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J-groups of orbit manifolds $D_p(4m+3,8n+7)$ of $S^{8m+7}\times S^{8n+7}$ by the dihedral group D_p

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Denote by D_p the dihedral group $\{g^it^j \mid g^p=t^2=gtgt=e\}$. Let $D_p(l,m)$ be an orbit manifold of a principal D_p-manifold $S^{2l+1}\times S^m$ with an action

$$g^it^j(z,x)=(\rho^ic^j(z),\ (-1)^jx),$$

where $\rho=\exp 2\pi\sqrt{-1}/p$ and $c(z)$ denotes the conjugate point of z [12]. The structure of the complex K-groups $K(D_p(l,m))$ are determined in [8] and [9]. In this paper we use the Atiyah-Hirzebruch spectral sequence to determine the real K-groups $\widetilde{KO}(D_p(4m+3,8n+7))$, and the result is applied to determine the structure of J-groups $J(D_p(4m+3,8n+7))$. Furthermore we discuss Thom complexes over $D_p(l,m)$ and the stable homotopy type of the stunted spaces $D_p(n,k)/D_p(n',k)$ and $D_p(n,k)/D_p(n,k')$.

1. The structure of $\widetilde{KO}(D_p(4m+3,8n+7))$, p: odd prime.

Let $L^m(p)=S^{2m+1}/Z_p$ be the standard lens space and S^k the k-sphere. $D_p(m,k)$ is homeomorphic to an orbit space of a principal Z_2-manifold $L^m(p)\times S^k$ with an action

$$t([z],x)=([\bar{z}],-x),\quad t \text{ the generator of } Z_2,$$

where $\bar{z}$ denotes the conjugate point of z. In [12] and [13], we have shown that the cochain groups $C^*(D_p(m,k);Z)$ are generated by $\{(c^i,d^j)\mid i=0,1,\dots,2m+1;\ j=0,1,\dots,k,\ \dim(c^i,d^j)=i+j\}$ and the coboundary operations are given by

$$\delta(c^{2i+1},d^j)=p(c^{2i+2},d^j)+\{(-1)^i+(-1)^j\}(c^{2i+1},d^{j+1}),$$
$$\delta(c^{2i},d^j)=\{(-1)^i+(-1)^{j+1}\}(c^{2i},d^{j+1})$$

and that the chain groups $C_*(D_p(m,k);Z)$ are generated by $\{(C_i,D_j)\mid i=0,1,\dots,2m+1;\ j=0,1,\dots,k,\ \dim(C_i,D_j)=i+j\}$ and the boundary oprations are given by

$$\partial(C_{2i+1},D_j)=\{(-1)^i+(-1)^{j+1}\}(C_{2i+1},D_{j-1}),$$

$$\partial(C_{2i}, D_j)=p(C_{2i-1}, D_j)+\{(-1)^i+(-1)^j\}(C_{2i}, D_{j-1}).$$

THEOREM 1.1. *$H^*(D_p(m,k); Z_2)$ is isomorphic to a polynomial ring of $x=(c^{2m+1}, d^0)$ and $y=(c^0, d^1)$ truncated by an ideal generated by y^{k+1} and x^2:*

$$H^*(D_p(m,k); Z_2)\cong Z_2[x,y]/(y^{k+1}, x^2).$$

PROOF. We obtain the diagonal chain map

$$\varDelta : C_*(D_p(m,k); Z_2)\to C_*(D_p(m,k); Z_2)\otimes C_*(D_p(m,k); Z_2)$$

defined by

$$\varDelta(C_{2i+1}, D_j)=\sum_{\substack{s+s'=i\\ t+t'=j}}(C_{2s+1}, D_t)\otimes(C_{2s'}, D_{t'})$$
$$+\sum_{\substack{s+s'=i\\ t+t'=j}}(C_{2s}, D_t)\otimes(C_{2s'+1}, D_{t'})$$

and

$$\varDelta(C_{2i}, D_j) = \sum_{\substack{s+s'=i\\ t+t'=j}}(C_{2s}, D_t)\otimes(C_{2s'}, D_{t'}).$$

$H^*(D_p(m,k); Z_2)$ are generated additively by $\{(c^0, d^j), (c^{2m+1}, d^j)\,|\,j=0,1, \dots, k\}$ and $H_*(D_p(m,k); Z_2)$ are generated additively by $\{(C_{2m+1}, D_j), (C_0, D_j)\,|\,j=0,1, \dots, k\}$. Observe that

$$<(c^{2m+1}, d^0)\cup(c^0, d^j), (C_i, D_l)>$$
$$=<\varDelta^*((c^{2m+1}, d^0)\otimes(c^0, d^j)), (C_i, D_l)>$$
$$=<(c^{2m+1}, d^0)\otimes(c^0, d^j), \varDelta_*(C_i, D_l)>$$
$$=\begin{cases}1 & \text{if } i=2m+1,\ l=j,\\ 0 & \text{otherwise.}\end{cases}$$

Hence, we obtain $(c^{2m+1}, d^0)\cup(c^0, d^j)=(c^{2m+1}, d^j)$, and by same procedure, $(c^0, d^j)\cup(c^0, d^{j'})=(c^0, d^{j+j'})$ and $(c^{2m+1}, d^0)\cup(c^{2m+1}, d^0)=0$. q. e. d.

PROPOSITION 1.2. (i) *$Sq^2(y^j)=\binom{j}{2}y^{j+2}$ for $y=(c^0, d^1)$.*

(ii) *$Sq^2(x)=0$, for $x=(c^{4m+3}, d^0)\in H^{4m+3}(D_p(2m+1, 4k+3); Z_2)$.*

PROOF. (i) is clear. (ii) : Suppose to the contrary that $Sq^2(x)\neq 0$. The differential $d_2^{4m+3,0}$ of the Atiyah-Hirzebruch spectral sequence is represented by the cohomology operation Sq^2 [**6**]. Therefore the assumption implies that $d_2^{4m+3,0}$ for $\widetilde{KO}^*(D_p(2m+1,4k+3))$ is non trivial, and the rank of $\widetilde{KO}^{4m+3}(D_p(2m+1, 4k+3))\otimes Q\leq 1$, for the free part of $\tilde{H}^*(D_p(2m+1, 4k+3); Z)$ is isomorphic to $Z(c^{4m+3}, d^0)\oplus Z(c^0, d^{4k+3})\oplus Z(c^{4m+3}, d^{4k+3})$. On the other hand, it follows from the Dold theorem [5] that $\widetilde{KO}^{4m+3}(D_p(2m+1, 4k+3))\otimes Q\cong Q\oplus Q$. Therefore $Sq^2(x)=0$. q. e. d.

Recall that $\tilde{H}^*(D_p(4m+3, 8n+7); Z)$ is isomorphic to $Z(c^0, d^{8n+7})\oplus Z(c^{8m+7}, d^0)\oplus Z(c^{8m+7}, d^{8n+7})\oplus\sum_j Z_2(c^0, d^{2j})\oplus\sum_j Z_2(c^{8m+7}, d^{2j})\oplus\sum_i Z_p(c^{4i}, d^0)\oplus$

$\sum_i Z_p(c^{4i}, d^{8n+7})$, where $0\leqq 2j\leqq 8n+7$ and $0\leqq 2i\leqq 4m+3$, ([13]).

In the spectral sequence $\{E_r^{p,q}, d_r^{p,q}\}$ for KO-theory, the differentials $d_2^{p,-8t}$, $d_2^{p,-8t-1}$ and $d_3^{p,-8t-2}$ are known as follows [6]:

$$(*)\begin{cases} d_2^{p,-8t}=Sq^2 \ : \ E_2^{p,-8t}\to E_2^{p+2,-8t-1} \\ d_2^{p,-8t-1}=Sq^2 \ : \ E_2^{p,-8t-1}\to E_2^{p+2,-8t-2} \\ d_3^{p,-8t-2}=\{\delta_2 Sq^2\} \ : \ E_3^{p,-8t-2}\to E_3^{p+3,-8t-4} \end{cases}$$

Let $KO^i=KO^i(a\ point)$. We now investigate the Atiyah-Hirzebruch spectral sequence $\{E_r^{p,q}, d_r^{p,q}\}$ for $\widetilde{KO}(D_p(4m+3, 8n+7))$.

(i) $E_2^{8t,-8t}$ is generated by $(c^0, d^{8t})\otimes\kappa_0$ and $(c^{8t}, d^0)\otimes\kappa_0$, where $E_2^{8t,-8t}\cong\tilde{H}^{8t}(D_p(4m+3, 8n+7); KO^{-8t})$ and κ_0 is a generator of $KO^{-8t}\cong Z$.

(ii) $E_2^{8t+1,-8t-1}\cong\tilde{H}^{8t+1}(D_p(4m+3, 8n+7); KO^{-8t-1})$ is generated by $y^{8t+1}\otimes\kappa_1$ and $x^{8m+7}y^{8t-8m-6}\otimes\kappa_1$, where κ_1 is the generator of $KO^{-8t-1}\cong Z_2$. Using (*) and Proposition 1.2, we find that

$$d_2^{8t+1,-8t-1}(x^{8m+7}y^{8t-8m-6}\otimes\kappa_1)=x^{8m+7}y^{8t-8m-4}\otimes\kappa_2,\quad d_2^{8t+1,-8t-1}(y^{8t+1}\otimes\kappa_1)=0,$$

where κ_2 is the generator of $KO^{-8t-2}\cong Z_2$. Hence, $E_3^{8t+1,-8t-1}$ is generated by $y^{8t+1}\otimes\kappa_1$.

(iii) $E_2^{8t+2,-8t-2}\cong\tilde{H}^{8t+2}(D_p(4m+3, 8n+7); KO^{-8t-2})$ is generated by $y^{8t+2}\otimes\kappa_2$ and $x^{8m+7}y^{8t-8m-5}\otimes\kappa_2$.

The differential

$$d_2^{8t,-8t-1}\colon E_2^{8t,-8t-1}\cong\tilde{H}^{8t}(D_p(4m+3, 8n+7); Z_2)\to E_2^{8t+2,-8t-2}$$

is trivial and $E_2^{8t+4,-8t-3}\cong 0$. Therefore $E_3^{8t+2,-8t-2}\cong E_2^{8t+2,-8t-2}$. From (*) and Proposition 1.2,

$$d_3^{8t+2,-8t-2}(x^{8m+7}y^{8t-8m-5}\otimes\kappa_2)=(c^{8m+7}, d^{8t-8m-2})\otimes\kappa_4$$

where κ_4 is a generator of $KO^{-8t-4}\cong Z$. This implies that $E_4^{8t+2,-8t-2}$ is generated by $y^{8t+2}\otimes\kappa_2$.

(iv) $E_2^{8t+3,-8t-3}\cong E_2^{8t+5,-8t-5}\cong E_2^{8t+6,-8t-6}\cong E_2^{8t+7,-8t-7}\cong 0$.

(v) $E_2^{8t+4,8t-4}$ is generated by $(c^{8t+4}, d^0)\otimes\kappa_4$ and $(c^0, d^{8t+4})\otimes\kappa_4$.

PROPOSITION 1.3. *The order of* $\widetilde{KO}(D_p(4m+3, 8n+7))\leqq 2^{\phi(8n+7)}p^{2m+1}$, *where* $\phi(s)$ *is the number of integers* s *with* $0<s\leqq 8n+7$ *and* $s\equiv 0, 1, 2, 4$ *modulo* 8.

PROOF. From the above observation of the spectral sequence for $\widetilde{KO}(D_p(4m+3, 8n+7))$ it follows that $\sum_j E_4^{j,-j}$ is generated by

$(c^0, d^{8t})\otimes\kappa_0$, $y^{8t+1}\otimes\kappa_1$, $y^{8t+2}\otimes\kappa_2$ and

$(c^0, d^{8t+4})\otimes\kappa_4$ which are order of 2,

and

$(c^{8t}, d^0)\otimes\kappa_0$, and $(c^{8t+4}, d^0)\otimes\kappa_4$ which are order of p. Therefore, Proposion 1.3 follows. q. e. d.

According to [7], there is an injective homomorphism

$$\theta : \widetilde{KO}(L^{4m+3}(p))\oplus\widetilde{KO}(RP^{8n+7})\to\widetilde{KO}(D_p(4m+3, 8n+7)),$$
$$\theta(x, y)=i_!(x)+p^!(y),$$

where $i_!$ is a homomorphism defined from the induced representation with respect to $Z_p\subset D_p$ and $p^!$ is a ring homomorphism induced from the natural projection

$p : D_p(4m+3, 8n+7)\to RP^{8n+7}$, $p([z, u])=[u]$. Note that the order of $\widetilde{KO}(L^{4m+3}(p))=p^{2m+1}$ [10], and the order of $\widetilde{KO}(RP^{8n+7})=2^{\phi(8n+7)}$ [1]. Finally, we obtain

THEOREM 1.4. $\theta : \widetilde{KO}(L^{4m+3}(p))\oplus\widetilde{KO}(RP^{8n+7})\to$ $\widetilde{KO}(D_p(4m+3, 8n+7))$ *is isomorphic.*

2. The structure of $J(D_p(4m+3, 8n+7))$, p : odd prime.

We use the important result due to Adams [2], [3] and Quillen [14]: $J(X)$ is isomorphic to $\widetilde{KO}(X)/V(X)$, where $V(X)=\cap\sum k^e(\psi^k-1)\ \widetilde{KO}(X)$, ψ^k : Adams operation.

Denote by ξ the canonical real line bundle over the real projective space and η the canonical complex line bundle over the lens space. Put $\sigma=\eta-1_C$ in $\widetilde{KO}(L^l(p))$ and let r be the real restriction homomorphism.

The following is well known.

THEOREM 2.1. (Adams [3], Kambe, Matsunaga and Toda [11])

(i) *$J(RP^n)$ is isomorphic to a cyclic group of order $2^{\phi(n)}$ generated by $J(\xi-1)$.*

(ii) *$J(L^l(p))$, $l\not\equiv 0$ mod. 4, is isomorphic to a cyclic group of order $p^{[l/(p-1)]}$ generated by $J(r(\sigma))$.*

We use maps $i : L^l(p)\to D_p(l, m)$, $i([z])=[z, e]$, $j : RP^m\to D_p(l, m)$, $j([x])=[e, x]$ and $p : D_p(l, m)\to RP^m$, $p([z, x])=[x]$, where $e=(1, 0, \dots, 0)$.

LEMMA 2.2. *The following diagram is commutative:*

$$\begin{array}{ccc}\widetilde{KO}(L^{4m+3}(p)) & \xrightarrow{i_!} & \widetilde{KO}(D_p(4m+3, 8n+7))\\ \downarrow\psi^k & & \downarrow\psi^k\\ \widetilde{KO}(L^{4m+3}(p)) & \xrightarrow{i_!} & \widetilde{KO}(D_p(4m+3, 8n+7)).\end{array}$$

PROOF. According to Proposition 2.2 of [7], $i^! i_! = 2$, and $i^! \psi^k i_! = i^! i_! \psi^k$. Therefore $\psi^k i_!(x) - i_! \psi^k(x)$ belongs to the kernel of $i^! : \widetilde{KO}(D_p(4m+3, 8n+7)) \to \widetilde{KO}(L^{4m+3}(p))$ and $\psi^k i_!(x) - i_! \psi^k(x) = 0$, for $\psi^k i_!(x) - i_! \psi^k(x)$ is contained in the p-group and $\mathrm{Ker}\, i^!$ is the 2-group from Theorem 1.4.

q. e. d.

By Lemma 2.2 we see that $i_!$ induces the homomorphism

$$i_! : J(L^{4m+3}(p)) \longrightarrow J(D_p(4m+3, 8n+7)).$$

THEOREM 2.3. *There exists an isomorphism*

$$\theta : J(D_p(4m+3, 8n+7)) \longrightarrow J(RP^{8n+7}) \oplus J(L^{4m+3}(p)),$$

which is defined by $\theta(\alpha) = (j^!(\alpha), i^!(\alpha))$.

PROOF. From Theorem 1.4, we have the exact sequence

$$\widetilde{KO}(RP^{8n+7}) \xrightarrow{p^!} \widetilde{KO}(D_p(4m+3, 8n+7)) \xrightarrow{i^!} \widetilde{KO}(L^{4m+3}(p)) \longrightarrow 0.$$

Since pj is the identity, $p^! : J(RP^{8n+7}) \longrightarrow J(D_p(4m+3, 8n+7))$ is injective and there exists a split short exact sequence

$$0 \longrightarrow J(RP^{8n+7}) \underset{j^!}{\overset{p^!}{\rightleftarrows}} J(D_p(4m+3, 8n+7)) \xrightarrow{i^!} J(L^{4m+3}(p)) \longrightarrow 0.$$

Then we obtain an isomorphism

$$\theta : J(D_p(4m+3, 8n+7)) \longrightarrow J(RP^{8n+7}) \oplus J(L^{4m+3}(p))$$

defined by $\theta(\alpha) = (j^!(\alpha), i^!(\alpha))$ with the inverse $\bar{\theta}$ defined by

$$\bar{\theta}(\beta, \gamma) = p^!(\beta) + i_!((1/2)\gamma).$$

q. e. d.

3. The stable homotopy type of $D_p(n, m)/D_p(n', m)$ and $D_p(n, m)/D_p(n, m')$.

Considering dihedral group actions on $S^{2l+1} \times S^m \times R$ and $S^{2l+1} \times S^m \times C$ by

$$t(z, x, y) = (\bar{z}, -x, -y), \quad g(z, x, y) = (\rho z, x, y), \quad (z, x, y) \in S^{2l+1} \times S^m \times R$$

and

$$t(z, x, u) = (\bar{z}, -x, \bar{u}), \quad g(z, x, y) = (\rho z, x, \rho y), \quad (z, x, u) \in S^{2l+1} \times S^m \times C,$$

we have a real line bundle

$$\xi_1 : (S^{2l+1} \times S^m \times R)/D_p \longrightarrow D_p(l, m)$$

and a real 2-plane bundle

$$\eta_1 : (S^{2l+1}\times S^m\times C)/D_p \longrightarrow D_p(l, m), \quad \text{(cf. [7])}.$$

Denote by X^η a Thom complex of a vector bundle η over X. Then we obtain

THEOREM 3.1. *$D_p(l, m)^{k\eta_1 \oplus n\xi_1}$ is homeomorphic to*
$$D_p(l+k, m+n)/(D_p(k-1, m+n) \cup D_p(l+k, n-1)).$$

PROOF. Taking spaces $D(k\eta_1 \oplus n\xi_1) = S^{2l+1}\times S^m\times D^{2k}\times D^n/\sim$ and $S(k\eta_1 \oplus n\xi_1) = S^{2l+1}\times S^m\times (S^{2k-1}\times D^n \cup D^{2k}\times S^{n-1})/\sim$, where $(z, x, u, y)\sim(\rho z, x, \rho u, y)$ and $(z, x, u, y)\sim(\bar{z}, -x, \bar{u}, -y)$, we see that Thom complex $D_p(l, m)^{k\eta_1\oplus n\xi_1}$ is homeomophic to $D(k\eta_1 \oplus n\xi_1)/S(k\eta_1 \oplus n\xi_1)$. Consider a map
$$f : D(k\eta_1 \oplus n\xi_1) \longrightarrow S^{2(l+k)+1}\times S^{m+n}$$
given by $f(z, x, u, y) = ((u, \sqrt{1-\|u\|^2}\cdot z), (y, \sqrt{1-\|y\|^2}\cdot x))$. Then $f(S(k\eta_1 \oplus n\xi_1)) \subset D_p(k-1, m+n) \cup D_p(l+k, n-1)$ are the f induces a homeomorphism
$$D_p(l, m)^{k\eta_1\oplus n\xi_1} \cong D_p(l+k, m+n)/(D_p(k-1, m+n) \cup D_p(l+k, n-1)).$$
q. e. d.

COROLLARY 3.2. *There are following homeomorphisms:*

(i) $D_p(l, n)/D_p(l, n-1) \cong S^n \wedge D_p(l, 0)^+$,

(ii) $D_p(k, m)/D_p(k-1, m) \cong S^k \wedge D_p(0, m)^{k\xi_1}$.

PROOF. (i) Taking $k=0$ and $m=0$ in Theorem 3.1, we have that $D_p(l, 0)^{n\xi_1}$ is homeomorphic to $D_p(l, n)/D_p(l, n-1)$. Since $H^1(D_p(l, 0); Z_2) \cong 0$ from Theorem 1.1, any line bundle over $D_p(l, 0)$ is trivial and (i) follows.

(ii) Taking $l=0$ and $n=0$ in Theorem 3.1, we have that $D_p(0, m)^{k\eta_1}$ is homeomorphic to $D_p(k, m)/D_p(k-1, m)$. Making use of Proposition 3.1 (ii) in [7], $\eta_1 \cong 1 \oplus \xi_1$, we obtain (ii). q. e. d.

PROPOSITION 3.3. *Let η_1 be the bundle over $D_p(4m+3, 8n+7)$. Then, $J(\eta_1-2)$ has order of $2^{\phi(8n+7)}p^{[(4m+3)/(p-1)]}$.*

PROOF. From Theorem 2.3, it follows that
$$\theta(J(\eta_1-2)) = (j^! J(\eta_1-2),\ i^! J(\eta_1-2)).$$

By making use of the fact that $j^! \eta_1 \cong 1 \oplus \xi$ (Proposition 3.1 (iv) of [7]) and $i^! \eta_1 = \eta$ (Proposition 3.1 (i) of [7]), we obtain $\theta(J(\eta_1-2)) = (J(\xi-1), J(\eta-2))$. And Theorem 2.1 completes the proof. q. e. d.

PROPOSITION 3.4. *Let ξ_1 be the bundle over $D_p(l, m)$. Then, $J(\xi_1-1)$ has oder of $2^{\phi(m)}$.*

PROOF. The natural injection $j : RP^m \longrightarrow D_p(l, m)$, $j([y]) = [e, y]$ and the natural projection $p : D_p(l, m) \longrightarrow RP^m$, $p([z, y]) = [y]$, satisfy $pj=$ identity. Therefore $p^! : J(RP^m) \to J(D_p(l, m))$ is injective. q. e. d.

THEOREM 3.5.

(i) $D_p(l, 2^{\phi(m-1)}q+n)/D_p(l, n-m+2^{\phi(m-1)}q) \simeq_S D_p(l, n)/D_p(l, n-m)$.

(ii) *If* $l-1\equiv 3$ *modulo* 4 *and* $m\equiv -1$ *modulo* 8, *then* $D_p(n+rq, m)/D_p(n+rq-l, m) \simeq_S D_p(n, m)/D_p(n-l, m)$, *where* $r=2^{\phi(m)}p^{[(l-1)/(p-1)]}$.

PROOF. (i) Let $t=2^{\phi(m-1)}q$. From Theorem 3.1, $D_p(l, t+n)/D_p(l, n-m+t)$ is homeomorphic to $D_p(l, m-1)^{(n-m+t+1)\xi_1}$. Proposition 3.4 and [4] implies that $D_p(l, m-1)^{(n-m+t+1)\xi_1} \simeq_S D_p(l, m-1)^{(n-m+1)\xi_1}$ and (i) follows.

(ii) From Theorem 3.1, $D_p(n+rq, m)/D_p(n+rq-l, m)$ is homeomorphic to $D_p(l-1, m)^{(n+rq-l+1)\eta_1}$. Proposition 3.3 implies that $J((n+rq-l+1)\eta_1) = J((n-l+1)\eta_1)$ and $D_p(l-1, m)^{(n+rq-l+1)\eta_1} \simeq_S D_p(l-1, m)^{(n-l+1)\eta_1}$ [4]. Using Theorem 3.1, we can see that $D_p(l-1, m)^{(n-l+1)\eta_1}$ is homeomorphic to $D_p(n, m)/D_p(n-l, m)$ and (ii) follows. q. e. d.

References

[1] J. F. ADAMS: *Vector fields on spheres*, Ann. of Math. 75(1962), 603-622.

[2] J. F. ADAMS: *On the groups $J(X)$-I*, Topology 2(1963), 181-195.

[3] J. F. ADAMS: *On the groups $J(X)$-II, -III*, Topology 3(1965), 137-172, 193-222.

[4] M. F. ATIYAH: *Thom complexes*, Proc. London Math. Soc. 11(1961), 291-310.

[5] A. DOLD: Lectures on general cohomology, Nordic Summer School in Math. Aarhus, 1968.

[6] M. FUJII: *K_0-group of projective spaces*, Osaka J. Math. 4(1967)141-149.

[7] T. FUJINO: *Immersions and embeddings of orbit manifolds $D_p(l, m)$ of $S^{2l+1}\times S^m$ by dihedral groups D_p*, Osaka J. Math. 12(1975), 299-304.

[8] T. FUJINO, N. ISHIKAWA and M. KAMATA: *On the complex K-groups of certain manifold*, Math. Rep. College General Ed. Kyushu Univ. 9(1973), 1-6.

[9] M. IMAOKA and M. SUGAWARA: *On the K-ring of the orbit manifold $(S^{2m+1}\times S^l)/D_n$ by the dihedral group D_n*, Hiroshima Math. J. 4(1974), 53-70.

[10] T. KAMBE: *The structure of K-rings of the lens space and their applications*, J. Math. Soc. Japan 18(1966), 135-146.

[11] T. KAMBE, H. MATSUNAGA and H. TODA: *A note on stunted lens space*, J. Math. Kyoto Univ. 5(1966), 143-149.

[12] M. KAMATA and H. MINAMI: *Bordism groups of dihedral groups*, J. Math. Soc. Japan 25(1973), 334-341.

[13] M. KAMATA: *On complex cobordism groups of classifying spaces for dihedral groups*, Osaka J. Math. 11(1974), 367-378.

[14] D. G. QUILLEN: *The Adams conjecture*, Topology 10(1971), 67-80.