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On fundamental of Rayleigh-Taylor turbulent mixing: correlations and fluctuations in statistically unsteady turbulent processes

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Content

Abstract

We present the physics-based theoretical analysis of the mechanisms and the properties of Rayleigh-Taylor turbulent mixing, and focus on the consideration of correlations and fluctuations of the statistically unsteady turbulent mixing dynamics. The analysis extends to non-canonical circumstances the ideas of Kolmogorov theory on symmetries of turbulent dynamics, accounts for the essentially multi-scale character of the flow evolution, and identifies the transport of momentum as a better indicator of turbulent mixing dynamics than the transport of energy. The invariance of the rate of momentum loss leads to essentially non-Kolmogorov invariant, scaling and spectral properties of turbulent mixing flow. Rayleigh-Taylor turbulent mixing exhibits more order compared to isotropic turbulence. Its viscous and dissipation scales are finite and set by the flow acceleration. We discuss the outcomes of theoretical results for practical applications and for the methods of flow mitigation and control.

Keywords

Turbulent mixing, Rayleigh-Taylor instability, invariants and scaling and symmetries, statistically unsteady process, stochastic modeling

I. Introduction

Turbulence is common to consider as the last unresolved problem of classical physics [1-16]. For years its complexity and universality assisted engineers and practitioners, nourished enthusiasm of scientists, and fascinated mathematicians [1-16]. Similarity and isotropy are fundamental hypotheses that advanced our understanding of turbulent processes. Still the problem withstands the efforts applied thus indicating a need in new concepts to better control the irregular dynamics [3,4]. Turbulent motions of realistic fluids are often characterized by non-equilibrium heat transport, sharp changes of density and pressure, and may be a subject to spatially varying and time-dependent acceleration and rotation [1-16]. Turbulent mixing induced by the Rayleigh-Taylor instability (RTI) is generic problem, which we encounter when trying to extend our knowledge of turbulent processes beyond the limit of idealized consideration [3,4].

Rayleigh-Taylor (RT) turbulent mixing is an extensive interfacial mixing process which develops when fluids of different densities are accelerated against a density gradient [1,2]. It governs a broad variety of natural phenomena spanning macroscopic to atomistic scales and high to low energy density regimes, and plays an important role in technological applications in aerodynamics and aeronautics [5-13]. Examples include instabilities of plasmas, light-material interaction, material transformation under high strain rate, atmospheric flows, shock-turbulence interaction, non-canonical wall-bounded flows, scramjet combustors, liquid atomization and free-space optical telecommunications [5-13]. Rayleigh-Taylor mixing is a multi-scale, heterogeneous, anisotropic and statistically unsteady turbulent process with non-local interactions among the many scales [3,4]. Its development is usually associated with the conditions of strong gradients of pressure and density and may also include spatially varying and time-dependent acceleration, diffusion of species, heat release, and chemical reactions [17-28,29-39]. These conditions depart from those under which canonical Kolmogorov turbulence is expected to occur [14-16,40]. Capturing the properties of Rayleigh-Taylor mixing can enable a better understanding of realistic turbulent flows and can further improve the methods of their mitigation and control [40]. Here we discuss the influence of momentum transport on fundamental properties of turbulent mixing, and outline some new ideas that may help to better control the mixing process in the applications [5-13].

Arising in a variety of diverse circumstances, RT flows exhibit some similar features of their evolution [3,4]. The mixing starts to develop when the fluid interface is slightly perturbed near its equilibrium state. The flow transitions from an initial stage, where the perturbation amplitude grows relatively quickly [e.g. exponentially in time, if the fluids are incompressible and immiscible and are to sustained acceleration or gravity $\mathbf{g}$], to a nonlinear stage, where the growth-rate slows and the interface is transformed into a composition of a large-scale coherent structure and small-scale irregular structures driven by shear, and then finally to a stage of turbulent mixing, whose dynamics is believed to be self-similar [41-58].

The large-scale coherent structure in RT flows is a periodic array of bubbles and spikes, with light (heavy) fluid with density $\rho_{l(h)}$ penetrating the heavy (light) fluid in bubbles (spikes) [3,4]. The dynamics of the structure is governed by two, in general independent, length scales: the amplitude $\tilde{h}$ in the direction of gravity and the spatial period λ in the normal plane [3,4,59-62]. The horizontal scale λ is set by the mode of fastest-growth or by the initial conditions [3,4]. It may increase, if the flow is two-dimensional and the initial perturbation is broad-band and incoherent [43-49]. The vertical scale $\tilde{h}$ grows as power-law with time, and it is believed that in the mixing regime $\tilde{h} \sim gt^2$, $|\mathbf{g}| = g$ [41-58]. This scale can be regarded as an integral scale, which represents cumulative contributions of small-scale structures in the flow dynamics [3,4,59-62]. The small-scale vortical structures are produced by the Kelvin Helmholtz instabilities at the fluid interface [17-39]. In miscible fluids, the small-scale structures diffuse from the interface into the bulk, and the mixing process is slowing down. Some other features are induced in the dynamics by compressibility, high energy density conditions and non-uniform acceleration [5-13].

To quantify RT mixing flow the observations were focused on diagnostics of the coarsest scales $\tilde{h}$ and ascertainment of dependency $\tilde{h} = \alpha A g t^2$ where α is a constant and $A = (\rho_h - \rho_l)/(\rho_h + \rho_l)$ is the Atwood number [17-39]. RT dynamics was characterized by period λ, and growth of this scale with $\lambda \sim \tilde{h} \sim gt^2$ was suggested as a primary mechanism of the mixing development [43-49]. To account for the time-dependence of the integral scale $\tilde{h} \sim gt^2$ and interpret experimental and numerical data in RTI in terms of turbulent power-laws, some modifications were applied to Kolmogorov theory, including an introduction of a virtual origin and a time-scale for transition to turbulence, a substitution of time-dependencies in Kolmogorov invariants, and a description of RTI by analogy with passive scalar mixing [50-57]. Some quantitative agreements were found between the observations data that spanned relatively short dynamic range and the models that used adjustable parameters [43-57]. Some qualitative features of the turbulent process still remain unclear, e.g. a relatively ordered character of RT flow at high Reynolds numbers [56]. To date, experiments and simulations did not provide a trustworthy guidance on whether the concepts of classical turbulence are applicable to an accelerating RT flow and whether the scaling $\tilde{h}/gt^2$ and λ/gt^2 are indeed universal.

We refer the reader to recent reviews [3,4,28,40] for a detailed discussion of the state of the art in theoretical, experimental, numerical and computational studies of Rayleigh-Taylor flows. This paper presents the physics-based analysis of the mechanisms and the properties of RT mixing suggested by the studies [3,4,59-62] and focuses on the consideration of correlations and fluctuations of the statistically unsteady turbulent dynamics. We extend to non-canonical circumstances of unsteady turbulent mixing the ideas of Kolmogorov theory [14-16] on symmetries of turbulent dynamics. Our consideration accounts for the essentially multi-scale character of RT evolution, and identifies the transport of momentum as a better indicator of RT mixing flow than the transport of energy [3,4,59-62]. The invariance of the rate of momentum loss leads to essentially non-Kolmogorov invariant, scaling and spectral properties of the turbulent mixing. The RT mixing exhibits more order compared to isotropic turbulence. Its viscous and dissipation scales are finite and set by the flow acceleration [3,59-62]. We discuss the outcomes of theoretical results for practical applications and for the methods of flow mitigation and control.

II. Mechanisms, symmetries, and invariants measures of turbulent processes.

As in any natural process, turbulent transports are governed by the conservations principles [14]. The conservations of mass and momentum have the form

$$\dot{\rho} + \nabla \cdot \rho \mathbf{V} = 0\,, \qquad \rho\left(\dot{\mathbf{V}} + (\mathbf{V} \cdot \nabla)\mathbf{V} - \mathbf{g}\right) + \nabla p + \mathbf{S} = 0\,, \tag{1}$$

where ρ, $\mathbf{V}$ and p are the fluid density, velocity and pressure, $\mathbf{S}$ denotes terms induced by viscous stress and other effects, and dot marks the partial derivative in time t. In RT flow the fluid interface is a discontinuity, and equations (1) yield also the boundary conditions at the interface which balance the transports of mass, momentum and energy of the fluids [14]. The system is spatially extended and has no mass sources.

2.1 Symmetries of turbulent processes.

A cornerstone of Kolmogorov theory is that the isotropic and homogeneous turbulent flow has a number of symmetries in statistical sense [15]. Indeed, for a homogeneous fluid with $\rho = const$, with neglected effects of gravity, viscous stress and other terms, $\mathbf{g} = \mathbf{S} = 0$, and asymptotically in time, system (1) describes canonical turbulent flow [15] that is invariant to Galilean transformation, to temporal translations, and to spatial translations, and spatial inversions and rotations. It is also scale-invariant with $L \to LK$, $t \to tK^{1-n}$ and $V \to VK^n$ for any n, where

$V=|\mathbf{V}|$, and L is a characteristic length scale [14-16]. Kolmogorov [15] found that for isotropic homogeneous turbulence $n=1/3$, and that the measure of the scaling symmetry is the rate of change of specific kinetic energy $\varepsilon \sim V^3/L$. Later it was shown that the arbitrariness of n reflects in turbulence multi-scaling and intermittency, see Refs [16,63-67] and references therein.

Similarly to Kolmogorov turbulence, Rayleigh-Taylor turbulent mixing has a number of symmetries [3,4,59-61]. Due to the presence of gravity, $\mathbf{g} \neq 0$, and non-inertial character of the dynamics, these symmetries are distinct from those of Kolmogorov turbulence. Rayleigh-Taylor mixing flow is invariant with respect to translations, inversions and rotations in the plane normal to $\mathbf{g}$, and to scaling transformation $L \to LK$, $t \to tK^{1-n}$ and $v \to vK^n$ with $n=1/2$. The measure of this scaling symmetry v^2/L has the same dimension as g and quantifies the rate of change of specific momentum, Table 1.

Table 1: Symmetries of turbulent processes

Kolmogorov turbulence	Kolmogorov turbulence is inertial and is invariant with respect to Galilean transformation, translations in time and 3D space, and spatial rotations and inversions. It is scale-invariant, $L \to LK$, $T \to TK^{1-n}$, $v \to vK^n$ with $n=1/3$.
RT turbulent mixing	Rayleigh-Taylor turbulent mixing is non-inertial and is invariant with respect to translation, rotations and inversions in the plane normal to gravity $\mathbf{g}$. It is scale invariant, $L \to LK$, $T \to TK^{1-n}$, $v \to vK^n$ with $n=1/2$.

2.2 Momentum-based consideration of turbulent mixing.

In RT mixing flow, the specific momentum is gained due to buoyancy and is lost due to dissipation. The dynamics of a parcel of fluid is governed by a balance per unit mass of the rate of momentum gain $\tilde{\mu}$ and the rate of momentum loss μ as

$$\dot{h}=v\,, \qquad \dot{v}=\tilde{\mu}-\mu \tag{2}$$

Here h is the vertical length scale, e.g. position of the center of mass of the fluid parcel, v is the corresponding velocity, and $\tilde{\mu}$ and μ are the absolute values of vectors pointed in opposite directions along the gravity $\mathbf{g}$ [3,4,59-62]. Eqs. (2) represent in a simplified dimensional-grounds-based form the conservation of mass and momentum (1).

The rate of momentum gain is the rate of change of specific momentum which can be gained due to buoyancy (e.g. the specific buoyant force), and $\tilde{\mu}=\tilde{\varepsilon}/v$, where $\tilde{\varepsilon}$ is the rate of energy gain (e.g. the rate of change of specific potential energy). The value $\tilde{\mu}=g\,f(A)$ with $f(A)$ being a function on the Atwood number, and it is rescaled hereafter as $g\,f(A) \to g$. The rate of momentum loss is the rate of change of specific momentum which is lost due to dissipation, and $\mu=\varepsilon/v$, where ε is the rate of change of specific kinetic energy. In the limit of vanishing viscosity on the basis of dimensional grounds $\varepsilon = C v^3/L$, where L is the characteristic length scale and $C=const$ [3,59-

62]. The ratio between $\tilde{\mu}, \tilde{\varepsilon}$ as well as μ, ε are the standard relations between the (specific) power and force [14].

As discussed in [59], asymptotic solutions for model (2) depend on whether the characteristic length scale of the flow is horizontal or vertical. If the characteristic length scale is horizontal, $L \sim \lambda$, then Eqs. (2) has steady solution with $v \sim \sqrt{g\lambda}$ and $h \sim t\sqrt{g\lambda}$, and the rates of momentum and energy are balanced: $\tilde{\mu} = \mu = g$ and $\tilde{\varepsilon} = \varepsilon = (g\lambda)^{3/2} / \lambda\sqrt{C}$. If the characteristic scale is vertical, $L \sim h$, then asymptotically in time, $h = a\,gt^2/2$ and $v = a\,gt$ with $a = (1 + 2C)^{-1}$. The rates of energy gain and dissipation are time-dependent, $\tilde{\varepsilon} = a\,g^2 t$ and $\varepsilon = (1 - a)a\,g^2 t$, and the rates of momentum gain and loss are time- and scale-invariant, $\tilde{\mu} = g$ and $\mu = C\,v^2/h$ [59-62]. As found in many observations, the values of a are rather small, $a \sim 0.05 - 0.15$ [17-39]. Thus, in the mixing flow almost all energy induced by the buoyancy dissipates, $\tilde{\varepsilon} \approx \varepsilon$ with $\varepsilon/\tilde{\varepsilon} = (1 - a)$, and the rates of momentum gain and loss slightly imbalance one another, $\tilde{\mu} \approx \mu$ with $(\tilde{\mu} - \mu)/\tilde{\mu} = a$. Self-similar mixing may develop when horizontal scale λ grows with time as $\lambda \sim h \sim gt^2$ [43-49], and when the vertical scale h, $h \sim \tilde{h}$, is the characteristic scale for energy dissipation that occur in the small-scale structures at the fluid interface [3,59-62].

2.3 Mechanisms of development of RT mixing.

Agreeing in certain limiting cases with principal results of the heuristic models [41-57], momentum consideration (2) identifies some new properties of the mixing flow [59-62]. It suggests that the accelerated turbulent mixing develops due the imbalance of gain and loss of specific momentum, $\mu \neq \tilde{\mu}$. This imbalance may occur when (i) the horizontal scale grows as $\lambda \sim gt^2$, and/or when (ii) the vertical scale h is a characteristic scale for energy dissipation, $\varepsilon = C\,v^3/L$ with $L \sim h$, and when it represents cumulative contributions of small-scale structures into the flow dynamics. Existence of two distinct mechanisms of the mixing development reconciles with one another the models [41-59]. It also agrees with results of theoretical studies [4], which found that the amplitude $\tilde{h}$ and period λ provide independent contributions to the nonlinear RT dynamics and that for highly coherent large-scale coherent structures the growth of horizontal scales may not occur.

2.4 Energy budget, transports of energy and momentum, position of the center of mass.

Turbulence is a property of dissipative systems and it decays unless it is driven [14-16,63-67]. Kolmogorov turbulence is driven by an external energy source, which supplies energy to the flow at a constant rate ε: Energy is injected at large scales by an external source, and then it is transferred without loss through the inertial interval and dissipates at small scales [14-16,63-67]. According to the momentum consideration [3,59-62], for Rayleigh-Taylor turbulent mixing an external energy source (other than gravity) is not required, and the specific momentum is gained due to buoyancy and is lost due to dissipation. In accelerated flow at any scale $\mu \neq \tilde{\mu}$ and $\varepsilon \neq \tilde{\varepsilon}$, and this imbalance indicates that the mean velocity of the center of mass of the fluid entrained in the motion is time-dependent, whereas in statistically steady turbulent flow it is invariable, Table 2.

Table 2: Energy source, transports of momentum and energy, center of mass position in the turbulent processes

Kolmogorov turbulence	Energy is injected at large scales by an external source, and is transferred without losses through the inertial interval and dissipates at small scales. Mean velocity of the center of mass of the fluid system is time-independent.
RT turbulent mixing	There is no external energy source other than gravity. Energy and momentum are gained due to buoyancy and are lost due to dissipation. In steady regime $\mu-\tilde{\mu}=0$ and $\varepsilon=\tilde{\varepsilon}=(g\lambda)^{3/2}/\lambda\sqrt{C}$. Accelerated turbulent mixing is driven by imbalance between the gain and loss of specific momentum, and at any scale $\mu\neq\tilde{\mu}$ and $\varepsilon\neq\tilde{\varepsilon}$. In accelerated mixing, the mean velocity of the center of mass of the fluid system is time-dependent.

2.5 Asymptotic states of turbulent processes in space and in time.

Statistically steady Kolmogorov turbulence is an asymptotic in time state, which is achieved when the memory of the initial conditions is completely lost, and when the boundaries of the outside domain do not influence the dynamics [15,63-67]. These conditions can be realized in a spatially extended system or in a finite-size domain, when the span of scales runs several decades from viscous to integral scale [15]. Implementation of these conditions in Rayleigh-Taylor turbulent mixing requires special attention [28]. In a finite-size domain, an asymptotic in time dynamics corresponds to a stable state with no motion at all: under the influence of gravity (directed from the top to the bottom) the system transits from an unstable configuration to a stable configuration (e.g. from an initial state with heavier fluid located at the top of the domain and lighter fluid - at the bottom to a reverse state), and the change in the system potential energy dissipates into heat. In a spatially extended system (e.g. in a large domain) the flow may accelerate, however at a certain time compressibility and stratification start to play a role and results in flow stabilization, as discussed in Ref. [14,59]. To allow for the development of Rayleigh-Taylor turbulent mixing and to enable its diagnostics over substantial span of scales, the size of the domain should be large enough yet not so large to prevent mixing stabilization by effects of compressibility and stratification.

2.6 Effective drag in the turbulent flows.

Regularization of accelerated turbulent mixing is at first glance an unusual concept. However, there is some evidence from previous studies that is does take place. For instance, re-laminarization of an accelerated flow is a well-known fluid dynamics phenomenon discovered in the works of Taylor [68] for flows in curved pipes and Sreenivasan [69] for boundary layers. Another indication of a more regular character of Rayleigh-Taylor mixing follows from the characteristic value of the flow drag. Coefficient C in the dependencies $\varepsilon=Cv^3/L$ and $\mu=Cv^2/L$ can be viewed as effective drag coefficient, which is related to the growth-rate $h=a\,gt^2/2$ via $a(1+2C)=1$ [59-62]. For $C\to 0$

(no drag) the solution is a free-fall with $a \to 1$, whereas for $C \to \infty$ (infinitely large drag) $a \to 0$ and the flow cannot accelerate. Experiments and simulations report relatively small values of $a \sim 0.05 - 0.15$ (with $\alpha \sim 0.03 - 0.07$ in the relation $h = \alpha A g t^2$ in [49]). These values correspond to drag coefficient of $C \sim 3 - 8$, indicating that flow may tend to be more laminar rather than turbulent [63-67]. In canonical Kolmogorov turbulence, the value of C is calculated from the third-order velocity structure function as $C = 5/4$ and $C \sim 1$ [15,63-67]. This may lead to $a = 2/7 \approx 0.3$ ($\alpha \sim 0.14$ in $h = \alpha A g t^2$ in [49]), which is significantly greater than the values actually observed.

2.7 Invariant measures of turbulent process.

In isotropic turbulence, the total momentum is zero because of isotropy. Time- and scale-invariance of the energy dissipation rate $\varepsilon \sim v^3/L$ implies that the energy injected at large scales, $\varepsilon \sim v^2 (v/L)$, is transferred without loss through the inertial range and is dissipated at small scales, $\varepsilon \sim (vL)(v/L)^2$ [15,16,63-67]. That is, time- and scale-invariance of the energy dissipation rate $\varepsilon \sim v^3/L$ is compatible with existence of inertial interval and non-dissipative energy transfer between the scales [15,16,63-67]. In accelerated Rayleigh-Taylor turbulent mixing, the rates of change of specific energy are time-dependent, the energy dissipation rate is time-dependent, $\tilde{\varepsilon} \sim \varepsilon \sim g^2 t$, and the specific momentum is imbalanced, $\tilde{\mu} \neq \mu$. Time- and scale invariance of $\mu \sim v^2/L$ implies that at any time and length scale the specific momentum is being lost at the same constant rate, and momentum transfer between the scales is non-dissipative [61]. Enstrophy is another invariant of isotropic turbulence [63-67], whereas in Rayleigh-Taylor mixing this value decays with time (thus providing another indication of a tendency of accelerated mixing flow to re-laminarize) [61]. In Rayleigh-Taylor flow, vortical structures form helixes not vortices. In a flow dominated by the growth of horizontal scales, $\lambda \sim h \sim g t^2$, the helicity is a statistically steady value and its steadiness may serve as an indicator of achieving a merger-driven self-similarity [61], Table 3.

Table 3: Some invariant measures of the turbulent process

Kolmogorov turbulence	Dynamics is statistically steady. Invariance of energy dissipation rates $\varepsilon \sim v^3/L$ is compatible with existence of inertial interval and energy cascade. Enstrophy and helicity are other invariants.
RT turbulent mixing	Dynamics is statistically unsteady. Invariance of rate of momentum loss $\mu \sim v^2/L$ leads to non-dissipative momentum transport between the scales. Energy dissipation rate and enstrophy are time-dependent, and helicity is invariant.

III. Correlations and fluctuations in turbulent processes.

3.1 Space-time scaling properties: correlations and fluctuations in the turbulent process.

For a description of scaling properties, let the length scale L and time scale T refer to large scales and times, let the characteristic velocity be v, let the characteristic velocity be v_l at a small length scales l, and let the characteristics velocity be v_τ on a short time-scale τ.

In Kolmogorov turbulence [14-16,63-67], the invariance of the energy dissipation rate $\varepsilon \sim v^3/L \sim v_l^3/l$ yields the velocity scaling $v_l/v \sim (l/L)^{1/3}$, N -th order velocity structure function $\sim (l\,\varepsilon)^{N/3}$, and velocity scaling with time $v_\tau/v \sim (\tau/T)^{1/3}$. The relative velocity of two parcels of fluids involved in these motions is $\sim (\varepsilon\tau)^{1/2}$ on a time delay τ, and it is substantially smaller than the velocity fluctuations $v_\tau \sim (\varepsilon v \tau)^{1/3}$ induced by turbulence. This well-known result means that in Kolmogorov turbulence, the main contribution to velocity fluctuations is provided by the turbulence not by the initial conditions [15], Tables 4,5.

Table 4: Spatial scaling of the velocity in the turbulent process

	Velocity scaling	**Velocity *N*th order structure function**
Kolmogorov turbulence	$v_l/v \sim (l/L)^{1/3}$ based on ε invariance	$\sim (l\,\varepsilon)^{N/3}$ based on ε invariance
RT turbulent mixing	$v_l/v \sim (l/L)^{1/2}$ based on μ invariance	$\sim (l\,\mu)^{N/2}$ based on μ invariance

In Rayleigh-Taylor turbulent mixing, the invariance of the rate of momentum loss $\mu \sim v^2/L \sim v_l^2/l$ yields the velocity scaling $v_l/v \sim (l/L)^{1/2}$, N -th order velocity structure function $\sim (l\,\mu)^{N/2}$, and the velocity scaling with time $v_\tau/v \sim (\tau/T)$. For two parcels of fluids involved in the motion with a time delay τ, their relative velocity is $\sim (\tilde{\mu}-\mu)\tau \sim g\tau$ and it is comparable to $v_\tau \sim \mu\tau$ induced by turbulent fluctuations, whereas their own velocities grow with time as $\sim gt$ and $\sim g(t-\tau)$ [14]. We see that in accelerated mixing flow, the velocity fluctuations are 'frozen' to the level of the initial conditions, and with time the contribution of fluctuations to the mixing dynamics is reduced, Tables 4,5.

Table 5: Temporal scaling of the velocity in the turbulent process

	Velocity scaling	**Velocity fluctuations**
Kolmogorov turbulence	$v_\tau/v \sim (\tau/T)^{1/3}$ based on ε invariance	$v_\tau \sim (\varepsilon v\tau)^{1/3}$ based on ε invariance and $(\varepsilon v\tau)^{1/3} >> (\varepsilon\tau)^{1/2}$
RT turbulent mixing	$v_\tau/v \sim (\tau/T)$ based on μ invariance	$v_\tau \sim \mu\tau$ based on μ invariance and $\mu\tau \sim (\tilde{\mu}-\mu)\tau \sim g\tau$

3.2 Reynolds number, viscous scale and integral scale in turbulent processes.

In Kolmogorov turbulence, Reynolds number is finite $\mathrm{Re} = vL/\nu = const$ and local Reynolds number $\mathrm{Re}_l = v_l l/\nu$ scales as $\mathrm{Re}_l \sim \mathrm{Re}(l/L)^{4/3}$ leading to the viscous length scale $l_\nu \sim (\nu^3/\varepsilon)^{1/4}$ and time-scale $\tau_\nu \sim (\nu/\varepsilon)^{1/2}$ for $\mathrm{Re}_l \sim 1$. In accelerated turbulent mixing the Reynolds number grows with time as $\mathrm{Re} = vL/\nu \sim g^2 t^3/\nu$ and the local Reynolds number $\mathrm{Re}_l = v_l l/\nu$ scales as $\mathrm{Re}_l \sim \mathrm{Re}(l/L)^{3/2}$. For $\mathrm{Re}_l \sim 1$ viscosity plays a dominant role, thus leading to viscous length-scale $l_\nu \sim (\nu^2/\mu)^{1/3}$ with the corresponding time scale $\tau_\nu \sim (\nu/\mu^2)^{1/3}$. The viscous length-scale is finite. It is set by the flow acceleration and are comparable to the wavelength of mode of fastest growth [1,2]. Thus despite in accelerated Rayleigh-Taylor mixing the Reynolds number can reach large values relatively quickly the flow viscous scale remains finite. An upper limit for Reynolds number $\mathrm{Re} \sim g^2 t^3/\nu$ can be estimated at a border of validity of incompressible approximation $gt \sim c$ as $\mathrm{Re}_c \sim c^3/g\nu$, where c is the sound speed, Table 6.

Table 6: Reynolds number, viscous scale, and integral scale in the turbulent process

	Reynolds number	**Viscous and integral scales**
Kolmogorov turbulence	$\mathrm{Re} = vL/\nu = const$. Invariance of ε leads to $\mathrm{Re}_l \sim (v_l l/\nu) \sim \mathrm{Re}(l/L)^{4/3}$.	Invariance of ε leads to $l_\nu \sim (\nu^3/\varepsilon)^{1/4}$ and $\tau_\nu \sim (\nu/\varepsilon)^{1/2}$. An integral scale is the scale at which energy is gained by the flow system
RT turbulent mixing	$\mathrm{Re} = vL/\nu \sim g^2 t^3/\nu$. Invariance of μ leads to $\mathrm{Re}_l \sim (v_l l/\nu) \sim \mathrm{Re}(l/L)^{3/2}$. For $gt \sim c$ upper limit is $\mathrm{Re}_c \sim c^3/g\nu$.	Invariance of μ leads to $l_\nu \sim (\nu^2/\mu)^{1/3}$ and $\tau_\nu \sim (\nu/\mu^2)^{1/3}$. An integral scale is the coarsest vertical scale representing cumulative contributions of small scale structures.

In Kolmogorov turbulence the integral scale is the scale, at which energy is gained by the flow system. For turbulent mixing this consideration may not be directly applicable. In Rayleigh-Taylor mixing, momentum and energy are gained and dissipated at any scale, and imbalance between the rate of momentum gain and loss leads to flow acceleration. The coarsest vertical scale in Rayleigh-Taylor flow can be regarded as an integral cumulative scale, which represents cumulative contributions of small-scale structures in the flow dynamics, Table 6.

3.3 Dimensional-analysis-based spectral properties of the turbulent process.

In isotropic turbulence, the invariance of energy dissipation rate leads to kinetic energy spectrum $E(k) \sim \varepsilon^{2/3} k^{-5/3}$ [15,63-67]. In Rayleigh-Taylor mixing accurate determination of spectra (and corresponding eigen-functions) is a formidable task because the dynamics is statistically unsteady. Dimensional analysis suggests that the spectrum of specific kinetic energy have the form

$E(k) \sim \mu k^{-2}$, which is steeper than Kolmogorov; similarly for the spectrum of specific momentum one obtains $M(k) \sim \mu^{1/2} k^{-3/2}$. In Kolmogorov turbulence $M(k) \equiv 0$ due to isotropy, Table 7, [61].

Table 7: Dimensional-analysis-based spectral properties of the turbulent process

	Spectrum of specific kinetic energy	**Spectrum of specific momentum**
Kolmogorov turbulence	$E(k) \sim \varepsilon^{2/3} k^{-5/3}$ set by invariance of ε.	$M(k) \equiv 0$ due to isotropy
RT turbulent mixing	$E(k) \sim \mu k^{-2}$ set by invariance of μ	$M(k) \sim \mu^{1/2} k^{-3/2}$ set by invariance of μ

3.4 Pressure fluctuations.

In Kolmogorov turbulence, pressure fluctuations are evaluated using fourth-order velocity structure function so that pressure fluctuates as $\sim \varepsilon^{4/3} l^{4/3}$ with spectrum $\sim \varepsilon^{4/3} k^{-7/3}$ [63-67]. For Rayleigh-Taylor mixing dimensional analysis suggests for pressure fluctuations $\sim \mu^2 l^2$ with spectrum $\sim \mu^2 k^{-3}$ which is steeper than in Kolmogorov turbulence, Table 8 [61].

Table 8: Dimensional-analysis-based properties of pressure fluctuations

	Scaling	**Spectrum**
Kolmogorov turbulence	$\sim \varepsilon^{4/3} l^{4/3}$ set by invariance of ε.	$\sim \varepsilon^{4/3} k^{-7/3}$ set by invariance of ε.
RT turbulent mixing	$\sim \mu^2 l^2$ set by invariance of μ	$\sim \mu^2 k^{-3}$ set by invariance of μ

3.5 Statistically steady and statistically unsteady turbulent mixing.

To conclude this section, we discuss in more details statistically steady and statistically unsteady regimes in Rayleigh-Taylor flows. In a steady regime, the flow can appear more coherent or more 'turbulent' depending on the Atwood number and the initial conditions [3,4]. For the steady flow, the rates of momentum gain and loss as well as energy gain and dissipation are balanced, $\tilde{\mu} = \mu$ and $\tilde{\varepsilon} = \varepsilon$, and the characteristic length scale of the flow λ is constant. The characteristic velocity is $v \sim \sqrt{g\lambda}$, the Reynolds number is $\mathrm{Re} = vL/\nu \sim \lambda\sqrt{g\lambda}/\nu$, and the energy dissipation rate is constant $\varepsilon \sim (g\lambda)^{3/2}/\lambda$. This formally corresponds to the viscous scale $(\nu^3/\varepsilon)^{1/4} \sim \left(\nu^3\lambda/(g\lambda)^{3/2}\right)^{1/4}$, which is smaller than the mode of fastest growth $(\nu^2/g)^{1/3}$ for $\lambda > (\nu^2/g)^{1/3}$. However, as $\lambda/\left(\nu^3\lambda/(g\lambda)^{3/2}\right)^{1/4} \sim \left(\lambda/(\nu^2/g)^{1/3}\right)^{9/8}$ and $9/8 \approx 1$, the characteristic span of scales in the steady flow is well captured by the ratio $\lambda/(\nu^2/g)^{1/3}$.

The flow acceleration increases the flow velocity, integral length scale, Reynolds numbers, and energy dissipation rate. At the first glance, this may lead to an appearance of high-Reynolds number turbulent flow with a significant span of scales [41-58]. Momentum consideration [59-62] suggests however that buoyancy-driven turbulent mixing is accelerated due to imbalance between the gain and loss of momentum and energy with $(\tilde{\mu}-\mu)/\tilde{\mu}=a$ and $(\tilde{\varepsilon}-\varepsilon)/\tilde{\varepsilon}=a$. In this flow the velocity $v \sim gt$, the length scale $h \sim gt^2$, the Reynolds number $\mathrm{Re} \sim g^2t^3/\nu$, and the span of scales $\sim gt^2/(\nu^2/g)^{1/3}$ indeed increases. Here the viscous scale is $\sim (\nu^2/g)^{1/3}$ and upper limit for the span of scales is $\sim c^2/g(\nu^2/g)^{1/3}$ for $c \sim gt$. However, compared to the case of statistically steady isotropic and homogeneous turbulence, the accelerated turbulent mixing exhibits stronger correlations, reduced contribution of fluctuations and steeper spectra and may tend to be more laminar [59-62], Table 9.

Table 9: Flow quantities in statistically steady and statistically unsteady Rayleigh-Taylor mixing

Steady RT flow	Balance of momentum and energy $\tilde{\mu}=\mu$ and $\tilde{\varepsilon}=\varepsilon$. Constant length scale λ, velocity $v \sim \sqrt{g\lambda}$, Reynolds number $\mathrm{Re} \sim \lambda\sqrt{g\lambda}/\nu$ and energy dissipation rate $\varepsilon \sim (g\lambda)^{3/2}/\lambda$ with corresponding viscous scale $\left(\nu^3\lambda/(g\lambda)^{3/2}\right)^{1/4}$ and span of scales $\left(\lambda/(\nu^2/g)^{1/3}\right)^{9/8}$.
Unsteady RT flow	Imbalance of momentum and energy $(\tilde{\mu}-\mu)/\tilde{\mu}=a$ and $(\tilde{\varepsilon}-\varepsilon)/\tilde{\varepsilon}=a$. Time-dependent length scale $h \sim gt^2$, velocity $v \sim gt$, Reynolds number $\mathrm{Re} \sim g^2t^3/\nu$ and energy dissipation rate $\varepsilon \sim g^2t$. Constant rate of momentum loss $\mu \sim g$ with corresponding viscous scale $\sim (\nu^2/g)^{1/3}$ and span of scales $\sim gt^2/(\nu^2/g)^{1/3}$ with upper limit $\sim c^2/g(\nu^2/g)^{1/3}$.

IV. Stochastic modeling of statistically unsteady turbulent mixing process

As in any turbulent process, RT mixing dynamics has a random character, which is resulted from contribution of small-scale structures and interactions of all the scales [60,63-67]. Capturing this randomness is a complex task. In Kolmogorov turbulence, random character of flow dissipation is induced by velocity fluctuations with the energy dissipation rate being a statistic invariant [63-67]. In RT mixing flow the velocity and the length scale both fluctuate and the energy dissipation rate grows with time. We account for the random character of dissipation in RT flow on the basis of idea that even in a statistically unsteady process [whose fluctuating quantities are time-dependent and non-Gaussian] there exist time- and scale-invariant values fluctuating about their means, particularly, the rate of momentum loss μ [60].

To study the effect of fluctuations on the mixing dynamics, Eqs.2 are represented in a differential form

$$dh = v dt, \qquad dv = \mathrm{d}\tilde{\mathrm{M}} - \mathrm{dM}, \tag{3a}$$

with differentials of momentum gain $\mathrm{d}\tilde{\mathrm{M}} = g\,dt$ and loss $\mathrm{dM} = C\left(v^2/h\right)dt$ and with C being a stochastic process [60]. This process is, in general, time-dependent, $C = C(t)$, and is characterized by a time-scale τ_C, showing how fast the distribution $C(t)$ approaches a stationary probability density function $p(C)$. The function $p(C)$ is non-symmetric, $C > 0$, with the mean value $\langle C \rangle$, with the mode $C_{\max}$ corresponding to the highest value of $p(C)$, and with the standard deviation σ, describing the fluctuations intensity. For stochastic processes with log-normal distribution $p(C) = e^{\left(-(\ln C - \ln C_0)^2/2\sigma^2\right)}/\sqrt{2\pi}\sigma C$, the mean $\langle C \rangle = C_0 \exp\left(\sigma^2/2\right)$, the mode $C_{\max} = C_0 \exp\left(-\sigma^2\right)$, and the set of stochastic differential equations in (3a) takes the form

$$dh = v dt, \qquad dv = g dt - C\frac{v^2}{h}dt, \qquad dC = -C\left(\ln\frac{C}{\langle C \rangle} - \frac{\sigma^2}{2}\right)\frac{dt}{\tau_C} + \sigma C\sqrt{\frac{2}{\tau_C}}dW, \tag{3b}$$

with dW being a standard Weiner process.

The stochastic modeling results indicate that fluctuations do not change the asymptotic time-dependence of the dynamics, so that $h \sim gt^2/2$ as $t/\tau \to \infty$, yet they influence significantly the coefficient $a = 2h/gt^2$ [60]. Depending on the shape of the distribution $p(C)$ and on the fluctuations intensity σ, the mean value of a may vary in several folds, and, furthermore, it saturates slowly with time for $(t/\tau) >> 1$. This result explains qualitatively the several-fold scatter in the values of a in the experiments and simulations [49]. It indicates that the growth-rate parameter a is sensitive to the dissipation statistics and it is a significant parameter not because it is "deterministic" or "universal," but because its value is rather small, $a << 1$ [50]. Found in many experiments and simulations, the small a implies that in RT flows almost all energy induced by the buoyant force dissipates, and a slight imbalance between the rates of momentum loss μ and gain $\tilde{\mu}$ is sufficient for the mixing development. We emphasize that the rate of momentum loss $\mu(t) = C v^2/h$ is relatively insensitive to the effect of fluctuations, and monitoring the momentum transport is thus has crucial importance for grasping the essentials of the mixing process.

V. Outcomes of theoretical analysis for mitigation and control of turbulent mixing process

To date, the design of experiments on RT mixing [17-39] employs the results of traditional models [41-58] suggesting the following scenarios for RT evolution. Initially, small perturbations at the interface with wavelength λ grow fast. In the nonlinear regime the velocity is $v \sim \sqrt{g\lambda}$ and amplitude is $h \sim vt \sim t\sqrt{g\lambda}$. Horizontal and vertical scales are strongly coupled, and self-similar growth of horizontal scales (e.g. bubble interaction and merge) leads to flow acceleration with $\lambda \sim h \sim gt^2$. In accelerated regime the scales grow as $h \sim gt^2$ and $\lambda \sim gt^2$, the Reynolds number and energy dissipation rate increase as $\mathrm{Re} \sim g^2 t^3/\nu$ and $\varepsilon \sim v^3/h \sim g^2 t$. This may be interpreted as the development of a turbulent state of the mixing flow, which is similar to isotropic and homogeneous turbulence and is independent of the initial conditions, and whose viscous scale decays as $(\nu^3/\varepsilon)^{1/4} \sim (\nu^3/g^2 t)^{1/4}$ and span of scales increases as $h/(\nu^3/\varepsilon)^{1/4} \sim g^{3/2} t^{9/4}/\nu^{3/4}$. Therefore, according to these scenarios, RT turbulent mixing, once it appears, cannot be controlled. To proceed to mixing regime faster, the initial perturbation may contain large wavelength modes. To suppress the mixing development, the interface should be 'finely polished' [41-58].

As discussed in the foregoing, some of results of traditional models [41-58] can be obtained within the frames of the theoretical analysis [59-62] with the use of additional adjustable parameters. Some other results of the traditional models [41-58] are known to have severe limitations [4].

The theoretical analysis [3,4,59-62] finds that in the nonlinear regime of RTI, horizontal and vertical scales contribute independently to the dynamics, and the rates of specific momentum and energy are balanced, $\tilde{\mu} = \mu$ and $\tilde{\varepsilon} = \varepsilon$. Accelerated turbulent mixing develops due to imbalance of specific momentum and energy with $(\tilde{\mu} - \mu)/\tilde{\mu} = a$ and $(\tilde{\varepsilon} - \varepsilon)/\tilde{\varepsilon} = a$, where $h = a\, g t^2/2$ (with 'effective' g accounting for the density ratio). There are two distinct mechanisms for the mixing development: (i) growth of horizontal scale (period) $\lambda \sim g t^2$ and (ii) dominance of vertical scale (amplitude) $L \sim h$ or energy dissipation. Bubble merge is possible but not a necessary condition for the mixing to occur. Compared to isotropic turbulence, RT turbulent mixing exhibits more order, steeper spectra, stronger correlations, and weaker contributions of fluctuations, which are 'frozen' to the initial conditions. In turbulent mixing flow the viscous scale is finite and is set by flow acceleration as $(\nu^2/\mu)^{1/3} \sim (\nu^2/g)^{1/3}$. The span of scales is $h/(\nu^2/g)^{1/3} \sim g t^2/(\nu^2/g)^{1/3}$ with the upper limit $c^2/g(\nu^2/g)^{1/3}$. In the mixing flow the rates of gain and loss of specific momentum are time- and scale-independent, $\tilde{\mu} \sim \mu \sim v^2/L \sim g$, the rates of energy gain and dissipation are time-dependent, $\tilde{\varepsilon} \sim \varepsilon \sim g^2 t$, and Reynolds number increases as $\mathrm{Re} \sim g^2 t^3/\nu$. Therefore, the theoretical analysis [3,4,59-62] suggests that RT mixing flow can in principle be controlled by means of initial perturbation and acceleration. Horizontal and vertical scales can be controlled independently, and initial perturbation with large wavelengths may not induce any turbulence [4,59-62]. For better control of RT mixing, one should impose proper (e.g. highly coherent) initial conditions in order to prevent bubble merge. Furthermore, one should choose very special initial conditions to force the flow to fluctuate [3,4,59-62].

It would be beneficial for the design of experiments on RT mixing to account for that in strongly fluctuating turbulent flows the Reynolds number is high; yet not in any high Reynolds number flow the fluctuations are strong [59-62]. Implementation of turbulent flows in experiments is an extremely challenging task [28], as good experiments on turbulence are the 'quantitative' experiments, which require accurate interpretation of the (bias-free) experimental noise. A qualitative experiment with a binary answer 'yes/no' may be a good solution the case of RT turbulent mixing. Such an experiment may involve a comparative study of RT mixing dynamics with various initial conditions, e.g. involving hexagonal grid, square grid, two-mode grid, and fractal grid [70] in case of three-dimensional spatially extended flows. According to the analysis [3,4,59-62], the expected results would be the following. For accurately implemented hexagonal grid, bubble merge may not occur, and the flow is 'regular' and is dominated by the coherent structure. For square grid, bubble merge may occur via multi-pole interactions. For two-mode grid with small and large wavelengths, bubble merge will develop faster compared to square grid. Fractal grid [70] may be the best to induce fluctuations and a 'turbulent-like' dynamics (these fluctuations may be dominated by the few modes and may not be stochastic [70]).

VI. Conclusion

We have considered the effect of momentum transport on scaling, invariant and statistical properties of Rayleigh-Taylor mixing flow [3,4,59-62]. It is shown that the rate of momentum loss is a better indicator of the unsteady turbulent dynamics than the rate of energy dissipation. Our consideration accounts for the multi-scale character of turbulent mixing dynamics and indicates two possible mechanisms for the mixing development. The first is the traditional "merge" associated with the growth of horizontal scales. The second is associated with the production of small-scale structures and with the growth of the vertical scale, which plays the role of the integral scale for energy dissipation. Based on invariance of the rate of momentum loss, we found that the fundamental properties of statistically unsteady Rayleigh-Taylor turbulent mixing depart substantially from classical Kolmogorov scenario. In particular, turbulent mixing flow exhibits more order compared to isotropic turbulence, and its viscous scale is set by the flow acceleration. The stochastic modeling results indicate that the growth-rate parameter of the mixing zone is a sensitive to statistical properties of dissipation. The momentum-based consideration of Rayleigh-Taylor mixing suggests a principal opportunity of mitigation and control of the statistically unsteady turbulent process.

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