wave decomposition which transforms the linearized problem (1.3) on Ω to the problem $\partial_t u + L_{\eta'} u = 0$ on Ω_{per} under Q-periodic boundary conditions in x'. Here $L_{\eta'}$ is the linear operator obtained by replacing the partial derivatives ∂_{x_j} ($j = 1, \cdots, n-1$) in L by $\partial_{x_j} + i\eta_j$ with parameter $\eta' = (\eta_1, \cdots, \eta_{n-1}) \in Q^*$, where Q^* is the dual cell defined by $Q^* = \prod_{j=1}^{n-1} [-\frac{\alpha_j}{2}, \frac{\alpha_j}{2})$.

When $|\eta'| \ll 1$, the operator $L_{\eta'}$ can be regarded as a perturbation of L_0; and the analytic perturbation theory is applied to show that

$$\rho(-L_{\eta'}) \supset \{\operatorname{Re} \lambda > -\beta_0\} \setminus \{\lambda_{\eta'}\} \text{ for some } \beta_0 > 0,$$

$$\sigma(-L_{\eta'}) \cap \{|\lambda| < \frac{\beta_0}{2}\} = \{\lambda_{\eta'}\},$$

where

$$\lambda_{\eta'} = -\frac{\gamma^2}{\nu} \sum_{i,j=1}^{n-1} a_{ij} \eta_i \eta_j + O(|\eta'|^3)$$

as $\eta' \to 0$. It then follows that this part of e^{-tL} behaves as in (1.4). As for the remaining part of η', we establish some estimates for a modified Stokes system (see section 4.3); and based on the established estimates we prove by an energy method that if $|\eta'| \geq r_0$ ($\eta' \in Q^*$), then

$$\rho(-L_{\eta'}) \supset \{\operatorname{Re} \lambda \geq -\beta_1\} \text{ for some } \beta_1 > 0,$$

and hence, this part of e^{-tL} decays exponentially. We note that we consider the linearized operator L as an operator on L^2 as in [6], which is in contrast to [7, 8] where the underlying space for the linearized operator is $H^1 \times L^2$. The L^2 setting will be useful for the stability analysis of stationary flows with nonzero velocity fields.

This paper is organized as follows. In section 2 we introduce some notations, function spaces and state some properties of the Bloch wave decomposition. In section 3 we state the main result of this paper. The proof of the main result is given in sections 4–5. In section 6 we give an outline of the proof of a lemma used in section 4.3.

2 Preliminaries

In this section we introduce the notations, function spaces and operators which will be used in this paper.

For a domain D and $1 \leq p \leq \infty$, the Lebesgue space over D is denoted by $L^p(D)$ and its norm is denoted by $\|\cdot\|_{L^p(D)}$. The symbol $W^{l,p}(D)$ stands for the l th order L^p Sobolev space and its norm is denoted by $\|\cdot\|_{W^{l,p}(D)}$. When $p = 2$, we denote $W^{l,2}(D)$ by $H^l(D)$ and its norm is denoted by$\|\cdot\|_{H^l(D)}$. We denote by $C_0^l(D)$ the set of all C^l function whose support is compact in D. The completion of $C_0^l(D)$ in $W^{l,p}(D)$ is denoted by $W_0^{l,p}(D)$. In particular, we write $W_0^{l,2}(D)$ as $H_0^l(D)$.

We simply denote by $L^p(D)$ the set of all vector fields $W = {}^\top(w^1, \cdots, w^n)$ on D whose components w^j $(j = 1, \cdots, n)$ belong to $L^p(D)$ and the norm is also denoted by $\|\cdot\|_{L^p(D)}$ if no confusion will occur. Similarly, the symbols $W^{l,p}(D)$ and $H^l(D)$ are used for vector fields.

For $u = {}^\top(\phi, w)$ with $\phi \in W^{k,p}(D)$ and $w = {}^\top(w^1, \cdots, w^n) \in W^{l,q}(D)$, we define the norm $\|u\|_{W^{k,p}(D)\times W^{l,q}(D)}$ by

$$\|u\|_{W^{k,p}(D)\times W^{l,q}(D)} = \|\phi\|_{W^{k,p}(D)} + \|w\|_{W^{l,q}(D)}.$$

We define the sets Q, Q^*, Ω_{per}, $\Sigma_{j,\pm}$ $(j = 1, \cdots, n-1)$ and Σ_n as follows:

$$Q := \textstyle\prod_{i=1}^{n-1}\left[-\frac{\pi}{\alpha_i}, \frac{\pi}{\alpha_i}\right), \quad Q^* := \prod_{i=1}^{n-1}\left[-\frac{\alpha_i}{2}, \frac{\alpha_i}{2}\right),$$

$$\Omega_{per} := \{x = (x', x_n)\, ;\, x' \in Q,\, \omega_1(x') < x_n < \omega_2(x')\},$$

$$\Sigma_{j,\pm} := \left\{x \in \overline{\Omega_{per}}\, ;\, x_j = \pm\tfrac{\pi}{\alpha_j}\right\},$$

$$\Sigma_n := \{x \in \partial\Omega\ ;\ x' \in Q,\, x_n = \omega_j(x')\ j = 1, 2\}.$$

In the case $D = \Omega_{per}$, we simply write $L^p(\Omega_{per})$ as L^p, and likewise, $W^{k,p}(\Omega_{per})$, $H^l(\Omega_{per})$ as $W^{k,p}$, H^l, respectively. Similarly, the norms are also abbreviated to $\|\cdot\|_{H^l}$, $\|\cdot\|_{W^{k,p}}$, and, in particular, we write $\|\cdot\|_{L^p(\Omega_{per})}$ as $\|\cdot\|_p$.

The inner product of $L^2(D)$ is defined by

$$(f, g)_{L^2(D)} = \int_D f(x)\overline{g(x)}dx, \quad f, g \in L^2(D).$$

When $D = \Omega_{per}$, we abbreviate it to (f, g). The dual space of $H_0^1(D)$ is denoted by $H^{-1}(D)$, and the pairing between $H^{-1}(D)$ and $H_0^1(D)$ is written as $[\cdot, \cdot]$. For $f \in L^2(\Omega_{per})$, its mean value over Ω_{per} is denoted by $< f >$, i.e.,

$$< f >= (f, 1) = \frac{1}{|\Omega_{per}|}\int_{\Omega_{per}} f(x)dx.$$

We often write $x \in \Omega$ as $x = {}^\top(x', x_n)$, $x' = {}^\top(x_1, \cdots, x_{n-1}) \in \mathbb{R}^{n-1}$. The partial derivatives of a function u are denoted by ∂_{x_j}, $\partial_{x_j}\partial_{x_k}$, and so on.

We will work in spaces of functions Q-periodic in x', and so, we introduce the function spaces $L^2_{per}(\Omega_{per})$, $C^\infty_{per}(\overline{\Omega_{per}})$, $C^\infty_{0,per}(\Omega_{per})$, $H^l_{per}(\Omega_{per})$, $H^l_{0,per}(\Omega_{per})$ that are defined by

$$L^2_{per}(\Omega_{per}) = \{u|_{\Omega_{per}}\, ;\, u \in L^2_{loc}(\overline{\Omega}),\, u(x' + \tfrac{2\pi}{\alpha_j}\boldsymbol{e}'_j, x_n) = u(x', x_n),\, (x', x_n) \in \Omega,\, 1 \leq j \leq n-1\},$$

$$C^\infty_{per}(\overline{\Omega_{per}}) = \{u|_{\overline{\Omega_{per}}}\,;\; u \in C^\infty(\overline{\Omega}),\; u(x'+\tfrac{2\pi}{\alpha_j}\boldsymbol{e}'_j, x_n) = u(x', x_n),\; (x', x_n) \in \Omega,\; 1 \le j \le n-1\},$$

$$C^\infty_{0,per}(\Omega_{per}) = \{u \in C^\infty_{per}(\overline{\Omega_{per}})\,;\; u = 0 \,\text{in a neighborhood of}\, \partial\Omega\},$$

$$H^l_{per}(\Omega_{per}) = \text{the closure of } C^\infty_{per}(\overline{\Omega_{per}}) \text{ in } H^l(\Omega_{per}),$$

$$H^l_{0,per}(\Omega_{per}) = \text{the closure of } C^\infty_{0,per}(\Omega_{per}) \text{ in } H^l(\Omega_{per}).$$

Observe that $L^2_{per}(\Omega_{per})$ can be identified with $L^2(\Omega_{per})$, and that

$$H^l_{per}(\Omega_{per}) = \{u \in H^l(\Omega_{per})\,;\; \partial^\beta_{x'} u|_{\Sigma_{j,-}} = \partial^\beta_{x'} u|_{\Sigma_{j,+}}\, 1 \le j \le n-1,\; |\beta| \le l-1\},$$

$$H^1_{0,per}(\Omega_{per}) = \{u \in H^1_{per}(\Omega_{per})\,;\; u|_{\Sigma_n} = 0\}.$$

We also set

$$L^2_{*,per}(\Omega_{per}) = \{f \in L^2_{per}(\Omega_{per})\,;\; < f >= 0\}$$

and

$$H^l_{*,per}(\Omega_{per}) = H^l_{per}(\Omega_{per}) \cap L^2_{*,per}(\Omega_{per}).$$

For $\eta' \in \mathbb{R}^{n-1}$ we denote

$$\tilde{\eta}' = {}^\top(\eta', 0) \in \mathbb{R}^n,$$

and $\nabla_{\eta'}$ is defined by

$$\nabla_{\eta'} = \nabla + i\tilde{\eta}'.$$

We further introduce the following notations

$$\nabla'_{\eta'} = \nabla' + i\eta', \quad \Delta_{\eta'} = \nabla_{\eta'} \cdot \nabla_{\eta'}, \quad \mathrm{div}\,_{\eta'} w = \nabla_{\eta'} \cdot w - i < \tilde{\eta}' \cdot w > .$$

Here ∇' denotes the gradient with respect to $x' = (x_1, \cdots, x_{n-1}) \in \mathbb{R}^{n-1}$. We note that $\Delta_{\eta'} = \nabla'_{\eta'} \cdot \nabla'_{\eta'} + \partial^2_{x_n}$.

We next introduce some operators. We denote by $\mathbb{P}_0$ and $\tilde{\mathbb{P}}$ the following $(n+1) \times (n+1)$ diagonal matrices:

$$\mathbb{P}_0 = \mathrm{diag}\,(1, 0, \cdots, 0), \quad \tilde{\mathbb{P}} = \mathrm{diag}\,(0, 1, \cdots, 1).$$

Note that $\mathbb{P}_0\, u = {}^\top(\phi, 0)$ and $\tilde{\mathbb{P}}\, u = {}^\top(0, w)$ for $u = {}^\top(\phi, w)$ with $w = {}^\top(w_1, \cdots, w_n)$.

We denote the kernel and range of an operator A by $\mathrm{Ker}\, A$ and $R(A)$, respectively.

For a function $f = f(x')$ $(x' \in \mathbb{R}^{n-1})$, we denote its Fourier transform by $\hat{f}$ or $\mathcal{F}[f]$:

$$\hat{f}(\xi') = \mathcal{F}[f](\xi') = \int_{\mathbb{R}^{n-1}} f(x') e^{-i\xi' \cdot x'} dx' \quad (\xi' \in \mathbb{R}^{n-1}).$$

The inverse Fourier transform $\mathcal{F}^{-1}$ is defined by

$$\mathcal{F}^{-1}[f](x') = (2\pi)^{-(n-1)} \int_{\mathbb{R}^{n-1}} f(\xi') e^{i\xi' \cdot x'} d\xi' \quad (x' \in \mathbb{R}^{n-1}).$$

We next introduce the Bloch wave decomposition. Let $\mathcal{S}(\mathbb{R}^{n-1})$ denote the Schwartz space on $\mathbb{R}^{n-1}$.

Definition 2.1. *We define the operator T by $(T\varphi)(x', \eta')$ $(x' \in \mathbb{R}^{n-1},\ \eta' \in \mathbb{R}^{n-1})$ for $\varphi \in \mathcal{S}(\mathbb{R}^{n-1})$, where*

$$\begin{aligned}
&(T\varphi)(x', \eta') \\
&= \frac{1}{(2\pi)^{\frac{n-1}{2}}|Q|^{\frac{1}{2}}} \sum_{(k_1,\cdots,k_{n-1})\in\mathbb{Z}^{n-1}} \hat{\varphi}(\eta' + \sum_{j=1}^{n-1} k_j\alpha_j \boldsymbol{e}'_j)\, e^{i\sum_{j=1}^{n-1} k_j\alpha_j x_j} \\
&= \frac{1}{|Q^*|^{\frac{1}{2}}} \sum_{(l_1,\cdots,l_{n-1})\in\mathbb{Z}^{n-1}} \varphi(x' + \sum_{j=1}^{n-1} l_j \frac{2\pi}{\alpha_j}\boldsymbol{e}'_j)\, e^{-i\eta'\cdot(x' + \sum_{j=1}^{n-1} l_j \frac{2\pi}{\alpha_j}\boldsymbol{e}'_j)}.
\end{aligned} \tag{2.1}$$

We also define the operator U as follows. For a function $\varphi(x', \eta') \in C^\infty(\mathbb{R}^{n-1}\times\mathbb{R}^{n-1})$ such that $\varphi(x', \eta')$ is Q-periodic in x' and $\varphi(x', \eta')e^{i\eta'\cdot x'}$ is Q^*-periodic in η', we define $(U\varphi)(x')$ $(x' \in \mathbb{R}^{n-1})$ by

$$(U\varphi)(x') = \frac{1}{|Q^*|^{\frac{1}{2}}} \int_{Q^*} \varphi(\eta', x')e^{i\eta'\cdot x'} d\eta'. \tag{2.2}$$

Note that $\varphi(x, \eta' + \alpha_j\boldsymbol{e}'_j) = \varphi(x', \eta')e^{-i\alpha_j\boldsymbol{e}'_j\cdot x'}$ $(j = 1, \cdots, n-1)$.

The operators T and U have the following properties. See, e.g., [21] for the details.

Proposition 2.2. (i) *$(T\varphi)(x', \eta')$ is Q-periodic in x' and $(T\varphi)(x', \eta')e^{i\eta'\cdot x'}$ is Q^*-periodic in η'.*

(ii) *T is uniquely extended to an isometric operator from $L^2(\mathbb{R}^{n-1})$ to $L^2(Q^*; L^2(Q))$.*

(iii) *U is the inverse operator of T.*

(iv) *Let ψ be Q-periodic in x'. Then it holds that $T(\psi\varphi) = \psi T(\varphi)$.*

(v) *$T(\partial_{x_j}\varphi) = (\partial_{x_j} + i\eta_j)T\varphi$ $(j = 1, \cdots, n-1)$ and T defines an isomorphism from $H^l(\mathbb{R}^{n-1})$ to $L^2(Q^*; H^l_{per}(Q))$. (Here $H^l_{per}(Q)$ denotes the space of Q-periodic functions belonging to $H^l(Q)$, as in the case of $H^l_{per}(\Omega_{per})$.)*

We next consider T as an operator acting on functions in $H^l(\Omega)$. Let $y = \Phi(x)$ be the following transformation

$$\begin{cases} y' = x', \\ y_n = \frac{1}{\omega_2(x') - \omega_1(x')}(x_n - \omega_1(x')). \end{cases}$$

Then Φ is a diffeomorphism from Ω to $\mathbb{R}^{n-1} \times (0, 1)$ and Φ transforms Q-periodic functions on Ω to those on $\mathbb{R}^{n-1} \times (0, 1)$. We denote the inverse transform of Φ by Ψ and we define the operators Φ^* and Ψ^* by $[\Phi^*u](x) = u(\Phi(x))$ and $[\Psi^*u](y) = u(\Psi(y))$, respectively. Then Φ^* is an isomorphism from $H^l(\Omega)$ to $H^l(\mathbb{R}^{n-1} \times (0, 1))$, and likewise, from $H^l_{per}(\Omega_{per})$ to $H^l_{per}(Q \times (0, 1))$, where $H^l_{per}(Q \times (0, 1))$ denotes the space of Q-periodic functions belonging to $H^l(Q \times (0, 1))$.

It is not difficult to see that Proposition 2.2 holds with $H^l(\mathbb{R}^{n-1})$ replaced by $H^l(\mathbb{R}^{n-1}\times(0,1))$, and likewise, with $H^l_{per}(Q)$ replaced by $H^l_{per}(Q\times(0,1))$. It then follows that $\Phi^*T\Psi^*$ is an isomorphism from $H^l(\Omega)$ to $L^2(Q^*;H^l_{per}(\Omega_{per}))$. Using the second expression of T in Definition 2.1 and the periodicity of ω_j $(j=1,2)$, one can see that $\Phi^*T\Psi^*u = Tu$ for functions u on Ω. Therefore, we will write $\Phi^*T\Psi^*u$ as Tu if no confusion will occur.

3 Main results

In this section we state the main results of this paper.

Let us consider the following linear problem

$$\partial_t u + Lu = 0, \quad u = {}^\top(\phi, w). \tag{3.1}$$

Here L is the operator on $L^2(\Omega)$ given by

$$L = \begin{pmatrix} 0 & \gamma\mathrm{div} \\ \gamma\nabla & -\nu\Delta - \tilde{\nu}\nabla\mathrm{div} \end{pmatrix} \tag{3.2}$$

with domain

$$D(L) = \left\{ u = {}^\top(\phi, w) \in L^2(\Omega); w \in H^1_0(\Omega),\ Lu \in L^2(\Omega) \right\}. \tag{3.3}$$

Our main issue is to investigate the spectral properties of the semigroup generated by $-L$. We first state that it generates a contraction semigroup.

Theorem 3.1. *The operator $-L$ generates a contraction C_0-semigroup e^{-tL} on $L^2(\Omega)$ and it holds that*

$$\|e^{-tL}u_0\|_{L^2(\Omega)} \le \|u_0\|_{L^2(\Omega)} \ (u_0 \in L^2(\Omega)).$$

The semigroup e^{-tL} has the following properties.

Theorem 3.2. *There is a bounded projection $\Pi : L^2(\Omega) \to L^2(\Omega)$ satisfying $\Pi L \subset L\Pi$ and $\Pi e^{-tL} = e^{-tL}\Pi$, and there hold the following estimates uniformly for $t > 0$ and $u_0 \in L^1(\Omega) \cap L^2(\Omega)$:*

(i) $\|e^{-tL}\Pi u_0\|_{L^2(\Omega)} \le C(1+t)^{-\frac{n-1}{4}}\|u_0\|_{L^1(\Omega)}$,

(ii) $\|e^{-tL}(I-\Pi)u_0\|_{L^2(\Omega)} \le Ce^{-\beta t}\|u_0\|_{L^2(\Omega)}$,

(iii) $\|e^{-tL}\Pi u_0 - [e^{-tH}\sigma_0]u^{(0)}\|_{L^2(\Omega)} \le Ct^{-\frac{n-1}{4}-\frac{1}{2}}\|u_0\|_{L^1(\Omega)}$.

Here β is a positive constant; e^{-tH} is the C_0-semigroup in $L^2(\mathbb{R}^{n-1})$ generated by the operator $-H$ defined by

$$H\sigma = -\frac{\gamma^2}{\nu}\sum_{i,j=1}^{n-1} a_{ij}\partial_{x_i}\partial_{x_j}\sigma \quad (\sigma \in D(H))$$

with domain $D(H) = H^2(\mathbb{R}^{n-1})$; and σ_0 and $u^{(0)}$ are given as follows:

$$\sigma_0 = \frac{|Q|}{|\Omega_{per}|}\int_{\omega_1(x')}^{\omega_2(x')} \phi_0(x', x_n)dx_n, \quad u^{(0)} = {}^\top(1, 0).$$

Here a_{ij} satisfies that

$$\sum_{i,j=1}^{n-1} a_{ij}\xi_i\xi_j \geq \kappa_0|\xi'|^2 \quad (\xi' = (\xi_1, \cdots, \xi_{n-1}) \in \mathbb{R}^{n-1})$$

with a constant $\kappa_0 > 0$ independent of ξ'. Furthermore, a_{ij} is given as $a_{ij} = (\nabla w^{(i)}, \nabla w^{(j)})_{L^2(\Omega_{per})}$ with ${}^\top(\phi^{(k)}, w^{(k)})$ $(k = 1, \cdots, n-1)$ satisfying the following Stokes system in Ω_{per}:

$$\begin{cases} \operatorname{div} w^{(k)} = 0, \\ -\Delta w^{(k)} + \nabla\phi^{(k)} = \boldsymbol{e}'_k, \\ w^{(k)}|_{\Sigma_{j,+}} = w^{(k)}|_{\Sigma_{j,-}}, \ \phi^{(k)}|_{\Sigma_{j,+}} = \phi^{(k)}|_{\Sigma_{j,-}}, \ w^{(k)}|_{\Sigma_n} = 0, \ <\phi^{(k)}>= 0 \end{cases} \tag{3.4}$$

for some $\phi^{(k)}$.

The proof of Theorem 3.2 will be given in sections 4 and 5. To prove Theorem 3.2, we will consider the resolvent problem $\lambda u + L_{\eta'}u = f$ on $L^2_{per}(\Omega_{per})$ with parameter $\eta' \in Q^*$. In the case of $|\eta'| \leq r_0$ for some small $r_0 > 0$, we regard $L_{\eta'}$ as a perturbation of L_0 and apply the analytic perturbation theory to study the spectrum of $-L_{\eta'}$. For $\eta' \in Q^*$ with $|\eta'| \geq r_0$, we establish estimates for a modified Stokes system and apply an energy method. Based on the analysis for $-L_{\eta'}$, we give a proof of Theorem 3.2.

4 Spectral properties of $L_{\eta'}$

In this section we investigate spectral properties of $L_{\eta'}$.

4.1 Formulation

Let us consider the resolvent problem for (3.1)

$$(\lambda + L)u = f, \quad u \in D(L). \tag{4.1}$$

Here $\lambda \in \mathbb{C}$ is a resolvent parameter.

Applying Ψ^* to (4.1), we have

$$(\lambda + \Psi^* L)\Psi^* u = \Psi^* f \text{ in } \mathbb{R}^{n-1} \times (0, 1). \tag{4.2}$$

Here $\Psi^* L$ is the differential operator of the form

$$\Psi^* L = \begin{pmatrix} 0 & \sum_{j=1}^n l_{12}^j(y', y_n)\partial_{y_j} \\ \sum_{j=1}^n l_{21}^j(y', y_n)\partial_{y_j} & \sum_{j,k=1}^n l_{22}^{j,k}(y', y_n)\partial_{y_j}\partial_{y_k} + \sum_{j=1}^n l_{22}^j(y', y_n)\partial_{y_j} \end{pmatrix}$$

with some l^j_{pq} and l^{jk}_{pq} $(p,q=1,2)$ being Q-periodic in y'. We next apply T to (4.2). It then follows from Proposition 2.2 (i), (iv) and (v) that (4.2) is transformed into the following problem on $Q\times(0,1)$:

$$(\lambda+\Psi^* L_{\eta'})T\Psi^* u = T\Psi^* f \quad (\eta'\in Q^*) \tag{4.3}$$

with Q-periodic boundary condition in y'. Applying Φ^* to (4.3) we arrive at

$$(\lambda+L_{\eta'})Tu = Tf \quad \text{on } \Omega_{per} \tag{4.4}$$

with the dual parameter $\eta'\in Q^*$, where $L_{\eta'}$ is the operator on $L^2_{per}(\Omega_{per})$ of the form

$$L_{\eta'} := \begin{pmatrix} 0 & \gamma^\top\nabla_{\eta'} \\ \gamma\nabla_{\eta'} & -\nu\Delta_{\eta'}-\tilde{\nu}\nabla_{\eta'}{}^\top\nabla_{\eta'} \end{pmatrix}$$

with domain $D(L_{\eta'})$

$$D(L_{\eta'}) = \{u={}^\top(\phi,w)\in L^2_{per}(\Omega_{per})\,;\, L_{\eta'}u\in L^2_{per}(\Omega_{per}),\ w\in H^1_{0,per}(\Omega_{per})\}.$$

It is not difficult to see that $D(L_{\eta'})=D(L_0)$ for all $\eta'\in Q^*$ and that $L_{\eta'}$ is a closed operator on $L^2_{per}(\Omega_{per})$.

If $\lambda\in\rho(-L_{\eta'})$, then, by (4.4), u is written as

$$u = U(\lambda+L_{\eta'})^{-1}Tf.$$

Therefore, to investigate the resolvent of $-L$, we will consider the problem for $-L_{\eta'}$:

$$\lambda u + L_{\eta'}u = f, \quad u\in D(L_0). \tag{4.5}$$

Before going further, we also introduce the adjoint operator of $L_{\eta'}$. We define the operator $L^*_{\eta'}$ by

$$L_{\eta'} := \begin{pmatrix} 0 & -\gamma^\top\nabla_{\eta'} \\ -\gamma\nabla_{\eta'} & -\nu\Delta_{\eta'}-\tilde{\nu}\nabla_{\eta'}{}^\top\nabla_{\eta'} \end{pmatrix}$$

with domain

$$D(L^*_{\eta'}) = \{u={}^\top(\phi,w)\in L^2_{per}(\Omega_{per})\,;\, L^*_{\eta'}u\in L^2_{per}(\Omega_{per}),\ w\in H^1_{0,per}(\Omega_{per})\}.$$

One can see that $D(L^*_{\eta'})=D(L^*_0)$ for all $\eta'\in Q^*$ and that $L^*_{\eta'}$ is the adjoint operator of $L_{\eta'}$.

4.2 The case $|\eta'|\le r_0$

In this subsection we consider (4.5) with $|\eta'|\le r_0$ for some sufficiently small $r_0>0$. It is convenient to write $L_{\eta'}$ in the form

$$L_{\eta'} := L_0 + \sum_{j=1}^{n-1}\eta_j L^{(1)}_j + \sum_{j,k=1}^{n-1}\eta_j\eta_k L^{(2)}_{jk},$$

where

$$L_0 := \begin{pmatrix} 0 & \gamma \mathrm{div} \\ \gamma \nabla & -\nu \Delta - \tilde{\nu} \nabla \mathrm{div} \end{pmatrix},$$

$$L_j^{(1)} := i \begin{pmatrix} 0 & \gamma {}^\top \boldsymbol{e}_j \\ \gamma \boldsymbol{e}_j & -2\nu \partial_{x_j} - \tilde{\nu} \boldsymbol{e}_j \mathrm{div} - \tilde{\nu} \nabla ({}^\top \boldsymbol{e}_j) \end{pmatrix},$$

$$L_{jk}^{(2)} := \begin{pmatrix} 0 & 0 \\ 0 & -\nu \delta_{jk} - \tilde{\nu} \boldsymbol{e}_j^T \boldsymbol{e}_k \end{pmatrix}.$$

We also set

$$M_{\eta'} = \sum_{j=1}^{n-1} \eta_j L_j^{(1)} + \sum_{j,k=1}^{n-1} \eta_j \eta_k L_{jk}^{(2)},$$

namely,

$$M_{\eta'} := \begin{pmatrix} 0 & i\gamma {}^\top \tilde{\eta}' \\ i\gamma \tilde{\eta}' & \nu(|\eta'|^2 - 2i\tilde{\eta}' \cdot \nabla) - i\tilde{\nu}\tilde{\eta}'^\top (\nabla + i\tilde{\eta}') - i\tilde{\nu}\nabla^\top \tilde{\eta}' \end{pmatrix}.$$

Similarly, we write $L_{\eta'}^*$ as

$$L_{\eta'}^* := L_0^* + \sum_{j=1}^{n-1} \eta_j L_j^{(1)*} + \sum_{j,k=1}^{n-1} \eta_j \eta_k L_{jk}^{(2)*},$$

where

$$L_0^* := \begin{pmatrix} 0 & -\gamma \mathrm{div} \\ -\gamma \nabla & -\nu \Delta - \tilde{\nu} \nabla \mathrm{div} \end{pmatrix},$$

$$L_j^{(1)*} := i \begin{pmatrix} 0 & -\gamma {}^\top \boldsymbol{e}_j \\ -\gamma \boldsymbol{e}_j & -2\nu \partial_{x_j} - \tilde{\nu} \boldsymbol{e}_j \mathrm{div} - \tilde{\nu} \nabla ({}^\top \boldsymbol{e}_j) \end{pmatrix},$$

$$L_{jk}^{(2)*} := \begin{pmatrix} 0 & 0 \\ 0 & -\nu \delta_{jk} - \tilde{\nu} \boldsymbol{e}_j^T \boldsymbol{e}_k \end{pmatrix}.$$

We begin with the resolvent estimates for the case $\eta' = 0$ which implies the generation of a contraction semigroup e^{-tL_0}.

In what follows we write

$$X = L^2_{per}(\Omega_{per})$$

for simplicity of notation.

Proposition 4.1. *It holds that* $\{\lambda; \mathrm{Re}\,\lambda > 0\} \subset \rho(-L_0)$, *and if* $\mathrm{Re}\,\lambda > 0$, *then*

$$\|(\lambda + L_0)^{-1} f\|_2 \leq \frac{1}{\mathrm{Re}\,\lambda} \|f\|_2,$$

$$\|\nabla \tilde{\mathbb{P}} (\lambda + L_0)^{-1} f\|_2 \leq \frac{1}{(\nu \mathrm{Re}\,\lambda)^{\frac{1}{2}}} \|f\|_2.$$

The same conclusion also holds for the adjoint operator L_0^*.

Proof. Let $\operatorname{Re}\lambda > 0$. Since

$$\operatorname{Re}((\lambda + L_0)u, u) = \nu\|\nabla w\|_2^2 + \tilde{\nu}\|\operatorname{div} w\|_2^2 + \operatorname{Re}\lambda\|u\|_2^2, \tag{4.6}$$

we see that if $(\lambda + L_0)u = 0$, then $u = 0$, and so, $\lambda + L_0$ is injective when $\operatorname{Re}\lambda > 0$. Observe also that if $\operatorname{Re}\lambda > 0$, then

$$\|u\|_2 \le \frac{1}{\operatorname{Re}\lambda}\|(\lambda + L_0)u\|_2, \tag{4.7}$$

$$\|\nabla w\|_2 \le \frac{1}{(\nu \operatorname{Re}\lambda)^{\frac{1}{2}}}\|(\lambda + L_0)u\|_2. \tag{4.8}$$

It follows from (4.7) that $R(\lambda + L_0)$ is a closed subspace of X. We note that these inequalities also holds with L_0 replaced by L_0^*. Let $v \in R(\lambda + L_0)^\perp$. Then, since $((\lambda+L_0)u, v) = 0$ for all $u \in D(L_0)$, we see that $v \in D(L_0^*)$ and $(\lambda+L_0^*)v = 0$. This, together with (4.7) with L_0 replaced by L_0^*, implies that $v = 0$. We thus conclude that $R(\lambda + L_0) = X$ and, hence, $\lambda + L_0$ is surjective. This completes the proof. □

The following estimates show that $-L_{\eta'}$ also generates a contraction semigroup.

Proposition 4.2. *It holds that* $\{\lambda : \operatorname{Re}\lambda > 0\} \subset \rho(-L_{\eta'})$ *and the following estimates hold for* $\operatorname{Re}\lambda > 0$*:*

$$\|(\lambda + L_{\eta'})^{-1}f\|_2 \le \frac{1}{\operatorname{Re}\lambda}\|f\|_2,$$

$$\|\nabla\tilde{\mathbb{P}}\,(\lambda + L_{\eta'})^{-1}f\|_2 \le \frac{C}{(\nu)^{\frac{1}{2}}}\left(\frac{1}{(\operatorname{Re}\lambda)^{\frac{1}{2}}} + \frac{1}{\operatorname{Re}\lambda}\right)\|f\|_2.$$

The same conclusion also holds for the adjoint operator $L_{\eta'}^*$

Proof. We have

$$\operatorname{Re}((\lambda + L_{\eta'})u, u) = \operatorname{Re}\lambda\|u\|_2^2 + \nu\|\nabla_{\eta'}w\|_2^2 + \tilde{\nu}\|\nabla_{\eta'}\cdot w\|_2^2.$$

It then follows that if $\operatorname{Re}\lambda > 0$, then

$$\|u\|_2 \le \frac{1}{\operatorname{Re}\lambda}\|(\lambda + L_{\eta'})u\|_2.$$

We also have

$$\operatorname{Re}((\lambda + L_{\eta'})u, u) \ge \nu\|\nabla w\|_2^2 + (\operatorname{Re}\lambda - C)\|u\|_2^2$$

for a constant $C > 0$ uniformly for $\eta' \in Q^*$. Therefore, we deduce that

$$\nu\|\nabla w\|_2^2 \le \|(\lambda + L_{\eta'})u\|_2\|u\|_2 + C\|u\|_2^2 \le C\left(\frac{1}{\operatorname{Re}\lambda} + \frac{1}{(\operatorname{Re}\lambda)^2}\right)\|(\lambda + L_{\eta'})u\|_2^2,$$

which gives

$$\|\nabla w\|_2 \le \frac{C}{\nu^{\frac{1}{2}}}\left(\frac{1}{(\operatorname{Re}\lambda)^{\frac{1}{2}}} + \frac{1}{\operatorname{Re}\lambda}\right)\|(\lambda + L_{\eta'})u\|_2.$$

As in the proof of Proposition 4.1, one can now obtain the desired results. This completes the proof. □

We next show that $\lambda = 0$ is a simple eigenvalue of $-L_0$.

Proposition 4.3. *There exists a constant $\beta_0 > 0$ such that $\rho(-L_0) \supset \{\lambda; \mathrm{Re}\,\lambda > -\beta_0,\ \lambda \neq 0\}$. Furthermore, $\lambda = 0$ is a simple eigenvalue of $-L_0$ and it holds that for λ satisfying $\mathrm{Re}\,\lambda > -\beta_0$ and $\lambda \neq 0$,*

$$(\lambda + L_0)^{-1} f = \frac{1}{\lambda}\Pi^{(0)} f + S_\lambda (I - \Pi^{(0)}) f,$$

and there hold the following estimates uniformly for λ satisfying $\mathrm{Re}\,\lambda > -\beta_0$:

$$\|S_\lambda (I - \Pi^{(0)}) f\|_2 \leq \frac{C}{\mathrm{Re}\,\lambda + \beta_0} \|f\|_2,$$

$$\|\nabla \tilde{\mathbb{P}}\, S_\lambda (I - \Pi^{(0)}) f\|_2 \leq \frac{C}{(\mathrm{Re}\,\lambda + \beta_0)^{\frac{1}{2}}} \|f\|_2.$$

Here $\Pi^{(0)}$ is the eigenprojection for the eigenvalue $\lambda = 0$ defined by

$$\Pi^{(0)} u = (u, u^{(0)*}) u^{(0)} = < \phi > u^{(0)},\ \ u = {}^\top(\phi, w),$$

where

$$u^{(0)} = {}^\top(1, 0), \quad u^{(0)*} = \frac{1}{|\Omega_{per}|} {}^\top(1, 0),$$

and S_λ is the operator defined by

$$S_\lambda := [(I - \Pi^{(0)})(\lambda + L_0)(I - \Pi^{(0)})]^{-1}.$$

The same conclusion also holds with L_0, S_λ, $\Pi^{(0)}$ replaced by L_0^*, S_λ^*, $\Pi^{(0)*}$, respectively where

$$S_\lambda^* := [(I - \Pi^{(0)*})(\lambda + L_0^*)(I - \Pi^{(0)*})]^{-1}, \ \ \Pi^{(0)*} u = (u, u^{(0)}) u^{(0)*}.$$

We give a proof of Proposition 4.3 only for L_0 since the case of L_0^* can be treated similarly. To prove Proposition 4.3, we prepare the following two lemmas.

Lemma 4.4. *It holds that $\mathrm{Ker}\, L_0 = \mathrm{span}\{u^{(0)}\}$ and $\Pi^{(0)}$ is a bounded projection on $L^2(\Omega_{per})$ that satisfies $\Pi^{(0)} X = \mathrm{Ker}\, L_0$, $\Pi^{(0)} L_0 \subset L_0 \Pi^{(0)} = 0$.*

Proof of Lemma4.4. Let $L_0 u = 0$. It then follows from (4.6) that $\nabla w = 0$, and hence, $\nabla \phi = 0$. This implies that $w = 0$ and $\phi =$ constant. This shows that $\mathrm{Ker}\, L_0 = \mathrm{span}\{u^{(0)}\}$. Clearly, $\Pi^{(0)}$ is a bounded projection onto $\mathrm{Ker}\, L_0$. For $u = {}^\top(\phi, w)$, we have $L_0 \Pi^{(0)} u = {}^\top(0, \gamma \nabla < \phi >) = 0$. On the other hand, for $u \in D(L_0)$, we have $\Pi^{(0)} L_0 u = < \gamma \mathrm{div}\, w > u^{(0)} = 0$. We thus conclude that $\Pi^{(0)} L_0 \subset L \Pi^{(0)} = 0$. This completes the proof. □

Lemma 4.5. *It holds that $\rho(-L_0|_{(I - \Pi^{(0)})X}) \supset \{\lambda; \mathrm{Re}\,\lambda > -\beta_0\}$ with a positive constant β_0, and the estimates for S_λ in Proposition 4.3 hold true.*

Proof of Lemma 4.5. In the proof we set $\mathcal{A} = -L_0|_{(I-\Pi^{(0)})X}$. Let us consider $\lambda u + \mathcal{A}u = f$. It is known that there exists a bounded linear operator $\mathcal{B} : L^2_*(\Omega_{per}) \to H^1_{0,per}(\Omega_{per})$ such that for any $g \in L^2_{*,per}(\Omega_{per})$ it holds that $\operatorname{div} \mathcal{B}g = g$ and $\|\nabla \mathcal{B}g\|_2 \le c_0\|g\|_2$ for some constant $c_0 > 0$. See [1, 2, 4] for the details.

We follow the argument in [6]. We introduce a new inner product

$$((u_1, u_2)) = (u_1, u_2) - \delta\{(w_1, \mathcal{B}\phi_2) + (\mathcal{B}\phi_1, w_2)\}$$

for $u_j = {}^\top(\phi_j, w_j)$ $(j = 1, 2)$ with a constant $\delta > 0$ to be determined later. This pairing $((u_1, u_2))$ defines an inner product on $L^2_{*,per}(\Omega_{per}) \times L^2(\Omega_{per})$ if $\delta > 0$ is sufficiently small. In fact, using the Poincaré inequality: $\|w\|_2 \le c_1\|\nabla w\|_2$, we see that there exists a constant $C > 0$ such that

$$((u, u)) = \|u\|_2^2 - \delta\{(w, \mathcal{B}\phi) + (\mathcal{B}\phi, w)\} \ge (1 - \delta c_0 c_1)\|u\|_2^2$$

and $((u, u)) \le (1 + \delta c_0 c_1)\|u\|_2^2$. Therefore, $((\cdot, \cdot))$ is an inner product and the norm defined by $((\cdot, \cdot))$ is equivalent to the norm $\|\cdot\|_2$ if $\delta > 0$ is taken sufficiently small.

We denote $\mathcal{A}u = {}^\top(\mathcal{A}_1 u, \mathcal{A}_2 u)$. Note that $\int_{\Omega_{per}} \mathcal{A}_1 u dx = 0$. We see that

$$\begin{aligned} ((\mathcal{A}u, u)) &= (L_0 u, u) - \delta\{(\mathcal{A}_2 u, \mathcal{B}\phi) + (\mathcal{B}(\mathcal{A}_1 u), w)\} \\ &\ge \nu\|\nabla w\|_2^2 + \tilde{\nu}\|\operatorname{div} w\|_2^2 + \frac{1}{2}\delta\gamma\|\phi\|_2^2 - \delta\{(\frac{\nu^2 c_0^2}{\gamma} + \gamma c_1^2)\|\nabla w\|_2^2 + \frac{\tilde{\nu}^2}{\gamma}\|\operatorname{div} w\|_2^2\} \\ &\ge \frac{1}{2}\nu\|\nabla w\|_2^2 + \frac{1}{2}\tilde{\nu}\|\operatorname{div} w\|_2^2 + \frac{1}{2}\delta\gamma\|\phi\|_2^2 \end{aligned}$$

if $\delta > 0$ is taken suitably small. Therefore, we have

$$(1-\delta c_0 c_1)\operatorname{Re}\lambda\|u\|_2 + \frac{1}{2}\nu\|\nabla w\|_2^2 + \frac{1}{2}\tilde{\nu}\|\operatorname{div} w\|_2^2 + \frac{1}{2}\delta\gamma\|\phi\|_2^2 \le \operatorname{Re}((f, u)) \le C\|f\|_2\|u\|_2.$$

Setting $\beta_0 = \frac{1}{2(1-\delta c_0 c_1)} \min\{\delta\gamma, \frac{\nu}{2c_1^2}\}$ we find by the Poincaré inequality that

$$(\operatorname{Re}\lambda + \beta_0)\|u\|_2 \le C\|f\|_2.$$

We thus conclude that if $\operatorname{Re}\lambda + \beta_0 > 0$, then

$$\|u\| \le \frac{C}{\operatorname{Re}\lambda + \beta_0}\|f\|_2$$

and

$$\|\nabla w\|_2 \le \frac{C}{(\operatorname{Re}\lambda + \beta_0)^{\frac{1}{2}}}\|f\|_2.$$

These estimates, together with Proposition 4.1, yield the desired results. This completes the proof. □

We are now in a position to prove Proposition 4.3.

Proof of Proposition 4.3. We set $X_0 = \Pi^{(0)}X$ and $X_1 = (I - \Pi^{(0)})X$. By Lemma 4.4, we have $X = X_0 \oplus X_1$ and $\rho(-L_0|_{X_0}) = \{\lambda : \lambda \neq 0\}$. This, together with Lemma 4.5, shows that $\{\lambda\,;\,\mathrm{Re}\,\lambda > -\beta_0,\ \lambda \neq 0\} \subset \rho(-L_0)$,

$$(\lambda + L_0)^{-1} f = \frac{1}{\lambda}\Pi^{(0)} f + S_\lambda (I - \Pi^{(0)}) f,$$

and S_λ satisfies the desired estimates. This completes the proof. □

We next derive the resolvent estimates for $-L_{\eta'}$ with $|\eta'| \leq r_0$.

Theorem 4.6. *There exists a constant $r_0 > 0$ such that if $\eta' \in Q^*$ satisfies $|\eta'| \leq r_0$, then*

$$\Sigma_1 := \{\lambda\ ;\ \mathrm{Re}\,\lambda \geq -\frac{3}{4}\beta_0\} \cap \{\lambda\ ;\ |\lambda| \geq \frac{\beta_0}{2}\} \subset \rho(-L_{\eta'}),$$

and the following estimates hold uniformly for $\lambda \in \Sigma_1$:

$$\|(\lambda + L_{\eta'})^{-1} f\|_2 \leq \frac{C}{\mathrm{Re}\,\lambda + \beta_0}\|f\|_2,$$

$$\|\nabla \tilde{\mathbb{P}}\,(\lambda + L_{\eta'})^{-1} f\|_2 \leq \frac{C}{(\mathrm{Re}\,\lambda + \beta_0)^{\frac{1}{2}}}\|f\|_2.$$

The same conclusion also holds with $L_{\eta'}$ replaced by $L^*_{\eta'}$.

Proof. Let $\lambda \in \Sigma_1$. By Proposition 4.3, we see that

$$\|(\lambda + L_0)^{-1} f\|_2 + \|\nabla \tilde{\mathbb{P}}\,(\lambda + L_0)^{-1} f\|_2 \leq C_1 \|f\|_2 \tag{4.9}$$

uniformly for $\lambda \in \Sigma_1$. Here C_1 is a constant depending only on β_0. It then follows that

$$\|L_j^{(1)} u\|_2 \leq C\{\|w\|_2 + \|\nabla w\|_2 + \|\phi\|_2\} \leq CC_1\|(\lambda + L_0)u\|_2, \tag{4.10}$$

$$\|L_{jk}^{(2)} u\|_2 \leq C\|w\|_2 \leq CC_1\|(\lambda + L_0)u\|_2 \tag{4.11}$$

uniformly for $\lambda \in \Sigma_1$ and $u \in D(L_0)$. We thus obtain

$$\|M_{\eta'}(\lambda + L_0)^{-1} f\|_2 \leq CC_1|\eta'|\|f\|_2 \ \ (\lambda \in \Sigma_1)$$

uniformly for $\lambda \in \Sigma_1$ and $f \in X$. Therefore, if $r_0 > 0$ is a constant satisfying $r_0 < \frac{1}{CC_1}$, then $\lambda \in \rho(-L_{\eta'})$ for $|\eta'| \leq r_0$ and it holds that

$$(\lambda + L_{\eta'})^{-1} = (\lambda + L_0)^{-1} \sum_{N=0}^{\infty} (-1)^N (M_{\eta'}(\lambda + L_0)^{-1})^N,$$

$$\|(\lambda + L_{\eta'})^{-1} f\|_2 \leq \frac{C}{\mathrm{Re}\,\lambda + \beta_0} \sum_{N=0}^{\infty} \|M_{\eta'}(\lambda + L_0)^{-1}\|^N \|f\|_2 \leq \frac{C}{\mathrm{Re}\,\lambda + \beta_0}\|f\|_2.$$

Similarly, $\|\nabla \tilde{\mathbb{P}}\,(\lambda + L_{\eta'})^{-1} f\|_2$ can be estimated as

$$\|\nabla \tilde{\mathbb{P}}\,(\lambda + L_{\eta'})^{-1} f\|_2 \leq \frac{C}{(\mathrm{Re}\,\lambda + \beta_0)^{\frac{1}{2}}}\|f\|_2.$$

The case of $L^*_{\eta'}$ can be proved similarly. This completes the proof. □

We now show that $\sigma(-L_{\eta'}) \cap \{\lambda; |\lambda| < \frac{\beta_0}{2}\}$ consists of a simple eigenvalue whose real part is negative and of order $O(|\eta'|^2)$ as $\eta' \to 0$.

Theorem 4.7. *There exists a constant $r_0 > 0$ such that if $|\eta'| \le r_0$, then $\sigma(-L_{\eta'}) \cap \{\lambda; |\lambda| < \frac{\beta_0}{2}\} = \{\lambda_{\eta'}\}$. Here $\lambda_{\eta'}$ is a simple eigenvalue that satisfies*

$$\lambda_{\eta'} = -\frac{\gamma^2}{\nu}\kappa(\eta') + O(|\eta'|^3) \quad (\eta' \to 0),$$

where

$$\kappa(\eta') = \sum_{j,k=1}^{n-1} a_{jk}\eta_j\eta_k, \quad a_{jk} = \frac{1}{|\Omega_{per}|}(\nabla w_1^{(j)}, \nabla w_1^{(k)}).$$

Here $w_1^{(k)}$ $(k = 1, \cdots, n-1)$ satisfy the Stokes system (3.4) *for some $\phi_1^{(k)}$; and $\kappa(\eta')$ satisfies $\kappa(\eta') \ge \kappa_0|\eta'|^2$ with some constant $\kappa_0 > 0$. As a result, it holds that* $\mathrm{Re}\,\lambda_{\eta'} \le -\frac{\kappa_0}{2}\frac{\gamma^2}{\nu}|\eta'|^2$.

Remark 4.8. A similar result holds for $L^*_{\eta'}$ with simple eigenvalue $\lambda^*_{\eta'} = \overline{\lambda}_{\eta'}$.

Remark 4.9. Since $\lambda_{\eta'} \to 0$ as $\eta' \to 0$, we see that for any $\beta \in (0, \frac{\beta_0}{2})$, there exists a constant $r = r(\beta) > 0$ such that if $|\eta'| \le r(\beta)$, then $|\lambda_{\eta'}| < \beta$ and $\{\lambda\,;\, \mathrm{Re}\,\lambda \ge -\frac{3}{4}\beta_0\} \cap \{\lambda\,;\, |\lambda| \ge \beta\} \subset \rho(-L_{\eta'})$.

Proof of Theorem 4.7. In view of Proposition 4.3, (4.10) and (4.11), we can apply the analytic perturbation theory to see that $\sigma(-L_{\eta'}) \cap \{\lambda; |\lambda| < \frac{\beta_0}{2}\}$ consists of a simple eigenvalue, say $\lambda_{\eta'}$, for sufficiently small η', and that $\lambda_{\eta'}$ is expanded as

$$\lambda_{\eta'} = \sum_{j=0}^{n-1} \lambda_j^{(1)}\eta_j + \sum_{j,k=0}^{n-1} \lambda_{jk}^{(2)}\eta_j\eta_k + O(|\eta'|^3)$$

with

$$\lambda_j^{(1)} = -(L_j^{(1)}u^{(0)}, u^{(0)*}),$$

$$\lambda_{jk}^{(2)} = -\frac{1}{2}((L_{jk}^{(2)} + L_{kj}^{(2)})u^{(0)}, u^{(0)*}) + \frac{1}{2}((L_j^{(1)}SL_k^{(1)} + L_k^{(1)}SL_j^{(1)})u^{(0)}, u^{(0)*}).$$

Here $S = S_\lambda|_{\lambda=0}$. See, e.g., [12, Chap. VII], [21, Chap. XII].

Let us compute $\lambda_j^{(1)}$. Since $(u, u^{(0)*}) =< \phi >$ for $u = {}^\top(\phi, w)$ and $L_j^{(1)}u^{(0)} = {}^\top(0, i\gamma \boldsymbol{e}_j)$, we have $\lambda_j^{(1)} = 0$.

As for $\lambda_{jk}^{(2)}$, since $L_{jk}^{(2)}u^{(0)} = 0$, we have $< L_{jk}^{(2)}u^{(0)} >= 0$. Furthermore,

$$\frac{1}{2}((L_j^{(1)}SL_k^{(1)} + L_j^{(1)}SL_k^{(1)})u^{(0)}, u^{(0)*}) = (L_j^{(1)}SL_k^{(1)}u^{(0)}, u^{(0)*}) =< L_j^{(1)}SL_k^{(1)}u^{(0)} > .$$

We compute $< L_j^{(1)}SL_k^{(1)}u^{(0)} >$. Set $u_1 = {}^\top(\phi_1, w_1) = SL_k^{(1)}u_0$. Then u_1 is a solution of

$$L_0u_1 = (I - \Pi^{(0)})L_k^{(1)}u^{(0)} = L_k^{(1)}u^{(0)}, \quad < \phi_1 >= 0,$$

that is,

$$\begin{cases} \gamma \mathrm{div}\, w_1 = 0, \\ -\nu \Delta w_1 + \gamma \nabla \phi_1 = i\gamma \boldsymbol{e}_k, \\ w_1|_{\Sigma_{j,+}} = w_1|_{\Sigma_{j,-}},\ \phi_1|_{\Sigma_{j,+}} = \phi_1|_{\Sigma_{j,-}},\ w_1|_{\Sigma_n} = 0,\ <\phi_1>= 0. \end{cases}$$

Lemma 4.5 implies that for each $k = 1, \cdots, n-1$, there exists a unique solution $\tilde{u}_1^{(k)} = {}^\top(\tilde{\phi}_1^{(k)}, \tilde{w}_1^{(k)})$ of this system. Let $u_1^{(k)} = {}^\top(\phi_1^{(k)}, w_1^{(k)})$ be a unique solution of (3.4) Then $\tilde{\phi}_1^{(k)} = i\phi_1^{(k)}$ and $\tilde{w}_1^{(k)} = \frac{i\gamma}{\nu} w_1^{(k)}$, and hence,

$$\begin{aligned} L_j^{(1)} S L_k^{(1)} u^{(0)} &= i \begin{pmatrix} 0 & \gamma^\top \boldsymbol{e}_j \\ \gamma \boldsymbol{e}_j & -2\nu \partial_{x_j} - \tilde{\nu} \boldsymbol{e}_j \mathrm{div} - \tilde{\nu} \nabla({}^\top \boldsymbol{e}_j) \end{pmatrix} \begin{pmatrix} \tilde{\phi}_1^{(k)} \\ \tilde{w}_1^{(k)} \end{pmatrix} \\ &= -\frac{\gamma^2}{\nu} \begin{pmatrix} \boldsymbol{e}_j \cdot w_1^{(k)} \\ * \end{pmatrix}. \end{aligned}$$

It then follows that

$$\begin{aligned} \lambda_{jk}^{(2)} &=< L_j^{(1)} S L_k^{(1)} u^{(0)} >= -\frac{\gamma^2}{\nu} < \boldsymbol{e}_j \cdot w_1^{(k)} > \\ &= -\frac{\gamma^2}{\nu} < (-\Delta w_1^{(j)} + \nabla \phi_1^{(j)}) \cdot w_1^{(k)} >= -\frac{\gamma^2}{\nu} \frac{1}{|\Omega_{per}|} (\nabla w_1^{(j)}, \nabla w_1^{(k)}). \end{aligned}$$

Let us show that the matrix $((\nabla w_1^{(j)}, \nabla w_1^{(k)}))$ is positive definite. We first observe that $w_1^{(1)}, \cdots, w_1^{(n-1)}$ are linearly independent. In fact, suppose that $w_1 = \sum_{j=1}^{n-1} c_j w_1^{(j)} = 0$. Then $\phi_1 = \sum_{j=1}^{n-1} c_j \phi_1^{(j)}$ satisfies $\nabla \phi_1 = \sum_{j=1}^{n-1} c_j \boldsymbol{e}_j$. Therefore, ϕ_1 is written as $\phi_1 = c + \sum_{j=1}^{n-1} c_j x_j$ with some constant c. Since ϕ_1 is Q-periodic in $x' = (x_1, \cdots, x_{n-1})$ and $<\phi_1>= 0$, we see that $c = c_1 = \cdots = c_{n-1} = 0$. We thus conclude that $w_1^{(1)}, \cdots, w_1^{(n-1)}$ are linearly independent.

Set $V = \mathrm{span}\,\{w_1^{(1)}, \cdots, w_1^{(n-1)}\}$ and take an orthonormal basis $\{f_1, \cdots, f_{n-1}\}$ of V as a subspace of $H^1_{0,per}(\Omega_{per})$ with respect to the inner product $(w, v)_{H^1_{0,per}} = (\nabla w, \nabla v)$. Then $w_1^{(m)}$ is written as $w_1^{(m)} = \sum_{k=1}^{n-1} b_{mk} f_k$ for $m = 1, \cdots, n-1$, and thus, $(w_1^{(1)}, \cdots, w_1^{(n-1)}) = (f_1, \cdots, f_{n-1})B$, where $B = (\boldsymbol{b}_1, \cdots, \boldsymbol{b}_{n-1})$ with $\boldsymbol{b}_m = {}^\top(b_{m1}, \cdots, b_{mn-1})$. It then follows that $\boldsymbol{b}_1, \cdots, \boldsymbol{b}_{n-1}$ are linearly independent. We have $(\nabla w_1^{(m)}, \nabla w_1^{(l)}) = (w_1^{(m)}, w_1^{(l)})_{H^1_{0,per}} = (BB^*)_{ml}$. Since BB^* is positive definite, so is the matrix $((\nabla w_1^{(m)}, \nabla w_1^{(l)}))$. It then follows that there is a constant $\kappa_0 > 0$ such that

$$\sum_{j,k=1}^{n-1} \lambda_{jk}^{(2)} \eta_j \eta_k = -\sum_{j,k=1}^{n-1} \frac{\gamma^2}{\nu} \frac{1}{|\Omega_{per}|} (\nabla w_1^{(j)}, \nabla w_1^{(k)}) \eta_j \eta_k = -\frac{\gamma^2}{\nu} |B^* \eta'|^2 \leq -\kappa_0 \frac{\gamma^2}{\nu} |\eta'|^2$$

for all $\eta' \in \mathbb{R}^{n-1}$. Therefore, there exists $r_0 > 0$ such that if $|\eta'| \leq r_0$, then $\mathrm{Re}\,\lambda_{\eta'} \leq -\frac{\kappa_0}{2} \frac{\gamma^2}{\nu} |\eta'|^2$. This completes the proof. □

Let $\Pi_{\eta'}$ be the eigenprojection for the eigenvalue $\lambda_{\eta'}$. Since $\Pi_{\eta'}e^{-tL}_{\eta'} = e^{\lambda_{\eta'}t}\Pi_{\eta'}$, we have the following estimate.

Theorem 4.10. *If $|\eta'| \le r_0$, then it holds that*

$$\|e^{-tL_{\eta'}}u_0 - e^{\lambda_{\eta'}t}\Pi_{\eta'}u_0\|_2 \le Ce^{-\frac{\beta_0}{2}t}\|u_0\|_2.$$

Theorem 4.10 follows from Theorems 4.6 and 4.7. See, e.g., [3, Chap. V, Theorem 1.11], [23].

We close this subsection with the estimates for the eigenprojections $\Pi_{\eta'}$ and $\Pi^*_{\eta'}$ for the eigenvalues $\lambda_{\eta'}$ and $\lambda^*_{\eta'}(=\overline{\lambda}_{\eta'})$ of $-L_{\eta'}$ and $-L^*_{\eta'}$, respectively.

Theorem 4.11. *For any nonnegative integer k, there exists a constant $r_k > 0$ such that the following estimates hold uniformly for $|\eta'| \le r_k$:*

(i) $\|\Pi_{\eta'}u\|_{H^k} \le C\|u\|_1$.

(ii) $\|(\Pi_{\eta'} - \Pi^{(0)})u\|_{H^k} \le C|\eta'|\|u\|_1$.

The same conclusion holds with $\Pi_{\eta'}$ replaced by $\Pi^*_{\eta'}$.

Proof. By Theorem 4.6 we have

$$\Pi_{\eta'} = \frac{1}{2\pi i}\int_{|\lambda|=\frac{\beta_0}{2}}(\lambda + L_{\eta'})^{-1}d\lambda, \quad \Pi^*_{\eta'} = \frac{1}{2\pi i}\int_{|\lambda|=\frac{\beta_0}{2}}(\lambda + L^*_{\eta'})^{-1}d\lambda.$$

Furthermore, $u_{\eta'} = \Pi_{\eta'}u^{(0)}$ and $u^*_{\eta'} = \Pi^*_{\eta'}u^{(0)^*}$ are eigenfunctions of $-L_{\eta'}$ and $-L^*_{\eta'}$ for the eigenvalues $\lambda_{\eta'}$ and $\lambda^*_{\eta'} = \overline{\lambda_{\eta'}}$, respectively; and it holds that

$$\Pi_{\eta'}u = \frac{(u, u^*_{\eta'})}{(u_{\eta'}, u^*_{\eta'})}u_{\eta'}.$$

Note that $u_{\eta'}|_{\eta'=0} = \Pi^{(0)}u^{(0)} = u^{(0)}$ and $u^*_{\eta'}|_{\eta'=0} = \Pi^{(0)^*}u^{(0)*} = u^{(0)*}$.

In view of (4.9), (4.10) and (4.11), we see that $(\lambda + L_{\eta'})^{-1}$ is expanded as

$$(\lambda + L_{\eta'})^{-1} = (\lambda + L_0)^{-1} - (\lambda + L_0)^{-1}\sum_{j=1}^{n-1}\eta_j L^{(1)}_j(\lambda + L_0)^{-1} + R_{\eta'}(\lambda)$$

and $R_{\eta'}(\lambda)$ is estimated as

$$\|R_{\eta'}(\lambda)f\|_2 \le C|\eta'|^2\|f\|_2, \ \|\nabla\tilde{\mathbb{P}}\, R_{\eta'}(\lambda)f\|_2 \le C|\eta'|^2\|f\|_2$$

uniformly for $|\eta'| \le r_0$ and $|\lambda| = \frac{\beta_0}{2}$. We write $u_{\eta'}$ as

$$\begin{aligned} u_{\eta'} \quad &= \quad u^{(0)} + \frac{1}{2\pi i}\int_{|\lambda|=\frac{\beta_0}{2}}\Big(-\sum_{j=1}^{n-1}\eta_j(\lambda + L_0)^{-1}L^{(1)}_j(\lambda + L_0)^{-1}u^{(0)}\Big)\,d\lambda \\ &\qquad +\frac{1}{2\pi i}\int_{|\lambda|=\frac{\beta_0}{2}}R_{\eta'}(\lambda)u^{(0)}\,d\lambda \\ &=: \quad u^{(0)} + \sum_{j=1}^{n-1}\eta_j u^{(1)}_j + u^{(2)}. \end{aligned}$$

Using (4.9), (4.10) and (4.11), we have

$$\|u_j^{(1)}\|_2 + \|\nabla\tilde{\mathbb{P}}\, u_j^{(1)}\|_2 \le C, \quad \|u^{(2)}\|_2 + \|\nabla\tilde{\mathbb{P}}\, u^{(2)}\|_2 \le C|\eta'|^2.$$

Similarly, we have the expression for $u_{\eta'}^*$:

$$u_{\eta'}^* = u^{(0)*} + \sum_{j=1}^{n-1} \eta_j u_j^{(1)*} + u^{(2)*},$$

with estimates

$$\|u_j^{(1)*}\|_2 + \|\nabla\tilde{\mathbb{P}}\, u_j^{(1)*}\|_2 \le C, \quad \|u^{(2)*}\|_2 + \|\nabla\tilde{\mathbb{P}}\, u^{(2)*}\|_2 \le C|\eta'|^2.$$

It then follows that

$$(u_{\eta'}, u_{\eta'}^*) = (u^{(0)}, u^{(0)*}) + (u_{\eta'} - u^{(0)}, u_{\eta'}^*) + (u^{(0)}, u_{\eta'}^* - u^{(0)*}) \ge 1 - C|\eta'| \ge \frac{1}{2}$$

for $|\eta'| \le r_0$ with $r_0 > 0$ replaced by a smaller one if necessary.

If we could have the estimates $\|u_{\eta'}^*\|_\infty \le C$ and $\|\partial_x^\alpha u_{\eta'}\|_2 \le C$, then it would hold that $\|\partial_x^\alpha \Pi_{\eta'} u\|_2 \le C\|u\|_1 \|u_{\eta'}^*\|_\infty \|\partial_x^\alpha u_{\eta'}\|_2 \le C\|u\|_1$. So we will deduce the estimates for $u_{\eta'}$ and $u_{\eta'}^*$, in other words, for $(\lambda + L_{\eta'})^{-1}u^{(0)}$ and $(\lambda + L_{\eta'}^*)^{-1}u^{(0)}$.

In the remaining we consider only $(\lambda + L_{\eta'})^{-1}u^{(0)}$ since $(\lambda + L_{\eta'}^*)^{-1}u^{(0)}$ can be estimated similarly. We also observe that the integral path of $u_{\eta'} = \frac{1}{2\pi i}\int_{|\lambda| = \frac{\beta_0}{2}} (\lambda + L_{\eta'})^{-1} u^{(0)} d\lambda$ can be deformed into $\{|\lambda| = \beta\} \subset \rho(-L_{\eta'})$.

We claim the following

Proposition 4.12. *Let m be a nonnegative integer. Then there exist constants $r_m > 0$ and $\beta_m > 0$ such that if $|\eta'| \le r_m$ and $\frac{\beta_m}{2} \le |\lambda| \le \beta_m$, then it holds that $(\lambda + L_{\eta'})^{-1}u^{(0)} \in H_{per}^{m+1}(\Omega_{per}) \times (H_{per}^{m+2} \cap H_{0,per}^1)(\Omega_{per})$ and*

$$\|(\lambda + L_{\eta'})^{-1}u^{(0)}\|_{H^{m+1}\times H^{m+2}} \le C$$

uniformly for $|\eta'| \le r_m$ and $\frac{\beta_m}{2} \le |\lambda| \le \beta_m$.

To prove Proposition 4.12, we employ the following lemma.

Lemma 4.13. *Let m be a nonnegative integer. Then there exists $\tilde{\beta}_m > 0$ such that if $|\lambda| \le \tilde{\beta}_m$, then it holds that $S_\lambda f \in H_{*,per}^{m+1}(\Omega_{per}) \times (H_{per}^{m+2} \cap H_{0,per}^1)(\Omega_{per})$ for any $f \in H_{*,per}^{m+1}(\Omega_{per}) \times H_{per}^m(\Omega_{per})$ and $S_\lambda f$ satisfies the estimates*

$$\|S_\lambda f\|_{H^{m+1}\times H^{m+2}} \le C\|f\|_{H^{m+1}\times H^m}.$$

uniformly for λ with $|\lambda| \le \tilde{\beta}_m$.

The proof of Lemma 4.13 will be given later.

Proof of Proposition 4.12. We prove by induction on m. We set $u := (\lambda + L_{\eta'})^{-1}u^{(0)}$. By Theorems 4.6 and 4.7, we have $\|u\|_{L^2\times H^1} \le C$ uniformly for $|\eta'| \le r_0$ and $\frac{\beta_0}{4} \le |\lambda| \le \frac{\beta_0}{2}$ with r_0 replaced by a smaller one if necessary.

We write $(\lambda + L_0)u = u^{(0)} - M_{\eta'}u$ and decompose $u = {}^\top(\phi, w)$ as $u =< \phi > u^{(0)} + u_1$, where $\Pi^{(0)}u =< \phi > u^{(0)}$ and $u_1 = (I - \Pi^{(0)})u$. Then we have

$$< \phi >= \frac{1}{\lambda}\{1 - i\gamma < \eta' \cdot w' >\},$$

$$(\lambda + L_0)u_1 = -(M_{\eta'}u_1 + < \phi > M_{\eta'}u^{(0)} - \Pi^{(0)}M_{\eta'}u).$$

It then follows that

$$| < \phi > | = \frac{1}{|\lambda|}\{1 + \gamma r_0\|w\|_2\} \leq C \tag{4.12}$$

uniformly for $|\eta'| \leq r_0$ and $\frac{\beta_1}{2} \leq |\lambda| \leq \beta_1$ with $\beta_1 > 0$ to be determined later. On the other hand, we have

$$\|M_{\eta'}u_1 + < \phi > M_{\eta'}u^{(0)} - \Pi^{(0)}M_{\eta'}u\|_{H^1\times L^2} \leq C\|u\|_{L^2\times H^1} \leq C$$

uniformly for $|\eta'| \leq r_0$ and $\frac{\beta_0}{4} \leq |\lambda| \leq \frac{\beta_0}{2}$. It then follows from Remark 4.9 and Lemma 4.13 that, with a suitable choice of $r_1 > 0$ and $\beta_1 > 0$, the estimate $\|u_1\|_{H^1\times H^2} \leq C$ holds uniformly for $|\eta'| \leq r_1$ and $\frac{\beta_1}{2} \leq |\lambda| \leq \beta_1$. This, together with (4.12), proves Proposition 4.12 for $m = 0$.

Assume that the proposition holds for $m = k$. We will show that the proposition holds for $m = k + 1$. By the inductive assumption, we have

$$\|M_{\eta'}u_1 + < \phi > M_{\eta'}u^{(0)} - \Pi^{(0)}M_{\eta'}u\|_{H^{k+2}\times H^{k+1}} \leq C\|u\|_{H^{k+1}\times H^{k+2}} \leq C$$

uniformly for $|\eta'| \leq r_k$ and $\frac{\beta_k}{2} \leq |\lambda| \leq \beta_k$. It then follows from Remark 4.9 and Lemma 4.13 that the estimate $\|u_1\|_{H^{k+2}\times H^{k+3}} \leq C$ holds uniformly for $|\eta'| \leq r_{k+1}$ and $\frac{\beta_{k+1}}{2} \leq |\lambda| \leq \beta_{k+1}$. Combining this with (4.12), we conclude that the proposition holds for $m = k + 1$. This completes the proof. □

We continue the proof of Theorem 4.11. Let m be a nonnegative integer. By Proposition 4.12, we see that

$$(\lambda + L_{\eta'})^{-1}u^{(0)} \in H^{m+1}_{per}(\Omega_{per}) \times (H^{m+2}_{per} \cap H^1_{0,per})(\Omega_{per}),$$

$$\|(\lambda + L_{\eta'})^{-1}u^{(0)}\|_{H^{m+1}\times H^{m+2}} \leq C$$

uniformly for $|\eta'| \leq r_m$ and $|\lambda| = \beta_m$. Deforming the integral path into $\{|\lambda| = \beta_m\}$, we thus deduce that $u_{\eta'} \in H^{m+1}_{per}(\Omega_{per}) \times H^{m+2}_{per}(\Omega_{per})$ and

$$\|u_{\eta'}\|_{H^{m+1}\times H^{m+2}} = \left\| \frac{1}{2\pi i}\int_{|\lambda|=\frac{\beta_m}{2}} (\lambda + L_{\eta'})^{-1}u^{(0)}d\lambda \right\|_{H^{m+1}\times H^{m+2}} \leq C. \tag{4.13}$$

Taking $m = k - 1$, we have $\|\partial_x^\alpha u_{\eta'}\|_{L^2} \leq C$ for $|\alpha| \leq k$ and $|\eta'| \leq r_k$. Similarly we can obtain (4.13) with $u_{\eta'}$ replaced by $u^*_{\eta'}$, and hence, $\|u^*_{\eta'}\|_\infty \leq C\|u^*_{\eta'}\|_{H^{[\frac{n}{2}]+1}} \leq C$. It then follows that

$$\|\partial_x^\alpha \Pi_{\eta'}u\|_2 \leq C\|u^*_{\eta'}\|_\infty\|\partial_x^\alpha u_{\eta'}\|_2\|u\|_1 \leq C\|u\|_1.$$

This proves (i).

Let us next consider (ii). We write $\Pi_{\eta'}u - \Pi^{(0)}u$ as

$$\Pi_{\eta'}u - \Pi^{(0)}u = \Big(\frac{1}{(u_{\eta'}, u^*_{\eta'})} - 1\Big)(u, u^{(0)*})u^{(0)} + \frac{1}{(u_{\eta'}, u^*_{\eta'})}\left\{(u, u^*_{\eta'})u_{\eta'} - (u, u^{(0)*})u^{(0)}\right\}$$
$$=: I_1 + I_2.$$

As for I_1, we have

$$|(u_{\eta'}, u^*_{\eta'}) - 1| = |(u_{\eta'} - u^{(0)}, u^*_{\eta'}) + (u^{(0)}, u^*_{\eta'} - u^{(0)*})|$$
$$\leq C\{\|u_{\eta'} - u^{(0)}\|_2 + \|u^*_{\eta'} - u^{(0)*}\|_2\}.$$

Since

$$u_{\eta'} - u^{(0)} = \frac{1}{2\pi i}\int_{|\lambda|=\beta_m}(\lambda + L_0)^{-1}\sum_{N=1}^{\infty}(-1)^N[M_{\eta'}(\lambda + L_0)^{-1}]^N u^{(0)}\, d\lambda,$$

we have $\|u_{\eta'} - u^{(0)}\|_2 \leq C|\eta'|$, and likewise, $\|u^*_{\eta'} - u^{(0)*}\|_2 \leq C|\eta'|$. We thus obtain

$$\|\partial_x^\alpha I_1\|_2 \leq C|\eta'||(u, u^{(0)*})\partial_x^\alpha u^{(0)}| \leq C|\eta'|\|u\|_1\|u^{(0)*}\|_\infty\|\partial_x^\alpha u^{(0)}\|_\infty \leq C|\eta'|\|u\|_1.$$

As for I_2, we have

$$\|\partial_x^\alpha\left\{(u, u^*_{\eta'})u_{\eta'} - (u, u^{(0)*})u^{(0)}\right\}\|_2$$
$$= \|(u, u^*_{\eta'} - u^{(0)*})\partial_x^\alpha u_{\eta'} + (u, u^{(0)*})\partial_x^\alpha(u_{\eta'} - u^{(0)})\|_2$$
$$\leq \|u\|_1\|u^*_{\eta'} - u^{(0)*}\|_\infty\|\partial_x^\alpha u_{\eta'}\|_2 + \|u\|_1\|u^{(0)*}\|_\infty\|\partial_x^\alpha(u_{\eta'} - u^{(0)})\|_2$$
$$\leq C\|u\|_1\{\|u^*_{\eta'} - u^{(0)*}\|_{H^{[\frac{n}{2}]+1}}\|u_{\eta'}\|_{H^k} + \|u_{\eta'} - u^{(0)}\|_{H^k}\}.$$

Since $\|M_{\eta'}u\|_{H^k\times H^{k-1}} \leq C|\eta'|\|u\|_{H^{k-1}\times H^k}$, with the aid of Lemma 4.13, we see that

$$\|(\lambda + L_0)^{-1}[M_{\eta'}(\lambda + L_0)^{-1}]^N u^{(0)}\|_{H^k\times H^k} \leq (C|\eta'|)^N$$

uniformly for $|\eta'| \leq r_k$ and $|\lambda| = \beta_k$. Taking $r_k > 0$ smaller if necessary, we obtain $\|u_{\eta'} - u^{(0)}\|_{H^k} \leq C|\eta'|$. Similarly, we can obtain $\|u^*_{\eta'} - u^{(0)*}\|_{H^{[\frac{n}{2}]+1}} \leq C|\eta'|$. It then follows that $\|\partial_x^\alpha I_2\|_2 \leq C|\eta'|\|u\|_1$ for $|\alpha| \leq k$. We thus conclude that

$$\|\partial_x^\alpha(\Pi_{\eta'} - \Pi^{(0)})u\|_2 \leq C|\eta'|\|u\|_1$$

for $|\alpha| \leq k$. This completes the proof. □

In the remaining of this subsection we give a proof of Lemma 4.13.

Proof of Lemma 4.13. We set $\check{u} = S_\lambda f$. Then

$$L_0\check{u} = f - \lambda\check{u},$$

which is regarded as an inhomogeneous Stokes system. This can be solved for $\check{u}$ if $|\lambda|$ is suitably small. In fact, let $f \in H^{m+1}_{*,per}(\Omega_{per}) \times H^m_{per}(\Omega_{per})$. Then, for each

$\check{v} \in H^{m+1}_{*,per}(\Omega_{per}) \times (H^{m+2}_{per} \cap H^1_{0,per})(\Omega_{per})$, there uniquely exists $\check{u} \in H^{m+1}_{*,per}(\Omega_{per}) \times (H^{m+2}_{per} \cap H^1_{0,per})(\Omega_{per})$ such that $L_0\check{u} = f - \lambda\check{v}$ and

$$\|\check{u}\|_{H^{m+1}\times H^{m+2}} \leq C|\lambda|\|\check{v}\|_{H^{m+1}\times H^{m+2}} + C\|f\|_{H^{m+1}\times H^m}.$$

See, e.g., [22, Chap. III, Theorem 1.5.3]. This estimate shows that the map $\check{v} \mapsto \check{u}$ is a contraction on $H^{m+1}_{*,per}(\Omega_{per}) \times (H^{m+2}_{per} \cap H^1_{0,per})(\Omega_{per})$ when $|\lambda| \leq \tilde{\beta}_m$ with suitably small $\tilde{\beta}_m$. This completes the proof. □

4.3 The case $|\eta'| \geq r_0$

In this subsection we investigate the spectrum of $-L_{\eta'}$ for $\eta' \in Q^*$ satisfying $|\eta'| \geq r_0$. We have already shown in Proposition 4.2 that $-L_{\eta'}$ generates a contraction semigroup $e^{-tL_{\eta'}}$. We will show that $e^{-tL_{\eta'}}$ has an exponential decay estimate uniformly for $\eta' \in Q^*$ with $|\eta'| \geq r_0$.

We first introduce an inner product of $H^1_{0,per}(\Omega_{per})$ in terms of $\nabla_{\eta'}$.

Proposition 4.14. *Let $\eta' \in Q^*$. Then $(\nabla_{\eta'}w, \nabla_{\eta'}v)$ defines an inner product of $H^1_{0,per}(\Omega_{per})$. Furthermore, $\|\nabla_{\eta'}w\|_2$ is equivalent to $\|w\|_{H^1}$ for $w \in H^1_{0,per}(\Omega_{per})$ and there holds the estimate*

$$C^{-1}\|w\|_{H^1} \leq \|\nabla_{\eta'}w\|_2 \leq C\|w\|_{H^1}$$

uniformly for $\eta' \in Q^*$ and $w \in H^1_{0,per}(\Omega_{per})$.

Proof. It suffices to show that $\|\nabla_{\eta'}w\|_2 = (\nabla_{\eta'}w, \nabla_{\eta'}w)$ is equivalent to $\|w\|_{H^1}$ for $w \in H^1_{0,per}(\Omega_{per})$. Let $w \in H^1_{0,per}(\Omega_{per})$. Then by using the Poincaré inequality, we see that

$$\|w\|_{H^1} \leq C\|\nabla w\|_2 \leq C'(\|\nabla'_{\eta'}w\|_2^2 + \|\partial_{x_n}w\|_2^2)^{\frac{1}{2}} = C'\|\nabla_{\eta'}w\|_2 \leq C''\|w\|_{H^1}.$$

This completes the proof. □

Before going further, we introduce some notations. We define $D_{\eta'}(w)$ by

$$\begin{aligned} D_{\eta'}(w) &:= \nu\|\nabla_{\eta'}w\|_2^2 + \tilde{\nu}\|\nabla_{\eta'}\cdot w\|_2^2 \\ &= \nu\|\nabla'_{\eta'}w\|_2^2 + \nu\|\partial_{x_n}w\|_2^2 + \tilde{\nu}\|\nabla'_{\eta'}\cdot w'\|_2^2 + \tilde{\nu}\|\partial_{x_n}w^n\|_2^2. \end{aligned}$$

In what follows we denote the projection $I - \Pi^{(0)}$ by Π_1:

$$\Pi_1 := I - \Pi^{(0)}.$$

To study problem (4.5) for $\eta' \in Q^*$ with $|\eta'| \geq r_0$, we decompose u into its $\Pi^{(0)}$-part and Π_1-part in X, namely,

$$u = \sigma u^{(0)} + u_1, \tag{4.14}$$

where $\sigma = (u, u^{(0)*}) =< \phi >\in \mathbb{C}$, $u^{(0)} = {}^\top(1,0)$ and $u_1 = {}^\top(\phi_1, w_1) \in X_1$. We note that

$$< \phi_1 >= 0. \tag{4.15}$$

It is easy to see that problem (4.5) is reduced to the following system

$$\begin{cases} \lambda\sigma + i\gamma < \tilde{\eta}' \cdot w_1 > = < f^0 >, \\ \lambda u_1 + L_{\eta'} u_1 - \Pi^{(0)} M_{\eta'} u_1 + M_{\eta'}(\sigma u^{(0)}) = f_1, \end{cases} \tag{4.16}$$

where $\sigma \in \mathbb{C}$, $u_1 = {}^\top(\phi_1, w_1) \in D(L_{\eta'}) \cap X_1$ and $f_1 = \Pi_1 f := {}^\top(f_1^0, \tilde{f}_1) \in X_1$. We observe that

$$\Pi^{(0)} M_{\eta'} u_1 = i\gamma < \tilde{\eta}' \cdot w_1 > u^{(0)}, \quad M_{\eta'}(\sigma u^{(0)}) = {}^\top(0, i\gamma \tilde{\eta}' \sigma).$$

We begin with the following

Proposition 4.15. *It holds that*

$$\mathrm{Re}\,\lambda(|\sigma|^2 + |u_1|^2) + D_{\eta'}(w_1) = \mathrm{Re}\{< f^0 > \overline{\sigma} + (f_1, u_1)\}. \tag{4.17}$$

Proof. Multiplying the first equation of (4.16) by $\overline{\sigma}$, we have

$$\lambda|\sigma|^2 + i\gamma < \tilde{\eta}' \cdot w_1 > \overline{\sigma} = < f^0 > \overline{\sigma}.$$

Taking the inner product of the second equation of (4.16) with u_1, we have

$$\lambda\|u_1\|_2^2 + (L_{\eta'} u_1, u_1) + (M_{\eta'}(\sigma u^{(0)}), u_1) - (\Pi^{(0)} M_{\eta'} u_1, u_1) = (f_1, u_1).$$

We add these two equations to obtain

$$\begin{aligned} &\lambda(|\sigma|^2 + \|u_1\|_2^2) + (L_{\eta'} u_1, u_1) + i\gamma < \tilde{\eta}' \cdot w_1 > \overline{\sigma} \\ &\qquad + (M_{\eta'}(\sigma u^{(0)}), u_1) - (\Pi^{(0)} M_{\eta'} u_1, u_1) = < f^{(0)} > \overline{\sigma} + (f_1, u_1). \end{aligned}$$

Since $\mathrm{Re}\,(L_{\eta'} u_1, u_1) = D_{\eta'}(w_1)$,

$$\mathrm{Re}\,\{i\gamma < \tilde{\eta}' \cdot w_1 > \overline{\sigma} + (M_{\eta'}(\sigma u^{(0)}), u_1)\} = \mathrm{Re}\,\{2i \mathrm{Im}\,(i\gamma < \tilde{\eta}' \cdot w_1 > \overline{\sigma})\} = 0$$

and

$$(\Pi^{(0)} M_{\eta'} u_1, u_1) = (i\gamma < \tilde{\eta}' \cdot w_1 >, \phi_1) = i\gamma < \tilde{\eta}' \cdot w_1 > < \overline{\phi}_1 > = 0,$$

we obtain

$$\mathrm{Re}\,\lambda(|\sigma|^2 + |u_1|^2) + D_{\eta'}(w_1) = \mathrm{Re}\{< f^0 > \overline{\sigma} + (f, u_1)\}.$$

This completes the proof. □

For later use, we next derive the estimate for λw_1.

Proposition 4.16. *It holds that*

$$\mathrm{Re}\,\overline{\lambda}\, D_{\eta'}(w_1) + |\lambda|^2 \|w_1\|_2^2 \le C\{\|f_1\|_2^2 + \|w_1\|_2^2 + | < f^0 > |^2 + \|\nabla_{\eta'} w_1\|_2^2\}. \tag{4.18}$$

Proof. We write the second equation of (4.16) as

$$\lambda\phi_1 + \gamma \mathrm{div}_{\eta'} w_1 = f_1^0, \tag{4.19}$$

$$\lambda w_1 - \nu\Delta_{\eta'} w_1 - \tilde{\nu}\nabla_{\eta'}(\nabla_{\eta'}\cdot w_1) + \gamma\nabla_{\eta'}\phi_1 + i\gamma\sigma\tilde{\eta'} = \tilde{f}_1. \tag{4.20}$$

We take the inner product of (4.20) with λw_1. Then the real part of the resulting equation yields

$$|\lambda|^2\|w_1\|_2^2 + \mathrm{Re}\,\overline{\lambda}\,D_{\eta'}(w_1) = \mathrm{Re}\,\{\gamma\overline{\lambda}(\phi_1, \nabla_{\eta'}\cdot w_1) - i\overline{\lambda}\gamma\sigma < \tilde{\eta'}\cdot\overline{w_1} > + \overline{\lambda}(\tilde{f}_1, w_1)\}.$$

The equation (4.19) gives that $\phi = \frac{1}{\lambda} f_1^0 - \frac{\gamma}{\lambda}\mathrm{div}_{\eta'} w_1$, and hence,

$$\begin{aligned} \mathrm{Re}\,|\lambda|^2\|w_1\|_2^2 + \mathrm{Re}\,\overline{\lambda}\,D_{\eta'}(w_1) \;=\; & \mathrm{Re}\,\Big\{\frac{\gamma\overline{\lambda}}{\lambda}(f_1^0, \nabla_{\eta'}\cdot w_1) - \frac{\overline{\lambda}}{\lambda}\gamma^2(\mathrm{div}_{\eta'} w_1, \nabla_{\eta'}\cdot w_1) \\ & - i\overline{\lambda}\gamma\sigma < \tilde{\eta'}\cdot\overline{w_1} > + \overline{\lambda}(\tilde{f}_1, w_1)\Big\}. \end{aligned} \tag{4.21}$$

By the first equation of (4.16), we have $\sigma = \frac{1}{\lambda} < f^0 > - \frac{i\gamma}{\lambda} < \tilde{\eta}', w_1 >$. Therefore, the right-hand side of (4.21) is estimated as

$$\begin{aligned} &\text{R.H.S. of (4.21)} \\ &= \mathrm{Re}\,\Big\{\frac{\gamma\overline{\lambda}}{\lambda}(f_1^0, \nabla_{\eta'}\cdot w_1) - \frac{\overline{\lambda}}{\lambda}\gamma^2(\mathrm{div}_{\eta'} w_1, \nabla_{\eta'}\cdot w_1) - i\gamma\frac{\overline{\lambda}}{\lambda} < f^0 >< \tilde{\eta'}\cdot\overline{w_1} > \\ &\qquad - \frac{\overline{\lambda}}{\lambda}\gamma^2| < \tilde{\eta'}\cdot w_1 > |^2 + \overline{\lambda}(\tilde{f}_1, w_1)\Big\} \\ &\le \epsilon|\lambda|^2\|w_1\|_2^2 + C\{\|\nabla_{\eta'} w_1\|_2^2 + | < f^0 > |^2 + \|f_1^0\|_2^2 + \frac{1}{\epsilon}\|\tilde{f}_1\|_2^2\} \end{aligned}$$

for any $\epsilon > 0$, where C is a positive constant independent of ϵ. Taking ϵ suitably small, we see that

$$|\lambda|^2\|w_1\|_2^2 + \mathrm{Re}\,\overline{\lambda}\,D_{\eta'}(w_1) \le C_1\{\|f_1\|_2^2 + | < f^0 > |^2 + \|\nabla_{\eta'} w_1\|_2^2\}.$$

This completes the proof. $\square$

We next derive the coercive estimate for σ.

Proposition 4.17. *It holds that*

$$\mathrm{Re}\,\lambda|\sigma|^2 + \frac{c_2\gamma^2}{2\nu}|\eta'|^2|\sigma|^2 \le C\{(1 + \frac{1}{|\eta'|^2})| < f^0 > |^2 + \|\tilde{f}_1\|_2^2 + |\lambda|^2\|w_1\|_2^2 + D_{\eta'}(w_1)\}, \tag{4.22}$$

where c_2 is a positive constant independent of γ, ν and $\eta' \in Q^*$.

To prove Proposition 4.17, we prepare several lemmas.

Lemma 4.18. *Let $f^0 \in L^2_{*,per}(\Omega_{per})$ and let $\tilde{f} \in H^{-1}_{per}(\Omega_{per})$. Then there uniquely exists ${}^\top(\phi, w) \in L^2_{*,per}(\Omega_{per}) \times H^1_{0,per}(\Omega_{per})$ satisfying*

$$\begin{cases} \mathrm{div}_{\eta'} w = f^0, \\ -\Delta_{\eta'} w + \nabla_{\eta'}\phi = \tilde{f}, \\ \phi|_{\Sigma_{j,+}} = \phi|_{\Sigma_{j,-}},\ w|_{\Sigma_{j,+}} = w|_{\Sigma_{j,-}},\ w|_{\Sigma_n} = 0. \end{cases} \tag{4.23}$$

Furthermore, it holds that

$$\|\phi\|_2 + \|\nabla_{\eta'} w\|_2 \leq C\{\|f^0\|_2 + \|\tilde{f}\|_{H^{-1}_{per}(\Omega_{per})}\}.$$

Lemma 4.18 can be proved in a similar manner to the proof of [22, Chap. III, Theorem 1.4.1]. An outline of the proof of Lemma 4.18 will be given in section 6.

Setting $f^0 = 0$ and $\tilde{f} = \boldsymbol{e}_k$ in Lemma 4.18, we have the following

Lemma 4.19. *Let* ${}^\top(\phi^{(1)}_{1,k,\eta'}, w^{(1)}_{1,k,\eta'}) \in L^2_{*,per}(\Omega_{per}) \times H^1_{0,per}(\Omega_{per})$ *be the pair of functions satisfying*

$$\begin{cases} \operatorname{div}_{\eta'} w^{(1)}_{1,k,\eta'} = 0, \\ -\Delta_{\eta'} w^{(1)}_{1,k,\eta'} + \nabla_{\eta'} \phi^{(1)}_{1,k,\eta'} = \boldsymbol{e}_k, \\ \phi_{1,k,\eta'}|_{\Sigma_{j,+}} = \phi_{1,k,\eta'}|_{\Sigma_{j,-}}, \ w^{(1)}_{1,k,\eta'}|_{\Sigma_{j,+}} = w^{(1)}_{1,k,\eta'}|_{\Sigma_{j,-}}, \ w_{1,k,\eta'}|_{\Sigma_n} = 0. \end{cases} \tag{4.24}$$

Then there exists a constant $C > 0$ such that the following inequalities

$$\|w^{(1)}_{1,k,\eta'}\|_{H^1} + \|\phi^{(1)}_{1,k,\eta'}\|_2 \leq C \quad (k = 1, \cdots, n-1)$$

hold uniformly for $\eta' \in Q^*$ with $|\eta'| \geq r_0$.

Lemma 4.20. *For each $k = 1, \cdots, n-1$, let* ${}^\top(\phi^{(1)}_{1,k,\eta'}, w^{(1)}_{1,k,\eta'}) \in L^2_{*,per}(\Omega_{per}) \times H^1_{0,per}(\Omega_{per})$ *be the pair of functions satisfying* (4.24)*. Then $w^{(1)}_{1,1,\eta'}, \cdots, , w^{(1)}_{1,n-1,\eta'}$ are linearly independent.*

Proof. Let $w := c_1 w^{(1)}_{1,1,\eta'} + \cdots + c_{n-1} w^{(1)}_{1,n-1,\eta'} = 0$. It then follows from (4.24) that $\nabla_{\eta'}\tilde{\phi} = \sum_{j=1}^{n-1} c_j \boldsymbol{e}_j$. Here $\tilde{\phi} = c_1\phi^{(1)}_{1,1,\eta'} + \cdots + c_{n-1}\phi^{(1)}_{1,n-1,\eta'}$. Since $|\eta'| \geq r_0$, there exists j such that $\eta_j \neq 0$. For this η_j, since $(\partial_{x_j} + i\eta_j)\tilde{\phi} = c_j$, we have $\partial_{x_j}(e^{i\eta_j x_j}\tilde{\phi}) = c_j e^{i\eta_j x_j}$. This implies that there exists a function $a(\check{x}_j)$ ($\check{x}_j = (x_1, \cdots, x_{j-1}, x_{j+1}, \cdots, x_{n-1})$) such that

$$e^{i\eta_j x_j}\tilde{\phi} = a(\check{x}_j) + \frac{c_j}{i\eta_j} e^{i\eta_j x_j},$$

namely,

$$\tilde{\phi} = a(\check{x}_j) e^{-i\eta_j x_j} + \frac{c_j}{i\eta_j}.$$

Since $\tilde{\phi}$ is Q-periodic in x', we see that $a(\check{x}_j)$ is Q-periodic and $a(\check{x}_j)e^{-i\frac{\pi}{\alpha_j}\eta_j} = a(\check{x}_j)e^{i\frac{\pi}{\alpha_j}\eta_j}$. Since $0 < |\eta_j| \leq \frac{\alpha_j}{2}$, we have $a(\check{x}_j) = 0$, and hence, $\tilde{\phi} = \frac{c_j}{i\eta_j}$. But, since $< \tilde{\phi} >= 0$, we have $c_j = 0$, and so $\tilde{\phi} = 0$. This implies that $\Sigma_{j=1}^{n-1} c_j \boldsymbol{e}_j = 0$. We thus conclude that $c_j = 0$ ($j = 1, \cdots, n-1$). This completes the proof. □

We are now in a position to prove Proposition 4.17.

Proof of Proposition 4.17. We multiply the first equation of (4.16) by $\bar{\sigma}$ and take the real part of the resulting equation to obtain

$$\operatorname{Re}\lambda|\sigma|^2 + \operatorname{Re}\{i\gamma < \tilde{\eta}' \cdot w_1 > \overline{\sigma}\} = \operatorname{Re}\{< f^0 > \overline{\sigma}\}. \tag{4.25}$$

Let us estimate $\mathrm{Re}\{i\gamma < \tilde{\eta}' \cdot w_1 > \overline{\sigma}\}$ on the left-hand side of (4.25). To do so, we decompose w_1 in the following way. In (4.19) and (4.20) we decompose ϕ_1 and w_1 as follows:

$$\phi_1 = \phi_1^{(1)} + \phi_1^{(2)},\ w_1 = w_1^{(1)} + w_1^{(2)},$$

where ${}^\top(\phi_1^{(1)}, w_1^{(1)})$ and ${}^\top(\phi_1^{(2)}, w_1^{(2)})$ satisfy the following systems, respectively:

$$\begin{cases} \mathrm{div}_{\,\eta'} w_1^{(1)} = 0, \\ -\Delta_{\eta'} w_1^{(1)} + \frac{\gamma}{\nu}\nabla_{\eta'}\phi_1^{(1)} = -\frac{i\gamma\sigma}{\nu}\tilde{\eta}', \\ \phi_1^{(1)}|_{\Sigma_{j,+}} = \phi_1^{(1)}|_{\Sigma_{j,-}},\ w_1^{(1)}|_{\Sigma_{j,+}} = w_1^{(1)}|_{\Sigma_{j,-}},\ w_1^{(1)}|_{\Sigma_n} = 0, \\ < \phi_1^{(1)} >= 0 \end{cases} \tag{4.26}$$

and

$$\begin{cases} \gamma \mathrm{div}_{\,\eta'} w_1^{(2)} = f_1^0 - \lambda\phi_1, \\ -\nu\Delta_{\eta'} w_1^{(2)} + \gamma\nabla_{\eta'}\phi_1^{(2)} = \tilde{f}_1 - \lambda w_1 + \tilde{\nu}\nabla_{\eta'}(\nabla_{\eta'}\cdot w_1), \\ \phi_1^{(2)}|_{\Sigma_{j,+}} = \phi_1^{(2)}|_{\Sigma_{j,-}},\ w_1^{(2)}|_{\Sigma_{j,+}} = w_1^{(2)}|_{\Sigma_{j,-}},\ w_1^{(1)}|_{\Sigma_n} = 0, \\ < \phi_1^{(2)} >= 0. \end{cases} \tag{4.27}$$

Let us estimate $i < \tilde{\eta}' \cdot w_1^{(1)} >$. We see from (4.26) that ${}^\top(\phi_1^{(1)}, w_1^{(1)})$ is written as

$$\begin{pmatrix} \phi_1^{(1)} \\ w_1^{(1)} \end{pmatrix} = -\frac{i\gamma}{\nu}\sigma\sum_{k=1}^{n-1}\eta_k \begin{pmatrix} \frac{\nu}{\gamma}\phi_{1,k,\eta'}^{(1)} \\ w_{1,k,\eta'}^{(1)} \end{pmatrix}.$$

Since $< \phi_{1,k,\eta'}^{(1)} >= 0$, we see that $0 = i < \tilde{\eta}' \cdot w_{1,k,\eta'}^{(1)} >< \overline{\phi_{1,k,\eta'}^{(1)}} >= (i < \tilde{\eta}' \cdot w_{1,k,\eta'}^{(1)} >, \phi_{1,k,\eta'}^{(1)})$, which implies that $(w_{1,k,\eta'}^{(1)}, \nabla_{\eta'}\phi_{1,k,\eta'}^{(1)}) = -(\mathrm{div}_{\,\eta'} w_{1,k,\eta'}^{(1)}, \phi_{1,k,\eta'}^{(1)}) = 0$. Taking this into account, we have

$$\begin{aligned} i < \tilde{\eta}' \cdot w_1^{(1)} > &= \frac{\gamma}{\nu}\sigma\sum_{j,k=1}^{n-1}\eta_j\eta_k(w_{1,j,\eta'}^{(1)}, \boldsymbol{e}_k) = \frac{\gamma}{\nu}\sigma\sum_{j,k=1}^{n-1}\eta_j\eta_k(w_{1,j,\eta'}^{(1)}, -\Delta_{\eta'}w_{1,k,\eta'}^{(1)} + \nabla_{\eta'}\phi_{1,k,\eta'}^{(1)}) \\ &= \frac{\gamma}{\nu}\sigma\sum_{j,k=1}^{n-1}\eta_j\eta_k(\nabla_{\eta'}w_{1,k,\eta'}^{(1)}, \nabla_{\eta'}w_{1,j,\eta'}^{(1)}). \end{aligned}$$

Let $\{f_{1,\eta'}, \cdots, f_{n-1,\eta'}\}$ be an orthonormal basis of span $\{w_{1,k,\eta'}^{(1)}\}_{k=1}^{n-1}$ in $H^1_{0,per}(\Omega_{per})$. Then $w_{1,k,\eta'}^{(1)}$ is written as $w_{1,k,\eta'}^{(1)} = \sum_{m=1}^{n-1} b_{km,\eta'} f_{m,\eta'}$, and therefore,

$$(\nabla_{\eta'}w_{1,j,\eta'}^{(1)}, \nabla_{\eta'}w_{1,k,\eta'}^{(1)}) = \sum_{l,m} b_{jm,\eta'}\overline{b_{kl,\eta'}}(\nabla_{\eta'}f_{m,\eta'}, \nabla_{\eta'}f_{l,\eta'}) = (B_{\eta'}B_{\eta'}^*)_{jk}.$$

Here $B_{\eta'}$ is the $(n-1)\times(n-1)$ matrix given by $B_{\eta'} = (b_{jk,\eta'})_{j,k=1}^{n-1}$. Note that $B_{\eta'}$ is nonsingular since $\{w_{1,k,\eta'}^{(1)}\}_{k=1}^{n-1}$ are linearly independent by Lemma 4.20. We thus find that $B_{\eta'}B_{\eta'}^*$ is positive definite for each η', and

$$i < \tilde{\eta}' \cdot w_1^{(1)} >= \frac{\gamma}{\nu}\sigma|B_{\eta'}^*\eta'|^2, \tag{4.28}$$

$$|B_{\eta'}^*\eta'|^2 \geq c_2|\eta'|^2 \tag{4.29}$$

uniformly for $\eta' \in Q^*$ with $|\eta'| \geq r_0$. Here c_2 is the number given by

$$c_2 = \inf_{\eta' \in Q^*, |\eta'| \geq r_0} c_{2,\eta'}$$

with

$$c_{2,\eta'} = \min\{\lambda\,;\, \lambda \text{ is an eigenvalue of } B_{\eta'}B_{\eta'}^*\} > 0.$$

Let us show that $c_2 > 0$. To do so, we first show that, for each $j, k = 1, \cdots, n-1$, $b_{jk,\eta'} = (\nabla_{\eta'} w_{1,j,\eta'}^{(1)}, \nabla_{\eta'} w_{1,k,\eta'}^{(1)})$ is continuous in η'. If this could be shown, then, by the continuity of the eigenvalue with respect to the components of matrix, we would have $c_2 > 0$.

Set $u_{1,k,\eta'}^{(1)} = {}^\top(\phi_{1,k,\eta'}^{(1)}, w_{1,k,\eta'}^{(1)})$. Then $u_{1,k,\eta'}^{(1)}$ satisfies $u_{1,k,\eta'}^{(1)} \in D(L_0) \cap X_1$ and $(L_0 + \Pi_1 M_{\eta'})u_{1,k,\eta'}^{(1)} = f_{1,k}$ with $f_{1,k} = {}^\top(0, \boldsymbol{e}_k)$. By Lemma 4.18, we see that $L_0 + \Pi_1 M_{\eta'}$ has a bounded inverse $(L_0 + \Pi_1 M_{\eta'})^{-1}$ on X_1 and it holds that

$$\|(L_0 + \Pi_1 M_{\eta'})^{-1} f\|_{L^2 \times H^1} \leq C\|f\|_2 \tag{4.30}$$

uniformly for $\eta' \in Q^*$ and $f \in X_1$. On the other hand, we see from (4.10) and (4.11) that

$$\|\Pi_1(M_{\eta'+h'} - M_{\eta'})u\|_2 \leq C|h'|\|(L_0 + \Pi_1 M_{\eta'})u\|_2$$

for $u \in D(L_0) \cap X_1$ and $h' \in \mathbb{R}^{n-1}$ with $|h'| \leq 1$. This, together with (4.30), implies that for each fixed $f \in X_1$, $(L_0 + \Pi_1 M_{\eta'})^{-1} f$ is analytic in $\eta' \in Q^*$ in $L^2_{*,per}(\Omega_{per}) \times H^1_{0,per}(\Omega_{per})$. Since $u_{1,k,\eta'}^{(1)} = (L_0 + \Pi_1 M_{\eta'})^{-1} f_{1,k}$, we find that $w_{1,k,\eta'}^{(1)}$ is analytic in $\eta' \in Q^*$ in $H^1_{0,per}(\Omega_{per})$. We thus see that $b_{jk,\eta'} = (\nabla_{\eta'} w_{1,j,\eta'}^{(1)}, \nabla_{\eta'} w_{1,k,\eta'}^{(1)})$ is continuous in η', and hence, the eigenvalues of $B_{\eta'}B_{\eta'}^*$ are continuous in η'. Since $c_{2,\eta'}$ is positive for each η' and is continuous in η', we deduce that

$$c_2 = \inf_{\eta' \in Q^*, |\eta'| \geq r_0} c_{2,\eta'} > 0.$$

By (4.28) and (4.29), we have

$$\mathrm{Re}\,\{i\gamma < \tilde{\eta}' \cdot w_1^{(1)} > \bar{\sigma}\} = \mathrm{Re}\,\{\frac{\gamma^2}{\nu}|B_{\eta'}^*\eta'|^2\sigma\bar{\sigma}\} \geq c_2\frac{\gamma^2}{\nu}|\eta'|^2|\sigma|^2.$$

As for $\mathrm{Re}\{i\gamma < \tilde{\eta}' \cdot w_1^{(2)} > \overline{\sigma}\}$, by Proposition 4.14, we have

$$\mathrm{Re}\,\{i\gamma < \tilde{\eta}' \cdot w_1^{(2)} > \bar{\sigma}\} \leq \epsilon\frac{\gamma^2}{\nu}|\eta'|^2|\sigma|^2 + \frac{C\nu}{\epsilon}\|\nabla_{\eta'} w_1^{(2)}\|_2^2$$

for all $\epsilon > 0$ with $C > 0$ independent of ϵ. On the other hand, using Lemma 4.18, we see from (4.27) that

$$\begin{aligned}\|\nabla_{\eta'} w_1^{(2)}\|_2 &\leq C\{\|\mathrm{div}\,_{\eta'} w_1\|_2 + \|\tilde{f}_1 - \lambda w_1 + \tilde{\nu}\nabla_{\eta'}(\nabla_{\eta'} \cdot w_1)\|_{H^{-1}_{per}(\Omega_{per})}\} \\ &\leq C\{D_{\eta'}(w_1) + \|\lambda w_1\|_2 + \|\tilde{f}_1\|_2\},\end{aligned}$$

and hence,

$$\mathrm{Re}\{i\gamma < \tilde{\eta}' \cdot w_1 > \overline{\sigma}\} \leq \epsilon \frac{\gamma^2}{\varepsilon}|\sigma|^2 + \frac{C}{\epsilon}\{D_{\eta'}(w_1) + \|\lambda w_1\|_2 + \|\tilde{f}_1\|_2\}.$$

Taking $\epsilon = \frac{1}{4}c_2$, we arrive at

$$\mathrm{Re}\,|\sigma|^2 + \frac{3}{4}\frac{\gamma^2 c_2}{\nu}|\sigma|^2|\eta'|^2 \leq C_3\{| < f^0 > ||\sigma| + \|\tilde{f}_1\|_2^2 + |\lambda|^2\|w_1\|_2^2 + D_{\eta'}(w_1)\},$$

which yields

$$\mathrm{Re}\,\lambda|\sigma|^2 + \frac{c_0\gamma^2}{2\nu}|\eta'|^2|\sigma|^2 \leq C_2\{(1 + \frac{1}{|\eta'|^2})| < f^0 > |^2 + \|\tilde{f}_1\|_2^2 + |\lambda|^2\|w_1\|_2^2 + D_{\eta'}(w_1)\}.$$

This completes the proof. □

We now establish the resolvent estimate for $-L_{\eta'}$ with $|\eta'| \geq r_0$.

Theorem 4.21. *Let $\eta' \in Q^*$ satisfy $|\eta'| \geq r_0$. Then there exists a constant $\beta_1 > 0$ such that $\{\lambda; \mathrm{Re}\,\lambda > -\beta_1\} \subset \rho(-L_{\eta'})$ and if $\mathrm{Re}\,\lambda > -\beta_1$, then*

$$\|(\lambda + L_{\eta'})^{-1}f\|_2 + \|\nabla\tilde{\mathbb{P}}\,(\lambda + L_{\eta'})^{-1}f\|_2 \leq \frac{C}{(\mathrm{Re}\,\lambda + \beta_1)^{\frac{1}{2}}}\|f\|_2.$$

The same conclusion holds with $L_{\eta'}$ replaced by $L_{\eta'}^*$.

Proof. Set $E[u] = (1 + b_2)|\sigma|^2 + \|u_1\|_2^2 + b_1 D_{\eta'}(w_1)$ with constants b_1, $b_2 > 0$ to be determined later. It suffices to show that

$$E[u] \leq \frac{C}{\mathrm{Re}\,\lambda + \beta_1}\{| < f^0 > |^2 + \|f_1^0\|_2^2 + \|\tilde{f}_1\|_2^2\}.$$

Consider (4.17) + (4.18) $\times\, b_1$. Then taking $b_1 > 0$ suitably small, we have

$$\begin{aligned}&\mathrm{Re}\,\lambda(|\sigma|^2 + \|u_1\|_2^2 + b_1 D_{\eta'}(w_1)) + \frac{1}{4}D_{\eta'}(w_1) + b_1|\lambda|^2\|w_1\|_2^2 \\ &\qquad \leq C\{| < f^0 > ||\sigma| + |(f_1^0, \phi_1)| + \|f_1\|_2^2 + | < f^0 > |^2\}.\end{aligned} \tag{4.31}$$

We next consider (4.22) $\times\, b_2$ + (4.31). Then with a suitably small $b_2 > 0$, we have

$$\begin{aligned}&\mathrm{Re}\,\lambda E[u] + \frac{1}{4}D_{\eta'}(w_1) + \frac{b_1}{2}|\lambda|^2\|w_1\|_2^2 + \frac{b_2}{4}\frac{c_0\gamma^2}{\nu}|\eta'|^2|\sigma|^2 \\ &\leq\; C\{\big(1 + \tfrac{1}{|\eta'|^2}\big)| < f^0 > |^2 + |(f_1^0, \phi_1)| + \|f_1\|_2^2\}.\end{aligned} \tag{4.32}$$

Since ${}^\top(\phi_1, w_1)$ satisfies ${}^\top(\phi_1, w_1) \in L^2_{*,per}(\Omega_{per}) \times H^1_{0,per}(\Omega_{per})$ and

$$\begin{cases} -\Delta_{\eta'}w_1 + \nabla_{\eta'}(\frac{\gamma}{\nu}\phi_1) = \frac{1}{\nu}\tilde{f}_1 - \frac{1}{\nu}\{\lambda w_1 - \tilde{\nu}\nabla_{\eta'}(\nabla_{\eta'} \cdot w_1) + i\gamma\sigma\tilde{\eta}'\}, \\ \mathrm{div}\,_{\eta'}w_1 = \frac{1}{\gamma}\{f_1^0 - \lambda\phi_1\}, \end{cases}$$

we see from Lemma 4.18 that

$$\begin{aligned}\|\phi_1\|_2^2 &\le C\tfrac{\nu^2}{\gamma^2}\{\|\mathrm{div}_{\eta'} w_1\|_2^2 + \tfrac{1}{\nu^2}\|\tilde{f}_1\|_2^2 + \tfrac{1}{\nu^2}|\lambda|^2\|w_1\|_2^2 \\ &\qquad + \tfrac{\tilde{\nu}^2}{\nu^2}\|\nabla_{\eta'}(\nabla_{\eta'}\cdot w_1)\|^2_{H^{-1}_{per}(\Omega_{per})} + \tfrac{\gamma^2}{\nu^2}|\eta'|^2|\sigma|^2\} \\ &\le C\big\{\tfrac{(\nu+\tilde{\nu})^2}{\gamma^2}\|\nabla_{\eta'} w_1\|_2^2 + \tfrac{1}{\gamma^2}\|\tilde{f}_1\|_2^2 + \tfrac{1}{\gamma^2}|\lambda|^2\|w_1\|_2^2 + |\eta'|^2|\sigma|^2\big\}.\end{aligned} \tag{4.33}$$

We consider (4.33) $\times\, b_3$ + (4.32). Taking $b_3 > 0$ suitably small, we have

$$\begin{aligned}&\mathrm{Re}\,\lambda E[u] + \tfrac{1}{8}D_{\eta'}(w_1) + \tfrac{b_1}{4}|\lambda|\|w_1\|_2^2 + \tfrac{b_3}{2}\|\phi_1\|_2^2 + \tfrac{b_2c_2}{8}\tfrac{\gamma^2}{\nu}|\eta'|^2|\sigma|^2 \\ &\le C\{\big(1+\tfrac{1}{|\eta'|^2}\big)|<f^0>|^2 + \|f_1^0\|_2^2 + \|\tilde{f}_1\|_2^2 + |(f_1^0,\phi_1)|\} \\ &\le \tfrac{b_3}{4}\|\phi_1\|_2^2 + C\{(1+\tfrac{1}{|\eta'|^2})|<f^0>|^2 + \|f_1^0\|_2^2 + \|\tilde{f}_1\|_2^2\},\end{aligned}$$

and hence,

$$\begin{aligned}&\mathrm{Re}\,\lambda E[u] + \tfrac{1}{8}D_{\eta'}(w_1) + \tfrac{b_1}{4}|\lambda|\|w_1\|_2^2 + \tfrac{b_3}{4}\|\phi_1\|_2^2 + \tfrac{b_2c_2}{8}\tfrac{\gamma^2}{\nu}|\eta'|^2|\sigma|^2 \\ &\quad \le C\{\big(1+\tfrac{1}{|\eta'|^2}\big)|<f^0>|^2 + \|f_1^0\|_2^2 + \|\tilde{f}_1\|_2^2\}.\end{aligned}$$

Using the Poincaré inequality, we have $\frac{1}{16}D_{\eta'}(w_1) + \frac{b_3}{4}\|\phi_1\|_2^2 + \frac{b_2c_2}{8}|\eta'|^2|\sigma|^2 \ge \beta_1 E[u]$ for some constant $\beta_1 = \beta_1(r_0) > 0$. We thus obtain

$$(\mathrm{Re}\,\lambda + \beta_1)E[u] + \frac{1}{16}D_{\eta'}(w_1) \le C\{|<f^0>|^2 + \|f_1^0\|_2^2 + \|\tilde{f}_1\|_2^2\}$$

for η' with $|\eta'| \ge r_0$. This completes the proof. □

We have already shown in Proposition 4.2 that $-L_{\eta'}$ generates a contraction semigroup $e^{-tL_{\eta'}}$. Theorem 4.21 implies that $e^{-tL_{\eta'}}$ decays exponentially for $\eta' \in Q^*$ with $|\eta'| \ge r_0$.

Theorem 4.22. *There holds the estimate*

$$\|e^{-tL_{\eta'}}u_0\|_2 \le Ce^{-\frac{\beta_1}{2}t}\|u_0\|_2$$

uniformly for $\eta' \in Q^*$ satisfying $|\eta'| \ge r_0$.

Theorem 4.22 follows from Theorem 4.21 and [3, Chap. V, Theorem 1.11].

5 Proof of Theorems 3.1 and 3.2

In this section we give proofs of Theorem 3.1 and 3.2.

Proof of Theorem 3.1. As in the proof of Proposition 4.1, one can show that $\{\lambda\,;\,\mathrm{Re}\,\lambda > 0\} \subset \rho(-L)$ and if $\mathrm{Re}\,\lambda > 0$, then

$$\begin{aligned}&\|(\lambda+L)^{-1}f\|_{L^2(\Omega)} \le \tfrac{1}{\mathrm{Re}\,\lambda}\|f\|_{L^2(\Omega)}, \\ &\|\nabla\tilde{\mathbb{P}}\,|(\lambda+L)^{-1}f\|_{L^2(\Omega)} \le \tfrac{1}{(\nu\mathrm{Re}\,\lambda)^{\frac{1}{2}}}\|f\|_{L^2(\Omega)}.\end{aligned} \tag{5.1}$$

Therefore, $-L$ generates a contraction semigroup e^{-tL} on $L^2(\Omega)$. This completes the proof. $\square$

Proof of Theorem 3.2. We set

$$\Pi := U\chi_0\Pi_{\eta'}T, \qquad \chi_0(\eta') = \begin{cases} 1 & |\eta'| \le r_0, \\ 0 & |\eta'| \ge r_0. \end{cases}$$

It then follows from Proposition 2.2 that $\Pi^2 = \Pi$. Furthermore, by Theorem 4.7, we have

$$e^{-tL}\Pi u_0 = U\chi_0 e^{-tL_{\eta'}}\Pi_{\eta'}Tu_0 = U\chi_0 e^{\lambda_{\eta'}t}\Pi_{\eta'}Tu_0.$$

Since

$$\sup_{\eta'\in Q^*}\|Tu_0\|_1 \le C\|u_0\|_{L^1(\Omega)},$$

we see from Theorems 4.7 and 4.11 that

$$\begin{aligned}\|e^{-tL}\Pi u_0\|_{L^2(\Omega)}^2 &\le C\int_{\eta'\in Q^*}\|\chi_0 e^{-tL_{\eta'}}\Pi_{\eta'}Tu_0\|_2^2\,d\eta' \le C\int_{|\eta'|\le r_0} e^{-\frac{\kappa_0}{2}\frac{\gamma^2}{\nu}|\eta'|^2t}\|\Pi_{\eta'}Tu_0\|_2^2\,d\eta' \\ &\le C\int_{|\eta'|\le r_0} e^{-\frac{\kappa_0}{2}\frac{\gamma^2}{\nu}|\eta'|^2t}\|Tu_0\|_1^2\,d\eta' \le Ct^{-\frac{n-1}{2}}\|u_0\|_{L^1(\Omega)}^2.\end{aligned}$$

On the other hand, we have

$$\|e^{-tL}\Pi u_0\|_{L^2(\Omega)}^2 \le C\int_{|\eta'|\le r_0} d\eta'\|u_0\|_{L^1(\Omega)}^2 \le C\|u_0\|_{L^1(\Omega)}^2.$$

We thus obtain

$$\|e^{-tL}\Pi u_0\|_{L^2(\Omega)} \le C(1+t)^{-\frac{n-1}{4}}\|u_0\|_{L^1(\Omega)}.$$

This proves (i) of Theorem 3.2.

As for the estimate for $e^{-tL}(I-\Pi)u_0$, we write it as

$$\begin{aligned}e^{-tL}(I-\Pi)u_0 &= U\chi_0 e^{-tL_{\eta'}}(I-\Pi_{\eta'})T + U(1-\chi_0)e^{-tL_{\eta'}}T \\ &= U\chi_0(e^{-tL_{\eta'}} - e^{\lambda_{\eta'}t}\Pi_{\eta'})T + U(1-\chi_0)e^{-tL_{\eta'}}T.\end{aligned}$$

It follows from Theorems 4.10 and 4.22 that

$$\|e^{-tL}(I-\Pi)u_0\|_{L^2(\Omega)} \le Ce^{-\beta t}\|u\|_{L^2(\Omega)},$$

where $\beta = \frac{1}{2}\min\{\beta_0, \beta_1\}$. This proves (ii) of Theorem 3.2.

Let us prove (iii) of Theorem 3.2. We write $e^{-tL}\Pi u_0$ as

$$e^{-tL}\Pi u_0 = U\chi_0 e^{\lambda\eta' t}\Pi^{(0)}Tu_0 + U\chi_0 e^{\lambda\eta' t}(\Pi_{\eta'} - \Pi^{(0)})Tu_0 =: J_1 + J_2.$$

For $\ell' = (\ell_1, \cdots, \ell_{n-1}) \in \mathbb{Z}^{n-1}$, we denote by $\Omega_{per,\ell'}$ the set $\{(x' + \sum_{j=1}^{n-1} \frac{2\pi}{\alpha_j} \boldsymbol{e}'_j, x_n, ; (x', x_n) \in \Omega_{per}\}$. By the definition of T, we have

$$\begin{aligned}
\Pi^{(0)} T u_0 &= \left[\int_{\Omega_{per}} < (T\phi_0)(x', \cdot) > \, dx \right] u^{(0)} \\
&= \left[\frac{1}{|\Omega_{per}||Q^*|^{\frac{1}{2}}} \sum_{\ell' \in \mathbb{Z}} \int_{\Omega_{per,\ell'}} \phi_0(x) e^{-i\eta' \cdot x'} \, dx \right] u^{(0)} \\
&= \left[\frac{1}{|\Omega_{per}||Q^*|^{\frac{1}{2}}} \int_{\Omega} \phi_0(x) e^{-i\eta' \cdot x'} \, dx \right] u^{(0)} = \frac{1}{(2\pi)^{\frac{n-1}{2}} |Q|^{\frac{1}{2}}} \hat{\sigma}_0(\eta') u^{(0)},
\end{aligned}$$

where

$$\sigma_0(x') = \frac{|Q|}{|\Omega_{per}|} \int_{\omega_1(x')}^{\omega_2(x')} \phi_0(x', x_n) \, dx_n.$$

It then follows that J_1 is written as

$$J_1 = \left[\frac{1}{(2\pi)^{n-1}} \int_{Q^*} \chi_0 e^{\lambda \eta' t} \hat{\sigma}_0(\eta') e^{i\eta' \cdot x'} d\eta' \right] u^{(0)} = [e^{-tH} \sigma_0(x')] u^{(0)} + J_1^{(1)} + J_1^{(2)}.$$

Here

$$J_1^{(1)} = \mathcal{F}^{-1} \left[(\chi_0 - 1) e^{-\frac{\gamma^2}{\nu} \kappa(\eta') t} \widehat{\sigma_0}(\eta') \right] u^{(0)},$$

$$J_1^{(2)} = \left[\frac{1}{(2\pi)^{n-1}} \int_{Q^*} \chi_0 (e^{\lambda_{\eta'} t} - e^{-\frac{\gamma^2}{\nu} \kappa(\eta') t}) \widehat{\sigma_0}(\eta') e^{i\eta' \cdot x'} \, d\eta' \right] u^{(0)}.$$

By the Plancherel Theorem, $J_1^{(1)}$ is estimated as

$$\begin{aligned}
\|J_1^{(1)}\|_{L^2(\Omega)}^2 &\le \bar{d} \|\mathcal{F}^{-1}[(\chi_0 - 1) e^{-\frac{\gamma^2}{\nu} \kappa(\eta') t} \widehat{\sigma_0}] u^{(0)}\|_{L^2(\mathbb{R}^{n-1})}^2 \\
&= (2\pi)^{-(n-1)} \bar{d} \|(\chi_0 - 1) e^{-\frac{\gamma^2}{\nu} \kappa(\eta') t} \widehat{\sigma_0}\|_{L^2(\mathbb{R}^{n-1})}^2
\end{aligned}$$

with $\bar{d} > 0$ given by $\bar{d} = \max_{x' \in \mathbb{R}^{n-1}} \{\omega_2(x') - \omega_1(x')\}$. Since $\mathrm{supp}\,(\chi_0 - 1) = \{|\eta'| \ge r_0\}$, we see that

$$\|(\chi_0 - 1) e^{-\frac{\gamma^2}{\nu} \kappa(\eta') t} \widehat{\sigma_0}\|_{L^2(\mathbb{R}^{n-1})}^2 \le C t^{-\frac{n-1}{2}} e^{-\frac{\gamma^2}{\nu} r_0^2 t} \|\phi_0\|_{L^1(\Omega)}^2,$$

and hence,

$$\|J_1^{(1)}\|_2 \le C t^{-\frac{n-1}{4}} e^{-\frac{\gamma^2}{2\nu} r_0^2 t} \|\phi_0\|_{L^1(\Omega)}.$$

As for $J_1^{(2)}$, we have

$$e^{\lambda_{\eta'} t} - e^{-\kappa(\eta') t} = (\lambda_{\eta'} + \kappa(\eta')) t \int_0^1 e^{-\kappa(\eta') t + \theta(\lambda_{\eta'} + \kappa(\eta')) t} d\theta.$$

Since $\lambda_{\eta'} = -\frac{\gamma^2}{\nu}\kappa(\eta') + O(|\eta'|^3)$, we obtain

$$|e^{\lambda_{\eta'}t} - e^{-\frac{\gamma^2}{\nu}\kappa(\eta')t}| \le C|\eta'|^3 t e^{-\frac{\kappa_0}{2}\frac{\gamma^2}{\nu}|\eta'|^2 t} \le C|\eta'| e^{-\frac{\kappa_0}{4}\frac{\gamma^2}{\nu}|\eta'|^2 t},$$

and hence,

$$\|J_1^{(2)}\|^2_{L^2(\Omega)} \le C\int_{|\eta'|\le r_0} |\eta'|^2 e^{-\frac{\kappa_0}{2}\frac{\gamma^2}{\nu}|\eta'|^2 t}\, d\eta' \left(\sup_{\eta'\in\mathbb{R}^{n-1}} |\widehat{\sigma}_0(\eta')|\right)^2 \le Ct^{-\frac{n-1}{2}-1}\|\phi_0\|^2_{L^1(\Omega)}.$$

Concerning J_2, we see from Theorem 4.11 that

$$\begin{aligned}\|J_2\|_{L^2(\Omega)} &\le C\|\chi_0 e^{\lambda_{\eta'}t}(\Pi_{\eta'} - \Pi^{(0)})Tu_0\|_{L^2(Q^*;L^2(\Omega_{per}))}\\ &\le C\|\chi_0|\eta'|e^{\lambda_{\eta'}t}\|Tu_0\|_1\|_{L^2(Q^*)} \le C(1+t)^{-\frac{n-1}{4}-\frac{1}{2}}\|u_0\|_{L^1(\Omega)}.\end{aligned}$$

We thus obtain the desired estimate. This completes the proof. □

6 Outline of proof of Lemma 4.18

In this section we give an outline of the proof of Lemma 4.18. We here only give several lemmas necessary for the proof since Lemma 4.18 can be proved by an argument similar to the proof of [22, Chap. III, Theorem 1.4.1], where the proof for the Stokes system (i.e., $\eta' = 0$) is given.

We begin by

Lemma 6.1. *There holds the estimate*

$$\|u\|_2 \le C\{\|\nabla_{\eta'}u\|_{H^{-1}_{per}(\Omega_{per})} + \|u\|_{H^{-1}_{per}(\Omega_{per})}\}$$

for $u \in L^2(\Omega_{per})$.

Lemma 6.1 can be proved in a similar manner to that of [19, Chap. 3, Lemma 7.1]. (Cf., [22, Chap. II, Lemma 1.1.3].)

Lemma 6.2. *There hold the following inequalities for* $u \in L^2_{*,per}(\Omega_{per})$:

$$\|u\|_2 \le C_1\|\nabla_{\eta'}u\|_{H^{-1}_{per}(\Omega_{per})} \le C_1C_2\|u\|_2.$$

Lemma 6.2 follows from Lemma 6.1 as in the proof of [22, Chap. II, Lemma 1.5.4].

Lemma 6.3. (i) *For every* $g \in L^2_{*,per}(\Omega_{per})$, *there exists* $w \in H^1_{0,per}(\Omega_{per})$ *satisfying*

$$\mathrm{div}_{\eta'} w = g, \quad \|\nabla_{\eta'}w\|_2 \le C\|g\|_2.$$

(ii) *For every* $f \in H^{-1}_{per}(\Omega_{per})$ *satisfying*

$$[f, w] = 0, \quad \textit{for all } w \in H^1_{0,per}(\Omega_{per}) \textit{ with } \mathrm{div}_{\eta'} w = 0,$$

there uniquely exists $p \in L^2_{*,per}(\Omega_{per})$ *such that*

$$\nabla_{\eta'}p = f, \quad \|p\|_2 \le C\|f\|_{H^{-1}_{per}(\Omega_{per})}.$$

One can prove Lemma 6.3 by using Lemma 6.2 as in the proof of [22, Chap. II, Lemma 2.1.1].

We define $H^1_{0,\sigma}(\Omega_{per})$ by

$$H^1_{0,\sigma}(\Omega_{per}) = \{w \in H^1_{0,per}(\Omega_{per}) \ ; \ \mathrm{div}_{\eta'} w = 0\}.$$

Lemma 6.4. *For every $f \in H^{-1}_{0,per}(\Omega_{per})$, there uniquely exists $w \in H^1_{0,\sigma}(\Omega_{per})$ satisfying*

$$(\nabla_{\eta'} w, \nabla_{\eta'} v) = [f, v]$$

for all $v \in H^1_{0,\sigma}(\Omega_{per})$, and

$$\|\nabla_{\eta'} w\|_2 \leq C \|f\|_{H^{-1}_{per}(\Omega_{per})}.$$

Furthermore, there uniquely exists $\phi \in L^2_{*,per}(\Omega_{per})$ such that

$$-\Delta_{\eta'} w - f = -\nabla_{\eta'} \phi$$

and

$$\|\phi\|_2 \leq C \|f\|_{H^{-1}_{per}(\Omega_{per})}.$$

Lemma 6.4 can be proved in a similar manner to the proof of [22, Chap. III, Theorem 1.3.1].

Lemma 4.18 follows from Lemmas 6.3 and 6.4 as in the proof of [22, Chap. III, Theorem 1.4.1]. □

Acknowledgements. The work of Y. Kagei was partly supported by JSPS KAKENHI Grant Number 24340028, 22244009, 24224003.

References

[1] M. E. Bogovskii, Solution of the first boundary value problem for an equation of continuity of an incompressible medium, Soviet Math. Dokl., **20** (1979), pp. 1094–1098.

[2] W. Borchers and H. Sohr, On the equations $\mathrm{rot}\, v = g$ and $\mathrm{div}\, u = f$ with zero boundary conditions, Hokkaido Math. J., **19** (1990), pp. 67–87.

[3] K.-J. Engel and R. Nagel, *One-Parameter Semigroups for Linear Evolution Equations*, Springer, New York (1999).

[4] G. P. Galdi, *An Introduction to the Mathematical Theory of the Navier-Stokes Equations*, Vol. 1, Springer-Verlag, New York (1994).

[5] D. Hoff and K. Zumbrun, Multi-dimensional diffusion waves for the Navier-Stokes equations of compressible flow, Indiana Univ. Math. J., **44** (1995), pp. 604–676.

[6] G. Iooss and M. Padula, Structure of the linearized problem for compressible parallel fluid flows, Ann. Univ. Ferrara, Sez. VII, **43** (1998), pp. 157–171.

[7] Y. Kagei, Resolvent estimates for the linearized compressible Navier-Stokes equation in an infinite layer, Funkcial. Ekvac., **50** (2007), pp. 287–337.

[8] Y. Kagei, Asymptotic behavior of the semigroup associated with the linearized compressible Navier-Stokes equation in an infinite layer, Publ. Res. Inst. Math. Sci., **43** (2007), pp. 763–794.

[9] Y. Kagei, Large time behavior of solutions to the compressible Navier-Stokes equation in an infinite layer, Hiroshima Math. J., **38** (2008), pp. 95–124.

[10] Y. Kagei, Asymptotic behavior of the solutions to the compressible Navier-Stokes equation around a parallel flow, Arch. Rational Mech. Anal., **205** (2012), pp.585–650.

[11] Y. Kagei, and T. Kobayashi, Asymptotic behavior of solutions to the compressible Navier-Stokes equations on the half space, Arch. Rational Mech. Anal., **177** (2005), pp. 231–330.

[12] T. Kato, *Perturbation Theory for Linear Operators*, Springer-Verlag, Berlin, Heidelberg, New York (1980).

[13] S. Kawashima, Systems of a hyperbolic-parabolic composite type, with applications to the equations of magnetohydrodynamics, Ph. D. Thesis, Kyoto University (1983).

[14] T. Kobayashi, Some estimates of solutions for the equations of motion of compressible viscous fluid in an exterior domain in $\mathbf{R}^3$, J. Differential Equations, **184** (2002), pp. 587–619.

[15] T. Kobayashi and Y. Shibata, Decay estimates of solutions for the equations of motion of compressible viscous and heat-conductive gases in an exterior domain in $\mathbf{R}^3$, Comm. Math. Phys. **200** (1999), pp. 621–659.

[16] A. Matsumura and T. Nishida, The initial value problem for the equations of motion of compressible viscous and heat-conductive fluids, Proc. Japan Acad. Ser. A, **55** (1979), pp. 337–342.

[17] A. Matsumura and T. Nishida, The initial value problems for the equations of motion of viscous and heat-conductive gases, J. Math. Kyoto Univ., **20** (1980), pp.67–104.

[18] A. Matsumura and T. Nishida, Initial boundary value problems for the equations of motion of compressible viscous and heat-conductive fluids, Comm. Math. Phys., **89** (1983), pp. 445–464.

[19] J. Necǎs, *Les méthodes directes en théorie des équations elliptiques*, Masson et Cie, Éditeurs, Paris, Academia, Éditeurs, Prague (1967).

[20] M. Okita, On the convergence rates for the compressible Navier- Stokes equations with potential force, to appear in Kyushu J. Math.

[21] M. Read and B. Simon, *Methods of Modern Mathematical Physics, IV: Analysis of Operators*, Academic Press, San Diego (1978).

[22] H. Sohr, *The Navier-Stokes equations : an elementary functional analytic approach*, Birkh´auser, Basel (2001).

[23] S. Ukai, Solutions of the Boltzmann equation, *Patterns and waves*, Stud. Math. Appl., **18**, North-Holland, Amsterdam (1986).

List of MI Preprint Series, Kyushu University

The Global COE Program
Math-for-Industry Education & Research Hub

MI

MI2008-1 Takahiro ITO, Shuichi INOKUCHI & Yoshihiro MIZOGUCHI
Abstract collision systems simulated by cellular automata

MI2008-2 Eiji ONODERA
The intial value problem for a third-order dispersive flow into compact almost Hermitian manifolds

MI2008-3 Hiroaki KIDO
On isosceles sets in the 4-dimensional Euclidean space

MI2008-4 Hirofumi NOTSU
Numerical computations of cavity flow problems by a pressure stabilized characteristic-curve finite element scheme

MI2008-5 Yoshiyasu OZEKI
Torsion points of abelian varieties with values in nfinite extensions over a p-adic field

MI2008-6 Yoshiyuki TOMIYAMA
Lifting Galois representations over arbitrary number fields

MI2008-7 Takehiro HIROTSU & Setsuo TANIGUCHI
The random walk model revisited

MI2008-8 Silvia GANDY, Masaaki KANNO, Hirokazu ANAI & Kazuhiro YOKOYAMA
Optimizing a particular real root of a polynomial by a special cylindrical algebraic decomposition

MI2008-9 Kazufumi KIMOTO, Sho MATSUMOTO & Masato WAKAYAMA
Alpha-determinant cyclic modules and Jacobi polynomials

MI2008-10 Sangyeol LEE & Hiroki MASUDA
Jarque-Bera Normality Test for the Driving Lévy Process of a Discretely Observed Univariate SDE

MI2008-11 Hiroyuki CHIHARA & Eiji ONODERA
A third order dispersive flow for closed curves into almost Hermitian manifolds

MI2008-12 Takehiko KINOSHITA, Kouji HASHIMOTO and Mitsuhiro T. NAKAO
On the L^2 a priori error estimates to the finite element solution of elliptic problems with singular adjoint operator

MI2008-13 Jacques FARAUT and Masato WAKAYAMA
Hermitian symmetric spaces of tube type and multivariate Meixner-Pollaczek polynomials

MI2008-14 Takashi NAKAMURA
Riemann zeta-values, Euler polynomials and the best constant of Sobolev inequality

MI2008-15 Takashi NAKAMURA
Some topics related to Hurwitz-Lerch zeta functions

MI2009-1 Yasuhide FUKUMOTO
Global time evolution of viscous vortex rings

MI2009-2 Hidetoshi MATSUI & Sadanori KONISHI
Regularized functional regression modeling for functional response and predictors

MI2009-3 Hidetoshi MATSUI & Sadanori KONISHI
Variable selection for functional regression model via the L_1 regularization

MI2009-4 Shuichi KAWANO & Sadanori KONISHI
Nonlinear logistic discrimination via regularized Gaussian basis expansions

MI2009-5 Toshiro HIRANOUCHI & Yuichiro TAGUCHII
Flat modules and Groebner bases over truncated discrete valuation rings

MI2009-6 Kenji KAJIWARA & Yasuhiro OHTA
Bilinearization and Casorati determinant solutions to non-autonomous 1+1 dimensional discrete soliton equations

MI2009-7 Yoshiyuki KAGEI
Asymptotic behavior of solutions of the compressible Navier-Stokes equation around the plane Couette flow

MI2009-8 Shohei TATEISHI, Hidetoshi MATSUI & Sadanori KONISHI
Nonlinear regression modeling via the lasso-type regularization

MI2009-9 Takeshi TAKAISHI & Masato KIMURA
Phase field model for mode III crack growth in two dimensional elasticity

MI2009-10 Shingo SAITO
Generalisation of Mack's formula for claims reserving with arbitrary exponents for the variance assumption

MI2009-11 Kenji KAJIWARA, Masanobu KANEKO, Atsushi NOBE & Teruhisa TSUDA
Ultradiscretization of a solvable two-dimensional chaotic map associated with the Hesse cubic curve

MI2009-12 Tetsu MASUDA
Hypergeometric τ-functions of the q-Painlevé system of type $E_8^{(1)}$

MI2009-13 Hidenao IWANE, Hitoshi YANAMI, Hirokazu ANAI & Kazuhiro YOKOYAMA
A Practical Implementation of a Symbolic-Numeric Cylindrical Algebraic Decomposition for Quantifier Elimination

MI2009-14 Yasunori MAEKAWA
On Gaussian decay estimates of solutions to some linear elliptic equations and its applications

MI2009-15 Yuya ISHIHARA & Yoshiyuki KAGEI
Large time behavior of the semigroup on L^p spaces associated with the linearized compressible Navier-Stokes equation in a cylindrical domain

MI2009-16 Chikashi ARITA, Atsuo KUNIBA, Kazumitsu SAKAI & Tsuyoshi SAWABE
Spectrum in multi-species asymmetric simple exclusion process on a ring

MI2009-17 Masato WAKAYAMA & Keitaro YAMAMOTO
Non-linear algebraic differential equations satisfied by certain family of elliptic functions

MI2009-18 Me Me NAING & Yasuhide FUKUMOTO
Local Instability of an Elliptical Flow Subjected to a Coriolis Force

MI2009-19 Mitsunori KAYANO & Sadanori KONISHI
Sparse functional principal component analysis via regularized basis expansions and its application

MI2009-20 Shuichi KAWANO & Sadanori KONISHI
Semi-supervised logistic discrimination via regularized Gaussian basis expansions

MI2009-21 Hiroshi YOSHIDA, Yoshihiro MIWA & Masanobu KANEKO
Elliptic curves and Fibonacci numbers arising from Lindenmayer system with symbolic computations

MI2009-22 Eiji ONODERA
A remark on the global existence of a third order dispersive flow into locally Hermitian symmetric spaces

MI2009-23 Stjepan LUGOMER & Yasuhide FUKUMOTO
Generation of ribbons, helicoids and complex scherk surface in laser-matter Interactions

MI2009-24 Yu KAWAKAMI
Recent progress in value distribution of the hyperbolic Gauss map

MI2009-25 Takehiko KINOSHITA & Mitsuhiro T. NAKAO
On very accurate enclosure of the optimal constant in the a priori error estimates for H_0^2-projection

MI2009-26 Manabu YOSHIDA
Ramification of local fields and Fontaine's property (Pm)

MI2009-27 Yu KAWAKAMI
Value distribution of the hyperbolic Gauss maps for flat fronts in hyperbolic three-space

MI2009-28 Masahisa TABATA
Numerical simulation of fluid movement in an hourglass by an energy-stable finite element scheme

MI2009-29 Yoshiyuki KAGEI & Yasunori MAEKAWA
Asymptotic behaviors of solutions to evolution equations in the presence of translation and scaling invariance

MI2009-30 Yoshiyuki KAGEI & Yasunori MAEKAWA
On asymptotic behaviors of solutions to parabolic systems modelling chemotaxis

MI2009-31 Masato WAKAYAMA & Yoshinori YAMASAKI
Hecke's zeros and higher depth determinants

MI2009-32 Olivier PIRONNEAU & Masahisa TABATA
Stability and convergence of a Galerkin-characteristics finite element scheme of lumped mass type

MI2009-33 Chikashi ARITA
Queueing process with excluded-volume effect

MI2009-34 Kenji KAJIWARA, Nobutaka NAKAZONO & Teruhisa TSUDA
Projective reduction of the discrete Painlevé system of type$(A_2 + A_1)^{(1)}$

MI2009-35 Yosuke MIZUYAMA, Takamasa SHINDE, Masahisa TABATA & Daisuke TAGAMI
Finite element computation for scattering problems of micro-hologram using DtN map

MI2009-36 Reiichiro KAWAI & Hiroki MASUDA
Exact simulation of finite variation tempered stable Ornstein-Uhlenbeck processes

MI2009-37 Hiroki MASUDA
On statistical aspects in calibrating a geometric skewed stable asset price model

MI2010-1 Hiroki MASUDA
Approximate self-weighted LAD estimation of discretely observed ergodic Ornstein-Uhlenbeck processes

MI2010-2 Reiichiro KAWAI & Hiroki MASUDA
Infinite variation tempered stable Ornstein-Uhlenbeck processes with discrete observations

MI2010-3 Kei HIROSE, Shuichi KAWANO, Daisuke MIIKE & Sadanori KONISHI
Hyper-parameter selection in Bayesian structural equation models

MI2010-4 Nobuyuki IKEDA & Setsuo TANIGUCHI
The Itô-Nisio theorem, quadratic Wiener functionals, and 1-solitons

MI2010-5 Shohei TATEISHI & Sadanori KONISHI
Nonlinear regression modeling and detecting change point via the relevance vector machine

MI2010-6 Shuichi KAWANO, Toshihiro MISUMI & Sadanori KONISHI
Semi-supervised logistic discrimination via graph-based regularization

MI2010-7 Teruhisa TSUDA
UC hierarchy and monodromy preserving deformation

MI2010-8 Takahiro ITO
Abstract collision systems on groups

MI2010-9 Hiroshi YOSHIDA, Kinji KIMURA, Naoki YOSHIDA, Junko TANAKA & Yoshihiro MIWA
An algebraic approach to underdetermined experiments

MI2010-10 Kei HIROSE & Sadanori KONISHI
Variable selection via the grouped weighted lasso for factor analysis models

MI2010-11 Katsusuke NABESHIMA & Hiroshi YOSHIDA
Derivation of specific conditions with Comprehensive Groebner Systems

MI2010-12 Yoshiyuki KAGEI, Yu NAGAFUCHI & Takeshi SUDOU
Decay estimates on solutions of the linearized compressible Navier-Stokes equation around a Poiseuille type flow

MI2010-13 Reiichiro KAWAI & Hiroki MASUDA
On simulation of tempered stable random variates

MI2010-14 Yoshiyasu OZEKI
Non-existence of certain Galois representations with a uniform tame inertia weight

MI2010-15 Me Me NAING & Yasuhide FUKUMOTO
Local Instability of a Rotating Flow Driven by Precession of Arbitrary Frequency

MI2010-16 Yu KAWAKAMI & Daisuke NAKAJO
The value distribution of the Gauss map of improper affine spheres

MI2010-17 Kazunori YASUTAKE
On the classification of rank 2 almost Fano bundles on projective space

MI2010-18 Toshimitsu TAKAESU
Scaling limits for the system of semi-relativistic particles coupled to a scalar bose field

MI2010-19 Reiichiro KAWAI & Hiroki MASUDA
Local asymptotic normality for normal inverse Gaussian Lévy processes with high-frequency sampling

MI2010-20 Yasuhide FUKUMOTO, Makoto HIROTA & Youichi MIE
Lagrangian approach to weakly nonlinear stability of an elliptical flow

MI2010-21 Hiroki MASUDA
Approximate quadratic estimating function for discretely observed Lévy driven SDEs with application to a noise normality test

MI2010-22 Toshimitsu TAKAESU
A Generalized Scaling Limit and its Application to the Semi-Relativistic Particles System Coupled to a Bose Field with Removing Ultraviolet Cutoffs

MI2010-23 Takahiro ITO, Mitsuhiko FUJIO, Shuichi INOKUCHI & Yoshihiro MIZOGUCHI
Composition, union and division of cellular automata on groups

MI2010-24 Toshimitsu TAKAESU
A Hardy's Uncertainty Principle Lemma in Weak Commutation Relations of Heisenberg-Lie Algebra

MI2010-25 Toshimitsu TAKAESU
On the Essential Self-Adjointness of Anti-Commutative Operators

MI2010-26 Reiichiro KAWAI & Hiroki MASUDA
On the local asymptotic behavior of the likelihood function for Meixner Lévy processes under high-frequency sampling

MI2010-27 Chikashi ARITA & Daichi YANAGISAWA
Exclusive Queueing Process with Discrete Time

MI2010-28 Jun-ichi INOGUCHI, Kenji KAJIWARA, Nozomu MATSUURA & Yasuhiro OHTA
Motion and Bäcklund transformations of discrete plane curves

MI2010-29 Takanori YASUDA, Masaya YASUDA, Takeshi SHIMOYAMA & Jun KOGURE
On the Number of the Pairing-friendly Curves

MI2010-30 Chikashi ARITA & Kohei MOTEGI
Spin-spin correlation functions of the q-VBS state of an integer spin model

MI2010-31 Shohei TATEISHI & Sadanori KONISHI
Nonlinear regression modeling and spike detection via Gaussian basis expansions

MI2010-32 Nobutaka NAKAZONO
Hypergeometric τ functions of the q-Painlevé systems of type $(A_2 + A_1)^{(1)}$

MI2010-33 Yoshiyuki KAGEI
Global existence of solutions to the compressible Navier-Stokes equation around parallel flows

MI2010-34 Nobushige KUROKAWA, Masato WAKAYAMA & Yoshinori YAMASAKI
Milnor-Selberg zeta functions and zeta regularizations

MI2010-35 Kissani PERERA & Yoshihiro MIZOGUCHI
Laplacian energy of directed graphs and minimizing maximum outdegree algorithms

MI2010-36 Takanori YASUDA
CAP representations of inner forms of $Sp(4)$ with respect to Klingen parabolic subgroup

MI2010-37 Chikashi ARITA & Andreas SCHADSCHNEIDER
Dynamical analysis of the exclusive queueing process

MI2011-1 Yasuhide FUKUMOTO& Alexander B. SAMOKHIN
Singular electromagnetic modes in an anisotropic medium

MI2011-2 Hiroki KONDO, Shingo SAITO & Setsuo TANIGUCHI
Asymptotic tail dependence of the normal copula

MI2011-3 Takehiro HIROTSU, Hiroki KONDO, Shingo SAITO, Takuya SATO, Tatsushi TANAKA & Setsuo TANIGUCHI
Anderson-Darling test and the Malliavin calculus

MI2011-4 Hiroshi INOUE, Shohei TATEISHI & Sadanori KONISHI
Nonlinear regression modeling via Compressed Sensing

MI2011-5 Hiroshi INOUE
Implications in Compressed Sensing and the Restricted Isometry Property

MI2011-6 Daeju KIM & Sadanori KONISHI
Predictive information criterion for nonlinear regression model based on basis expansion methods

MI2011-7 Shohei TATEISHI, Chiaki KINJYO & Sadanori KONISHI
Group variable selection via relevance vector machine

MI2011-8 Jan BREZINA & Yoshiyuki KAGEI
Decay properties of solutions to the linearized compressible Navier-Stokes equation around time-periodic parallel flow
Group variable selection via relevance vector machine

MI2011-9 Chikashi ARITA, Arvind AYYER, Kirone MALLICK & Sylvain PROLHAC
Recursive structures in the multispecies TASEP

MI2011-10 Kazunori YASUTAKE
On projective space bundle with nef normalized tautological line bundle

MI2011-11 Hisashi ANDO, Mike HAY, Kenji KAJIWARA & Tetsu MASUDA
An explicit formula for the discrete power function associated with circle patterns of Schramm type

MI2011-12 Yoshiyuki KAGEI
Asymptotic behavior of solutions to the compressible Navier-Stokes equation around a parallel flow

MI2011-13 Vladimír CHALUPECKÝ & Adrian MUNTEAN
Semi-discrete finite difference multiscale scheme for a concrete corrosion model: approximation estimates and convergence

MI2011-14 Jun-ichi INOGUCHI, Kenji KAJIWARA, Nozomu MATSUURA & Yasuhiro OHTA
Explicit solutions to the semi-discrete modified KdV equation and motion of discrete plane curves

MI2011-15 Hiroshi INOUE
A generalization of restricted isometry property and applications to compressed sensing

MI2011-16 Yu KAWAKAMI
A ramification theorem for the ratio of canonical forms of flat surfaces in hyperbolic three-space

MI2011-17 Naoyuki KAMIYAMA
Matroid intersection with priority constraints

MI2012-1 Kazufumi KIMOTO & Masato WAKAYAMA
Spectrum of non-commutative harmonic oscillators and residual modular forms

MI2012-2 Hiroki MASUDA
Mighty convergence of the Gaussian quasi-likelihood random fields for ergodic Levy driven SDE observed at high frequency

MI2012-3 Hiroshi INOUE
A Weak RIP of theory of compressed sensing and LASSO

MI2012-4 Yasuhide FUKUMOTO & Youich MIE
Hamiltonian bifurcation theory for a rotating flow subject to elliptic straining field

MI2012-5 Yu KAWAKAMI
On the maximal number of exceptional values of Gauss maps for various classes of surfaces

MI2012-6 Marcio GAMEIRO, Yasuaki HIRAOKA, Shunsuke IZUMI, Miroslav KRAMAR, Konstantin MISCHAIKOW & Vidit NANDA
Topological Measurement of Protein Compressibility via Persistence Diagrams

MI2012-7 Nobutaka NAKAZONO & Seiji NISHIOKA
Solutions to a q-analog of Painlevé III equation of type $D_7^{(1)}$

MI2012-8 Naoyuki KAMIYAMA
A new approach to the Pareto stable matching problem

MI2012-9 Jan BREZINA & Yoshiyuki KAGEI
Spectral properties of the linearized compressible Navier-Stokes equation around time-periodic parallel flow

MI2012-10 Jan BREZINA
Asymptotic behavior of solutions to the compressible Navier-Stokes equation around a time-periodic parallel flow

MI2012-11 Daeju KIM, Shuichi KAWANO & Yoshiyuki NINOMIYA
Adaptive basis expansion via the extended fused lasso

MI2012-12 Masato WAKAYAMA
On simplicity of the lowest eigenvalue of non-commutative harmonic oscillators

MI2012-13 Masatoshi OKITA
On the convergence rates for the compressible
Navier- Stokes equations with potential force

MI2013-1 Abuduwaili PAERHATI & Yasuhide FUKUMOTO
A Counter-example to Thomson-Tait-Chetayev's Theorem

MI2013-2 Yasuhide FUKUMOTO & Hirofumi SAKUMA
A unified view of topological invariants of barotropic and baroclinic fluids and their application to formal stability analysis of three-dimensional ideal gas flows

MI2013-3 Hiroki MASUDA
Asymptotics for functionals of self-normalized residuals of discretely observed stochastic processes

MI2013-4 Naoyuki KAMIYAMA
On Counting Output Patterns of Logic Circuits

MI2013-5 Hiroshi INOUE
RIPless Theory for Compressed Sensing

MI2013-6 Hiroshi INOUE
Improved bounds on Restricted isometry for compressed sensing

MI2013-7 Hidetoshi MATSUI
Variable and boundary selection for functional data via multiclass logistic regression modeling

MI2013-8 Hidetoshi MATSUI
Variable selection for varying coefficient models with the sparse regularization

MI2013-9 Naoyuki KAMIYAMA
Packing Arborescences in Acyclic Temporal Networks

MI2013-10 Masato WAKAYAMA
Equivalence between the eigenvalue problem of non-commutative harmonic oscillators and existence of holomorphic solutions of Heun's differential equations, eigenstates degeneration, and Rabi's model

MI2013-11 Masatoshi OKITA
Optimal decay rate for strong solutions in critical spaces to the compressible Navier-Stokes equations

MI2013-12 Shuichi KAWANO, Ibuki HOSHINA, Kazuki MATSUDA & Sadanori KONISHI
Predictive model selection criteria for Bayesian lasso

MI2013-13 Hayato CHIBA
The First Painleve Equation on the Weighted Projective Space

MI2013-14 Hidetoshi MATSUI
Variable selection for functional linear models with functional predictors and a functional response

MI2013-15 Naoyuki KAMIYAMA
The Fault-Tolerant Facility Location Problem with Submodular Penalties

MI2013-16 Hidetoshi MATSUI
Selection of classification boundaries using the logistic regression

MI2014-1 Naoyuki KAMIYAMA
Popular Matchings under Matroid Constraints

MI2014-2 Yasuhide FUKUMOTO & Youichi MIE
Lagrangian approach to weakly nonlinear interaction of Kelvin waves and a symmetry-breaking bifurcation of a rotating flow

MI2014-3 Reika AOYAMA
Decay estimates on solutions of the linearized compressible Navier-Stokes equation around a Parallel flow in a cylindrical domain

MI2014-4 Naoyuki KAMIYAMA
The Popular Condensation Problem under Matroid Constraints

MI2014-5 Yoshiyuki KAGEI & Kazuyuki TSUDA
Existence and stability of time periodic solution to the compressible Navier-Stokes equation for time periodic external force with symmetry

MI2014-6 This paper was withdrawn by the authors.

MI2014-7 Masatoshi OKITA
On decay estimate of strong solutions in critical spaces for the compressible Navier-Stokes equations

MI2014-8 Rong ZOU & Yasuhide FUKUMOTO
Local stability analysis of azimuthal magnetorotational instability of ideal MHD flows

MI2014-9 Yoshiyuki KAGEI & Naoki MAKIO
Spectral properties of the linearized semigroup of the compressible Navier-Stokes equation on a periodic layer

List of MI Preprint Series, Kyushu University

The Global COE Program
Math-for-Industry Education & Research Hub

MI

MI2008-1 Takahiro ITO, Shuichi INOKUCHI & Yoshihiro MIZOGUCHI
Abstract collision systems simulated by cellular automata

MI2008-2 Eiji ONODERA
The intial value problem for a third-order dispersive flow into compact almost Hermitian manifolds

MI2008-3 Hiroaki KIDO
On isosceles sets in the 4-dimensional Euclidean space

MI2008-4 Hirofumi NOTSU
Numerical computations of cavity flow problems by a pressure stabilized characteristic-curve finite element scheme

MI2008-5 Yoshiyasu OZEKI
Torsion points of abelian varieties with values in nfinite extensions over a p-adic field

MI2008-6 Yoshiyuki TOMIYAMA
Lifting Galois representations over arbitrary number fields

MI2008-7 Takehiro HIROTSU & Setsuo TANIGUCHI
The random walk model revisited

MI2008-8 Silvia GANDY, Masaaki KANNO, Hirokazu ANAI & Kazuhiro YOKOYAMA
Optimizing a particular real root of a polynomial by a special cylindrical algebraic decomposition

MI2008-9 Kazufumi KIMOTO, Sho MATSUMOTO & Masato WAKAYAMA
Alpha-determinant cyclic modules and Jacobi polynomials

MI2008-10 Sangyeol LEE & Hiroki MASUDA
Jarque-Bera Normality Test for the Driving Lévy Process of a Discretely Observed Univariate SDE

MI2008-11 Hiroyuki CHIHARA & Eiji ONODERA
A third order dispersive flow for closed curves into almost Hermitian manifolds

MI2008-12 Takehiko KINOSHITA, Kouji HASHIMOTO and Mitsuhiro T. NAKAO
On the L^2 a priori error estimates to the finite element solution of elliptic problems with singular adjoint operator

MI2008-13 Jacques FARAUT and Masato WAKAYAMA
Hermitian symmetric spaces of tube type and multivariate Meixner-Pollaczek polynomials

MI2008-14 Takashi NAKAMURA
Riemann zeta-values, Euler polynomials and the best constant of Sobolev inequality

MI2008-15 Takashi NAKAMURA
Some topics related to Hurwitz-Lerch zeta functions

MI2009-1 Yasuhide FUKUMOTO
Global time evolution of viscous vortex rings

MI2009-2 Hidetoshi MATSUI & Sadanori KONISHI
Regularized functional regression modeling for functional response and predictors

MI2009-3 Hidetoshi MATSUI & Sadanori KONISHI
Variable selection for functional regression model via the L_1 regularization

MI2009-4 Shuichi KAWANO & Sadanori KONISHI
Nonlinear logistic discrimination via regularized Gaussian basis expansions

MI2009-5 Toshiro HIRANOUCHI & Yuichiro TAGUCHII
Flat modules and Groebner bases over truncated discrete valuation rings

MI2009-6 Kenji KAJIWARA & Yasuhiro OHTA
Bilinearization and Casorati determinant solutions to non-autonomous 1+1 dimensional discrete soliton equations

MI2009-7 Yoshiyuki KAGEI
Asymptotic behavior of solutions of the compressible Navier-Stokes equation around the plane Couette flow

MI2009-8 Shohei TATEISHI, Hidetoshi MATSUI & Sadanori KONISHI
Nonlinear regression modeling via the lasso-type regularization

MI2009-9 Takeshi TAKAISHI & Masato KIMURA
Phase field model for mode III crack growth in two dimensional elasticity

MI2009-10 Shingo SAITO
Generalisation of Mack's formula for claims reserving with arbitrary exponents for the variance assumption

MI2009-11 Kenji KAJIWARA, Masanobu KANEKO, Atsushi NOBE & Teruhisa TSUDA
Ultradiscretization of a solvable two-dimensional chaotic map associated with the Hesse cubic curve

MI2009-12 Tetsu MASUDA
Hypergeometric τ-functions of the q-Painlevé system of type $E_8^{(1)}$

MI2009-13 Hidenao IWANE, Hitoshi YANAMI, Hirokazu ANAI & Kazuhiro YOKOYAMA
A Practical Implementation of a Symbolic-Numeric Cylindrical Algebraic Decomposition for Quantifier Elimination

MI2009-14 Yasunori MAEKAWA
On Gaussian decay estimates of solutions to some linear elliptic equations and its applications

MI2009-15 Yuya ISHIHARA & Yoshiyuki KAGEI
Large time behavior of the semigroup on L^p spaces associated with the linearized compressible Navier-Stokes equation in a cylindrical domain

MI2009-16 Chikashi ARITA, Atsuo KUNIBA, Kazumitsu SAKAI & Tsuyoshi SAWABE
Spectrum in multi-species asymmetric simple exclusion process on a ring

MI2009-17 Masato WAKAYAMA & Keitaro YAMAMOTO
Non-linear algebraic differential equations satisfied by certain family of elliptic functions

MI2009-18 Me Me NAING & Yasuhide FUKUMOTO
Local Instability of an Elliptical Flow Subjected to a Coriolis Force

MI2009-19 Mitsunori KAYANO & Sadanori KONISHI
Sparse functional principal component analysis via regularized basis expansions and its application

MI2009-20 Shuichi KAWANO & Sadanori KONISHI
Semi-supervised logistic discrimination via regularized Gaussian basis expansions

MI2009-21 Hiroshi YOSHIDA, Yoshihiro MIWA & Masanobu KANEKO
Elliptic curves and Fibonacci numbers arising from Lindenmayer system with symbolic computations

MI2009-22 Eiji ONODERA
A remark on the global existence of a third order dispersive flow into locally Hermitian symmetric spaces

MI2009-23 Stjepan LUGOMER & Yasuhide FUKUMOTO
Generation of ribbons, helicoids and complex scherk surface in laser-matter Interactions

MI2009-24 Yu KAWAKAMI
Recent progress in value distribution of the hyperbolic Gauss map

MI2009-25 Takehiko KINOSHITA & Mitsuhiro T. NAKAO
On very accurate enclosure of the optimal constant in the a priori error estimates for H_0^2-projection

MI2009-26 Manabu YOSHIDA
Ramification of local fields and Fontaine's property (Pm)

MI2009-27 Yu KAWAKAMI
Value distribution of the hyperbolic Gauss maps for flat fronts in hyperbolic three-space

MI2009-28 Masahisa TABATA
Numerical simulation of fluid movement in an hourglass by an energy-stable finite element scheme

MI2009-29 Yoshiyuki KAGEI & Yasunori MAEKAWA
Asymptotic behaviors of solutions to evolution equations in the presence of translation and scaling invariance

MI2009-30 Yoshiyuki KAGEI & Yasunori MAEKAWA
On asymptotic behaviors of solutions to parabolic systems modelling chemotaxis

MI2009-31 Masato WAKAYAMA & Yoshinori YAMASAKI
Hecke's zeros and higher depth determinants

MI2009-32 Olivier PIRONNEAU & Masahisa TABATA
Stability and convergence of a Galerkin-characteristics finite element scheme of lumped mass type

MI2009-33 Chikashi ARITA
Queueing process with excluded-volume effect

MI2009-34 Kenji KAJIWARA, Nobutaka NAKAZONO & Teruhisa TSUDA
Projective reduction of the discrete Painlevé system of type$(A_2 + A_1)^{(1)}$

MI2009-35 Yosuke MIZUYAMA, Takamasa SHINDE, Masahisa TABATA & Daisuke TAGAMI
Finite element computation for scattering problems of micro-hologram using DtN map

MI2009-36 Reiichiro KAWAI & Hiroki MASUDA
Exact simulation of finite variation tempered stable Ornstein-Uhlenbeck processes

MI2009-37 Hiroki MASUDA
On statistical aspects in calibrating a geometric skewed stable asset price model

MI2010-1 Hiroki MASUDA
Approximate self-weighted LAD estimation of discretely observed ergodic Ornstein-Uhlenbeck processes

MI2010-2 Reiichiro KAWAI & Hiroki MASUDA
Infinite variation tempered stable Ornstein-Uhlenbeck processes with discrete observations

MI2010-3 Kei HIROSE, Shuichi KAWANO, Daisuke MIIKE & Sadanori KONISHI
Hyper-parameter selection in Bayesian structural equation models

MI2010-4 Nobuyuki IKEDA & Setsuo TANIGUCHI
The Itô-Nisio theorem, quadratic Wiener functionals, and 1-solitons

MI2010-5 Shohei TATEISHI & Sadanori KONISHI
Nonlinear regression modeling and detecting change point via the relevance vector machine

MI2010-6 Shuichi KAWANO, Toshihiro MISUMI & Sadanori KONISHI
Semi-supervised logistic discrimination via graph-based regularization

MI2010-7 Teruhisa TSUDA
UC hierarchy and monodromy preserving deformation

MI2010-8 Takahiro ITO
Abstract collision systems on groups

MI2010-25 Toshimitsu TAKAESU
On the Essential Self-Adjointness of Anti-Commutative Operators

MI2010-26 Reiichiro KAWAI & Hiroki MASUDA
On the local asymptotic behavior of the likelihood function for Meixner Lévy processes under high-frequency sampling

MI2010-27 Chikashi ARITA & Daichi YANAGISAWA
Exclusive Queueing Process with Discrete Time

MI2010-28 Jun-ichi INOGUCHI, Kenji KAJIWARA, Nozomu MATSUURA & Yasuhiro OHTA
Motion and Bäcklund transformations of discrete plane curves

MI2010-29 Takanori YASUDA, Masaya YASUDA, Takeshi SHIMOYAMA & Jun KOGURE
On the Number of the Pairing-friendly Curves

MI2010-30 Chikashi ARITA & Kohei MOTEGI
Spin-spin correlation functions of the q-VBS state of an integer spin model

MI2010-31 Shohei TATEISHI & Sadanori KONISHI
Nonlinear regression modeling and spike detection via Gaussian basis expansions

MI2010-32 Nobutaka NAKAZONO
Hypergeometric τ functions of the q-Painlevé systems of type $(A_2 + A_1)^{(1)}$

MI2010-33 Yoshiyuki KAGEI
Global existence of solutions to the compressible Navier-Stokes equation around parallel flows

MI2010-34 Nobushige KUROKAWA, Masato WAKAYAMA & Yoshinori YAMASAKI
Milnor-Selberg zeta functions and zeta regularizations

MI2010-35 Kissani PERERA & Yoshihiro MIZOGUCHI
Laplacian energy of directed graphs and minimizing maximum outdegree algorithms

MI2010-36 Takanori YASUDA
CAP representations of inner forms of $Sp(4)$ with respect to Klingen parabolic subgroup

MI2010-37 Chikashi ARITA & Andreas SCHADSCHNEIDER
Dynamical analysis of the exclusive queueing process

MI2011-1 Yasuhide FUKUMOTO& Alexander B. SAMOKHIN
Singular electromagnetic modes in an anisotropic medium

MI2011-2 Hiroki KONDO, Shingo SAITO & Setsuo TANIGUCHI
Asymptotic tail dependence of the normal copula

MI2011-3 Takehiro HIROTSU, Hiroki KONDO, Shingo SAITO, Takuya SATO, Tatsushi TANAKA & Setsuo TANIGUCHI
Anderson-Darling test and the Malliavin calculus

MI2011-4 Hiroshi INOUE, Shohei TATEISHI & Sadanori KONISHI
Nonlinear regression modeling via Compressed Sensing

MI2011-5 Hiroshi INOUE
Implications in Compressed Sensing and the Restricted Isometry Property

MI2011-6 Daeju KIM & Sadanori KONISHI
Predictive information criterion for nonlinear regression model based on basis expansion methods

MI2011-7 Shohei TATEISHI, Chiaki KINJYO & Sadanori KONISHI
Group variable selection via relevance vector machine

MI2011-8 Jan BREZINA & Yoshiyuki KAGEI
Decay properties of solutions to the linearized compressible Navier-Stokes equation around time-periodic parallel flow
Group variable selection via relevance vector machine

MI2011-9 Chikashi ARITA, Arvind AYYER, Kirone MALLICK & Sylvain PROLHAC
Recursive structures in the multispecies TASEP

MI2011-10 Kazunori YASUTAKE
On projective space bundle with nef normalized tautological line bundle

MI2011-11 Hisashi ANDO, Mike HAY, Kenji KAJIWARA & Tetsu MASUDA
An explicit formula for the discrete power function associated with circle patterns of Schramm type

MI2011-12 Yoshiyuki KAGEI
Asymptotic behavior of solutions to the compressible Navier-Stokes equation around a parallel flow

MI2011-13 Vladimír CHALUPECKÝ & Adrian MUNTEAN
Semi-discrete finite difference multiscale scheme for a concrete corrosion model: approximation estimates and convergence

MI2011-14 Jun-ichi INOGUCHI, Kenji KAJIWARA, Nozomu MATSUURA & Yasuhiro OHTA
Explicit solutions to the semi-discrete modified KdV equation and motion of discrete plane curves

MI2011-15 Hiroshi INOUE
A generalization of restricted isometry property and applications to compressed sensing

MI2011-16 Yu KAWAKAMI
A ramification theorem for the ratio of canonical forms of flat surfaces in hyperbolic three-space

MI2011-17 Naoyuki KAMIYAMA
Matroid intersection with priority constraints

MI2012-1 Kazufumi KIMOTO & Masato WAKAYAMA
Spectrum of non-commutative harmonic oscillators and residual modular forms

MI2012-2 Hiroki MASUDA
Mighty convergence of the Gaussian quasi-likelihood random fields for ergodic Levy driven SDE observed at high frequency

MI2012-3 Hiroshi INOUE
A Weak RIP of theory of compressed sensing and LASSO

MI2012-4 Yasuhide FUKUMOTO & Youich MIE
Hamiltonian bifurcation theory for a rotating flow subject to elliptic straining field

MI2012-5 Yu KAWAKAMI
On the maximal number of exceptional values of Gauss maps for various classes of surfaces

MI2012-6 Marcio GAMEIRO, Yasuaki HIRAOKA, Shunsuke IZUMI, Miroslav KRAMAR, Konstantin MISCHAIKOW & Vidit NANDA
Topological Measurement of Protein Compressibility via Persistence Diagrams

MI2012-7 Nobutaka NAKAZONO & Seiji NISHIOKA
Solutions to a q-analog of Painlevé III equation of type $D_7^{(1)}$

MI2012-8 Naoyuki KAMIYAMA
A new approach to the Pareto stable matching problem

MI2012-9 Jan BREZINA & Yoshiyuki KAGEI
Spectral properties of the linearized compressible Navier-Stokes equation around time-periodic parallel flow

MI2012-10 Jan BREZINA
Asymptotic behavior of solutions to the compressible Navier-Stokes equation around a time-periodic parallel flow

MI2012-11 Daeju KIM, Shuichi KAWANO & Yoshiyuki NINOMIYA
Adaptive basis expansion via the extended fused lasso

MI2012-12 Masato WAKAYAMA
On simplicity of the lowest eigenvalue of non-commutative harmonic oscillators

MI2012-13 Masatoshi OKITA
On the convergence rates for the compressible
Navier- Stokes equations with potential force

MI2013-1 Abuduwaili PAERHATI & Yasuhide FUKUMOTO
A Counter-example to Thomson-Tait-Chetayev's Theorem

MI2013-2 Yasuhide FUKUMOTO & Hirofumi SAKUMA
A unified view of topological invariants of barotropic and baroclinic fluids and their application to formal stability analysis of three-dimensional ideal gas flows

MI2013-3 Hiroki MASUDA
Asymptotics for functionals of self-normalized residuals of discretely observed stochastic processes

MI2013-4 Naoyuki KAMIYAMA
On Counting Output Patterns of Logic Circuits

MI2013-5 Hiroshi INOUE
RIPless Theory for Compressed Sensing

MI2013-6 Hiroshi INOUE
Improved bounds on Restricted isometry for compressed sensing

MI2013-7 Hidetoshi MATSUI
Variable and boundary selection for functional data via multiclass logistic regression modeling

MI2013-8 Hidetoshi MATSUI
Variable selection for varying coefficient models with the sparse regularization

MI2013-9 Naoyuki KAMIYAMA
Packing Arborescences in Acyclic Temporal Networks

MI2013-10 Masato WAKAYAMA
Equivalence between the eigenvalue problem of non-commutative harmonic oscillators and existence of holomorphic solutions of Heun's differential equations, eigenstates degeneration, and Rabi's model

MI2013-11 Masatoshi OKITA
Optimal decay rate for strong solutions in critical spaces to the compressible Navier-Stokes equations

MI2013-12 Shuichi KAWANO, Ibuki HOSHINA, Kazuki MATSUDA & Sadanori KONISHI
Predictive model selection criteria for Bayesian lasso

MI2013-13 Hayato CHIBA
The First Painleve Equation on the Weighted Projective Space

MI2013-14 Hidetoshi MATSUI
Variable selection for functional linear models with functional predictors and a functional response

MI2013-15 Naoyuki KAMIYAMA
The Fault-Tolerant Facility Location Problem with Submodular Penalties

MI2013-16 Hidetoshi MATSUI
Selection of classification boundaries using the logistic regression

MI2014-1 Naoyuki KAMIYAMA
Popular Matchings under Matroid Constraints

MI2014-2 Yasuhide FUKUMOTO & Youichi MIE
Lagrangian approach to weakly nonlinear interaction of Kelvin waves and a symmetry-breaking bifurcation of a rotating flow

MI2014-3 Reika AOYAMA
Decay estimates on solutions of the linearized compressible Navier-Stokes equation around a Parallel flow in a cylindrical domain

MI2014-4 Naoyuki KAMIYAMA
The Popular Condensation Problem under Matroid Constraints

List of MI Preprint Series, Kyushu University

The Global COE Program
Math-for-Industry Education & Research Hub

MI

MI2008-1 Takahiro ITO, Shuichi INOKUCHI & Yoshihiro MIZOGUCHI
Abstract collision systems simulated by cellular automata

MI2008-2 Eiji ONODERA
The intial value problem for a third-order dispersive flow into compact almost Hermitian manifolds

MI2008-3 Hiroaki KIDO
On isosceles sets in the 4-dimensional Euclidean space

MI2008-4 Hirofumi NOTSU
Numerical computations of cavity flow problems by a pressure stabilized characteristic-curve finite element scheme

MI2008-5 Yoshiyasu OZEKI
Torsion points of abelian varieties with values in nfinite extensions over a p-adic field

MI2008-6 Yoshiyuki TOMIYAMA
Lifting Galois representations over arbitrary number fields

MI2008-7 Takehiro HIROTSU & Setsuo TANIGUCHI
The random walk model revisited

MI2008-8 Silvia GANDY, Masaaki KANNO, Hirokazu ANAI & Kazuhiro YOKOYAMA
Optimizing a particular real root of a polynomial by a special cylindrical algebraic decomposition

MI2008-9 Kazufumi KIMOTO, Sho MATSUMOTO & Masato WAKAYAMA
Alpha-determinant cyclic modules and Jacobi polynomials

MI2008-10 Sangyeol LEE & Hiroki MASUDA
Jarque-Bera Normality Test for the Driving Lévy Process of a Discretely Observed Univariate SDE

MI2008-11 Hiroyuki CHIHARA & Eiji ONODERA
A third order dispersive flow for closed curves into almost Hermitian manifolds

MI2008-12 Takehiko KINOSHITA, Kouji HASHIMOTO and Mitsuhiro T. NAKAO
On the L^2 a priori error estimates to the finite element solution of elliptic problems with singular adjoint operator

MI2008-13 Jacques FARAUT and Masato WAKAYAMA
Hermitian symmetric spaces of tube type and multivariate Meixner-Pollaczek polynomials

MI2008-14 Takashi NAKAMURA
Riemann zeta-values, Euler polynomials and the best constant of Sobolev inequality

MI2008-15 Takashi NAKAMURA
Some topics related to Hurwitz-Lerch zeta functions

MI2009-1 Yasuhide FUKUMOTO
Global time evolution of viscous vortex rings

MI2009-2 Hidetoshi MATSUI & Sadanori KONISHI
Regularized functional regression modeling for functional response and predictors

MI2009-3 Hidetoshi MATSUI & Sadanori KONISHI
Variable selection for functional regression model via the L_1 regularization

MI2009-4 Shuichi KAWANO & Sadanori KONISHI
Nonlinear logistic discrimination via regularized Gaussian basis expansions

MI2009-5 Toshiro HIRANOUCHI & Yuichiro TAGUCHII
Flat modules and Groebner bases over truncated discrete valuation rings

MI2009-6 Kenji KAJIWARA & Yasuhiro OHTA
Bilinearization and Casorati determinant solutions to non-autonomous 1+1 dimensional discrete soliton equations

MI2009-7 Yoshiyuki KAGEI
Asymptotic behavior of solutions of the compressible Navier-Stokes equation around the plane Couette flow

MI2009-8 Shohei TATEISHI, Hidetoshi MATSUI & Sadanori KONISHI
Nonlinear regression modeling via the lasso-type regularization

MI2009-9 Takeshi TAKAISHI & Masato KIMURA
Phase field model for mode III crack growth in two dimensional elasticity

MI2009-10 Shingo SAITO
Generalisation of Mack's formula for claims reserving with arbitrary exponents for the variance assumption

MI2009-11 Kenji KAJIWARA, Masanobu KANEKO, Atsushi NOBE & Teruhisa TSUDA
Ultradiscretization of a solvable two-dimensional chaotic map associated with the Hesse cubic curve

MI2009-12 Tetsu MASUDA
Hypergeometric τ-functions of the q-Painlevé system of type $E_8^{(1)}$

MI2009-13 Hidenao IWANE, Hitoshi YANAMI, Hirokazu ANAI & Kazuhiro YOKOYAMA
A Practical Implementation of a Symbolic-Numeric Cylindrical Algebraic Decomposition for Quantifier Elimination

MI2009-14 Yasunori MAEKAWA
On Gaussian decay estimates of solutions to some linear elliptic equations and its applications

MI2009-15 Yuya ISHIHARA & Yoshiyuki KAGEI
Large time behavior of the semigroup on L^p spaces associated with the linearized compressible Navier-Stokes equation in a cylindrical domain

MI2009-16 Chikashi ARITA, Atsuo KUNIBA, Kazumitsu SAKAI & Tsuyoshi SAWABE
Spectrum in multi-species asymmetric simple exclusion process on a ring

MI2009-17 Masato WAKAYAMA & Keitaro YAMAMOTO
Non-linear algebraic differential equations satisfied by certain family of elliptic functions

MI2009-18 Me Me NAING & Yasuhide FUKUMOTO
Local Instability of an Elliptical Flow Subjected to a Coriolis Force

MI2009-19 Mitsunori KAYANO & Sadanori KONISHI
Sparse functional principal component analysis via regularized basis expansions and its application

MI2009-20 Shuichi KAWANO & Sadanori KONISHI
Semi-supervised logistic discrimination via regularized Gaussian basis expansions

MI2009-21 Hiroshi YOSHIDA, Yoshihiro MIWA & Masanobu KANEKO
Elliptic curves and Fibonacci numbers arising from Lindenmayer system with symbolic computations

MI2009-22 Eiji ONODERA
A remark on the global existence of a third order dispersive flow into locally Hermitian symmetric spaces

MI2009-23 Stjepan LUGOMER & Yasuhide FUKUMOTO
Generation of ribbons, helicoids and complex scherk surface in laser-matter Interactions

MI2009-24 Yu KAWAKAMI
Recent progress in value distribution of the hyperbolic Gauss map

MI2009-25 Takehiko KINOSHITA & Mitsuhiro T. NAKAO
On very accurate enclosure of the optimal constant in the a priori error estimates for H_0^2-projection

MI2009-26 Manabu YOSHIDA
Ramification of local fields and Fontaine's property (Pm)

MI2009-27 Yu KAWAKAMI
Value distribution of the hyperbolic Gauss maps for flat fronts in hyperbolic three-space

MI2009-28 Masahisa TABATA
Numerical simulation of fluid movement in an hourglass by an energy-stable finite element scheme

MI2009-29 Yoshiyuki KAGEI & Yasunori MAEKAWA
Asymptotic behaviors of solutions to evolution equations in the presence of translation and scaling invariance

MI2009-30 Yoshiyuki KAGEI & Yasunori MAEKAWA
On asymptotic behaviors of solutions to parabolic systems modelling chemotaxis

MI2009-31 Masato WAKAYAMA & Yoshinori YAMASAKI
Hecke's zeros and higher depth determinants

MI2009-32 Olivier PIRONNEAU & Masahisa TABATA
Stability and convergence of a Galerkin-characteristics finite element scheme of lumped mass type

MI2009-33 Chikashi ARITA
Queueing process with excluded-volume effect

MI2009-34 Kenji KAJIWARA, Nobutaka NAKAZONO & Teruhisa TSUDA
Projective reduction of the discrete Painlevé system of type$(A_2 + A_1)^{(1)}$

MI2009-35 Yosuke MIZUYAMA, Takamasa SHINDE, Masahisa TABATA & Daisuke TAGAMI
Finite element computation for scattering problems of micro-hologram using DtN map

MI2009-36 Reiichiro KAWAI & Hiroki MASUDA
Exact simulation of finite variation tempered stable Ornstein-Uhlenbeck processes

MI2009-37 Hiroki MASUDA
On statistical aspects in calibrating a geometric skewed stable asset price model

MI2010-1 Hiroki MASUDA
Approximate self-weighted LAD estimation of discretely observed ergodic Ornstein-Uhlenbeck processes

MI2010-2 Reiichiro KAWAI & Hiroki MASUDA
Infinite variation tempered stable Ornstein-Uhlenbeck processes with discrete observations

MI2010-3 Kei HIROSE, Shuichi KAWANO, Daisuke MIIKE & Sadanori KONISHI
Hyper-parameter selection in Bayesian structural equation models

MI2010-4 Nobuyuki IKEDA & Setsuo TANIGUCHI
The Itô-Nisio theorem, quadratic Wiener functionals, and 1-solitons

MI2010-5 Shohei TATEISHI & Sadanori KONISHI
Nonlinear regression modeling and detecting change point via the relevance vector machine

MI2010-6 Shuichi KAWANO, Toshihiro MISUMI & Sadanori KONISHI
Semi-supervised logistic discrimination via graph-based regularization

MI2010-7 Teruhisa TSUDA
UC hierarchy and monodromy preserving deformation

MI2010-8 Takahiro ITO
Abstract collision systems on groups

MI2010-9 Hiroshi YOSHIDA, Kinji KIMURA, Naoki YOSHIDA, Junko TANAKA & Yoshihiro MIWA
An algebraic approach to underdetermined experiments

MI2010-10 Kei HIROSE & Sadanori KONISHI
Variable selection via the grouped weighted lasso for factor analysis models

MI2010-11 Katsusuke NABESHIMA & Hiroshi YOSHIDA
Derivation of specific conditions with Comprehensive Groebner Systems

MI2010-12 Yoshiyuki KAGEI, Yu NAGAFUCHI & Takeshi SUDOU
Decay estimates on solutions of the linearized compressible Navier-Stokes equation around a Poiseuille type flow

MI2010-13 Reiichiro KAWAI & Hiroki MASUDA
On simulation of tempered stable random variates

MI2010-14 Yoshiyasu OZEKI
Non-existence of certain Galois representations with a uniform tame inertia weight

MI2010-15 Me Me NAING & Yasuhide FUKUMOTO
Local Instability of a Rotating Flow Driven by Precession of Arbitrary Frequency

MI2010-16 Yu KAWAKAMI & Daisuke NAKAJO
The value distribution of the Gauss map of improper affine spheres

MI2010-17 Kazunori YASUTAKE
On the classification of rank 2 almost Fano bundles on projective space

MI2010-18 Toshimitsu TAKAESU
Scaling limits for the system of semi-relativistic particles coupled to a scalar bose field

MI2010-19 Reiichiro KAWAI & Hiroki MASUDA
Local asymptotic normality for normal inverse Gaussian Lévy processes with high-frequency sampling

MI2010-20 Yasuhide FUKUMOTO, Makoto HIROTA & Youichi MIE
Lagrangian approach to weakly nonlinear stability of an elliptical flow

MI2010-21 Hiroki MASUDA
Approximate quadratic estimating function for discretely observed Lévy driven SDEs with application to a noise normality test

MI2010-22 Toshimitsu TAKAESU
A Generalized Scaling Limit and its Application to the Semi-Relativistic Particles System Coupled to a Bose Field with Removing Ultraviolet Cutoffs

MI2010-23 Takahiro ITO, Mitsuhiko FUJIO, Shuichi INOKUCHI & Yoshihiro MIZOGUCHI
Composition, union and division of cellular automata on groups

MI2010-24 Toshimitsu TAKAESU
A Hardy's Uncertainty Principle Lemma in Weak Commutation Relations of Heisenberg-Lie Algebra

MI2011-5 Hiroshi INOUE
Implications in Compressed Sensing and the Restricted Isometry Property

MI2011-6 Daeju KIM & Sadanori KONISHI
Predictive information criterion for nonlinear regression model based on basis expansion methods

MI2011-7 Shohei TATEISHI, Chiaki KINJYO & Sadanori KONISHI
Group variable selection via relevance vector machine

MI2011-8 Jan BREZINA & Yoshiyuki KAGEI
Decay properties of solutions to the linearized compressible Navier-Stokes equation around time-periodic parallel flow
Group variable selection via relevance vector machine

MI2011-9 Chikashi ARITA, Arvind AYYER, Kirone MALLICK & Sylvain PROLHAC
Recursive structures in the multispecies TASEP

MI2011-10 Kazunori YASUTAKE
On projective space bundle with nef normalized tautological line bundle

MI2011-11 Hisashi ANDO, Mike HAY, Kenji KAJIWARA & Tetsu MASUDA
An explicit formula for the discrete power function associated with circle patterns of Schramm type

MI2011-12 Yoshiyuki KAGEI
Asymptotic behavior of solutions to the compressible Navier-Stokes equation around a parallel flow

MI2011-13 Vladimír CHALUPECKÝ & Adrian MUNTEAN
Semi-discrete finite difference multiscale scheme for a concrete corrosion model: approximation estimates and convergence

MI2011-14 Jun-ichi INOGUCHI, Kenji KAJIWARA, Nozomu MATSUURA & Yasuhiro OHTA
Explicit solutions to the semi-discrete modified KdV equation and motion of discrete plane curves

MI2011-15 Hiroshi INOUE
A generalization of restricted isometry property and applications to compressed sensing

MI2011-16 Yu KAWAKAMI
A ramification theorem for the ratio of canonical forms of flat surfaces in hyperbolic three-space

MI2011-17 Naoyuki KAMIYAMA
Matroid intersection with priority constraints

MI2012-1 Kazufumi KIMOTO & Masato WAKAYAMA
Spectrum of non-commutative harmonic oscillators and residual modular forms

MI2012-2 Hiroki MASUDA
Mighty convergence of the Gaussian quasi-likelihood random fields for ergodic Levy driven SDE observed at high frequency

MI2012-3 Hiroshi INOUE
A Weak RIP of theory of compressed sensing and LASSO

MI2012-4 Yasuhide FUKUMOTO & Youich MIE
Hamiltonian bifurcation theory for a rotating flow subject to elliptic straining field

MI2012-5 Yu KAWAKAMI
On the maximal number of exceptional values of Gauss maps for various classes of surfaces

MI2012-6 Marcio GAMEIRO, Yasuaki HIRAOKA, Shunsuke IZUMI, Miroslav KRAMAR, Konstantin MISCHAIKOW & Vidit NANDA
Topological Measurement of Protein Compressibility via Persistence Diagrams

MI2012-7 Nobutaka NAKAZONO & Seiji NISHIOKA
Solutions to a q-analog of Painlevé III equation of type $D_7^{(1)}$

MI2012-8 Naoyuki KAMIYAMA
A new approach to the Pareto stable matching problem

MI2012-9 Jan BREZINA & Yoshiyuki KAGEI
Spectral properties of the linearized compressible Navier-Stokes equation around time-periodic parallel flow

MI2012-10 Jan BREZINA
Asymptotic behavior of solutions to the compressible Navier-Stokes equation around a time-periodic parallel flow

MI2012-11 Daeju KIM, Shuichi KAWANO & Yoshiyuki NINOMIYA
Adaptive basis expansion via the extended fused lasso

MI2012-12 Masato WAKAYAMA
On simplicity of the lowest eigenvalue of non-commutative harmonic oscillators

MI2012-13 Masatoshi OKITA
On the convergence rates for the compressible
Navier- Stokes equations with potential force

MI2013-1 Abuduwaili PAERHATI & Yasuhide FUKUMOTO
A Counter-example to Thomson-Tait-Chetayev's Theorem

MI2013-2 Yasuhide FUKUMOTO & Hirofumi SAKUMA
A unified view of topological invariants of barotropic and baroclinic fluids and their application to formal stability analysis of three-dimensional ideal gas flows

MI2013-3 Hiroki MASUDA
Asymptotics for functionals of self-normalized residuals of discretely observed stochastic processes

MI2013-4 Naoyuki KAMIYAMA
On Counting Output Patterns of Logic Circuits

MI2013-5 Hiroshi INOUE
RIPless Theory for Compressed Sensing

MI2013-6 Hiroshi INOUE
Improved bounds on Restricted isometry for compressed sensing

MI2013-7 Hidetoshi MATSUI
Variable and boundary selection for functional data via multiclass logistic regression modeling

MI2013-8 Hidetoshi MATSUI
Variable selection for varying coefficient models with the sparse regularization

MI2013-9 Naoyuki KAMIYAMA
Packing Arborescences in Acyclic Temporal Networks

MI2013-10 Masato WAKAYAMA
Equivalence between the eigenvalue problem of non-commutative harmonic oscillators and existence of holomorphic solutions of Heun's differential equations, eigenstates degeneration, and Rabi's model

MI2013-11 Masatoshi OKITA
Optimal decay rate for strong solutions in critical spaces to the compressible Navier-Stokes equations

MI2013-12 Shuichi KAWANO, Ibuki HOSHINA, Kazuki MATSUDA & Sadanori KONISHI
Predictive model selection criteria for Bayesian lasso

MI2013-13 Hayato CHIBA
The First Painleve Equation on the Weighted Projective Space

MI2013-14 Hidetoshi MATSUI
Variable selection for functional linear models with functional predictors and a functional response

MI2013-15 Naoyuki KAMIYAMA
The Fault-Tolerant Facility Location Problem with Submodular Penalties

MI2013-16 Hidetoshi MATSUI
Selection of classification boundaries using the logistic regression

MI2014-1 Naoyuki KAMIYAMA
Popular Matchings under Matroid Constraints

MI2014-2 Yasuhide FUKUMOTO & Youichi MIE
Lagrangian approach to weakly nonlinear interaction of Kelvin waves and a symmetry-breaking bifurcation of a rotating flow

MI2014-3 Reika AOYAMA
Decay estimates on solutions of the linearized compressible Navier-Stokes equation around a Parallel flow in a cylindrical domain

MI2014-4 Naoyuki KAMIYAMA
The Popular Condensation Problem under Matroid Constraints

MI2014-5 Yoshiyuki KAGEI & Kazuyuki TSUDA
Existence and stability of time periodic solution to the compressible Navier-Stokes equation for time periodic external force with symmetry

MI2014-6 This paper was withdrawn by the authors.

MI2014-7 Masatoshi OKITA
On decay estimate of strong solutions in critical spaces for the compressible Navier-Stokes equations

MI2014-8 Rong ZOU & Yasuhide FUKUMOTO
Local stability analysis of azimuthal magnetorotational instability of ideal MHD flows

MI2014-9 Yoshiyuki KAGEI & Naoki MAKIO
Spectral properties of the linearized semigroup of the compressible Navier-Stokes equation on a periodic layer