Optimized Schwarz Domain Decomposition Methods

Magoulès, Frédéric
Ecole Centrale Paris

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Frédéric Magoulès**

**Ecole Centrale Paris

Applied Mathematics and Systems Laboratory, Grande Voie des Vignes

92295 Châtenay-Malabry Cedex, France

E-mail: frederic.magoules@hotmail.com

Summary:
In this paper, the general principles of asynchronous iterative algorithms and their extension to optimized Schwarz methods are presented. The mathematical framework needed for such algorithms is briefly introduced and the convergence of the asynchronous Schwarz method is given. The experimental results, performed on large scale engineering problems, demonstrate that the proposed asynchronous optimized Schwarz algorithms are as fast as their synchronous counterparts in all cases and become faster when the number of cores increases.

Key words: domain decomposition methods, Schwarz methods, parallel computing, asynchronous iterations

Abstract
Domain Decomposition methods are well suited for parallel computations. Indeed, the division of a problem into smaller subproblems, through artificial subdivisions of the domain, is a means for introducing parallelism. Domain Decomposition strategies include in one way or another the following ingredients: - a decomposer to split a mesh into subdomains ; - local solvers to find solutions for the subdomains for specific boundary conditions on the interface ; - interface conditions enforcing compatibility and equilibrium between overlapping or non-overlapping subdomains ; - a solution strategy for the interface problem. The differences between the methods lies in how those ingredients are actually put to work and how they are combined to form an efficient solution strategy for the problem at hand.

This paper focuses on the Schwarz domain decomposition method. It shows how these methods have efficiently evolved over the years to so called Optimized Schwarz methods by using specially designed boundary conditions on the interface. These optimized interface conditions specially designed to take into account the heterogeneity between the subdomains on each sides of the interfaces (in porous media), or the propagation of the wave through the interfaces (in acoustics), for instance, lead to robust and efficient algorithms.
This paper outlines the evolution of these optimized interfaces conditions over the years, and introduces how such methods can now be tuned for extremely large scale simulation on Petascale and Exascale computers. The proposed idea is based on the extension of optimized Schwarz methods to asynchronous context. The word asynchronous is here related to the iterations of the mathematical algorithm. Numerical experiments performed on large scale engineering problems illustrate the robustness and efficiency of the proposed method when solving engineering problems in fluid dynamics on parallel computers.

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