that lies at the bottom of all things; but that this is so far from being the final one, that even now the fourth stage is on its heels. In the fourth stage the conception of Force disappears, and whatever happens is regarded as a deed. The object of the discourse was to explain the nature of this transition, and to introduce certain conceptions which might serve to prepare the way for it.

There are, then, to be considered two different answers to the question, 'What is it that lies at the bottom of things?' The two answers correspond to two different ways of stating the question; namely, first, 'Why do things happen?' and, secondly, 'What is it precisely that does happen?' The speaker maintained that the first question is external to the province of science altogether, and science has nothing to do with it; but that the second is exactly the question to which science is always trying to find the answer. It may be doubted whether the first question is within the province of human knowledge at all. For it is as necessary that a question should *mean something*, in order to be a real question, as that an answer should mean something, in order to be a real answer. And it is quite possible to put words together with a note of interrogation after them without asking any real question thereby. Whether the phrase, 'Why do things happen?' as applied to physical phenomena, is a phrase of this kind or no, is not here to be considered. But that to the scientific enquirer there is not any '*why*' at all, and that if he ever uses the word it is always in the sense of *what*, the speaker regarded as certain. In order to show what sort of way an exact knowledge of the facts would supersede the enquiry

after the cause of them, he then made use of the hypothesis of continuity; showing, in the following manner, that it involves such an interdependence of the facts of the universe as forbids us to speak of one fact or set of facts as the *cause* of another fact or set of facts.]

The hypothesis of the continuity of space and time is explained, and the alternative hypothesis is formulated.

From the hypothesis of the complete continuity of time-changes, a knowledge of the entire history of a single particle is shown to be involved in a complete knowledge of its state at any moment.

Things frequently move. Some things move faster than others. Even the same thing moves faster at one time than it does at another time. When you say that you are walking four miles an hour, you do not mean that you actually walk exactly four miles in any particular hour; you mean that if anybody did walk for an hour, keeping all the time exactly at the rate at which you are walking, he would in that hour walk four miles. But now suppose that you start walking four miles an hour, and gradually quicken your pace, until you are walking six miles an hour. Then this question may be asked: Suppose that anybody chose a particular number between four and six, say four and five-eighths, is it perfectly certain that at some instant or other during that interval you were walking at the rate of four miles and five-eighths in the hour? Or, to put it more accurately, suppose that we have a vessel containing four pints of water exactly, and that somebody adds to it a casual quantity of water less than two pints. Then is it perfectly certain that between these two times, when you were walking at four miles an hour, and when you were walking six miles an hour, there was some particular instant at which you were walking exactly as many miles and fractions of a mile an hour as there are pints and fractions of a pint of water in the vessel? The hypothesis of continuity says that the answer to this question is yes; and this is the answer which everybody gives nowadays; which everybody has given mostly since the invention of the differential calculus.

But this is a question of fact, and not of calculation. Let us, therefore, try and imagine what the contrary hypothesis would be like.

You know what a 'wheel of life' is. There is a cylinder with slits in its side, which can be spun round rapidly; and you look through the slits at the pictures opposite. The result is that you see the pictures moving; moreover, you see them move faster or slower according as you turn the cylinder faster or slower. This is what you see, and what appears to happen; but now let us consider what actually does happen. I remember in particular a picture of a man rolling a ball down an inclined plane towards you; he was standing at the farther edge of the inclined plane, as it were behind a counter, and he picked up the balls one by one and rolled them towards you. But now when you took out the strips of paper on which the pictures were drawn, you found that they were really pictures of this man and his ball in a graduated series of positions. Each picture, of course, was perfectly still in itself, a mere drawing on the paper. The first one represented him with his hand below the counter; just picking up the ball; in the next, he had the ball in his hand,

drawn back ready to roll down; in the next, the hand was thrown forwards with the ball in it; in the next, the ball had just left his hand and rolled a little way down; in the next farther, and so on. Now, these pictures being put in the inside of the cylinder which is turning round, come opposite you one by one. But you do not look directly at them; there are slits interposed. The effect of that is, that if you look straight at a certain portion of the opposite picture you can only see it for a very small interval of time; that, namely, during which the slit is passing in front of your eye. Now let us carefully examine what happens. When the slit passes, it goes so quickly that you get, as it were, almost an instantaneous photograph on your eye of the opposite picture; say of the man with his hand below the counter. Then this is effaced, and you see absolutely nothing until the next slit passes. But by the time the next slit comes, another picture has got opposite to you; so that you get an instantaneous photograph this time of the man with his hand drawn back and the ball in it. Then this in its turn is effaced, for a time you see nothing, and then you are given an instantaneous glimpse of the hand thrown forward. In this way, what you really see is darkness relieved by regularly-recurring glimpses of the figure in different positions. Now, this experience that you get is obviously consistent with the hypothesis that the man goes on moving all the time when he is hidden from you; so as to be in exactly that series of positions when you do catch a glimpse of him. And, in fact, you do instinctively, by an inevitable habit, admit this hypothesis, not merely into your mind as a speculation, but into your very sensation as an observed phenomenon. You simply see the man move; and, except for a certain weariness in the eyes, there is nothing to distinguish this perception of movement from any other perception of movement. At the same time we do know very distinctly, and beyond the shadow of a doubt, that there is no continuity in the picture at all: that, in fact, you do not see the same picture twice following, but a new one every time till the cycle is completed; and that the picture never is in any position intermediate between two successive ones of those which you see. Here then is an apparently continuous motion which is really discontinuous; and moreover there is an apparently continuous perception of it which is really discontinuous—that is, it seems to be gradually changed, while it really goes by little jumps.

I suppose very few people have looked at this toy without wondering whether it is not actually and truly a wheel of life, without any joke at all. I mean, that it is very natural for the question to present itself, Do I ever really see anything move? May not all my apparently continuous perceptions be ultimately made up of little jumps, which I run together by this same inevitable instinct? There is another way in which this is sometimes suggested. If you move your hand quickly, you can see a continuous line of light, because the image of every position of your hand lingers a little while upon the retina. But now, if you do this in a room lighted only by an electric spark which is not going very fast, so that the general result is darkness broken by nearly instantaneous flashes at regular intervals; then, instead of seeing a continuous line of light,

you will see a distinct series of different hands, perhaps about an inch apart, if the electric spark is going very slowly, and you move your hand very quickly. But now make the spark go quicker, or your hand slower; the distances between these several hands will gradually diminish, till—you do not know how—the continuous line of light is restored. And the question inevitably presents itself—is not every case of apparently continuous perception really a case of successive distinct images very close together?

That is to say, for instance, if I move my hand so in front of me, and apparently see it take up in succession every possible position on its path between the two extreme positions; do I really see this, or do I only see my hand in a certain very large number of distinct positions, and not at all in the intervening spaces?

I have no doubt whatever myself, that the latter alternative is the true one, and that the wheel of life is really an illustration and type of every moment of our existence. But I am not going to give my reasons for this opinion, because it is quite a different question from the one I am trying to get at. The question, namely, is this. What I see, or fancy I see, is quite consistent with the hypothesis that my hand really does go on moving continuously all the time, and takes up an infinite number of positions between the two extreme ones. But if this hypothesis is not true, what is true? and how are we to imagine any other state of things than that supposed by the hypothesis of continuity?

I draw here two rows of points. The upper row of points is to represent a series of positions in space which it is conceivable that a certain thing might take up.

The lower row of points is to represent a series of instants in time at which it is conceivable that the same thing might exist. Suppose now that at the instant of time represented by the first point of the lower row, the thing held the position in space represented by the first point of the upper row. Suppose that it only existed there for that instant, and then disappeared utterly, so that at these succeeding instants where the lower points have no points directly above them the thing is nowhere at all. Lastly, suppose that at this instant of time which *has* a space-dot above it, the thing existed in that space-position; and so on all through, the thing only existing at those instants whose representative points have a space-dot exactly above them, and being then in the space-position signified by such dots. Then we may call this a discontinuous motion; a motion because the thing is in different places at different times, though it is not at all times that it exists at all; and a discontinuous motion because the thing passes from one position to another *distant* from it without going through any intermediate position.

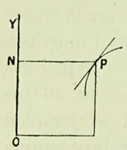
Now imagine that in each of these two series the dots are very close together indeed, and very great in number; so that, however small one made them on the paper, the lines would look as if they were continuous lines. And let the thing be a white speck travelling along the upper line in the manner I have described; namely, existing only when there is one dot exactly over another; only that as the lower dots represent instants of time, we may make some definite supposition and assume that one inch of them represents a second.

Then it is clear that if the dots were taken close enough together, and enough of them, the appearance would be precisely what we ordinarily see when a white speck moves along a line. That is to say, we have got some sort of representation of what we might have to suppose, if we did not assume the truth of the law of continuity.

You must here notice in particular that I suppose the series of positions denoted by the upper dots to be *all* the positions that are between the two end ones; that is, I suppose the path from one of these end ones to the other to be made up of a series of discrete positions. And similarly I suppose the series of instants denoted by the lower dots to be *all* the time that elapses between the two end ones; that is, I suppose the interval of time to contain a perfectly definite number of instants, these being further indivisible. Or we may say that on this alternative hypothesis space and time are discontinuous; that is, they are in separate parts which do not hold together. Now I must beg you to remember for a little while what the hypothesis of continuity is *not*, for I shall have to refer to this point again subsequently. In this kind of jumping motion that we have been imagining, the rate of motion of a thing could only be measured by the size of one of its jumps; that is, by the number of positions it passed over between two existences compared with the number of instants passed over. And this rate might obviously change by jumps as violent and sudden as those of the thing itself; at any instant when the thing was non-existent its rate would be non-existent, and whenever the thing came into existence its rate would suddenly have a value depending on how far off its last position was. In this case, therefore, our question about the intermediate rate—whether between walking four miles an hour and walking six miles an hour you must necessarily walk at all intermediate paces—must be answered in the negative. Now then, at last, let us investigate some consequences of supposing that motion is really continuous as it seems to be.

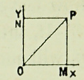
First, how to measure the rate at which a thing is moving? This was done experimentally by Galileo in the case of falling bodies, and I shall have to speak again of the results which he obtained. But at present I want to speak not of an experimental method of finding the rate, but of a theoretical method of representing it, invented by Newton, and called the curve of velocities.

Suppose that a point N is going along the line O Y, sometimes fast and sometimes slow; and that a point M



is going along the line O X always at the same rate. Also somebody always holds a stick N P so as to move with the point N, and be horizontal; and somebody holds a stick M P so as to move with the point M, and be vertical; and a third person keeps a pencil pressed in the corner where the two sticks cross at P. Then when the points M and N move, the point P will move too; and its motion will depend on that of the two other points. For instance, if the point N moves always exactly as fast as the point M, then the point P will go along the line O P midway between the lines O X O Y. If N moves twice as fast as M always, the point P will go along a line

nearer O Y; and if N moves only half as fast as M, then P will go along a line nearer O X. And in general, the faster N moves, the more the line will be tilted up; and if the rate at which N goes is changeable, the direction of P's motion will be changeable, and P will then describe a curve, which



will be very steep when N is going fast, and more flat when N is going slow. So that the steepness of this curve is now a visible measure of the rate at which N is going, and the *curvature* of it is a visible expression of the fact that the rate is changeable. Now the hypothesis of continuity in the motion of N asserts not merely that N itself moves without any jumps, but that the rate at which N is going changes gradually without any jumps, and consequently that the direction of P's motion changes gradually; or that the curve described by P cannot have a sharp point like this. But it asserts a great deal more besides this, which I shall now endeavour to explain. Let us imagine a new point N_1, so moving that whenever the old N is going at four inches a second, N_1 shall be four inches from O; and when N is going at two inches a second, N_1 shall be two inches from O, and so on, the distance of N_1 from O being always exactly as far as N would go in a second if it went at the rate at which it was moving at that instant. Then the distance O N_1 measures the *rate* at which N is going, or the *velocity* of N. If, for example, there was a thing like a thermometer hung up in a train, so that the height of the mercury always indicated how fast the train was going; when the train was going 17 miles an hour, the mercury stood at 17 inches, and so on; then the top of the mercury would behave towards the train exactly as I want the point N_1 to behave towards the point N. It is to indicate by its height how fast N is going.

If, then, the velocity of N is changeable, the point N_1 will move up and down; and the rate at which N_1 moves up or down is clearly the rate at which the velocity of N is increasing or diminishing. This rate at which the velocity of N changes is called its acceleration. To return to our gauge inside of a train, if in the course of a minute it went up from 17 to 19, the train would be said to have an acceleration of two miles an hour per minute.

Now I shall take another point N_2, which is to behave towards N_1 exactly as N_1 behaves towards N; namely, the distance of N_2 from O_2 is to be always equal to the number of inches which N_1 is going in a second. And then I shall take a point N_3, related in just this same way to N_2, and so on, until I come to a point that does not move at all; and that I might never come to, so that I should have to go on taking new points for ever. But suppose now that I have got this series of points, and that they are all moving together. Then first of all there is my point N, which moves anyhow. Next there is N_1, such that $O_1 N_1$ is the velocity of N, or the rate of change of N's position. Next there is N_2, such that $O_2 N_2$ is the acceleration of N, or the rate of change of the rate of change of N's position. Then again $O_3 N_3$ is the change of the acceleration of N, or the rate of change of the rate of change of the rate of change of N's position, and so on. We may, if we like, agree to call the velocity of N the change of the first order, the velocity of N_1 the change of the second order, and so on.

Then the hypothesis of the perfect continuity of N's motion asserts that all these points move continuously without any jumps. Now, a jump made by any one of these points, being a finite change made in no time, would be a change made at an infinite rate; the next point, therefore, and all after it, would go right away from O, and disappear altogether. We may thus express the law of continuity also in this form; that there is no infinite change of any order.

Now, observe further that the rate at which anything is going is a property of the thing at that instant, and exists whether the thing goes any more or not. If I drop a marble on the floor, it goes faster and faster till it gets there, and then stops; but at the instant when it hit the floor it was going at a perfectly definite rate, which can be calculated, though it did not actually go any more.

In the same way the configuration of all these points which depend on the point N is a property of its motion at any given instant, quite independent of the continuance of that motion. I want you to take particular notice of this fact, that as the point N moves about, the whole set of points connected with it moves too; and that you may regard them as connected by some machine, which you may stop at any moment to contemplate the simultaneous positions of all these points; and that this set of simultaneous positions belongs just simply to that one position of the point N, and therefore to one instant of time.

Now I am going to state to you dogmatically a certain mathematical theorem, called Taylor's theorem; whereby you will see the very remarkable consequences of this hypothesis that we have made.

Namely, there is a certain rule whereby when the positions of all these points are known for any particular instant of time, then their positions at any other instant of time may be calculated from these; and it is impossible that they should have at that other instant any other positions than those so calculated. Provided always that there is no infinite change of any order; that is to say, that no one of the points has taken a sudden jump and sent all the points after it away to an infinite distance from O at any instant between the one for which the positions are given and the one for which they are calculated.

Remember that the positions of all the derivative points are mere properties of the motion of the point N at any instant; that in fact we must know them all in order to know completely the state of the point N at that instant. And then observe the result that we have arrived at. From the knowledge of the complete state at any instant of a thing whose motion obeys the law of continuity, we can calculate where it was at any past time, and where it will be at any future time. Now the hypothesis of continuity, of which we have only got disjointed fragments hitherto, is this; that the motion of every particle of the whole universe is entirely continuous. It follows from this hypothesis that the state at this moment of any detached fragment—say a particle of matter at the tip of my tongue—is an infallible record of the eternal past, an infallible prediction of the eternal future.

This is not the same as the statement that a complete knowledge of the position and velocity of every body in the universe at a given moment would suffice

to determine the position at any previous or subsequent moment. That depends on an entirely different hypothesis, and relates to the whole, while this proposition that I am now expounding relates to every several part however small. Now reflect upon the fact that for a single particle—quite irrespective of everything else—the history of eternity is contained in every second of time; and then try if you can find room in this one stifling eternal fact for any secondary causes and the question why? Why does the moon go round the earth? When the Solar system was nebulous, anybody who knew all about some one particle of nebulous vapour might have predicted that it would at this moment form part of the moon's mass, and be rotating about the earth exactly as it does. But why with an acceleration inversely as the square of the distance? There is no why; the fact is probably equivalent to saying that the continuous motion of one body is such as not to interfere with the continuous motion of another. If once so, then always; the cause is only the fact that at some moment the thing is so,—or rather, the facts of one time are not the cause of the facts of another, but the facts of all time are included in one statement, and rigorously bound up together.

Parallel, however, with this hypothesis of temporal continuity, there is another hypothesis, not so universally held, of a continuity in space; for which indeed I hope to make more room presently. And out of this it appears that as the history of eternity is written in every second of time, so the state of the universe is written in every point of space.