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Novel examples of the application of gauge/gravity duality

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Abstract

When we work on an intractable non-perturbative problem of strongly interacting gauge field theory, the method of the gauge/gravity duality is a very useful tool. In this approach, we can treat the full quantum field theory in flat d-dimensional Minkowski spacetime as the dual of a classical gravitational theory in curved $D\,(> d)$ dimensional spacetime. In this thesis we study two types of the problems that are very difficult to investigate in the ordinary field theoretical approach, namely the physics of an accelerated quark in a scale dependent strongly interacting field theory and a QCD-like model with a finite chemical potential.

We first review the idea of gauge/gravity duality briefly and introduce the models that we consider in this thesis. Next, we discuss a quark accelerated with a constant acceleration in a scale dependent strongly interacting gauge theory by using the duality introduced in the review. We study thermal aspects for an accelerated quark (Unruh effect) by applying the gauge/gravity duality in the co-moving frame of the quark. There is a screening of the color force due to the thermal effect of the acceleration and the thermal effect is anisotropic in contrast to the ordinary thermal effect.

We also study a QCD-like model with a finite chemical potential by using Sakai-Sugimoto model. In our approach, an infinite number of baryons are introduced as the sum of an infinite number of solitons of $SU\,(N_f)$ gauge field on N_f probe D8-branes. This "dilute gas approximation" of solitons makes the action simple. We get non-trivial results for the relation between the baryon number density n_B and the baryon chemical potential μ_B; we find a first-order phase transition from $n_B = 0$ phase (the vacuum) to finite n_B phase (infinite nuclear matter) when the chemical potential is increased. We then extract the equation of state of neutrons from this result and apply it to neutron stars. The radius and the mass which are obtained from our calculation is, however, much smaller than the empirical values. The results implies that our approach should be improved by taking the contributions from the repulsive force into account.

Contents

Chapter 1

Introduction

1.1 Overview

In elementary particle physics, the field theoretical approach is a fundamental method for clear understanding of nature. Quantum electrodynamics(QED) is a prominent example of the success of the approach. The theory predicts many physical values from only two values, the electron mass m_e and the electric charge e. The fact that many physical quantities are predicted by only a few parameters is marvelous.

Though quantum field theories is successful for a wide region of elementary particle physics, it is not always possible to obtain useful predictions for all of physical quantities. It usually depends on perturbation theory that works only when the coupling constants are small. Moreover couplings depend on the energy scale of the process in question. Thus the perturbative expansion is not possible in all the scales. For example, at low energies, the perturbation theory works in QED and the theory predicts the correct values, the method however does not work in Quantum chromodynamics(QCD) that describes the physics of hadrons. We are unable to make predictions for low-energy quantities by using perturbation theory. This problem can occur in any "asymptotic free" theory in which couplings increase with the decreasing energy scale. It is therefore important to understand non-perturbative aspects of quantum field theory.

When we cannot use perturbation theory, we must choose other methods for the calculation. The numerical simulations using lattice field theory and the calculation by using a gauge/gravity duality are two of the main non-perturbative methods. Lattice field theory is defined on a discretized Euclidean spacetime and may be used as a definition of a theory of continuous spacetime in an appropriate limit. This approach is very clear, but it costs us much time and money to get results with high accuracy. The method using a gauge/gravity duality, which is a duality between a gauge theory with a large number of colors N_c with a gravitational theory, is a another viable non-perturbative one. In this approach, we can calculate quantities easier than in lattice gauge theory. However it has several problems—it is applied to gauge theory with large N_c that is quite far from QCD and the correspondence between a physical quantity in a gauge theory and that in the dual gravitational theory is not very clear in general. Moreover the gauge/gravity duality is just a conjecture and it has not been proven.

Even though gauge/gravity duality has some problems, it is very powerful for analyzing a complicated system of strongly interacting gauge theory. Since we can analyze a full quantum field theory by examining a corresponding classical gravitational theory by using a gauge/gravity duality, we can easily calculate a physical quantity of the gauge theory. For example, we can relatively easily calculate physical quantities of a theory with a finite chemical potential by using gauge/gravity duality, while it is very difficult to do it in lattice gauge theory simulation because of the sign problem. The gauge/gravity duality is a good approach for analyzing systems which is intractable in other approaches.

1.2 Outline of my work

1.2.1 Background and motivations

In this thesis, I discuss two different types of systems which are intractable in the gauge field theory.

1. A quark accelerated with an uniform acceleration a in the strongly interacting field theory with the non-conformal Yang-Mills coupling constant.

2. A system with finite baryon density, represented as the Sakai-Sugimoto model with a baryon chemical potential μ_B.

Both of these are very awkward systems for the usual technologies of field theory because of the following reasons. First there are problems in strongly interacting field theory. Thus it is impossible to treat these problems by using a perturbative method. Second there are additional difficulties such as the non-trivial time-dependent motion of the quark and non-zero chemical potential for the baryon. These additional hurdles make the problem intractable for any other methods. Therefore the method of the gauge/gravity duality plays an important role for studying these systems.

The accelerated quark and Unruh effect When we consider physics for an accelerated object, there is very mysterious phenomenon called Unruh effect[1, 2]. The statement is "an observer which accelerate with a constant acceleration a in Minkowski spacetime feels a thermal bath with the temperature $T = a/2\pi$". This effect occurs very commonly and is independent of the details of the interaction among the fields and whether fields are scalar fields or fermion fields or others.

The non-zero chemical potential system The other complicated system is QCD with a finite chemical potential. The ordinary lattice QCD with a vanishing chemical potential can be studied by the numerical calculation because the calculation can be performed by integrating the QCD functional integral with real positive convergent measure. The system with non-zero chemical potential, however, cannot be calculated by the same way due to the sign problem. By introducing a chemical potential, the integrand of the QCD partition function is no longer a real function and thus the usual Monte Carlo simulation with a positive definite probability for the Markov process cannot apply. Thus it is very difficult for the lattice QCD approach to studying such system.

Neutron stars are mostly constructed of neutrons. Their mass and radius may be calculated from the Tolman-Oppenheimer-Volkoff (TOV) equation by specifying the equation of state (EoS) of neutron. Because the EoS of the neutron matter should be derived from QCD at finite baryon density (with a finite chemical potential), the validity of the calculational method directly affects the results of the analysis of TOV equations via the EoS.

1.2.2 Structure of the thesis

The structure of this thesis and the summary of each chapter are the following.

Chapter 2 In Chap. 2 we briefly review the gauge/gravity duality. We introduce the four types of the gauge/gravity duality models, that is, the original AdS/CFT correspondence, the SAdS/finite temperature QFT correspondence, the dAdS/non-conformal QFT correspondence and the Sakai-Sugimoto model. We also introduce the dualities of physical quantities, such as Wilson loops, temperature, the friction constant, the baryon number density and the baryon chemical potential etc.. We explain the concept of gauge/gravity duality by reviewing the AdS/CFT correspondence. The SAdS model is introduced to explain how we treat the theory at finite temperature in the gauge/gravity duality, and in this model the Wilson loop and the drag force are affected by the thermal effect. The dAdS model and Sakai-Sugimoto model are introduced as the preparation for Chap. 3 and Chap. 4 respectively. The dAdS model is one of the simplest deformation of the AdS model and it realizes the non-conformal YM coupling. The quark is confined by the color force in this model. This model is used to discuss the physics of the accelerated quark in Chap. 3. Sakai-Sugimoto model is the most close to QCD among the gauge/gravity duality models. In the model the baryons are introduced as solitons of $SU(N_f)$ gauge field on N_f D8-branes. We use the model for constructing a system with a finite chemical potential in Chap. 4.

Chapter 3 In Chap. 3, we discuss the physics of a quark accelerated with a uniform acceleration a in the non-conformal gauge field theory. First we review the preceding study in the AdS model which has no scale parameter. In the model, the accelerated string which corresponds to the accelerated quark has a horizon on its induced metric at $r_c = R^2 a$. To move to the co-moving frame (Rindler frame) of this string, the ERT is introduced. The ERT maps the part of the string above r_c into the Rindler frame and the part of the string is static in the frame. In the Rindler frame, there is the horizon which may be interpreted as temperature. The temperature is coincide with the Unruh temperature $T = a/2\pi$.

The analysis similar to the AdS model is applied to the dAdS model. In this model, we can also find the same type of the solution as in the case of the AdS model. The horizon in the induced metric of the string is slightly shifted upward $r_c > R^2 a$. Then we apply the ERT to this model. We study the potential between a quark and an anti-quark and the drag force acting on a quark moving with a constant velocity in this transformed coordinate system. Because of the energy scale dependence of the YM coupling, several characteristic thermal features are observed. First, since the dAdS model exhibits confinement, it is nontrivial that we find an isolated quark in Rindler frame. However we should not regard the existence of an isolated quark in the Rindler frame as the existence of the confinement-deconfinement phase transition of the Minkowski vacuum, because the original Minkowski vacuum does not experience the phase transition. Second, this thermal effect is quite anisotropic. Even though there also exist anisotropic features in AdS case, these are more drastic in our case. For example, there is no static state such that the isolated quark moving with constant velocity to the direction of the acceleration in Rindler frame, while there are in the Rindler-AdS frame.

Chapter 4 In Chap. 4, we discuss the system with finite baryon density by using the Sakai-Sugimoto model. We first review the one-baryon and two-baryon systems in the Sakai-Sugimoto model and then generalize them to the system with an infinite number of baryons. Then applying the dilute gas approximation for $SU(N_f)$ gauge field and the mean field approximation for $U(1)$, we make the system of an infinite number of the baryons tractable. Comparing the energy at finite baryon number density with that at zero baryon number density, under the fixed chemical potential condition, we find a first-order phase transition from zero baryon number density phase to finite baryon number density phase as expected. However the critical value of the baryon number density is considerably larger than the value of the normal nuclear density.

We also study neutron stars to check the validity of our model. The EoS which is calculated in the field theory side affects the radius and the mass of neutron stars. We extract the EoS from our model by assuming several statistical-mechanical relations for fermions and the momentum dependent mass which reflects the non-trivial interactions among baryons. Then we solve the TOV equations and study neutron stars. The upper bound of the mass and the radius of neutron stars, however, is much smaller than the empirical values.

Chapter 2

Gauge/gravity duality

In this section, we review the concept of the gauge/gravity duality and display some realization of that.

2.1 Overview:Gauge/gravity duality

Since gauge theory and gravity are quite different theories, there seems to be no relationship between them. But it is not generally true.

For example, the relationship between these theories can be seen in large N_c gauge theory. The action of large N_c gauge theory is written schematically as

$$S \sim \frac{N_c}{\lambda}\mathrm{Tr}\int d^4x \left[A\partial^2 A + A^2\partial A + A^4\right],$$

where $\lambda \equiv g_{YM}^2 N_c$ and A denotes $SU(N_c)$ gauge field which index is suppressed. Then Feynman rules of this theory is written as follows,

- Relate each propagator of A with the factor $\frac{\lambda}{N_c}$, because the propagator is the inverse of the quadratic differential operator $\frac{N_c}{\lambda}\partial^2$
- Relate each loop with the factor N_c, because N_c types of color indices run in the loop.
- Relate each vertex with $\frac{N_c}{\lambda}$, because all coupling constants are this value.

The factor of the diagram with P propagators, L loops and V vertices is

$$\left(\frac{\lambda}{N_c}\right)^P N_c^L \left(\frac{N_c}{\lambda}\right)^V = \lambda^{P-V}N_c^{\chi}, \quad \chi \equiv -P + L + V.$$

In the large N_c limit, the contribution of the diagram with smaller χ is not important. χ is coincide with the Euler index of the polygon with P edges, L faces and V vertices and the diagram is just the polygon when we regard the propagators, the loops and vertices of the

	$SU(N_c)$	Local Lorentz
Transformation	$\Phi \to e^{i\theta^a T_a}\Phi$	$\Phi \to e^{\frac{1}{2}\epsilon^{\alpha\beta}S_{\alpha\beta}}\Phi$
Connection	$A_\mu = A_\mu^a T_a$	$\omega_\mu = \frac{1}{2}\omega^{\alpha\beta}{}_\mu S_{\alpha\beta}$
Covariant derivative	$D_\mu = \partial_\mu - igA_\mu$	$D_\mu = \partial_\mu + \omega_\mu = \partial_\mu + \frac{1}{2}\omega^{\alpha\beta}{}_\mu S_{\alpha\beta}$
Curvature	$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right]$	$\frac{1}{2}R^{\alpha\beta}{}_{\mu\nu}S_{\alpha\beta} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu + [\omega_\mu, \omega_\nu]$

Table 2.1: The theoretical structures of $SU(N_c)$ gauge theory and the theory with local Lorentz symmetry.Φ is a some representation of the gauge group or of the Lorentz group, A_μ^a, T^a and g are the gauge field, the generator and the coupling constant of $SU(N_c)$ gauge theory, respectively. $\omega^{\alpha\beta}{}_\mu$ and $S_{\alpha\beta}$ are the spin connection and the generators of local Lorentz group. Here $\alpha, \beta = 0, \cdots, D-1$ represent indices for the local Lorentz coordinate and $\mu, \nu = 0, \cdots, D-1$ represent indices for the general coordinate.

diagram as the edges, the faces and the vertices of the polygon respectively. This observation implies the existence of some relationship between the gauge theory and gravity, because the strength of the contribution of diagrams of large N_c gauge theory can be determined by the topology of diagrams.

Another example of a relationship between these theories is the formalism. We can confirm that $SU(N_c)$ gauge theory with $SU(N_c)$ gauge symmetry and gravitational theory with local Lorentz symmetry has the common theoretical structures as in Table 2.1. Replacing θ^a, T_a, $-ig$, A_μ and $F_{\mu\nu}$ with $\epsilon^{\alpha\beta}$, $S_{\alpha\beta}$, 1, ω_μ and $\frac{1}{2}R^{\alpha\beta}{}_{\mu\nu}S_{\alpha\beta}$, respectively, we find a complete analogy between the theoretical structures of these two theories.

So there must be some relationships between these two different theories. One of the realizations of this is the gauge/gravity duality. The gauge/gravity duality is a concept which stems from superstring theory. In string theory, there are open strings which contain gauge fields and closed strings which contain gravitons. It is natural to think that gauge theory is related to gravity. In Appendix A, we argue that in the $l_s \to 0$ limit the low energy effective action of the closed string becomes the one of ten dimensional gravity as in Eq. (A.24) and the Dp-brane action contains the $(p+1)$-dimensional gauge theory as in Eq. (A.31). Since the gauge theory is on D-branes, they play a crucial role of linking the gauge theory with gravity. The reason why gauge fields are localized on D-branes is clear. The gauge field is a set of massless quanta of oscillations of an open string which ends on D-branes and the masses of the lowest excitations of strings are proportional to the length of the string. In order for the excitations to be massless, the string should not have length so that it is localized on a D-brane. On the other hand, the graviton is a set of massless quanta of oscillation of closed string, so they can be anywhere. We will explain the more detailed idea of the gauge/gravity duality in the following sections.

2.2 AdS/CFT correspondence

The AdS/CFT correspondence is one of the realizations of gauge/gravity duality and it is the clearest model to explain the idea of the duality. AdS stands for the gravitational theory on Anti-de Sitter spacetime and CFT for $\mathcal{N}=4$ super Yang-Mills theory which is a conformal field theory. We will explain how the duality realizes this correspondence.

First we explain why gravity on AdS spacetime corresponds to CFT. This may be done by considering the system of N_c D3-branes from two different points of view. The action of this system is given by

$$S = S_{bulk} + S_{D3} + S_{int},$$

where S_{bulk}, S_{D3} and S_{int} are the action on ten-dimensional spacetime , that on D3-branes and the action that describes interactions between the bulk modes and the brane modes.

In the low energy limit $l_s \to 0$, the effective action in Eq. (A.31) is valid. Then in the limit $S_{int} \to 0$ S_{bulk} becomes the action of the ten-dimensional gravity in a flat spacetime, S_{SUGRA} and S_{D3} becomes the action of $\mathcal{N}=4$ $U(N_c)$ gauge theory in $3+1$ dimensions, S_{CFT}. So in this limit we have two decoupled systems, S_{SUGRA} and S_{CFT}.

On the other hand, we can describe the same system in a different fashion. Since D-branes are massive charged objects which act as sources for the graviton or other fields in supergravity, the ten dimensional spacetime geometry is changed from the flat geometry. Especially in the limit $l_s \to 0$, the ten-dimensional Newton constant $G_{10} \sim l_s^8 \to 0$ and the supergravity description of the D-brane action, Eq. (A.24), is valid. Then D3-brane solution of supergravity takes the form,

$$ds^2 = \left(1+\frac{R^4}{r^4}\right)^{-1/2} \eta_{\mu\nu}dx^\mu dx^\nu + \left(1+\frac{R^4}{r^4}\right)^{1/2} \left(dr^2 + r^2 d\Omega_5^2\right), \tag{2.1}$$

where $R^4 \equiv 4\pi g_s \alpha'^2 N_c$ and $\mu,\nu = 0,1,2,3$. The metric Eq. (2.1) is decomposed into two parts,

$$\begin{aligned} ds^2_{AdS} &= \frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2}\left(dr^2 + r^2 d\Omega_5^2\right) \quad (r \ll R), \\ ds^2_{flat} &= \eta_{MN}dx^M dx^N \quad (r \gg R), \end{aligned} \tag{2.2}$$

where $M,N = 0,\cdots,9$. The metric Eq. (2.2) represents the geometry of $\mathrm{AdS}_5 \times S^5$ spacetime. In the limit $l_s \to 0$ with $\lambda = g_{YM}^2 N_c \sim g_s N_c$ being fixed, R goes to zero and the region of Eq. (2.2) shrinks to a single point $r=0$ and all other region becomes Minkowski spacetime. In this limit, we get two separated theories described by the action $S = S_{AdS} + S_{\text{SUGRA}}$.

Now we have two descriptions for the same system, N_c D3-branes in flat spacetime, and they share the same action S_{SUGRA}. So we conclude that the residual part of the action describes same physics. This is a simple reason why we consider gravity on AdS spacetime corresponds to CFT. We summarize the features of the AdS/CFT correspondence in Table 2.2.

	theory	dimensionality	coordinates	$N_c \sim \infty$
Gravity	SUGRA in $\mathrm{AdS}_5 \times S^5$	(effectively) 5	t, x, y, z, r	# of D5-branes
QFT	$\mathcal{N}=4$ SYM in flat 4D space	4	t, x, y, z	the gauge group $SU(N_c)$

	Interaction	parameters	symmetry
Gravity	weak(classical)	$R,\ \alpha',\ g_s,\ N_c$	$SO(3,2) \times SO(6)$
QFT	strong(full quantum)	$\lambda \equiv \frac{R^4}{2\alpha'^2} = 2\pi g_s N_c,\ N_c$	$SO(3,1) \times$R symmetry

Table 2.2: The summary of the AdS/CFT correspondence. In reality, the dimensionality of the gravity theory is 10. However it reduces to 5 due to the rotational symmetry of the S^5 part of the geometry. Four of the remaining five coordinates of the ten-dimensional spacetime is the coordinates of 4D Minkowski spacetime common to the dual gauge theory. The fifth coordinate r of the gravity side is related to the energy scale.

In order for the arguments given above to be valid, several conditions must be satisfied. First the limit $g_s \to 0$ must be taken for the string perturbation to be valid. Secondly we need to take the limit $N_c \to \infty$ with fixed $\lambda \sim g_s N_c$. Finally we need $(R/l_s)^4 \sim \lambda \gg 1$ for the classical gravity description to be valid. From these it is seen that the validity of the above arguments is guaranteed when the conditions $N_c \gg \lambda \gg 1$ is required.

The evidence of the AdS/CFT correspondence is the equivalence of the symmetries on the both sides. On the CFT side, there is the conformal symmetry (ten Poincaré transformations $M_{\mu\nu},\ P_\mu$ and four conformal boost K_μ and one dilatation D) and the conformal algebra is,

$$
\begin{aligned}
[M_{\mu\nu}, P_\rho] &= -i\left(\eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu\right), \\
[M_{\mu\nu}, M_{\rho\sigma}] &= -i\eta_{\mu\rho}M_{\nu\sigma} + i\eta_{\nu\rho}M_{\mu\sigma} - i\eta_{\nu\sigma}M_{\mu\rho} - i\eta_{\mu\sigma}M_{\nu\rho}, \\
[D, P_\mu] &= -iP_\mu, \quad [D, K_\mu] = iK_\mu, \\
[M_{\mu\nu}, K_\rho] &= -i\left(\eta_{\mu\rho}K_\nu - \eta_{\nu\rho}K_\mu\right), \\
[P_\mu, K_\nu] &= 2iM_{\mu\nu} - 2i\eta_{\mu\nu}D,
\end{aligned}
$$

with all other commutators vanishing. On the AdS side, there is the isometry $SO(3,2)$ (which has fifteen generators $J_{MN},\ (M, N = 0, 1, \cdots, 5)$) and the $SO(3,2)$ algebra is,

$$
[J_{MN}, J_{PQ}] = -i\eta'_{MP}J_{NQ} + i\eta'_{NP}J_{MQ} - i\eta'_{NQ}J_{MP} - i\eta'_{MQ}J_{NP},
$$

where $\eta'_{MN} = \mathrm{diag}(-1, 1, 1, 1, -1)$ and all other commutators vanish. The generators of the conformal group may be identified with those of the $SO(3,2)$ isometry group;

$$
J_{\mu\nu} = M_{\mu\nu}, \quad J_{\mu 4} = \frac{1}{2}\left(K_\mu - P_\mu\right), \quad J_{\mu 5} = \frac{1}{2}\left(K_\mu + P_\mu\right), \quad J_{45} = D.
$$

In addition, the isometry S^5 on the AdS side plays a role of R-symmetry on the CFT side. Thus the symmetries on both sides are the same.

On the AdS side, there are five coordinates $x^\mu,\ r$. The coordinates x^μ have $SO(3,1)$ isometry, so it is natural that they are interpreted as the coordinates of the $(3+1)$-dimensional gauge theory. However there is an additional fifth coordinate r which do not exist in the CFT. What is the meaning of this coordinates? Note that the metric given in Eq. (2.2) is invariant under the transformation,

$$x^\mu \to kx^\mu,\ r \to r/k.$$

This symmetry represents the scale invariance of the CFT, and the fifth coordinate r has the same scaling property as that of the energy. Thus we may interpret this coordinate as the energy scale.

2.2.1 Wilson-loop

In gauge theory, Wilson loops are important operator in analyzing the theory. A Wilson loop is defined as

$$W(C) = \mathrm{Tr}\left[P\exp\left(i\oint_C A\right)\right],$$

where P denotes path ordering and the trace is taken over the gauge indices. C is a closed path in four dimensional spacetime. When the path C is the rectangular one with the sides of the time directions of length T and the sides of a space direction of length L, and we take the limit $T\to\infty$, the expectation value of the Wilson loop behaves as

$$\langle W\rangle \sim e^{-TV(L)},$$

where $V(L)$ represents the potential energy between quark and anti-quark with interval L. The information on $V(L)$ is important because it can be used as the criterion whether the theory exhibits confinement or not. If $V(L)$ has an upper bound, we can separate a quark and an anti-quark as far as we want, that is, the color is deconfined. Conversely, if $\frac{\partial}{\partial L}V$ does not approaches zero and has a positive value when L increases, we need infinite energy to separate a quark from an anti-quark, then the color is confined.

Using the AdS/CFT correspondence, a Wilson loop may be obtained by calculating the minimal surface of the string world sheet which boundary depict the closed path C on AdS boundary $(r=\infty)$[3]. This correspondence is quite natural. C may be interpreted as the path of a heavy quark anti-quark pair in the gauge theory. Here by a quark we mean a fundamental representation of $SU(N_c)$. An open string which ends on N_c D-branes is also a fundamental representation of $SU(N_c)$ in the string theory. However when we represent the D3 brane action as the supergravity action, two strings which had been ending on D3-branes lose their endpoints and they must be connected each other. The image of this correspondence is shown in Fig. 2.1.

In a curved spacetime, Nambu-Goto string action (A.1) is written as

$$S_{NG} = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma\sqrt{-\det\left(\frac{\partial X^M}{\partial\sigma^a}\frac{\partial X^N}{\partial\sigma^b}G_{MN}\right)},$$

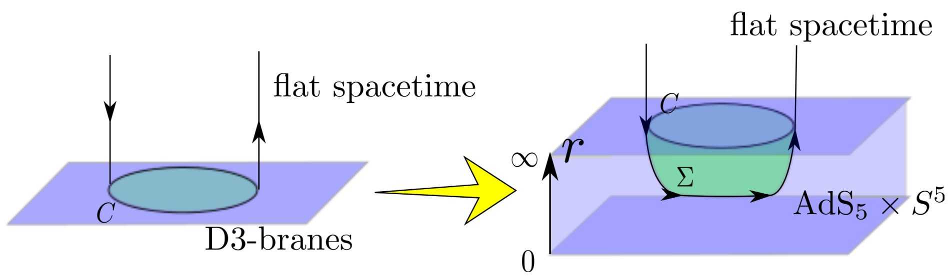


Figure 2.1: The image of the correspondence between the Wilson loop in the CFT and the minimal surface Σ of the string world-sheet in $\mathrm{AdS}_5 \times S^5$ space. On the left hand side, two end points of two strings which have different orientation represents two different fundamental representations of $SU(N_c)$ in the CFT on N_c D3-branes. Then the closed loop of the end points of the strings C should represent a Wilson loop with closed path C. In the appropriate limit, namely in the decoupling limit $\alpha' \to 0$ and the near horizon limit, this situation can be regarded as the minimal surface Σ which satisfies $\partial\Sigma = C$ in $\mathrm{AdS}_5 \times S^5$ space.

where G_{MN} is the metric of ten dimensional spacetime (here, $\mathrm{AdS}_5 \times S^5$ spacetime). By setting $\sigma^0 = t,\ \sigma^1 = r,\ X^1 = x(r)$, the action may be rewritten as

$$S_{NG} = -\frac{T}{2\pi\alpha'}\int dr\sqrt{1+\left(\frac{r}{R}\right)^4 x'^2}, \tag{2.3}$$

where $x' \equiv \partial_r x$. The action does not depend on x, so that there is a conserved momentum

$$p_x \equiv -2\pi\alpha'\partial_{x'}\mathcal{L} = \frac{\left(\frac{r}{R}\right)^4 x'}{\sqrt{1+\left(\frac{r}{R}\right)^4 x'^2}} = c, \tag{2.4}$$

where c is a constant. Because we want a static string solution which corresponds with a rectangular closed path C on the AdS boundary, we search for the solution which has a turning point $r_b : x'(r_b) = \infty$. Then we determine the constant c as $c = \left(\frac{r_b}{R}\right)^2$, solve Eq. (2.4) for x', and get

$$x' = \pm\frac{\left(\frac{R}{r}\right)^2}{\sqrt{\left(\frac{r}{r_b}\right)^4 - 1}}. \tag{2.5}$$

Setting $x(r_b) = 0,\ x(r = \infty) = L/2$ and integrating the both sides of equation, we get the relation between L and r_b

$$\begin{aligned}
\frac{L}{2} &= \int_{r_b}^{\infty} dr \frac{\left(\frac{R}{r}\right)^2}{\sqrt{\left(\frac{r}{r_b}\right)^4 - 1}} \\
&= \frac{R^2}{r_b} \int_1^{\infty} dy \frac{1}{y^2\sqrt{y^4 - 1}} \\
&= \frac{R^2}{r_b} \frac{\sqrt{2}\pi^{3/2}}{\Gamma\left(\frac{1}{4}\right)^2} \ . \qquad (2.6)
\end{aligned}$$

Substituting Eq. (2.5) into Eq. (2.3), we get

$$S = -\frac{2r_b T}{2\pi\alpha'} \int_1^{\infty} dy \frac{y^2}{\sqrt{y^4 - 1}}. \qquad (2.7)$$

The integral diverges because the integrand $\frac{y^2}{\sqrt{y^4-1}}$ goes like ~ 1 at large y. The divergence comes from the contributions originated from the infinitely heavy masses of the quarks. Thus we should subtract this contribution,

$$m_q \equiv \frac{1}{2\pi\alpha'} \int_0^{\infty} dr = \frac{r_m}{2\pi\alpha'} \left(\int_1^{y} dy + 1 \right),$$

from Eq. (2.7). Using Eq. (2.6), we finally get the quark anti-quark potential

$$\begin{aligned}
V(L) &= -\frac{S}{T} - 2m_q \\
&= \frac{2r_b}{2\pi\alpha'} \left\{ \int_1^{\infty} dy \left(\frac{y^2}{\sqrt{y^4 - 1}} - 1 \right) - 1 \right\} \\
&= -\frac{4\pi^2 (2\lambda)^{1/2}}{\Gamma\left(\frac{1}{4}\right)^4} \frac{1}{L} \ .
\end{aligned}$$

This coulomb-like potential is expected because the CFT has no scale, so L^{-1} is needed for this quantity to have the right dimension. In this type of the potential, quarks are not confined because they can go away from the potential force with a finite energy.

2.3 SAdS/finite temperature QFT

One of the simple modification of the AdS/CFT correspondence is adding the temperature to the these theories.

2.3.1 Black hole and Thermodynamics

In gravitational theory, temperature comes in when one considers a black hole. Thermodynamics is described by a few quantities, temperature T, entropy S etc.. On the other hand a black hole is also characterized by a few quantities, the black hole mass M, the black hole charge Q and the angular momentum J. Let us assume that it is a spherical symmetric and chargeless black hole in 4 dimensional spacetime, i.e., a Schwarzschild black hole. There are few quantities to characterize the black hole, the mass M, the area of the horizon A and the surface gravity κ which is defined as the acceleration of a hypothetical particles on the horizon. In reality these quantities, however, are not independent and can be written with M only. The area A is determined by the mass M,

$$A = 4\pi r_H^2 = 16\pi G^2 M^2, \tag{2.8}$$

where $r_H = 2GM$ is the Schwarzschild radius and G is the 4-dimensional Newton constant. This quantity only increases and it never decreases classically, because no particles can escape from the inside of the black hole. Thus it may be regarded as an analog of entropy. Then differentiating both sides of Eq. (2.8), we get

$$dM = \frac{1}{32\pi G^2 M} dA. \tag{2.9}$$

This relation looks like the first low of thermodynamics, $dE = TdS$. When the metric of the black hole is represented as

$$ds^2 = -f(r)\, dt^2 + \frac{1}{f(r)} dr^2 + \cdots, \tag{2.10}$$

where $f(r_H) = 0$. The temperature may be calculated by the following procedure. After making a Wick rotation from t to the Euclidean time t_E by replacing $t \to -it_E$, impose the periodic condition on t_E to avoid the conical singularity,

$$\begin{aligned} ds^2 &= \frac{1}{f(r)} dr^2 + f(r)\, dt_E^2 + \cdots \\ &\sim \frac{1}{f'(r_H)(r - r_H)} dr^2 + f'(r_H)(r - r_H)\, dt_E^2 + \cdots \\ &= d\rho^2 + \rho^2 d\left(\frac{f'(r_H)}{2} t_E\right)^2 + \cdots, \end{aligned}$$

where we introduce a new coordinate $\rho = 2\sqrt{\frac{(r-r_H)}{f'(r_H)}}$. Thus we require that the period is given by $\delta t_E = \frac{4\pi}{f'(r_H)}$. The Hawking temperature is coincide with the inverse of this period, namely

$$T_H = \frac{f'(r_H)}{4\pi}.$$

For a Schwarzschild black hole, the temperature becomes $T_H = \frac{1}{8\pi GM}$, so the first law of the black hole dynamics Eq. (2.9) may be rewritten as

$$dE = T_H dS,$$

where

$$E = M, \quad T_H = \frac{1}{8\pi GM}, \quad S = \frac{A}{4G}.$$

As we have seen above, the black hole represents a thermodynamic system, however, its entropy is not proportional to the volume of the black hole, but to the area of its surface. This holographic feature is compatible with the gauge/gravity duality.

2.3.2 SAdS/finite temperature QFT

As we have discussed in section 2.3.1, the gravitational theory at finite temperature has the black hole geometry. The metric of such a geometry is written as

$$ds^2 = -\left(\frac{r}{R}\right)^2 f(r)\, dt^2 + \frac{dr^2}{\left(\frac{r}{R}\right)^2 f(r)} + \left(\frac{r}{R}\right)^2 \left(dx^2 + dy^2 + dz^2\right), \tag{2.11}$$

where $f(r) = 1 - \left(\frac{r_H}{r}\right)^4$ and there is a horizon at $r = r_H$. This geometry is called Schwarzschild-AdS (SAdS) black hole. The black hole has the Hawking temperature which is defined by using the surface gravity on the horizon $r = r_H$ as

$$T_H = \frac{\kappa}{2\pi} = \frac{\partial_r \left(-G_{tt}\right)|_{r=r_H}}{4\pi} = \frac{r_H}{\pi R^2},$$

where $G_{tt} = -\left(\frac{r}{R}\right)^2 f(r)$.

2.3.3 Wilson loop

Wilson loops in this background are studied in Ref. [4]. We will review the arguments briefly.

In this background, the string action which represents a Wilson loop is written as

$$S_{NG} = -\frac{1}{2\pi\alpha'} \int d\tau dr \sqrt{1 + \left(\frac{r}{R}\right)^4 f(r)\, x'^2},$$

where $x(r)$ is the function which characterize the configuration of the string and $x' \equiv \partial_r x$. We can extract the information on the quark-antiquark potential V from this action by solving x with appropriate boundary conditions and identifying $-S_{NG}/T$ with the potential energy including the contribution of two quark masses, where T and L are the time and the space intervals of the Wilson loop, respectively.

Then conserved canonical momentum conjugate to x is

$$p_x \sim \frac{\left(\frac{r}{R}\right)^4 f(r)\, x'}{\sqrt{1+\left(\frac{r}{R}\right)^4 f(r)\, x'^2}} = c = \left(\frac{r_b}{R}\right)^2 \sqrt{f(r_b)},$$

where c is determined by imposing the condition that the string has the bottom at $r = r_b = br_H > r_H$, namely $x'(r_b) = \infty$.[1] We also impose the condition $x(r_b) = 0$. Solving this equation for x', we get

$$x'(r) = \pm\frac{1}{\sqrt{\left(\left(\frac{r}{r_b}\right)^4 \frac{f(r)}{f(r_b)} - 1\right)\left(\frac{r}{R}\right)^4 f(r)}}. \tag{2.12}$$

Then space interval L which corresponds to the distance between the quark and the antiquark is calculated as

$$\begin{aligned} \frac{L}{2} &= \int_{r_b}^{\infty} dr\; x'(r) \\ &= \frac{R^2}{r_H}\sqrt{b^4-1}\int_b^{\infty} dy \frac{1}{\sqrt{(y^4-b^4)(y^4-1)}}, \end{aligned} \tag{2.13}$$

and the quark-antiquark potential V is

$$\begin{aligned} V &= -\frac{S_{NG}-2S_{free}}{T} \\ &= \frac{r_H}{2\pi\alpha'}\int_b^{\infty} dy \sqrt{\frac{y^4-1}{y^4-b^4}} - \frac{2r_H}{2\pi\alpha'}\int_1^{\infty} dy 1 \end{aligned} \tag{2.14}$$

where $S_{free}/T = -\frac{r_H}{2\pi\alpha'}\int_1^{\infty} dy 1$ represents a contribution of a free quark mass. Evaluating Eq. (2.13) and Eq. (2.14) for each r_b with fixed r_H numerically, we obtain the relation between V and L as shown in Fig. 2.2.

As it can be seen in Fig. 2.2, there are two special point L_{max} and L_*. U-shaped strings cannot extend more than the length L_{max} and we have two string configurations in $L < L_{max}$. One of these configurations has the energy which is higher than the other configuration. The higher energy configuration at $L = 0$ agrees with the twice of the energy of an isolated string which corresponds to quark mass. In addition, two U-shaped string solutions with $L > L_*$ have the energy higher than that of two isolated string. We can confirm that $L_* < L_{max}$ from Fig. 2.2. When there are some solutions with the same physical condition (it means the same value of L in this case), we should choose the solution of the lowest energy. Then the quark-antiquark potential $V(L)$ vanishes at $L > L_*$. The feature of this behavior of V-L relation can be interpreted as the existence of the screening of the color force in the region $L > L_*$. It is characteristic of the theory at finite temperature.

[1] If we chose $r_b < r_H$, x' and x would be pure imaginary. Because they have a meaning of the position of the string, we must choose $r_b > r_H$.

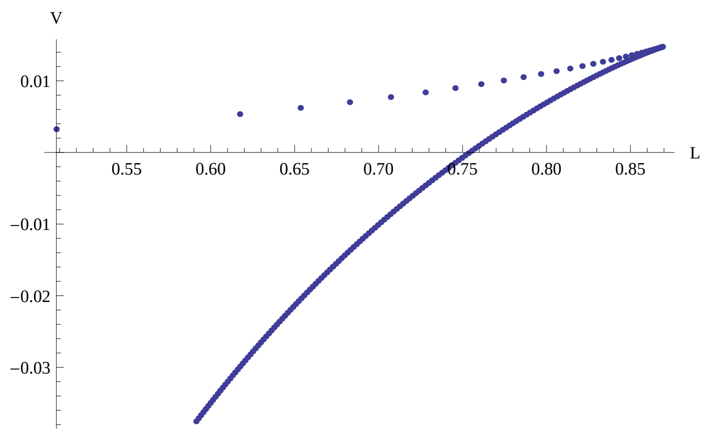


Figure 2.2: V-L relation for finite temperature theory with $\alpha' = 1$, $R = 1$ and $r_H = 1$. Because we normalize the potential by subtracting two quark masses, we should consider that the result can be useful while $V(L) < 0$ is satisfied and we regard $V(L)$ as zero in the regions of $V(L) > 0$ or the region there are no solution which satisfies the condition (2.12).

2.3.4 Drag force

Since the SAdS space has a temperature, there are various thermodynamic features. One of such quantities is a drag force [5]. In the thermal theory, an individual quark moving with an uniform velocity feels resistance from the thermal background. An individual quark is represented by a string hanging down into the horizon from the AdS boundary in the context of gauge/gravity duality. So we should analyze a string which hangs down from the AdS boundary, moving with a uniform velocity

$$x(t, r) = vt + \xi(r). \tag{2.15}$$

Since we want to know the resistance in equilibrium, $\xi(r)$ should not depend on t. When the metric is diagonal, the action of this string and the equation of motion become

$$S_{NG} = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-g}, \tag{2.16}$$

$$\partial_r \left(\frac{\xi' |G_{tt}| G_{xx}}{\sqrt{-g}} \right) = \partial_\xi \left(\sqrt{-g} \right), \tag{2.17}$$

where $\sqrt{-g} \equiv \sqrt{|G_{tt}| G_{rr} - v^2 G_{xx} G_{rr} + \xi'^2 |G_{tt}| G_{xx}}$. In the SAdS spacetime, $|G_{tt}| = \left(\frac{r}{R}\right)^2 f(r)$, $G_{rr} = \frac{1}{\left(\frac{r}{R}\right)^2 f(r)}$, $G_{xx} = \left(\frac{r}{R}\right)^2$, $f(r) = 1 - \left(\frac{r_H}{r}\right)^4$ and the action given in Eq. (2.16) is rewritten as

$$S_{NG} = -\frac{1}{2\pi\alpha'} \int dt dr \sqrt{1 - v^2 \frac{1}{f(r)} + \xi'^2 \left(\frac{r}{R}\right)^4 f(r)}.$$

Because it does not depend on ξ, the conjugate momentum

$$p_\xi \equiv -2\pi\alpha' \frac{\partial \mathcal{L}_{NG}}{\partial \xi'} = \frac{\left(\frac{r}{R}\right)^4 f(r)\,\xi'}{\sqrt{1 - v^2 \frac{1}{f(r)} + \xi'^2 \left(\frac{r}{R}\right)^4 f(r)}} = C \tag{2.18}$$

does not depend on r. Solving this equation for ξ', we get

$$\xi' = \pm \frac{1}{f(r)} \left(\frac{R}{r}\right)^2 \sqrt{\frac{f(r) - v^2}{\left(\frac{r}{R}\right)^4 f(r)/C^2 - 1}}, \tag{2.19}$$

where the sign must be the same as that of the velocity because the string in the bulk is trailed by the end point on the boundary $r = \infty$. Since ξ is a spacetime coordinate, it and its derivative should be real. Thus, to keep the quantity inside the root positive, we impose the following conditions

$$f(\overline{r}) = v^2, \quad \left(\frac{\overline{r}}{R}\right)^4 f(\overline{r}) - C^2 = 0,$$

where $\overline{r}$ is the turning point where the both signs of the denominator and the numerator inside the root change. These conditions imply that

$$C = \pm \frac{v}{\sqrt{1-v^2}} \frac{r_H^2}{R^2}, \tag{2.20}$$

where the sign convention is the same as that of Eq. (2.19). Then plugging Eq. (2.20) into Eq. (2.19), we get

$$\begin{aligned} \xi' &= v \frac{r_H^2 R^2}{r^4 - r_H^4}, \\ \xi &= -\frac{R^2}{2r_H} v \left(\tan^{-1} \frac{r}{r_H} + \ln \sqrt{\frac{r + r_H}{r - r_H}} \right). \end{aligned}$$

A typical configuration of this string is displayed in Fig. 2.3.

Actually $-\frac{1}{2\pi\alpha'} p_\xi$ represents a (r, x) component of a conserved world sheet current of the spacetime energy-momentum $P^r_{\ x} \equiv \frac{\partial \mathcal{L}_{NG}}{\partial(\partial_r x)}$. Thus the quark which corresponds to the end of this string loses the quantity of the momentum per unit time. Therefore the force of resistance acting on the quark due to the thermal disturbance is

$$\begin{aligned} \frac{dp}{dt} &= -\frac{1}{2\pi\alpha'} \frac{v}{\sqrt{1-v^2}} \frac{r_H^2}{R^2} \\ &= -\frac{\sqrt{2\lambda}}{2} \pi T_H^2 \frac{p}{m_q} \\ &\equiv -\eta_{SAdS} \frac{p}{m_q}, \end{aligned}$$

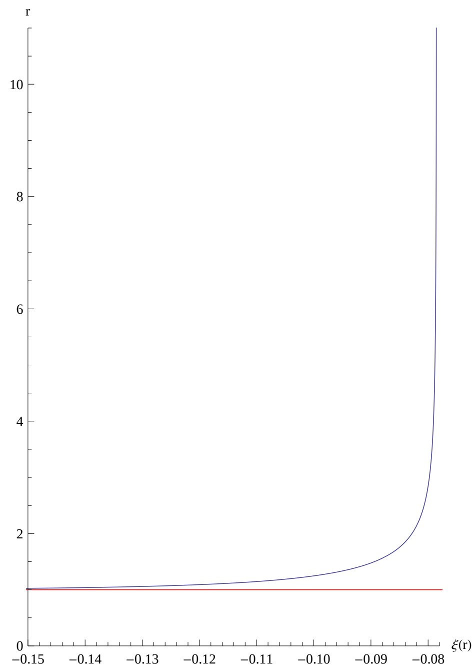


Figure 2.3: A typical configuration of the trailing string with $\alpha' = 1,\ R = 1,\ r_H = 1$ and $v = 0.1$. The horizontal line represents the horizon at $r = r_H$.

where m_q is a (formally infinitely heavy) quark mass, $\lambda = g_{YM}^2 N_c = R^4/2\alpha'^2$ is the 't Hooft coupling, $T_H = \frac{r_H}{\pi R^2}$ is the temperature of the dual field theory and the friction constant for a dual thermal QFT of Schwarzschild-AdS η_{SAdS} is

$$\eta_{SAdS} = \frac{\sqrt{2\lambda}}{2}\pi T_H^2 \tag{2.21}$$

This temperature dependence of η_{SAdS} is peculiar to the field theory equivalent to the gravitational theory on the SAdS spacetime. The SAdS spacetime is generalization of the AdS spacetime with the scale r_H. Thus the corresponding field theory becomes the generalization of the CFT with temperature T_H. Because the theory contains only one scale T_H, quantities that have mass dimension +2 like η must proportional to T_H^2.

2.4 dAdS/non-conformal QFT

Another way of introducing the scale into the theory is making the YM coupling g_{YM} dependent on the scale. A simple model in which such a behavior can be seen is given in Ref. [6, 7]. The model can be obtained by considering 10D IIB supergravity with the self dual five-form field strength F_5, the dilaton Φ, and the axion χ. This supergravity is the low energy limit of the D3-D(-1) (or D-instanton) system. The solution is

$$ds^2 = e^{\Phi/2}\left\{\frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2}dr^2 + d\Omega_5^2\right\}, \quad e^\Phi = 1 + \frac{q}{r^4}, \tag{2.22}$$

where $R^4 = 4\pi g_s N_c$ and q denotes the vacuum expectation value of the gauge fields condensate. Since the metric Eq. (2.22) is the same as the metric of the AdS space up to the deformation of the dilaton factor $e^{\Phi/2}$, we call the spacetime of this metric deformed-AdS spacetime or dAdS spacetime in this thesis.

The form of the low energy effective action of D-brane, Eq. (A.31), suggests that we should identify the YM gauge coupling g_{YM}^2 as

$$g_{YM,p}^2 = \frac{g_s e^\Phi}{T_{Dp}\left(2\pi\alpha'\right)^2} \tag{2.23}$$

in the non-constant dilaton case. If the dilaton field Φ depends on the fifth coordinate r, Eq. (2.23) implies scale dependence of the YM coupling. The model described by the metric in Eq. (2.22) is just such a case. In this model, thus, quarks would be confined by this scale dependent color force. In fact, we can find the quark confinement because we find a linear rising potential between a quark and an antiquark with the tension $\sim \sqrt{q}/\lambda^{1/2}$. However, chiral symmetry is preserved because the vacuum expectation value of the order parameter is zero. In other words, the dynamical mass generation of massless quarks does not occur.

This model corresponds to $\mathcal{N} = 2$ supersymmetric Yang-Mills theory [7].

2.4.1 Wilson loop

Because of the scale dependence of the YM coupling, the Wilson loop in this theory is different from the one in CFT. In this model the action given in Eq. (2.3) and the conserved momentum, Eq. (2.4), are modified as

$$S = -\frac{T}{2\pi\alpha'}\int dr e^{\Phi/2}\sqrt{1 + \left(\frac{r}{R}\right)^4 x'^2},$$

$$p_x \equiv -2\pi\alpha'\partial_{x'}\mathcal{L} = \frac{e^{\Phi/2}\left(\frac{r}{R}\right)^4 x'}{\sqrt{1 + \left(\frac{r}{R}\right)^4 x'^2}} = \left(\frac{r_b}{R}\right)^2 e^{\Phi(r_b)/2},$$

where r_b is a bottom point of the string which satisfies $x'(r_b) = \infty$, $r_b \neq 0$. Then the separation between a quark and an antiquark L and the quark-antiquark potential may be estimated as

$$\begin{aligned} L/2 &= \int_{r_b}^{\infty} dr x' \\ &= e^{\Phi(r_b)/2} L_0/2, \end{aligned} \tag{2.24}$$

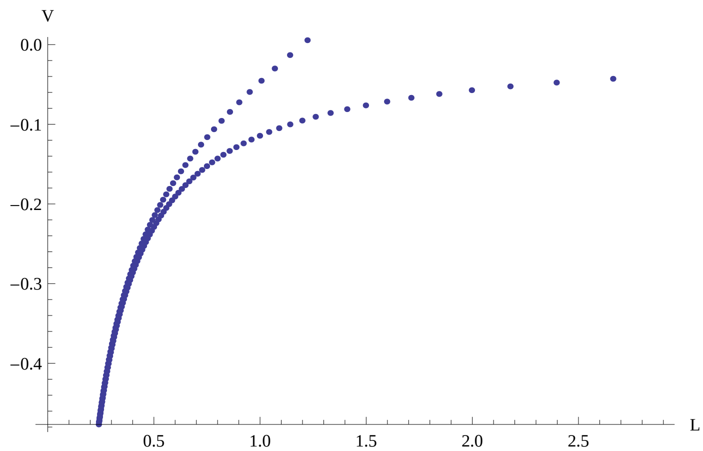


Figure 2.4: V-L relation for non-conformal theory. The parameters are set as $\alpha' = 1,\ R = 1$. We display the data for $q = 4$ (upper plots) and $q = 0$ (lower plots). The potential for $q = 4$ is linear rising in the large L region, while the potential for $q = 0$ is coulomb-like.

and

$$\begin{aligned} V(L) &= -\frac{S}{T} - 2m_q \\ &= \frac{2r_b}{2\pi\alpha'}\left\{\int_1^\infty dy \frac{y^2}{\sqrt{y^4-1}}\left(1+\frac{q}{r_b^4}\frac{1}{y^4}\right) - 1\right\}, \end{aligned} \tag{2.25}$$

where $L_0/2 = \frac{R^2}{r_b}\frac{\sqrt{2}\pi^{3/2}}{\Gamma\left(\frac{1}{4}\right)^2}$ is the separation in the AdS/CFT case, Eq. (2.6), and the quark mass m_q is defined by $m_q \equiv \frac{1}{2\pi\alpha'}\int_0^\infty dr$.[2] The relation between the quark-antiquark potential V and L are shown in Fig.2.4. Taking limit $r_b \to 0$, Eq. (2.24) and Eq.(2.25) become

$$\begin{aligned} L/2 &\sim \sqrt{q}\frac{R^2}{r_b^3}\frac{\sqrt{2}\pi^{3/2}}{\Gamma\left(\frac{1}{4}\right)^2}, \\ V &\sim \frac{2q}{2\pi\alpha' r_b^3}\frac{\sqrt{2}\pi^{3/2}}{\Gamma\left(\frac{1}{4}\right)^2}. \end{aligned}$$

Thus the relation between the potential V and the separation L is given by

$$V(L) \sim \frac{\sqrt{q}}{2\pi\alpha'^2\sqrt{2\lambda}}L \tag{2.26}$$

[2]This "quark mass" is just a regularization of the potential Eq. (2.25) and it is not real isolated physical quark mass. The natural definition of the real quark mass

$$m_q = \frac{1}{2\pi\alpha'}\int_0^\infty dr e^{\Phi/2}$$

has the IR divergence. It can be interpreted as the absence of the isolated quark in the dAdS model.

in $r_b \sim 0$ region. Eq. (2.26) means that the potential increases as a linear function of L with the coefficient $\frac{\sqrt{q}}{2\pi\alpha'^2\sqrt{2\lambda}}$ in large L region. We can regard this observation as the sign of the confinement.

2.5 Sakai-Sugimoto model

Another holographic model is the Sakai-Sugimoto model [8, 9]. The model is constructed by placing N_f probe D8-branes into a D4 background. The D4 background[10] is a solution of the field equations of the supergravity that corresponds to a system of N_c D4-branes and one of the coordinates τ compactified on S^1,

$$\begin{aligned} ds_{10}^2 &= \left(\frac{U}{R}\right)^{3/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f\left(U\right)d\tau^2\right) \\ &\quad + \left(\frac{R}{U}\right)^{3/2}\left(\frac{dU^2}{f\left(U\right)} + U^2 d\Omega_4^2\right), \end{aligned} \tag{2.27}$$

$$e^\phi = g_s\left(\frac{U}{R}\right)^{3/4}, \quad F_4 = dC_3 = \frac{6\pi N_c}{8\pi^2}\epsilon_4, \quad f\left(U\right) = 1 - \frac{U_{KK}^3}{U^3}, \quad R^3 = \pi g_s N_c l_s^3,$$

where $d\Omega_4^2,\ \epsilon_4$ are the line element and the volume form, respectively. x^μ $(\mu = 0, 1, 2, 3)$ and τ are the direction along which the D4-brane is extended. U_{KK} is a constant parameter. In order to avoid the conical singularity at $U = U_{KK}$, τ must be a periodic variable with

$$\tau \sim \tau + \delta\tau, \quad \delta\tau \equiv \frac{4\pi}{3}\frac{R^{3/2}}{U_{KK}^{1/2}}.$$

The Yang-Mills couping g_{YM} at the Kalzu-Klein mass scale $M_{KK} = \frac{2\pi}{\delta\tau}$ is

$$g_{YM}^2 = \frac{\left(2\pi\right)^2 g_s l_s}{\delta\tau}.$$

Embedding a D8-brane in the D4 background (2.27) with the D8-brane's coordinates

$$\left(\xi^\mu, \xi^\tau, \xi^U, \xi^{\Omega_4}\right) = \left(x^\mu, \tau, U\left(\tau\right), \xi_0^{\Omega_4} = \text{const.}\right),$$

we get the induced metric on the D8-brane as

$$\begin{aligned} ds_{D8}^2 &= \left(\frac{U}{R}\right)^{3/2}\eta_{\mu\nu}dx^\mu dx^\nu + \left(\left(\frac{U}{R}\right)^{3/2} f\left(U\right) + \left(\frac{R}{U}\right)^{3/2}\frac{U'^2}{f\left(U\right)}\right)d\tau^2 \\ &\quad + \left(\frac{R}{U}\right)^{3/2} U^2 d\Omega_4^2, \end{aligned} \tag{2.28}$$

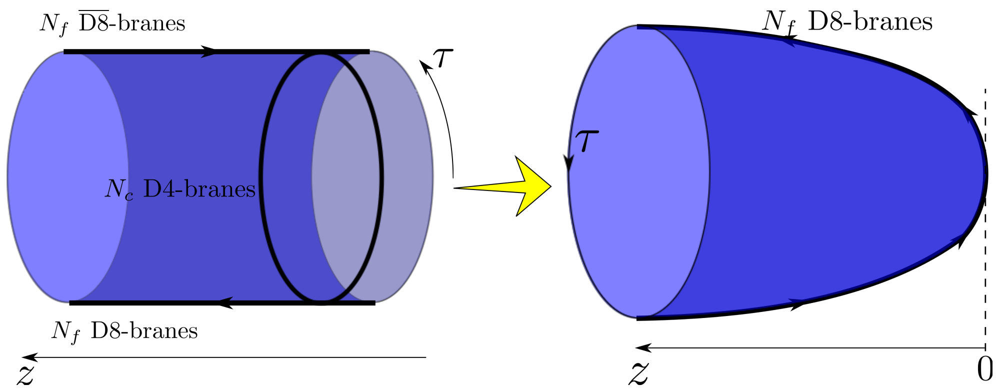


Figure 2.5: A brief sketch of the Sakai-Sugimoto model. On the left side, N_c D4-branes and $N_f\,(\ll N_c)$ D8-branes are embedded in flat spacetime. After taking some appropriate limit, the contributions of N_c D4-branes are replaced by the geometry with the metric, Eq. (2.27), and N_f D8 and $\overline{\text{D8}}$-branes are connected with each other.

where $U' = \frac{d}{d\tau}U(\tau)$. Imposing appropriate boundary conditions as $U(0) = U_{KK},\ U'(0) = 0$, the induced metric on the D8-brane (2.28) is reduced to

$$ds^2_{D8} = \left(\frac{U}{R}\right)^{3/2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{4}{9}\left(\frac{R}{U}\right)^{3/2}\frac{U_{KK}}{U}dz^2 + R^{3/2}U^{1/2}d\Omega_4^2, \tag{2.29}$$

$$U^3 = U_{KK}^3 + U_{KK}z^2.$$

A brief sketch of this system is shown in Fig.2.5 . Open strings which are regarded as quarks in holographic description can end on N_f D8-branes, so the quark has $U(N_f)$ flavor symmetry. When we describe the system in terms of supergravity, N_f D8-branes and N_f $\overline{\text{D8}}$-branes which are originally distinct objects will connect and become a single component of N_f D8-branes. This can be interpreted as the holographic description of the spontaneous breaking of the $U(N_f)_L \times U(N_f)_R$ chiral symmetry.

The gauge field on this D8-brane configuration has nine components, $A_\mu(\mu = 0,1,2,3)$, A_z and A_α ($\alpha = 5,6,7,8$, the coordinates on the S^4). Since the brane configuration has $SO(5)$ isometry which corresponds to the rotation of x^5, x^6, x^7, x^8, x^9, we assume $A_\mu = A_\mu(x^\mu, z),\ A_z = A_z(x^\mu, z)$ and $A_\alpha = 0$. Then the DBI part of the D8-brane action becomes

$$\begin{aligned} S_{D8} &= -T_8\int d^9x e^{-\phi}\sqrt{-\det\left(G_{ab} + 2\pi\alpha' F_{ab}\right)} \\ &= -\tilde{T}_8\int d^4x dz\left[\frac{R^3}{4U}\eta^{\mu\nu}\eta^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} + \frac{9}{8}\frac{U^3}{U_{KK}}\eta^{\mu\nu}F_{\mu z}F_{\nu z}\right] \\ &\quad + S_{D8}^{(0)} + \mathcal{O}\left(\alpha'^3\right), \end{aligned} \tag{2.30}$$

where $\tilde{T}_8 \equiv 2^4\pi^2R^{3/2}U_{KK}^{1/2}T_{D8}\left(2\pi\alpha'\right)^2/3^2g_s$, $S_{D8}^{(0)} \equiv -T_8\int d^9xe^{-\phi}\sqrt{-\det G_{ab}}$.

2.5.1 Baryons in Sakai-Sugimoto model

A baryon in terms of holography is represented as a D-brane wrapped around a sphere.[11] In the Sakai-Sugimoto model, it is a D4-brane wrapped around the S^4. This corresponds to a soliton of the five dimensional gauge theory on D8-branes. We can confirm that these two descriptions of the baryon are related each other by calculating their mass. The mass of D4-brane wrapped around the S^4 can be extracted from the action of a D4-brane,

$$\begin{aligned} S_{D4} &= -T_4 \int dt \int d\Omega_4 \, e^{-\Phi} \sqrt{-G_{tt} G_{(S^4)}}\Big|_{U=U_{KK}} \\ &= -T_4 \left(\frac{8\pi^2}{3} \right) g_s^{-1} R^3 U_{KK} \int dt. \end{aligned}$$

Thus the D4-brane mass is

$$m_{D4} = T_4 \left(\frac{8\pi^2}{3} \right) g_s^{-1} R^3 U_{KK} = \frac{1}{27\pi} M_{KK} \lambda N_c.$$

On the other hand, the minimum contribution of gauge field in D8-brane action in Eq. (2.30) can be estimated as

$$\begin{aligned} S_{D8} - S_{D8}^{(0)} &= -\tilde{T}_8 \int d^4x dz \left[\frac{R^3}{4U} F_{ij}^2 + \frac{9}{8} \frac{U^3}{U_{KK}} F_{iz}^2 \right] \\ &\geq -\frac{3R^{3/2}\tilde{T}_8}{4} \int d^4x dz \sqrt{U_{KK} \left(1 + \left(\frac{z}{U_{KK}} \right)^2 \right)^{2/3}} \left| \epsilon^{ijk} F_{jk} F_{iz} \right| \\ &\geq -\frac{1}{27\pi} M_{KK} \lambda N_c N_B \int dt \end{aligned}$$

where we use relations $\left(\epsilon^{ijk} F_{jk}\right)^2 = 2F_{ij}^2$, $(V_i - W_i)^2 \geq 0$ for the first inequality and $1 + \left(\frac{z}{U_{KK}}\right)^2 \geq 1$ and $N_B \equiv \frac{1}{8\pi^2} \int d^3x dz \epsilon^{ijk} F_{jk} F_{iz}$ for the second inequality. The minimum contribution from gauge field with $N_B = 1$ is coincide with the D4-brane mass,

$$m_{soliton} \equiv \frac{1}{27\pi} M_{KK} \lambda N_c = m_{D4}.$$

The fact that the mass of soliton with the soliton number $N_s = 1$ coincide with that of a D4-brane wrapped around the S^4 once can be regarded as an evidence of the equivalence between these two descriptions of the baryon.

Chapter 3

Holographic Unruh effect

3.1 Introduction of this chapter

Unruh effect is the thermal effect for an accelerated observer. The statement is "An observer, who is accelerated with a constant acceleration a in the Minkowski spacetime, would see a thermal bath of the temperature $a/2\pi$". A brief explanation of this effect is given in Appendix E.1.

A similar situation has been studied for the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in the context of the holography [12, 13, 14, 15]. In the approach of Refs. [14, 15], an accelerated quark has been introduced as a string solution of the Nambu-Goto action which is embedded in the AdS_5 (anti-de Sitter) background dual to the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory. The solution in this background has been found by Xiao [14], and one finds an event horizon in the induced metric (in its world sheet) of this string configuration. The position of this horizon is specified by the fifth coordinate of the bulk.

Xiao proposed an extended form of Rindler transformation (ERT) to move to a co-moving frame of the accelerated quark. Performing this ERT, the event horizon appears in the bulk. Thus the theory dual to the geometry after the ERT is considered as a Yang-Mills theory at a finite temperature. The temperature is given by the Rindler temperature $T_R = a/2\pi$. At the same time, the position of the bulk horizon can be put at the same fifth-coordinate point with the one of the world sheet horizon of the accelerated string. As a result, in the new coordinate, one finds a static string which connects the boundary and the event horizon of the bulk.

This is nothing but a free quark-string configuration in the Rindler vacuum. Since the theory dual to the AdS_5 is in the deconfinement phase, there is also a free quark in the Minkowski vacuum at zero temperature. However we should notice that the free quark in a vacuum is not the same as the one in another vacuum, because the static free quark in the Minkowski vacuum cannot be transformed to the one of the Rindler vacuum by the ERT, and vice versa. In both theories dual to AdS_5 and to the one transformed by the ERT, the quarks are not confined. So the confinement-deconfinement transition has not been regarded as a thermal effect in the Rindler vacuum. Thus, it remains an important point to study this transition for the gauge theory in the confinement phase in the Minkowski vacuum. There had been no such attempt until we did it.

We consider a confining Yang-Mills theory in the Minkowski vacuum in order to examine properties of its Rindler vacuum, which is obtained by performing the ERT. As a concrete model, we consider a supersymmetric background solution of type IIB theory introduced in Sec. 2.4. This background is dual to the $\mathcal{N}=2$ supersymmetric Yang-Mills theory, and the quark is confined in this theory [7, 16].

We look for a solution of the equation of motion for the Nambu-Goto action that is similar to the one found by Xiao. Then the original coordinates with the Minkowski vacuum are transformed to the co-moving coordinates of the accelerated string solution by the ERT given by Xiao. After performing this transformation, we could find the free quark-string configuration in the Rindler vacuum. This implies that this Rindler vacuum is in the quark deconfinement phase. However, we should again notice that the "quark" in the Rindler vacuum is different from the quark in the Minkowski vacuum. Then this phase change between the Minkowski and Rindler vacuum cannot be interpreted as the phase transition, which is seen in the usual finite temperature theory.

In the vacuum defined by the new coordinates, the dual theory can be regarded as the thermal Yang-Mills theory with the Rindler temperature. We examine its thermal properties and assure that the confinement has been lost at any finite value of the Rindler temperature. So, there is no critical temperature in this case. On the other hand, some remnants of the confining force are seen in various quantities. The situation is similar to the case of the finite temperature theory dual to the AdS_5-Schwarzschild background.

an accelerated solution In the case of AdS/CFT correspondence[14], it is suggested that the Unruh effect is observed by analyzing the system in which a quark and an anti-quark are uniformly accelerated along mutually opposite directions. In the AdS/CFT context, the motion of the quark corresponds to the motion of the end of the string which moves in the bulk. So such a string solution $x\,(t,r)$ should behave

$$x\,(t,r)^2 - t^2 = \frac{1}{a^2}, \quad r \to \infty,$$

and satisfy the equation of motion

$$\partial_t \left(\frac{\partial \mathcal{L}_{NG}}{\partial_t x} \right) + \partial_r \left(\frac{\partial \mathcal{L}_{NG}}{\partial_r x} \right) = 0,$$

where

$$\begin{aligned}
\mathcal{L}_{NG} &= \sqrt{-\det\begin{pmatrix} G_{00}+G_{11}\dot{x}^2 & G_{11}\dot{x}x' \\ G_{11}\dot{x}x' & G_{11}x'^2+G_{rr}\end{pmatrix}} \\
&= \sqrt{\left(|G_{00}|-G_{11}\dot{x}^2\right)\left(G_{11}x'^2+G_{rr}\right)+G_{11}^2\dot{x}^2x'^2} \\
&= G_{11}\sqrt{x'^2-\dot{x}^2\frac{G_{rr}}{G_{11}}+\frac{G_{rr}}{G_{11}}},
\end{aligned} \tag{3.1}$$

and $-G_{00}=G_{11},\ G_{rr}$ are the diagonal components of the background spacetime metric which depends only on r. Then the equation of motion (which is also the momentum conservation law) is given as

$$\partial_t p_t + \partial_r p_r = 0,$$

where

$$\begin{aligned}
p_t &\equiv \frac{\partial \mathcal{L}_{NG}}{\partial \dot{x}} = \frac{-G_{rr}\dot{x}}{\sqrt{x'^2-\frac{G_{rr}}{G_{11}}\dot{x}^2+\frac{G_{rr}}{G_{11}}}}, \\
p_r &\equiv \frac{\partial \mathcal{L}_{NG}}{\partial x'} = \frac{G_{11}x'}{\sqrt{x'^2-\frac{G_{rr}}{G_{11}}\dot{x}^2+\frac{G_{rr}}{G_{11}}}}.
\end{aligned}$$

If we use an ansatz

$$x(t,r) = \sqrt{t^2+f(r)}, \tag{3.2}$$

the equation of motion becomes

$$-\frac{G_{rr}}{\sqrt{\left(\frac{1}{2}f'(r)\right)^2+\frac{G_{rr}}{G_{11}}f(r)}} + \partial_r\left(\frac{G_{11}\frac{1}{2}f'(r)}{\sqrt{\left(\frac{1}{2}f'(r)\right)^2+\frac{G_{rr}}{G_{11}}f(r)}}\right) = 0. \tag{3.3}$$

The induced metric on this string is written as

$$ds^2 = \left(G_{00}+G_{11}\dot{x}^2\right)dt^2 + 2G_{11}\dot{x}x'dtdr + \left(G_{11}x'^2+G_{rr}\right)dr^2.$$

Then the trajectory of light ($ds^2=0$) on this metric is represented as

$$dr = \frac{-2G_{11}\dot{x}x' \pm \sqrt{\left(2G_{11}\dot{x}x'\right)^2 - 4G_{11}\left(G_{11}x'^2+G_{rr}\right)\left(\dot{x}^2-1\right)}}{2\left(G_{11}x'^2+G_{rr}\right)}dt,$$

where the plus sign denotes a trajectory of light which initially goes up to the positive direction of r, and the minus sign denotes that initially falls down. When $\dot{x}=1$, i.e., $f(r_c)=0$, $dr=0$ or negative, any information on the lower part ($r<r_c$) of this solution cannot propagate to the upper part ($r_c<r$) across r_c. The existence of such a point implies

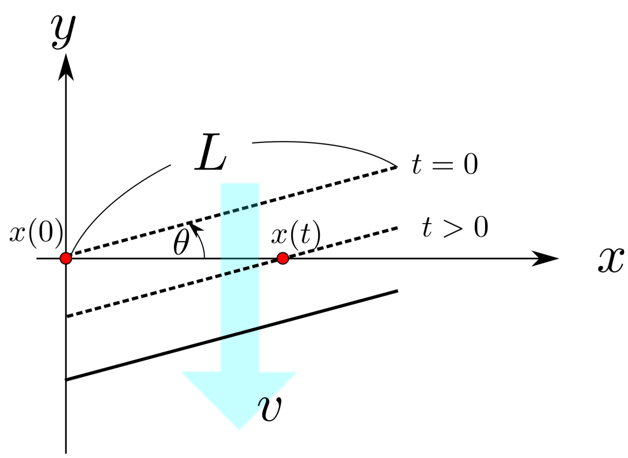


Figure 3.1: A straight bar moving along with the y axis.

that the physics of the upper part can be represented only by the upper part and do not need the information on the lower part.

What happen in the region $r < r_c$? At $r = r_c$, $\dot{x}(r_c)$ reaches the speed of light, $c = 1$. It implies that $\dot{x}(r)$ in $r < r_c$ may exceeds c. Does this fact cause any problem? If we expand $f(r)$ near $r = r_c$ as

$$f(r) = f(r_c) + f'(r_c)(r - r_c) + \cdots ,$$

the leading order of the equation of motion in $r - r_c$ expansion leads to

$$f'(r_c) = \frac{4G_{rr}(r_c)}{G'_{11}(r_c)},$$

where $G'_{11} \equiv \partial_r G_{11}$. This quantity is positive when $G'_{11}(r_c) > 0,\ G_{rr}(r_c) > 0$. This condition is satisfied in Eqs. (2.2), (2.11), (2.22), (2.27) and (2.29). Thus, indeed, $\dot{x}$ exceeds the speed of light c,

$$\dot{x} = \frac{t}{\sqrt{t^2 + f(r)}} > 1, \quad \text{for } r < r_c. \tag{3.4}$$

However it does not mean that true string speed exceeds c. Because string is extended object, the intersection at which string across the hyperplane at $r = r_c$ can exceeds the speed of light even if the actual speed is smaller than c.

Considering a following situation is useful to understand above statement. Imagine a straight bar moving along with the y axis with a constant velocity $v < c$ in flat Euclidean space (x, y),

$$x(\sigma, t) \;=\; L\cos\theta\sigma, \quad y(\sigma, t) = L\sin\theta\sigma - vt, \quad (0 < \sigma < 1).$$

It is shown in Fig. 3.1. Then we consider an intersection $x(t)$ where the bar crosses the hyperplane at $y = 0$. It can be written as

$$x(t) = \frac{\cos\theta}{\sin\theta} vt, \quad 0 < t < \frac{\sin\theta}{v} L.$$

And then, the speed of this intersection becomes

$$\dot{x}(t) = \frac{1}{\tan\theta} v.$$

This can exceeds c if

$$\theta < \tan^{-1}\frac{v}{c}.$$

However the actual speed of this object v is clearly smaller than c. This is what happens in the region Eq. (3.4). So there is no strange point in the string solution Eq. (3.2).

3.2 Holographic Unruh effect in AdS/CFT

In the case of AdS/CFT correspondence, substituting $-G_{00} = G_{11} = \frac{r^2}{R^2},\ G_{rr} = \frac{R^2}{r^2}$ into Eq. (3.3), we get the equation of motion

$$-\frac{\frac{R^2}{r^2}}{\sqrt{\left(\frac{1}{2}f_0'(r)\right)^2 + \frac{R^4}{r^4}f_0(r)}} + \partial_r\left(\frac{\frac{r^2}{R^2}\frac{1}{2}f_0'(r)}{\sqrt{\left(\frac{1}{2}f_0'(r)\right)^2 + \frac{R^4}{r^4}f_0(r)}}\right) = 0. \qquad (3.5)$$

This equation is satisfied by $f_0(r) = \frac{1}{a^2} - \frac{R^4}{r^2}$. Thus the accelerated string solution, Eq. (3.2), is given as

$$x(t,r) = \sqrt{t^2 + \frac{1}{a^2} - \frac{R^4}{r^2}}. \qquad (3.6)$$

This solution approaches the ordinary trajectory of an accelerated object in the AdS boundary $r \to \infty$,

$$x(t, r\to\infty) = \sqrt{t^2 + \frac{1}{a^2}},$$

and also has the turning point at $r_b = \frac{R^2}{\sqrt{t^2+\frac{1}{a^2}}}$,

$$x'(t, r_b) = \frac{\frac{R^4}{r_b^3}}{\sqrt{t^2 + \frac{1}{a^2} - \frac{R^4}{r_b^2}}} = \infty.$$

Thus this solution represents a string, one of whose ends is accelerated to the positive x-direction and the other is accelerated to the negative x-direction with the acceleration a. A brief sketch of this solution is displayed in Fig. 3.2.

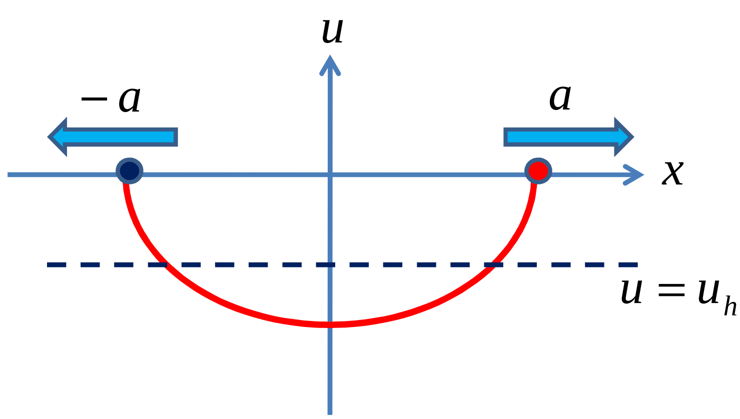


Figure 3.2: A brief sketch of the solution Eq. (3.2). The string has the induced metric horizon at $u\left(\equiv \frac{r}{R^2}\right) = u_h$.

To move to the co-moving coordinate of the accelerated quark, we perform the transformation which is introduced by Xiao [14] defined as

$$\begin{aligned} x &= \sqrt{\frac{1}{a^2} - \frac{1}{s^2}} e^{a\beta} \cosh\left(a\tau\right), \\ t &= \sqrt{\frac{1}{a^2} - \frac{1}{s^2}} e^{a\beta} \sinh\left(a\tau\right), \\ r &= sR^2 e^{-a\beta}, \end{aligned} \tag{3.7}$$

where $-\infty < \beta, \tau < \infty$, $a < s < \infty$. This transformation is the 5-dimensional extension of the 4-dimensional Rindler transformation. The usual Rindler transformation is the transformation concerning two coordinates in the 4-dimensional spacetime, time and the one of the accelerated direction, but the extended one contains one additional direction of the fifth coordinate of AdS_5 spacetime. We call this as the extended Rindler transformation (ERT). Performing the ERT, the metric Eq. (2.2) becomes

$$ds^2 = R^2 \left[\frac{ds^2}{s^2 - a^2} - \left(s^2 - a^2\right) d\tau^2 + s^2 \left(d\beta^2 + e^{-2a\beta}\left(dy^2 + dz^2\right) + d\Omega_5^2\right) \right],$$

and the string configuration Eq. (3.6) is mapped to

$$\beta = 0.$$

In the Rindler coordinates, the string configuration is static and an end of the string fall into the horizon at $s = a$. Then the quark that corresponds to the other end on the AdS boundary does not feel the existence of the anti-quark that corresponds to the end falling into the horizon. Because of the existence of the horizon, the quark feels the temperature

$$T = \frac{a}{2\pi}.$$

This temperature can be calculated by the same method introduced in Sec. 2.3. This finite temperature effect is similar to the situation in Appendix E.1, so we regard this effect as the holographic description of the Unruh effect.

3.3 Holographic Unruh effect in dAdS/non-conformal QFT

Here we perform similar analysis to that in Sec. 3. First we find the solution for a string accelerated in the bulk background, Eq. (2.22), dual to the $\mathcal{N}=2$ supersymmetric confining Yang-Mills theory. It can be obtained by solving the equation of motion, Eq. (3.3), and it has the induced metric horizon at $r_c > R^2 a$. Second we perform the ERT to this system in order to move to a co-moving frame of this accelerated string solution, the Rindler frame. Then in this frame we investigate two quantities, the drag force and the Wilson loop, which are sensitive to temperature of the system.

3.3.1 Accelerating string solution

We assume that the solution of the accelerated string is written as in Eq. (3.2) and then the equation of motion for the string is represented as in Eq. (3.3) with $-G_{tt} = G_{xx} = e^{\Phi/2}\frac{r^2}{R^2}$, $G_{rr} = e^{\Phi/2}\frac{R^2}{r^2}$ and $e^{\Phi} = 1 + \frac{q}{r^4}$, namely

$$-\frac{e^{\Phi/2}R^2}{r^2\sqrt{\left(\frac{1}{2}f'(r)\right)^2 + \frac{R^4}{r^4}f(r)}} + \partial_r\left(\frac{e^{\Phi/2}r^2\frac{1}{2}f'(r)}{R^2\sqrt{\left(\frac{1}{2}f'(r)\right)^2 + \frac{R^4}{r^4}f(r)}}\right) = 0. \tag{3.8}$$

It is very difficult to solve this differential equation analytically, but we can solve this numerically by imposing two appropriate boundary conditions, $f(r=\infty) = 1/a^2$, $f'(r_b) = 0$, where r_b is the solution of $f(r_b) = 0$. The former condition means that we consider the quark that is accelerated with a constant acceleration a, and the latter one is necessary to consider the string, both ends of which come back to the boundary. So it turns out that this gravitational system represents that the quark and the anti-quark are accelerated with the same acceleration a back to back in the non-conformal confining quantum field theory. The numerical result of $f(r)$ for $a=1$ and several values of q are shown in Fig. 3.3.

3.3.2 Extended Rindler transformation and geometry

Then we move to a co-moving frame of the accelerated quark. In the transformed metric, the quark should not depend on the time coordinate. We again employ the ERT, Eq. (3.7), as such a transformation. In this case, spacetime metric becomes

$$ds^2 = e^{\Phi/2}R^2\left[\frac{ds^2}{s^2-a^2} - \left(s^2-a^2\right)d\tau^2 + s^2\left(d\beta^2 + e^{-2a\beta}\left(dy^2+dz^2\right) + d\Omega_5^2\right)\right], \tag{3.9}$$

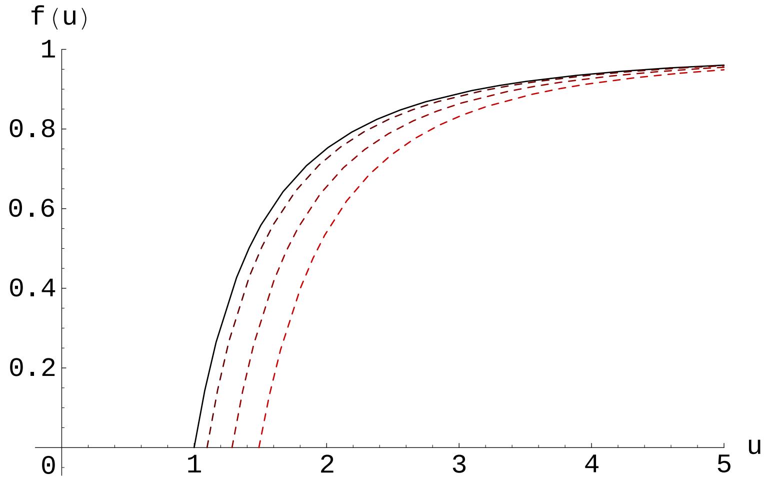


Figure 3.3: The numerical results of $f(u)$ ($u \equiv r/R^2$) for $a = 1$ and $q = 0,\ 0.5,\ 3.0$ and 10 are shown from left to right. The zero point of the solution moves to the right with increasing q.

$$e^{\Phi} = 1 + \frac{q e^{4a\beta}}{s^4 R^8},$$

and the mapping of the string solution Eq. (3.2) is given by

$$x^2 - t^2 = f(r) \rightarrow \left(\frac{1}{a^2} - \frac{1}{s^2}\right) e^{2a\beta} = f\left(sR^2 e^{-a\beta}\right).$$

The string configuration in this Rindler spacetime is shown in Fig. 3.4. We note that the point r_c, that is the solution of $f(r_c) = 0$ on the string, is mapped to $s_c = a$. Thus the boundary, on which the information on the lower part of the string can propagate, agrees with the horizon of the spacetime metric. The temperature of this metric can be calculated by performing the same procedure as in the AdS case. The result is

$$T_H = \frac{a}{2\pi}. \tag{3.10}$$

In the case of the non-conformal confining theory, we again get the same Unruh temperature. This result is caused by ERT, Eq. (3.7). We employ the transformation to reproduce the correct Unruh temperature Eq. (3.10). Whether the transformation is the unique transformation to reproduce the result Eq. (3.10) or not has not been understood. We summarize the relation between transformations and temperatures for these systems in Appendix E.2.

3.3.3 Thermal features of the emergent Rindler spacetime

In this section, we study thermal properties of the Rindler spacetime Eq. (3.9) by applying gauge/gravity duality. Resultant properties may be interpreted as the physics that the accelerated quark feels. It is interesting to see whether the confinement persists in the

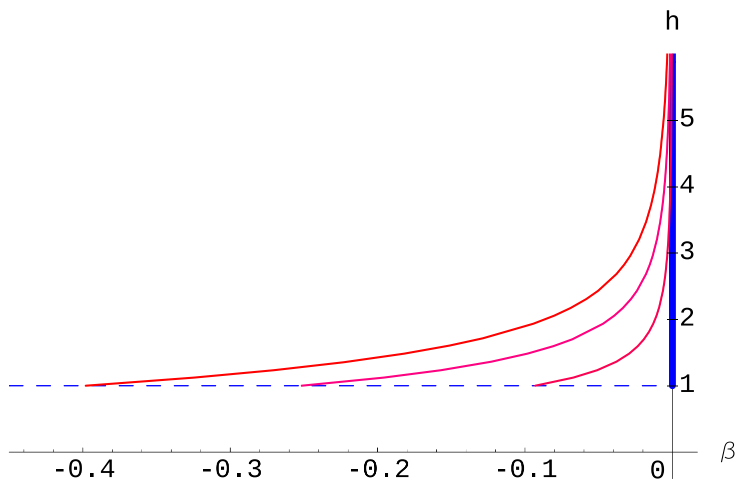


Figure 3.4: Examples of the string solution $\beta\,(s)$ for $q = 0$ (straight line) and $q = 0.5, 3, 10$ (from right to left) with $a = 1$. Solutions for finite q are bent due to the Yang-Mills force expressed by the dilaton. Then the larger q becomes, the larger the deformation of the solution grows.

Rindler-dAdS space or not. We are also interested in the difference between the statistical features along the accelerated direction β and that along another direction y in 4-dimensional spacetime.

Drag force Let us first examine the drag force acting on the quark moving with a constant velocity in this thermal background. It can be studied by considering a string solution of Eq. (2.15). It represents a quark moving with the constant velocity in an equilibrium state in a thermal medium. We obtain the value of the friction constant of the quark in the thermal medium from this analysis.

Strings moving to the longitudinal direction β A quark moving along β axis appears to be described by the solution,

$$\beta\,(\tau, s) = v\tau + \xi\,(s)\,.$$

However such solution is not allowed. The equation of motion may be obtained by substituting

$$\begin{aligned} G_{tt} &\rightarrow G_{\tau\tau} = -e^{\Phi/2} R^2 \left(s^2 - a^2\right), \\ G_{rr} &\rightarrow G_{ss} = e^{\Phi/2} R^2 \frac{1}{s^2 - a^2}, \\ G_{xx} &\rightarrow G_{\beta\beta} = e^{\Phi/2} R^2 s^2, \end{aligned}$$

into Eq. (2.17), as

$$\partial_s \left(e^{\Phi/2} \frac{\xi' s^2 \left(s^2 - a^2\right)}{\sqrt{-\tilde{g}}} \right) - \partial_\xi \left(e^{\Phi/2}\right) \sqrt{-\tilde{g}} = 0,$$

where $\sqrt{-\tilde{g}} \equiv \sqrt{1-\frac{v^2 s^2}{s^2-a^2}+\xi'^2 s^2\left(s^2-a^2\right)}$. This equation is rewritten as

$$
\begin{aligned}
\partial_s\left(\frac{\xi' s^2\left(s^2-a^2\right)}{\sqrt{-\tilde{g}}}\right)\frac{\sqrt{-\tilde{g}}}{1-\frac{v^2 s^2}{s^2-a^2}} &= \frac{1}{2}\partial_\xi\left(\ln e^\Phi\right) \\
&= \frac{2aq}{s^4 R^8}\frac{e^{4a\beta}}{1+\frac{qe^{4a\beta}}{s^4R^8}}.
\end{aligned}
$$

Since the left-hand side of this equation is independent of τ, the equation of motion is reduced to

$$
e^{-4av\tau}=e^{4a\xi}\left(C-\frac{q}{s^4R^8}\right)\ ,
$$

where C is a constant. This equation has no solution when ξ does not depend on τ. Thus we conclude that there is no equilibrium state for a quark with a constant velocity in the dual gauge theory to the geometry of Eq. (3.9). The reason why this situation arises is the β dependence of the dilaton factor $e^\Phi=1+\frac{qe^{4a\beta}}{s^4R^8}$.

If we consider the case of Rindler-AdS spacetime, i.e., the $q=0$ case, the equation of motion becomes

$$
\partial_s p_\xi=0, \quad p_\xi\equiv-2\pi\alpha'\frac{\partial L}{\partial\xi'}=\frac{R^2\xi' s^2\left(s^2-a^2\right)}{\sqrt{1-\frac{v^2s^2}{s^2-a^2}+\xi'^2s^2\left(s^2-a^2\right)}}
$$

By following parallel arguments to that in Sec. 2.3.4, we get the relation

$$
\frac{dp}{dt} = -2\pi\sqrt{2\lambda}\frac{T_U^2}{\sqrt{1-v^2}}\frac{p}{m_q}
$$

where $p/m_q=\frac{v}{\sqrt{1-v^2}}$, $\lambda=g_{YM}^2N_c=R^4/2\alpha'^2$ is the 't Hooft coupling, $T_U=\frac{a}{2\pi}$ is the Unruh temperature. For $v\sim 0$, the friction constant is given by

$$
\eta^\beta_{RAdS}=2\pi\sqrt{2\lambda}T_U^2. \tag{3.11}
$$

Strings moving to the transverse direction y When we choose the coordinate y as the moving direction, the string solution is supposed to be

$$
y\left(\tau,s\right)=v\tau+\xi\left(s\right),\ \beta=\beta\left(s\right).
$$

Since the metric Eq. (3.9) depends on β, we should keep the dependence of β when we consider the string configuration. Then the induced metric of this string and the action are written as

$$
g_{\tau\tau}=-R^2e^{\Phi/2}\left(\left(s^2-a^2\right)-v^2e^{-2a\beta}s^2\right), \quad g_{\tau s}=vR^2e^{\Phi/2}e^{-2a\beta}s^2\xi',
$$

$$
g_{ss}=R^2e^{\Phi/2}\left(\frac{1}{s^2-a^2}+e^{-2a\beta}s^2\xi'^2+s^2\beta'^2\right),
$$

and

$$\begin{aligned}
S &= \int d\tau ds \mathcal{L} \\
&= -\frac{1}{2\pi\alpha'}\int d\tau ds \times \\
& \quad e^{\Phi/2}R^2\sqrt{\left(1-\frac{v^2e^{-2a\beta}s^2}{s^2-a^2}\right)\left(1+s^2\left(s^2-a^2\right)\beta'^2\right)+\left(s^2-a^2\right)e^{-2a\beta}s^2\xi'^2}.
\end{aligned}$$

Note that the induced metric on the string has a horizon at $\overline{s}$ that satisfies $(\overline{s}^2-a^2)-v^2e^{-2a\beta(\overline{s})}\overline{s}^2=0$. We want to know the conserved momentum p_ξ to derive the drag force for the quark moving along y axis with a constant velocity. It is defined as

$$p_\xi \equiv -2\pi\alpha'\frac{\partial\mathcal{L}}{\partial\xi'} = e^{\Phi/2}R^2\frac{\left(s^2-a^2\right)e^{-2a\beta}s^2\xi'}{\sqrt{-\tilde{g}}} = C, \tag{3.12}$$

$$p_\beta \equiv -2\pi\alpha'\frac{\partial\mathcal{L}}{\partial\beta'} = e^{\Phi/2}R^2\frac{\left(\left(s^2-a^2\right)-v^2e^{-2a\beta}s^2\right)s^2\beta'}{\sqrt{-\tilde{g}}}, \tag{3.13}$$

where

$$\sqrt{-\tilde{g}} \equiv \sqrt{\left(1-\frac{v^2e^{-2a\beta}s^2}{s^2-a^2}\right)\left(1+s^2\left(s^2-a^2\right)\beta'^2\right)+\left(s^2-a^2\right)e^{-2a\beta}s^2\xi'^2},$$

and the equation of motion is written as

$$\partial_s p_\xi = 0, \quad \partial_s p_\beta = -2\pi\alpha'\frac{\partial\mathcal{L}}{\partial\beta}. \tag{3.14}$$

Solving Eq. (3.12) for ξ', we get

$$\xi' = \pm\sqrt{\frac{\left(1-\frac{v^2e^{-2a\beta}s^2}{s^2-a^2}\right)\left(1+s^2\left(s^2-a^2\right)\beta'^2\right)C^2}{\left(s^2-a^2\right)e^{-2a\beta}s^2\left(e^{\Phi}R^4\left(s^2-a^2\right)e^{-2a\beta}s^2-C^2\right)}}, \tag{3.15}$$

where the sign should be chosen appropriately. Since $y=vt+\xi$ is the position of the string, ξ and ξ' are real. So the numerator and the denominator inside the square root must vanish simultaneously on the induced metric horizon $s=\overline{s}$. The condition leads to $C^2=e^{\Phi(\overline{s})}R^4v^2e^{-4a\beta(\overline{s})}\overline{s}^4$, as is immediately seen from Eq. (3.13) by assuming $\beta'(\overline{s})<\infty$. We can solve the string configuration $(\beta(s),\xi(s))$ numerically by using Eqs. (3.13) and (3.14). Typical configurations are shown in Fig. 3.5.

Then the canonical momentum conjugate to y is

$$\begin{aligned}
p_\xi &= v\sqrt{\left(e^{-a\beta(\overline{s})}\overline{s}\right)^4R^4+\frac{q}{R^4}} \\
&= \frac{v}{R^2}\sqrt{r\left(\overline{s},\beta\left(\overline{s}\right)\right)^4+q}.
\end{aligned}$$

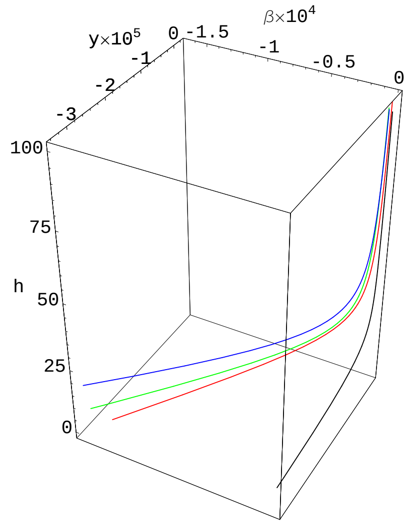


Figure 3.5: Strings trailing along with y axis for $v = 0.1$, $a = 1$, $R = 1$ and $q = 0$ (on $\beta = 0$ plane), $q = 1$ (front), $q = 2$ (middle), $q = 5$ (back). These strings that have finite q are curved to axis due to the existence of dilaton and the curve is sharper with the larger value of q.

Thus the drag force for y direction is defined as

$$\begin{aligned} \frac{dp}{dt} &= -\frac{1}{2\pi\alpha'}\sqrt{1-v^2}\sqrt{\frac{r\left(\bar{s}, \beta\left(\bar{s}\right)\right)^4 + q}{R^4}}\frac{p}{m_q} \\ &\sim -\eta^y \frac{p}{m_q} \qquad (\text{for small } v)\,, \end{aligned}$$

where

$$\eta^y \equiv \frac{R^2 a^2}{2\pi\alpha'}\sqrt{\frac{r_c^4 + q}{R^8 a^4}}$$

is the friction constant in this "thermal" medium due to acceleration and r_c is a point which satisfies $f\left(r_c\right) = 0$. In the limit $q \to 0$, this quantity becomes

$$\eta^y_{RAdS} \equiv 2\pi\sqrt{2\lambda}T_U^2. \tag{3.16}$$

Thus, in the Rindler-AdS case, the friction constants for two different directions agree when v is sufficiently small. The difference between Eq. (3.11) and Eq. (3.16) is evaluated as

$$\eta^\beta_{RAdS} - \eta^y_{RAdS} \sim 2\pi\sqrt{2\lambda}T_U^2 v^2 \qquad (\text{for small } v)\,.$$

This result implies that "thermal" effect from a constant acceleration is anisotropic. The situation is quite different from the ordinary isotropic thermal effect. This anisotropy is referred to in the work [17] with the field theoretical approach.

We also notice that the factors of the friction constants in Rindler-AdS system η^{β}_{RAdS}, η^{y}_{RAdS} are different from that in Schwarzschild-AdS system η_{SAdS} , Eq. (2.21),

$$\eta^{\beta}_{RAdS} = \eta^{y}_{RAdS} = 4\eta_{SAdS}.$$

Wilson loop Next we consider the potential between a quark and an antiquark. It is obtained by considering the Wilson loop operator. The expectation value of the operator corresponds to a static string with the both ends on the boundary. The behavior of the Wilson loop in the theory at a finite temperature will be quite different from that in the theory at zero temperature. Especially the confinement properties would be lost from the theory because the infrared strong-force is screened by the fluctuations of the thermal matter. This situation also arises when the theory is at any finite value of the Unruh temperature.

In the original background Eq. (2.22), the gauge coupling constant is defined by $g^2_{YM} = 2\pi g_s e^{\Phi}$ and depends on β and s as follows,

$$\frac{g^2_{YM}}{2\pi g_s} = 1 + \frac{q e^{4a\beta}}{s^4 R^8} \tag{3.17}$$

This implies that the Yang-Mills force depends on the energy scale s and also on the coordinate β in the real three space. The Yang-Mills force between a quark and an antiquark is completely screened when they are separated by the distance (L) larger than a critical value (L_*) because of the temperature, as we have seen in Sec. 2.3.3.[1] Namely the quark is free from the antiquark which is separated by the distance $L > L_*$, but we know that the quark can feel the force from the antiquark in the region of $L < L_*$ and this force is nearly equivalent to the one given at zero temperature.

In the present case, we find a linear rising potential in the region of $L_0 < L < L_*$, where $L < L_0$ defines the ultraviolet region of the conformally symmetric limit. And we find the tension parameter Eq. (2.26)

$$\tau_{eff} = \frac{\sqrt{q}}{2\pi\alpha' R^2}$$

at zero temperature in the present model. Then we expect the tension parameter in the Rindler coordinate would be given by

$$\tau_R = \frac{\sqrt{q} e^{2a\beta}}{2\pi\alpha' R^2} \tag{3.18}$$

which is however coordinate dependent. We can assure this point through the Wilson-Loop calculation given below. Therefore we study the dynamical properties in this vacuum in two cases, a string which stretches along the longitudinal direction and that stretches along transverse direction in three dimensional space in the new coordinate.

[1]Properly speaking, we should subtract the contributions of the isolated quark and the anti-quark from the potential energy to obtain the contribution from interaction between them. There is however only one configuration of the isolated quark and the anti-quark when L is larger than L_*. Thus pointing out the existence of the maximum value of the separation L_* is sufficient to confirm the screening of the color force.

From the gauge coupling given above Eq. (3.17), we can say that the force between the quark and the antiquark would depend on a and also on β. In our original metric, the quarks are confined due to the strong infrared gauge coupling constant. Namely, it diverges for $r \to 0$. In the new Rindler coordinate system, the infrared strong force would be screened by the fluctuations of the thermal matter with the temperature $T = a/2\pi$. The situation would be parallel to the case of the AdS-Schwarzschild background which is dual to the high temperature gauge field theory. Because of this screening, we would find the deconfinement phase in the Rindler vacuum. This is the Unruh effect in the confinement theory. In order to assure this point, we study the force between the quark and the antiquark, which are represented by the static strings in the Rindler vacuum Eq. (3.9).

In this analysis, we consider two types of the string configuration, the string extended to the longitudinal (β) direction and the one extended to the transverse (y) direction.. These two configurations correspond to two types of the Wilson loop operator in the field theory. Because of the anisotropy between these two directions Eq. (3.9), It is expected that the qualitative behavior of these two types of Wilson loop are quite different.

We will use the Hamilton formalism which is explained in Appendix B to analyze the equation of motion of these string configurations. The formalism is useful for the numerical calculation.

Strings stretched to the longitudinal (β) direction First we consider a string configuration which is extended to the longitudinal direction β. An ansatz for this type of the string configuration is given in the following form,

$$(\tau, s, \beta) = (\tau, s(\sigma), \beta(\sigma))$$

and the other coordinates is set to zero because the metric Eq. (3.9) is symmetric under the ERT. Then the string action becomes

$$S = -\int d\tau U$$

$$U = \frac{R^2}{2\pi\alpha'}\int d\sigma \mathcal{L} = \frac{R^2}{2\pi\alpha'}\int d\sigma e^{\Phi/2}\sqrt{s'^2 + s^2\left(s^2 - a^2\right)\beta'^2}$$

where the prime denotes the derivative with respect to σ, namely, $s' = \partial_\sigma s,\ \beta' = \partial_\sigma \beta$. Since there are no τ dependence in U, we can use the technique introduced in Appendix B by regarding U as the action in Eq. (B.1).

We define the conjugate momenta of s and β as

$$\begin{aligned} p_s &\equiv \frac{\partial \mathcal{L}}{\partial s'} = e^{\Phi/2}\frac{s'}{\sqrt{s'^2 + s^2\left(s^2 - a^2\right)\beta'^2}}, \\ p_\beta &\equiv \frac{\partial \mathcal{L}}{\partial \beta'} = e^{\Phi/2}\frac{s^2\left(s^2 - a^2\right)\beta'}{\sqrt{s'^2 + s^2\left(s^2 - a^2\right)\beta'^2}}. \end{aligned} \tag{3.19}$$

Then the Hamiltonian $\mathcal{H} \equiv s' p_s + \beta' p_\beta - \mathcal{L}$ becomes

$$\mathcal{H} = \Delta \tilde{\mathcal{H}},$$

where

$$\Delta \equiv 2e^{-\Phi/2}\sqrt{s'^2 + s^2\left(s^2 - a^2\right)\beta'^2}, \quad \tilde{\mathcal{H}} = \frac{1}{2}\left[p_s^2 + \frac{p_\beta^2}{s^2\left(s^2 - a^2\right)} - e^{\Phi}\right]. \tag{3.20}$$

We may confirm the relation

$$\tilde{\mathcal{H}} = 0 \tag{3.21}$$

by substituting Eq. (3.19) into Eq. (3.20). Considering the Hamilton's equations of motion for $\tilde{\mathcal{H}}$, we get

$$s' = p_s, \quad \beta' = \frac{p_\beta}{s^2\left(s^2 - a^2\right)},$$
$$p_s' = \frac{2s^2 - a^2}{s^3\left(s^2 - a^2\right)^2} p_\beta^2 - \frac{2qe^{4a\beta}}{s^5 R^8}, \quad p_\beta' = \frac{2aqe^{4a\beta}}{s^4 R^8}. \tag{3.22}$$

The equations of motions Eq. (3.22) are solved with appropriate boundary conditions. Since there are four functions s, β, p_s, p_β, four first order differential equations, and one constraint Eq. (3.21), we need three boundary conditions. We choose these boundary conditions as

$$s_{max} = s\left(0\right) = s_0 > a, \ \beta\left(0\right) = \beta_0, \ p_s\left(0\right) = 0, \tag{3.23}$$

where s_0 denotes the bottom of the string and then constraint Eq. (3.21) determines

$$p_\beta\left(0\right) = s_0\sqrt{\left(s_0^2 - a^2\right)\left(1 + \frac{qe^{4a\beta_0}}{s_0^4 R^4}\right)}. \tag{3.24}$$

In these conditions Eqs. (3.23) and (3.24), we get the string configuration which two ends approach to boundary s_{max}. Thus we have two values $(\sigma_1, \ \sigma_2)$ of σ which correspond to the two end points at the boundary. Such σ_1 and σ_2 satisfy the condition

$$s_{max} = s\left(\sigma_1\right) = s\left(\sigma_2\right). \tag{3.25}$$

The distance between the quark and the antiquark, L, is defines as

$$L \equiv \left|\beta\left(\sigma_2\right) - \beta\left(\sigma_1\right)\right|.$$

There are an infinite number of configurations with distance L. Since we are interested in the quark at $\beta = 0$, we choose the configurations that satisfies $\beta\left(\sigma_1\right) = 0$. Adjusting the value β_0, we set the point $\beta\left(\sigma_1\right)$ to zero. Then the quark-antiquark potential becomes

$$E\left(L\right) = \frac{R^2}{2\pi\alpha'}\int_{\sigma_1}^{\sigma_2} d\sigma e^{\Phi/2}\sqrt{s'^2 + s^2\left(s^2 - a^2\right)\beta'^2}$$

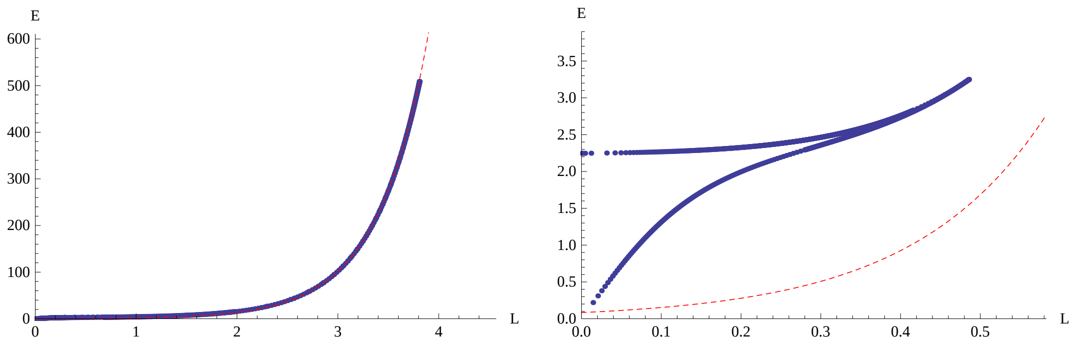


Figure 3.6: E-L relations for a quark and an antiquark in the longitudinally extended case for $s_{max} = 10,\ R = 1,\ q = 10$, and $a = 1$ (left), $a = 3$ (right). The (red) dashed curves represent Eq. (3.26), which represent as the effect of the color force existing in the confinement phase.

where $s(\sigma), \beta(\sigma)$ are solutions of the equations of motion. In our analysis, we set $s_{max} = 10$ to regularize the potential $E(L)$. The results are shown in Fig. 3.6. For sufficiently large q compared to the temperature $a/2\pi$, we can see the color force with a definite tension before the screening effects become dominant. The energy could be estimated as follows,

$$E = \frac{R^2}{2\pi\alpha'}\int_0^L d\beta \simeq \frac{\sqrt{q}e^{2aL}}{4a\pi\alpha' R^2}. \tag{3.26}$$

Actually, we obtain a good fit with this curve for low temperature case $a = 1.0$ (See the left figure of Fig. 3.6).

Strings stretched to the longitudinal (y) direction Next we consider a string extended to the y direction which is transverse to the acceleration direction. Because the metric depends on β, the string configuration must depend on β even if the position of the end points of the string do not depend on β on the boundary. Therefore an ansatz of the string configuration should be of the form

$$(\tau, s, \beta, y) = (\tau, s(\sigma), \beta(\sigma), y(\sigma)).$$

In this case the action becomes

$$S = -\int d\tau U,$$

where

$$U = \frac{R^2}{2\pi\alpha'}\int d\sigma e^{\Phi/2}\sqrt{s'^2 + s^2\left(s^2 - a^2\right)\beta'^2 + y'^2 e^{-2a\beta}s^2\left(s^2 - a^2\right)},$$

and a dot denotes the derivative with respect to the parameter σ. The canonical momenta conjugate to $s,\ \beta,\ y$ and Hamiltonian are defined as

$$
\begin{aligned}
p_s &\equiv \frac{\partial\mathcal{L}}{\partial s'}=e^{\Phi/2}\frac{s'}{\sqrt{s'^2+s^2\left(s^2-a^2\right)\beta'^2+y'^2e^{-2a\beta}s^2\left(s^2-a^2\right)}},\\
p_\beta &\equiv \frac{\partial\mathcal{L}}{\partial \beta'}=e^{\Phi/2}\frac{s^2\left(s^2-a^2\right)\beta'}{\sqrt{s'^2+s^2\left(s^2-a^2\right)\beta'^2+y'^2e^{-2a\beta}s^2\left(s^2-a^2\right)}},\\
p_y &\equiv \frac{\partial\mathcal{L}}{\partial y'}=e^{\Phi/2}\frac{e^{-2a\beta}s^2\left(s^2-a^2\right)y'}{\sqrt{s'^2+s^2\left(s^2-a^2\right)\beta'^2+y'^2e^{-2a\beta}s^2\left(s^2-a^2\right)}},
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{H} &\equiv s'p_s+\beta'p_\beta+y'p_y-\mathcal{L}\\
&= \Delta\tilde{\mathcal{H}},
\end{aligned}
$$

where

$$
\begin{aligned}
\tilde{\mathcal{H}} &\equiv \frac{1}{2}\left[p_s^2+\frac{p_\beta^2}{s^2\left(s^2-a^2\right)}+\frac{e^{2a\beta}p_y^2}{s^2\left(s^2-a^2\right)}-e^{\Phi}\right]=0, && (3.27)\\
\Delta &\equiv 2e^{-\Phi/2}\sqrt{s'^2+s^2\left(s^2-a^2\right)\beta'^2+y'^2e^{-2a\beta}s^2\left(s^2-a^2\right)}. && (3.28)
\end{aligned}
$$

The Hamilton's equations of motion are then

$$
s'=p_s,\quad \beta'=\frac{p_\beta}{s^2\left(s^2-a^2\right)},\quad y'=\frac{e^{2a\beta}p_y}{s^2\left(s^2-a^2\right)},
$$

$$
\begin{aligned}
p_s' &= \frac{2s^2-a^2}{s^3\left(s^2-a^2\right)^2}\left(p_\beta^2+e^{2a\beta}p_y^2\right)-\frac{2qe^{4a\beta}}{s^5R^8},\\
p_\beta' &= -\frac{ae^{2a\beta}p_y^2}{s^2\left(s^2-a^2\right)}+\frac{2aqe^{4a\beta}}{s^4R^8},\quad p_y'=0. && (3.29)
\end{aligned}
$$

When we solve the above equations, we impose the following boundary conditions,

$$
\beta\left(0\right)=\beta_0,\quad s\left(0\right)=s_0>a,\quad x\left(0\right)=0,
$$

$$
p_\beta\left(0\right)=0,\quad p_s\left(0\right)=0,
$$

and the constraint Eq. (3.27) leads to

$$
p_y\left(0\right)=s_0e^{a\beta_0}\sqrt{\left(s_0^2-a^2\right)\left(1+\frac{qe^{4a\beta_0}}{s_0^4R^4}\right)}.
$$

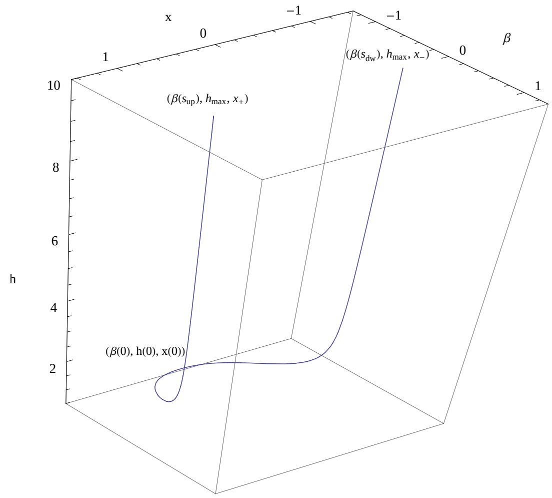


Figure 3.7: 3D string configuration stretched in the y direction. As y decreases the bottom of the string is stretched to direction.

On the boundary $s = s_{max}$, β and y represent the position of the quark (β_{bdy}, y_1) and the antiquark (β_{bdy}, y_2),

$$\beta(\sigma_1) = \beta(\sigma_2) = \beta_{bdy}, \quad y(\sigma_1) = y_1, \quad y(\sigma_2) = y_2$$

where $\sigma_1,\ \sigma_2$ is defined as in Eq. (3.25). The distance between the quark and the antiquark, L, is defined as

$$L = |y_2 - y_1| \,.$$

When all string configurations with the same L, β_{bdy} and with the different quark position $y(\sigma_1)$ represent the same physics because the metric Eq. (3.9) and this configuration have the symmetry under the translation of the y coordinate. Now we would like to see the force in the y direction through the solution of Eq. (3.29). So we solve these equations by imposing the condition that the end point coordinate β_{bdy} is fixed at a certain value. So here we must tune the boundary values, β_0 and h_0 in order to realize the same β_{bdy} for each solution. Since the ERT is defined as in Eq. (3.7), we choose the solution for

$$\beta_{bdy} = 0,$$

and a typical string solution in the present case is shown in Fig. 3.7.

We can see the linear rising part before the screening takes place. This linearly rising part is fitted by the formula

$$E = \frac{\sqrt{q} e^{2a\overline{\beta}} R^2}{2\pi\alpha' R^2} L + \text{const.}\ , \tag{3.30}$$

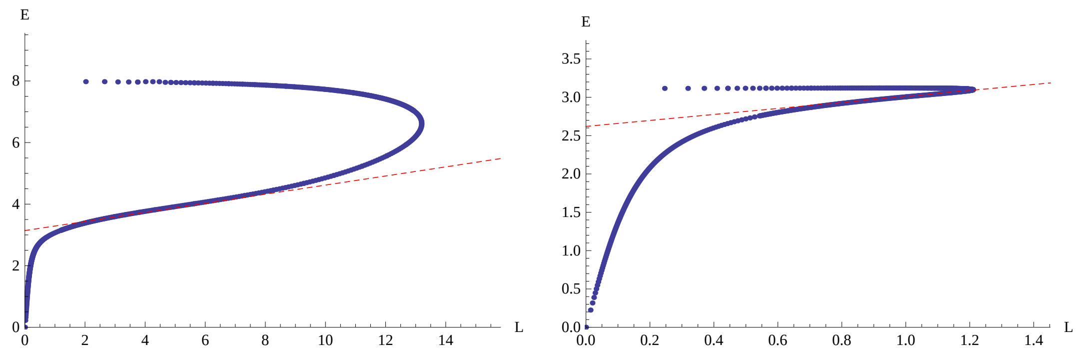


Figure 3.8: Typical E-L relations for quark and antiquark for $a = 0.1$ (left) and $a = 1.0$ (right), $s_{max} = 10$, $R = 10$ and $q = 10$. The dashed (red) line represents the tension of the potential of linear rising part, given by Eq. (3.30) and Eq. (3.31), which is expected as the effect of the color force existing in the confinement phase.

where $\overline{\beta}$ is approximately given as,

$$\overline{\beta} \approx \frac{\beta\,(s = a)}{3}. \tag{3.31}$$

As for the upper part of the E-L relation, the curve increases with decreasing L. This may be understood as follows. The upper part represents the string configurations whose bottom points are pulled to near the horizon. Then the lower part of the string grows to the direction of β as shown in Fig. 3.7 and the energy of the string becomes large. This kind of behavior cannot be seen in the case of the AdS-Schwarzschild background.

The quark-antiquark potential becomes

$$E\,(L) = \frac{R^2}{2\pi\alpha'}\int_{\sigma_1}^{\sigma_2} d\sigma e^{\Phi/2}\sqrt{s'^2 + s^2\left(s^2 - a^2\right)\beta'^2 + y'^2 e^{-2a\beta} s^2\left(s^2 - a^2\right)}$$

where $s\,(\sigma)\,,\ \beta\,(\sigma)\,,\ y\,(\sigma)$ are solutions of the equations of motion, Eq. (3.29). The relation between the energy E and distance L are shown in Fig.3.8

We again come upon the anisotropy of the Unruh effect. The quark-antiquark potential $E\,(L)$ for the β direction is quite different from that for the y direction especially in the small a region.

3.4 Relation to the 4D field theory

Here we give comments on the statement about the Unruh effect given in Ref. [2], where the analysis is performed within the 4D field theory. The main result is that any Green ’ s functions in the vacuum of Minkowski space-time are the same as those of the Rindler space-time when the calculation is restricted to the same Rindler wedge in the Minkowski coordinates. So one may think that the phase of the Minkowski vacuum cannot be changed in the Rindler vacuum since the VEV of any order parameter would be the same in both vacuums.

However, in the present paper, we show that the vacuum of the Rindler space-time is in the quark deconfinement phase in spite of the fact that the original theory in the Minkowski vacuum is in the confinement phase. Then our calculation seems to be inconsistent with the statement of Ref. [2]. However this point would be resolved as follows.

Let us consider the VEV of the Wilson loop operator that determines whether confinement takes place or not. In the field theory side, the operator is given by,

$$\mathcal{O}(C) = \mathrm{tr}\left(\mathcal{P}\exp\left[ig\oint_C A_\mu(x)\,dx^\mu\right]\right),$$

where $\mathcal{P}$ denotes the path ordering of a closed path C in the line integration for the gauge field $A_\mu(x)$. Its VEV is written as

$$A(C) = \langle 0_M|\,\mathcal{O}(C)\,|0_M\rangle$$

for the Minkowski vacuum $|0_M\rangle$, and

$$B(C) = \frac{\mathrm{Tr}\left(e^{-H^R/T}\mathcal{O}(C)\right)}{\mathrm{Tr}\left(e^{-H^R/T}\right)}$$

for the finite temperature (T) Rindler vacuum, respectively. The statement in Ref. [2] implies the equivalence of A and B when they are calculated within the same Rindler wedge.

Our calculation of A and B do not lead to $A = B$ due to the following reasons. First, the paths used in A and B are not related by the ERT. In order to see the potential between the quark and the antiquark, we have performed the calculations for the rectangular path in the t-x plane for A and for the one in the τ-y (or τ-β) plane for B respectively. The rectangular path used in A, C_A, cannot be transformed to the one used in B by the ERT since it must be transformed to the τ dependent path. Actually fixed $x\,(=x_1)$ path in Minkowski coordinates is written as

$$\begin{aligned}
\beta &= \frac{1}{a}\ln\left[a^2\left(x_1^2 - t^2\right)\right], \\
\tau &= \frac{1}{a}\tanh^{-1}\left(\frac{t}{x_1}\right),
\end{aligned}$$

in the Rindler coordinates. Moreover, a fixed $\beta\,(=\beta_1)$ path, $C_{B,\beta}$, or a fixed $y\,(=y_1)$ path, $C_{B,y}$, in Rindler coordinates are written as

$$\begin{aligned}
x &= \frac{1}{a}e^{a\beta_1}\cosh(a\tau), \\
t &= \frac{1}{a}e^{a\beta_1}\sinh(a\tau),
\end{aligned}$$

for fixed $\beta\,(=\beta_1)$ path and,

$$
\begin{aligned}
x &= \frac{1}{a}\cosh\left(a\tau\right), \\
y &= y_1 \\
t &= \frac{1}{a}\sinh\left(a\tau\right),
\end{aligned}
$$

for fixed $y\,(=y_1)$ path with $\beta=0$ in Minkowski coordinates. Thus our Wilson-loop calculations in $A\,(C_A)$ and $B\,(C_B)$ are not related by the ERT. This is the reason why we could obtain different results from the calculation of A and B.

Second, let us comment on the quark string configuration in the Rindler vacuum. This point is also related to the fact that the quark in the Rindler vacuum is different from the one in the Minkowski vacuum. In estimating B, the essential string solution responsible for the proof of the deconfinement is the one which connects the boundary and the event horizon, because this solution can be interpreted as the free quark and this is possible only for the deconfinement phase. We could find such a solution only in the Rindler vacuum. The interesting point is that this free-quark configuration in the Rindler vacuum is obtained by the ERT from the constantly accelerated quark string configuration given in the Minkowski vacuum as shown above. However this configuration is not used in the evaluation of A in our theoretical scheme. Because of these reasons, we do not obtain the relation $A=B$. We are examining the parts, of A and B which cannot be related by the ERT. Then our statement does not contradict with the one in Ref. [2].

Let us comment on other possible choices of the coordinate transformation than that adopted above. For example, the coordinate transformation given in Ref. [2] which does not include the fifth coordinate of the bulk may be considered as such a transformation. This is different from the ERT used here. In the latter case, the transformation is performed in three dimensional coordinate including the fifth one. As a result, the event horizon appears in the bulk, then the infrared region is cut off in the dynamics of the dual 4D theory. Then the dynamical properties responsible for the long-range force would be lost in the vacuum of the new coordinate system.

We should also study the VEV of other physical quantities. In this context, we notice other papers [18, 19] from a different 4D non-perturbative approach, and the author demonstrates chiral symmetry restoration in the Rindler vacuum. So it would be necessary to proceed in this direction in order to deepen the knowledge of the Rindler vacuum.

3.5 Summary:Holographic Unruh effect

We give here a constantly accelerated quark as a string solution of the Nambu-Goto action which is embedded in the supergravity ($\mathrm{dAdS}_5\times S^5$) background dual to the confining Yang-Mills theory. For the string solution given in the zero-temperature Minkowski spacetime, we find an event horizon in its induced metric. This horizon is also found in the case of AdS_5

background dual to the non-confining theory. In any case, this fact can be considered as a clear signal of the radiation of gluons due to the acceleration of the color charged quark.

We consider the extended Rindler transformation proposed by Xiao in order to move to the co-moving frame of the accelerated quark and to study the properties of the Rindler vacuum. The coordinate transformation is generalized to the 5D bulk theory, but the boundary is described by the usual 4D Rindler metric. The dual theory is found in a thermal medium with the Rindler temperature $T = a/2\pi$, so it is expected that the theory is in a different vacuum from the one in the inertial frame. To study this point, we have examined several dynamical properties of the new vacuum to compare them to those observed in the inertial coordinate. Then we find that the vacuum properties are changed from those seen in the inertial frame. In the Rindler vacuum, the friction arises from the thermal medium when a quark moves and the color force is screened by the thermal effect at long distances.

We discuss the drag force to see the thermal effects in the Rindler vacuum. We recognize the anisotropy between the friction constants for the longitudinal (β) direction and for the transverse (y) direction. In the Rindler-AdS$_5$ background, the anisotropy appears in the friction constants as the different dependence on the velocity v, while in the Rindler-dAdS background we have more a drastic result. Namely we cannot find any static states which corresponds to a quark moving with the constant velocity v in the field theory dual to the Rindler-dAdS background. We also find the difference of factor four between the friction constant in the Rindler vacuum and the one in the AdS$_5$-Schwarzschild background.

We also discuss the Wilson loop to study the color force screening by the thermal effect. As a result, we have the potential between the quark and antiquark in Rindler space. In the Rindler coordinates, the color force is screened by the thermal effect at long distances and the confining property is lost. The screening length depends on the temperature and also on the direction in the three space, namely, if it is accelerated in the longitudinal direction y, z or in the transverse directions β. As for the temperature dependence, we find some difference from the one observed previously in the AdS-Schwarzschild background, especially for the screening in the longitudinal direction. The reason of this anisotropy in the 3D space is the behavior of the dilaton. The dilaton expresses the gauge coupling constant, and it is deformed in the longitudinal direction as in Eq. (3.17) due to the extended Rindler transformation. Then the potential between the quark and the antiquark has different forms in the longitudinal and in the transverse directions. This point has been assured through the direct observation of the potential between the quark and the antiquark.

Even if we calculate the force in the transverse direction, we can see the effect of the longitudinally deformed dilaton. For example, consider an excited bound state of the quark and the antiquark in the Rindler coordinates. Then we could find a small repulsion in the direction (y) transverse to the accelerated direction (β). This repulsion is understood from the string configuration in the 3D space β-y-s. When the distance y decreases, the string is stretched in the β direction near the horizon $s \simeq a$. This phenomenon is understood as the remnant of the strong confining force near the horizon. On the other hand, the attractive force is exponentially enhanced in the longitudinal direction at fairly large distance. This behavior is caused by the deformed dilaton in the Rindler coordinate in the longitudinal direction.

Finally, we give our main conclusion that we could find a new vacuum when we move to the co-moving coordinate of an accelerated quark by the extended Rindler transformation considered here and find a kind of a high temperature theory in a deconfinement phase. Then, in the new vacuum, the properties given by the long range force in the inertial frame is lost. Thus our calculation seems to be inconsistent with the claim given in Ref. [2]. However, this point is resolved as explained in the previous section. The main reason is that we are considering the different Green 's functions to decide the phase of the vacuum.

Chapter 4

Holographic cold nuclear matter

Abstract of this chapter We study cold nuclear matter based on the Sakai-Sugimoto model, where baryons are introduced as solitons on probe D8/$\overline{\text{D8}}$ branes. Within the dilute gas approximation of solitons, we search for stable states by using the variational method. We find a first-order phase transition from the vacuum[1] to the nuclear matter phase as we increase the chemical potential. At the critical chemical potential, we could see a jump in the baryon density from zero to a finite value. The baryon number density is rather large compared to the normal nuclear density. The result is caused by our dilute gas approximation, and it may be improved by introducing the contribution of two-body interaction of baryons. We also study the neutron star by solving the Tolman-Oppenheimer-Volkoff (TOV) equations [20, 21]. We read off the equation of state (EoS) of our nuclear matter by introducing Fermi momentum. Then, we apply our model to the neutron star. We solve the TOV equations by plugging in the EoS obtained from the model to get the maximum values of the mass and the radius. They are considerably smaller than the empirical ones. We suspect that the discrepancy comes from the dilute gas approximation.

4.1 Finite baryon density system

It is very difficult to study dense nuclear matter from first-principle calculations in non-perturbative QFTs like QCD. For example, it is difficult to study that from lattice gauge theory simulations because of "sign problem" when we introduce the chemical potential to the theory[22, 23]. So we hope that gauge/gravity duality works in this subject and it may give some non-trivial results. It is also challenging to make clear the properties of the nuclear matter from the viewpoint of holographic gauge theory. It may be useful to improve the understanding of holographic approaches.

In this chapter we use the Sakai-Sugimoto model introduced in Sec. 2.5, which is a gauge/gravity duality model very close to QCD. In this model, baryons are represented as the solitons on N_f probe flavor branes[8, 24, 25, 26]. The baryon number N_B is identified

[1] Here, "the vacuum" means the state with the lowest energy at the vanishing chemical potential.

with the soliton number of the solution N_s and the number of the probe flavor branes N_f corresponds to the number of flavors of quarks. We set $N_f = 2$ throughout our analysis for simplicity. The solution with the soliton number $N_s = 1$ is given as the BPST instanton solution of $SU(N_f = 2)$ YM theory in the flat five dimensional spacetime on the probe branes [24]. The solution is characterized by a scale parameter which corresponds to the baryon size. This size is determined dynamically so that the action of the D8/$\overline{\text{D8}}$, which is expressed as the sum of the Dirac-Born-Infeld (DBI) and Chern-Simons (CS) term, is minimized. This analysis for the system with the baryon number $N_B = 1$ may be generalized to the system with $N_B \geq 2$ by using the soliton solution with $N_s \geq 2$. Then the CS term plays a crucial role when we introduce the chemical potential.

There are several papers in which the nuclear system with the chemical potential of baryons is studied [27, 28, 29, 30, 31, 32, 33, 34]. However we feel dissatisfaction with the papers. For example, in Ref. [28], the solitons of the flavor gauge fields are introduced as a delta-function type source only in the CS term, though the soliton size is finite and the contribution must also be included in the DBI term. As a result, the authors of Ref. [28] have observed a gapless transition from the vacuum to the nuclear matter phase at zero temperature. However this observation is in contradiction with the ones obtained in other theories (See for example Ref. [35]).

On the other hand, the authors of Ref. [30] keep the flavor gauge fields in the DBI action as well as in the CS term. So they find a first-order phase transition at finite baryon density. However, the configuration used for the flavored gauge fields has infinite baryon number density, so it is difficult to study the finite baryon number density. Furthermore, they do not check whether their D8-brane configuration is the lowest-energy solution for the equations of motion for the D-branes or not, though it may be dependent on the $U(N_f)$ gauge fields via the interactions on D8-branes.

Here we introduce an explicit form of soliton solution which has a smooth profile in the the direction of the fifth coordinate instead of the delta function form. Furthermore, we keep the flavored Yang-Mills fields in the DBI action of the D8 probe brane up to the square of the field strength. This is crucial for finding a gap of the baryon density at the transition point as in Ref. [30]. In contrast to the case in Ref. [30], in our approach, the flavored YM field is solved by determining the remaining size parameter of the soliton with fixed chemical potential. After that, the physical quantities, such as chemical potential and baryon number density, are obtained and used for the search for the phase transition.

We should fix the value of the position of the D8/$\overline{\text{D8}}$-brane because it determines a certain feature of the dual gauge theory. Here we restrict the profile of the D8/$\overline{\text{D8}}$-brane to the antipodal solution. Then, we can use a simple D8-brane profile [24], which is obtained without any solitons as an antipodal U-shaped configuration. In general, the lowest energy solution for some finite baryon density n_B is not equivalent to the simple solution mentioned above, since it is not antipodal. However, the solution is always approximated by the simple one with a negligible correction for any value of n_B. This is the reason why we restrict the profile to the antipodal solution. Another reason is for the simplicity of the present analysis.

Through our analysis, we find a first-order phase transition, which is expressed in n_B-μ_B plane, where μ_B denote the baryon chemical potential. At the transition point, the baryon number density jumps from zero to a finite value, which corresponds to the transition from the vacuum to a nuclear matter phase. By adjusting the parameters, at the transition point, we observe the baryon mass as $\mu_B \simeq 2.3$ GeV and the baryon number density as $n_B \simeq 0.70$ fm^{-3}. These values are considerably large compared to the realistic nucleon mass and the normal nuclear density. Furthermore, at this transition point, the size of the baryon are rather small.

In this section, we first review baryons in the Sakai-Sugimoto model, and then introduce the dilute gas model of solitons. Finally the results obtained in the model are discussed.

4.1.1 Action and 1-soliton solution

The action of the Sakai-Sugimoto model with $N_f = 2$ is written as follow[24],

$$S = S_{YM} + S_{CS},$$

$$\begin{aligned} S_{YM} &= -a\lambda N_c \int d^4x dz \ \mathrm{tr}\left[\frac{1}{2}h(z)F_{\mu\nu}^2 + k(z)F_{\mu z}^2\right] \\ &\quad -\frac{a\lambda N_c}{2}\int d^4xdz \ \left[\frac{1}{2}h(z)\hat{F}_{\mu\nu}^2 + k(z)\hat{F}_{\mu z}^2\right] + \mathcal{O}\left(\lambda^{-1}\right), \end{aligned} \tag{4.1}$$

$$S_{CS} = \frac{N_c}{64\pi^2}\epsilon_{MNPQ}\int d^4xdz\left[\hat{A}_0 \mathrm{tr}\left(F_{MN}F_{PQ}\right) + \cdots\right], \tag{4.2}$$

where $a^{-1} \equiv 216\pi^3$, $k(z) = 1 + z^2$, $h(z) = k^{-1/3}$, $M, N = 1, 2, 3, z$ and $\epsilon_{123z} = +1$. A_M, $\hat{A}_M$ denote $SU(N_f), U(1)$ gauge fields respectively. The ellipsis in S_{CS} stands for terms that are irrelevant to the discussion below. Rescaling the fields and integration variables as

$$x^M \to \lambda^{-1/2}x^M, \ (A_M, A_0) \to \left(\lambda^{1/2}A_M, A_0\right), \tag{4.3}$$

we can rewrite the YM action Eq. (4.1) as

$$\begin{aligned} S_{YM} &= -aN_c\int d^4dz \ \mathrm{tr}\left[\frac{\lambda}{2}F_{MN}^2 + \left(-\frac{z^2}{6}F_{ij}^2 + z^2F_{iz}^2 - F_{0M}^2\right)\right] \\ &\quad -\frac{aN_c}{2}\int d^4dz \ \left[\frac{\lambda}{2}\hat{F}_{MN}^2 + \left(-\frac{z^2}{6}\hat{F}_{ij}^2 + z^2\hat{F}_{iz}^2 - \hat{F}_{0M}^2\right)\right], \end{aligned}$$

while S_{CS} takes the same form as the one in Eq. (4.2). Then the leading contributions of the equations of motion for their gauge fields in λ^{-1} expansion are,

$$\begin{aligned} D_M F_{0M} + \frac{1}{64\pi^2 a}\epsilon_{MNPQ}\hat{F}_{MN}F_{PQ} &= 0, \\ D_N F_{MN} &= 0, \\ \partial_M \hat{F}_{0M} + \frac{1}{64\pi^2 a}\epsilon_{MNPQ}\left\{\mathrm{tr}\left(F_{MN}F_{PQ}\right) + \frac{1}{2}\hat{F}_{MN}\hat{F}_{PQ}\right\} &= 0, \\ \partial_N \hat{F}_{MN} &= 0. \end{aligned} \tag{4.4}$$

These equations can be solved as,

$$F_{ij} = \frac{2\rho^2}{\left((x-x_0)^2+\rho^2\right)^2}\epsilon_{ija}\tau^a \quad F_{zj} = \frac{2\rho^2}{\left((x-x_0)^2+\rho^2\right)^2}\tau_j, \quad A_0 = 0, \tag{4.5}$$

$$\hat{A}_M = 0, \quad \hat{A}_0 = \frac{1}{8\pi^2 a}\frac{1}{(x-x_0)^2}\left[1-\frac{\rho^4}{\left((x-x_0)^2+\rho^2\right)^2}\right], \tag{4.6}$$

where τ_i are Pauli matrices and $(x-x_0)^2 = z^2 + (\vec{x}-\vec{x_0})^2$. This $SU(2)$ gauge field may be written as

$$A_M = -if(x)\, g\partial_M g^{-1},$$

where

$$f(x) = \frac{(x-x_0)^2}{(x-x_0)^2+\rho^2}, \quad g(x) = \frac{(z-z_0)-i(\vec{x}-\vec{x_0})\cdot\vec{\tau}}{\sqrt{(x-x_0)^2}}.$$

x_0 and ρ denote the center position and the size of this soliton, respectively. The parameter ρ is determined by requiring that the action is minimized with respect to the variation of it. In fact, the variables and the fields in Eqs. (4.5) and (4.6), x^M , F_{MN} and $\hat{A}_0$, are rescaled as in Eq. (4.3). Thus in order to restore the original variables and fields, we must rescale the fields and variables as,

$$x^M \to \lambda^{+1/2}x^M, \ (A_M, A_0) \to \left(\lambda^{-1/2}A_M, A_0\right). \tag{4.7}$$

Performing the rescaling in Eq. (4.7), we assure that Eq. (4.5) is maintained when we use the original variables and field strengths.

The baryon number is defined by

$$N_B \equiv \frac{1}{32\pi^2}\int d^3x dz\ \epsilon_{MNPQ}\mathrm{tr}\left(F_{MN}F_{PQ}\right), \tag{4.8}$$

and the solution Eq. (4.5) gives $N_B = 1$. Substituting Eqs. (4.5), (4.6) into the action, Eqs. (4.1), (4.2), the soliton mass M is obtained as $S = -\int dtM$,

$$M = M_0\left[1+\lambda^{-1}\left(\frac{\rho^2}{6}+\frac{1}{320\pi^4 a^2}\frac{1}{\rho^2}+\frac{z_0^2}{3}\right)+\mathcal{O}\left(\lambda^{-2}\right)\right], \quad M_0 \equiv 8\pi^2 a\lambda N_c. \tag{4.9}$$

The values of ρ and Z is determined by minimizing M,

$$\rho^2 = 8\pi^2 a\sqrt{\frac{6}{5}}, \quad z_0 = 0. \tag{4.10}$$

Note that the baryon size Eq. (4.10) is very smaller than an expected value. Since we rescaled the parameter as Eq. (4.3), we should evaluate the size of baryon as

$$\rho^2 = \frac{8\pi^2 a}{\lambda M_{KK}^2}\sqrt{\frac{6}{5}},$$

when we compare ρ with the real value of the baryon size. Setting $\lambda = 16.6$ and $M_{KK} = 949\text{MeV}$,[2] we find

$$\rho = 5.88 \times 10^{-3}\text{fm}. \tag{4.11}$$

This value is much smaller than the expected value of the ordinary baryon size $\sim 1\text{fm}$.

4.1.2 2-soliton solution and repulsive force

A two-baryon solution can be constructed by ADHM construction in [26]. It is parametrized by four quaternionic parameters

$$\boldsymbol{X}_i = z_{0,i} + i\vec{x}_{0,i}\cdot\vec{\tau}, \quad \boldsymbol{y}_i = y_i^4 + i\vec{y}_i\cdot\vec{\tau}, \quad (i = 1,2).$$

$x_{0,i}^M = (\vec{x}_{0,i}, z_{0,i})$ corresponds to the position of the baryons, $\rho_i \equiv \sqrt{y_i^I y_i^I}$ is the size of each baryon and $\boldsymbol{a}_i \equiv \boldsymbol{y}_i/\rho_i$ is the $SU(2)$ orientation of each soliton. In this setup, the potential of this system U $\left(S = -\int dtU\right)$ is given as,

$$U = \sum_{i=1}^{2} M_i + H_{pot}^{(SU(2))} + H_{pot}^{(U(1))} + \mathcal{O}\left(\lambda^{-1}\right), \tag{4.12}$$

where

$$M_i = M_0\left[1 + \lambda^{-1}\left(\frac{\rho_i^2}{6} + \frac{1}{320\pi^4 a^2}\frac{1}{\rho_i^2} + \frac{z_{0,i}^2}{3}\right)\right],$$

$$H_{pot}^{(SU(2))} = \frac{4\pi^2 a N_c}{3}\rho_1^2\rho_2^2\frac{r^a r^b}{|r|^4}\mathrm{tr}\left(i\tau^a\boldsymbol{a}_2^{-1}\boldsymbol{a}_1\right)\mathrm{tr}\left(i\tau^b\boldsymbol{a}_2^{-1}\boldsymbol{a}_1\right), \tag{4.13}$$

$$H_{pot}^{(U(1))} \sim \frac{N_c}{8\pi^2 a}\frac{1}{|r|^2}\left[\frac{1}{2} + \frac{2\left(\boldsymbol{a}_1\cdot\boldsymbol{a}_2\right)^2 - 1}{5}\left(\frac{\rho_2^2}{\rho_1^2} + \frac{\rho_1^2}{\rho_2^2}\right)\right], \tag{4.14}$$

, $r^M = X_1^M - X_2^M\,(M = 1,2,3,z)$ and $|r| = \sqrt{r^M r^M}$. These expressions are valid in the region of $\mathcal{O}(1/M_{KK}) < |r| < \mathcal{O}\left(\sqrt{\lambda}/M_{KK}\right)$. The dependence on $+|r|^{-2}$ of this potential implies a repulsive force from the nucleon-nucleon interaction.

To minimize the potential energy of this system, we should take $z_{0,i} = 0$. Thus $SU(2)$ soliton is placed on the bottom point of the D8-brane configuration. Then $|r|$ and ρ_i become the separation between baryons and the size of each baryon in three dimensional space of ordinary Minkowski spacetime, respectively. It is natural to assume that each baryon has the same size, namely $\rho_1 = \rho_2 = \rho$.

[2] These values of parameters are determined by fitting the rho meson mass and pion decay constant [9].

In this setup, we can rewrite Eq. (4.12) as

$$
\begin{aligned}
U &= 2M_0 + \tilde{H}_{pot}^{(SU(2))} + \tilde{H}_{pot}^{(U(1))}, \\
\tilde{H}_{pot}^{(SU(2))} &= 2\frac{4\pi^2 a N_c}{3}\rho^2 \left(1 + \frac{2c_1\left(\hat{r}, \boldsymbol{a}_1, \boldsymbol{a}_2\right)}{k^2}\right), \qquad (4.15) \\
\tilde{H}_{pot}^{(U(1))} &\sim \frac{54\pi N_c}{5}\frac{1}{\rho^2}\left(1 + \frac{1}{4}\frac{1 + 8c_2\left(\boldsymbol{a}_1, \boldsymbol{a}_2\right)}{k^2}\right), \qquad (4.16)
\end{aligned}
$$

where we denote $\vec{r} = k\rho\hat{r}$, $\hat{r}^2 = 1$ and

$$
c_1\left(\hat{r}, \boldsymbol{a}_1, \boldsymbol{a}_2\right) \equiv \left(\hat{r}\cdot\left(\boldsymbol{a}_2^{-1}\boldsymbol{a}_1\right)\right)^2, \quad c_2\left(\boldsymbol{a}_1, \boldsymbol{a}_2\right) \equiv \left(\boldsymbol{a}_1\cdot\boldsymbol{a}_2\right)^2 \qquad (4.17)
$$

are constants which depend on $SU\left(2\right)$ orientations and they satisfy $0 \leq c_1, c_2 \leq 1$. We notice that when $|r| \gg \rho$, namely $k \gg 1$ in the expression Eqs. (4.15), (4.16), the second term of each potential can be neglected compared with the first term and then the net contribution of these potential agrees with twice of that of one-baryon, Eq. (4.9).

4.1.3 Multi-soliton solution

Dilute gas approximation and mean field approximation When we want to analyze a system with finite baryon density in 4-dimensional spacetime, we should search for solutions of the equations of motion Eq. (4.4) with $N_B = \infty$. However it is very difficult to find such solutions. Then we employ the dilute gas approximation; we treat a $SU\left(2\right)$ gauge strength of the multi-soliton solution as the sum of gauge field strengths of one-soliton solutions,

$$
F_{ij} = \sum_{m=1}^{N_B} F_m \epsilon_{ija}\tau^a \quad F_{zj} = \sum_{m=1}^{N_B} F_m \tau_j\ , \qquad (4.18)
$$

$$
F_m \equiv \frac{2\rho^2}{\left(\left(x - \left(x_0\right)_m\right)^2 + \rho^2\right)^2},
$$

and we ignore the product of one-soliton solutions at two different positions, namely

$$
F_m F_{n\neq m} \sim 0. \qquad (4.19)
$$

Then the right hand side of Eq. (4.8) becomes,

$$
\begin{aligned}
\frac{1}{32\pi^2}\int d^3x dz\ \epsilon_{MNPQ}\mathrm{tr}\left(F_{MN}F_{PQ}\right) &= \frac{4! \times 2}{32\pi^2}\sum_{m=1}^{N_B}\int d^3x dz\ F_m^2 \\
&= N_B.
\end{aligned}
$$

Thus we retain the correct baryon number. We use this solution as a trial function which is supposed to be a solution of the system. This approximation should be valid when the distance between baryons are sufficiently larger than their baryon size , $\left|(x_0)_i - (x_0)_j\right| \gg \rho$. It can be confirmed in the case of two-baryon system (see Sec. 4.1.2).

We assume that the mean field approximation is able to apply the $U(1)$ gauge field, namely $\hat{A}_0(\vec{x}, z)$ is approximated as $\hat{A}_0(z)$.[3] We suppose that this approximation will be valid when baryon number density is finite, namely $N_B = \infty$.

Back reaction to D8-branes Because of the infinite number of baryons, N_f D8/$\overline{\text{D8}}$-branes may feel the back reaction from the baryons. If this is true, the configurations of N_f D8/$\overline{\text{D8}}$-branes may be changed and the action Eq. (4.1) is no longer a correct description. So we should reconsider D8/$\overline{\text{D8}}$-branes configurations as their induced metric is

$$ds_9^2 = \frac{3}{2}\left(\frac{4}{9}k^{1/2}\eta_{\mu\nu}dx^\mu dx^\nu + g(\tau)\frac{4}{9}k^{-5/6}dz^2 + k^{1/6}d\Omega_4^2\right),$$

$$g(\tau) = 1 + z^2 k^{1/3}\tau'^2, \quad k = 1 + z^2,$$

where $\tau' = \frac{\partial\tau}{\partial z}$ and τ has a periodicity $\tau \sim \tau + 2\pi$.

Then, we can write the action as

$$S = -2\kappa V_3 \int dt \int_{z_0}^{\infty} dz L(n_B), \tag{4.20}$$

$$\begin{aligned} L(n_B) &= k^{5/6}\sqrt{g(\tau)k^{-1/3} - \left(\frac{1}{2}\frac{27\pi}{2\lambda}\partial_z\hat{A}_0\right)^2 Q_1} \\ &\quad -6\left(\frac{1}{2}\frac{27\pi}{2\lambda}\hat{A}_0\right)n_B\left(\frac{9\pi}{\lambda}\right)^2\overline{q}^2, \end{aligned} \tag{4.21}$$

where

$$Q_1 = 1 + \frac{3}{2}n_B\left(\frac{9\pi}{\lambda}\right)^2\overline{q}^2\left(k^{-1} + \frac{k^{1/3}}{g(\tau)}\right), \quad \overline{q}^2 = \frac{9}{8}\frac{\pi^2\rho^4}{\left((z-z_0)^2+\rho^2\right)^{5/2}}$$

n_B and V_3 are the baryon number density and volume of 3-dimensional space, $n_B \equiv \frac{N_B}{V_3}$, $V_3 \equiv \int d^3x$, respectively. We only expand the Lagrangian in terms of $SU(2)$ field and leave $U(1)$ field inside the square root in Eq. (4.21). This is necessary to obtain the correct results

[3] It is natural that $SU(2)$ gauge field depends on 4-dimensional Minkowski coordinates x^μ, while $U(1)$ gauge field is not. In the one-soliton case, x^μ dependence on their field strength is

$$F_{MN} \sim |x|^{-4}, \; \hat{F}_{0M} \sim |x|^{-3}, \quad \text{for } |x| \equiv \sqrt{x^i x^i} \gg \rho.$$

So the decrease of the value of $U(1)$ gauge field strength is slower than that of $SU(2)$ gauge field strength. This fact implies that our mean field approximation is valid only for $U(1)$.

(See Appendix D). The bottom of D8 branes may shift from $z=0$ to $z=z_0$ because of the existence of N_B baryons. Then the equations of motion for $\hat{A}_0(z)$ and $\tau(z)$ are given by

$$\partial_z\left(\frac{1}{2}\frac{27\pi}{2\lambda}\partial_z\hat{A}_0\frac{k^{5/6}}{G}Q_1\right) = 6\left(\frac{9\pi}{\lambda}\right)^2 n_B\overline{q}^2, \tag{4.22}$$

$$\partial_z\left(z^2\tau'\frac{k^{5/6}}{G}Q_1\right) = 3\left(\frac{9\pi}{\lambda}\right)^2 n_B\partial_z\left(k^{3/2}G\frac{\overline{q}^2z^2\tau'}{g^2(\tau)}\right), \tag{4.23}$$

where $G\equiv\sqrt{g(\tau)\,k^{-1/3}-\left(\frac{1}{2}\frac{27\pi}{2\lambda}\partial_z\hat{A}_0\right)^2}$. These equations can be reduced to

$$\frac{z^2k\tau'}{g^{1/2}}\left\{H^{-1}Q_1-H\frac{3\left(\frac{9\pi}{\lambda}\right)^2n_Bk^{1/3}\overline{q}^2}{g}\right\}=\left[zQ_1k^{3/4}\right]_{z=z_0}, \tag{4.24}$$

where

$$H\equiv\frac{\frac{k^{5/6}Q_1}{d}}{\sqrt{1+\left(\frac{k^{5/6}Q_1}{d}\right)^2}}.$$

We can solve Eq. (4.24) numerically. Considering only the class of solutions which satisfy $\tau(\infty)=\pi/2$ (antipodal solution), we can confirm numerically that the lowest energy solution may be approximated as

$$\tau'(z)=0 \quad \text{for} \quad z>z_0 \quad \text{and} \quad z_0\to 0.$$

Therefore we can conclude that the change of the configuration of D8-branes by the back reaction is negligible. So we can set $g(\tau)=1$ and can solve Eq. (4.22) as

$$\hat{A}_0 = \left(\frac{1}{2}\frac{27\pi}{2\lambda}\right)^{-1}\int_{z_0}^{z}dz\frac{k^{-1/6}d}{\sqrt{Q_0^2k^{5/3}+d^2}}, \tag{4.25}$$

where $Q_0=Q_1|_{g=1}=1+\frac{3}{2}n_B\left(\frac{9\pi}{\lambda}\right)^2\overline{q}^2\left(k^{-1}+k^{1/3}\right)$ and $d\equiv 6\left(\frac{9\pi}{\lambda}\right)^2n_B\int_{z_0}^{z}dz\overline{q}^2$.

Chemical potential The CS term Eq. (4.2) plays a crucial role when we introduce the chemical potential. In the field theory side, the chemical potential for the baryon is introduced in the partition function as

$$Z[\mu_B]=\int D\Psi_BD\overline{\Psi_B}\cdots\exp\left[-S\left[\Psi_B,\overline{\Psi_B},\cdots\right]+\int dt\;\mu_BN_B\right],$$

where $\Psi_B,\overline{\Psi_B}$ are the fermionic baryon fields and $N_B\equiv\int d^3x\overline{\Psi_B}\gamma^0\Psi_B$ is the baryon number operator. Note that the CS term Eq. (4.2) is similar to the definition of baryon number Eq. (4.8). It implies that the quark chemical potential $\mu_q=\frac{\mu_B}{N_c}$ should be defined as

$$\mu_q=\hat{A}_0(\infty). \tag{4.26}$$

Note that this should be the quark chemical potential rather than the baryon chemical potential, because if we set $\hat{A}_0 = \mu_q$ the CS term Eq. (4.2) becomes

$$\begin{aligned} S_{CS} &= \mu_q \frac{N_c}{32\pi^2} \epsilon_{MNPQ} \int d^4x \int_0^\infty dz \, \mathrm{tr}\left(F_{MN} F_{PQ}\right) \\ &= \mu_q N_q, \end{aligned}$$

where $N_q \equiv N_B N_c$ is the number of the valence quarks.

4.1.4 Energy density and phase transition

We define the energy density of the D8-brane as

$$\epsilon\left(\mu_B, n_B, \rho\right) = \int dz \; L\left(\mu_B, n_B, \rho\right), \tag{4.27}$$

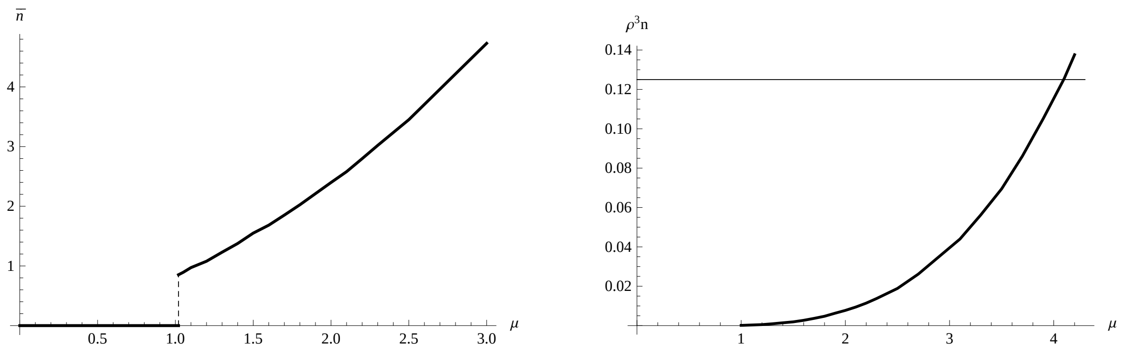
where $L\left(\mu_B, n_B, \rho\right)$ is the Lagrangian Eq. (4.21) with the solution Eq. (4.25) and the condition Eq. (4.26). It can be evaluated numerically once $U(1)$ gauge field $\hat{A}_0$ is solved as in Eq. (4.25). We should determine the parameters (n_B, ρ) by requiring that these minimize the energy density Eq. (4.27) with fixed μ_B. Because μ_B is the parameter which characterizes the field theory, we should not vary the chemical potential when we minimize the energy density. Then we get a set $\{(\mu, n(\mu), \rho(\mu))\}$ for each μ. We will call the minimum value of Eq. (4.27) for each μ as $\epsilon(\mu, n_B)$.

Then we compare the energy density $\epsilon(\mu, n_B)$ with $\epsilon(\mu, n_B = 0) \equiv \int dz k^{1/3}$ for various values of chemical potential μ to probe the phase structure of this finite baryon system. If $\epsilon(\mu, n_B) > \epsilon(\mu, 0)$, $n_B = 0$ phase will be realized. On the other hand, if $\epsilon(\mu, n_B) < \epsilon(\mu, 0)$, the phase with finite baryon density will be realized.

4.1.5 Results

We find a phase transition at $\mu_{cr} = 1.0176$ (See Fig. 4.1a and its caption). The phase transition is of first order and the value of the baryon number density jumps from zero up to a finite value at the transition point. Our model remains in the state with zero baryon number for the values of chemical potential smaller than the critical value. This behavior of the system with nonzero chemical potential also occurs in QCD with a finite chemical potential and known as the "silver braze problem" (See Ref.[35]).

The fact that a system with finite baryon number is stabler than a system with vanishing baryon number density implies that there may be attractive force between baryons, which has not been seen in Sec. 4.1.2. This contribution comes from the interaction between $U(1)$ gauge field inside the square root and $SU(2)$ gauge field in the Lagrangian Eq. (4.21). The analyses in Sec. 4.1.1 and Sec. 4.1.2 are performed by keeping only the second order of the $U(1)$ field strength and $SU(2)$ field strength. Thus the attractive force cannot be found in these analyses.



(a) The diagram of the relationship between $\mu \equiv \frac{1}{2}\frac{27\pi}{2\lambda}\mu_q$ and $\overline{n} = \frac{9\pi^2}{2}n$. The first-order phase transition, from vacuum to the nuclear matter phase at $\mu_{cr} = 1.0176$, are seen.

(b) The relation μ-$\rho^3 n$. The horizontal line at $\rho^3 n = \frac{1}{8}$ corresponds to a rough accommodation limit of the dilute gas approximation. The intersection point at about $\mu \sim 3.1(\mu_B \sim 6.91\text{GeV})$ denotes the useful limit of the model.

Figure 4.1: The numerical results for $\lambda = 16.6$, $M_{KK} = 0.949\text{GeV}$

Choosing parameters as $M_{KK} = 0.949\text{GeV}$ and $\lambda = 16.6$ which are same as [8, 24] (these values are determined by fitting the rho meson mass and the pion decay constant), we get

$$\mu_B \simeq 2.3\text{GeV}, \quad n_B \simeq 0.70\text{fm}^{-3}, \quad \rho \simeq 0.079\text{fm}. \tag{4.28}$$

This value of n_B is larger than the density of the normal nuclear matter, ~0.16fm^{-3}. It can be interpreted as the effect of a lack of the repulsive force between baryons. The value of baryon size ρ is slightly improved compare to the result of the one-baryon case Eq. (4.11), but it is still too small.

4.2 Application to neutron stars

We derive the relation between the baryon chemical potential μ_B and the baryon number density n_B in the previous section. Then it is natural to apply the n_B-μ_B relation to some baryonic system. In this section we will construct a compact star constructed by our baryonic matter. Since the baryon considered here is neutral, we may think that such a star as a neutron star. One of our motivations for constructing a neutron star is a test of our baryonic matter model. To compare our result with the real physical values is useful for noticing how we should improve the model.

To analyze a neutron star, we must solve the TOV equations,

$$\begin{aligned} \frac{dm}{dr} &= 4\pi r^2 \rho(r), \\ \frac{dp}{dr} &= -\frac{G\rho(r)\, m(r)}{r^2}\left[1+\frac{p(r)}{\rho(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{m(r)}\right]\left[1-\frac{2Gm(r)}{r}\right]^{-1}, \end{aligned} \tag{4.29}$$

where $m(r)$ is a mass inside the sphere of the radius r, $\rho(r)$ and $p(r)$ are a energy density and pressure at r respectively, and G is the newton constant. There are three unknown functions $m(r)$, $\rho(r)$ and $p(r)$, though there are only two equations Eq. (4.29). Thus, in solving the TOV equations we must know the relation of the energy density ρ and pressure p of the nuclear matter, namely, the EoS, to eliminate one of the unknown functions. So we should extract the information from our holographic model.

In general, ρ and p are holographically obtained by using the bulk metric according to holographic renormalization [36, 37]. In the present case, however, the bulk metric is independent of the soliton gas since the flavor branes are treated as the probe. An attempt to solve the TOV equations has been performed by using a holographic EoS obtained in a context of holographic framework [38], however it leads to a large radius.

Then, we propose an alternative way to get the relation between ρ and p. We require that our holographic baryonic matter model satisfies the ordinary statistical mechanics for a fermionic system with zero-temperature. According to the Luttinger's theorem for an interacting fermion system [39], the volume enclosed by a Fermi surface is directly proportional to the particle density. In general, the shape of the Fermi surface may be changed by the interaction. In this analysis, however, we assume that, instead of considering a change of the baryon's Fermi surface, we take into account the effect of the interaction among fermions by introducing a momentum dependent mass. This momentum dependence of the fermion mass reflects a non-trivial interaction among the fermions, and this momentum dependent mass is determined from our holographic model. After that, we obtain the EoS of our holographic model, and then the TOV equations are solved.

4.2.1 Momentum dependent mass and EoS

As mentioned above, the Luttinger theorem and our assumption imply that the baryon number density can be written in the term of the Fermi momentum as

$$n_B = \frac{gS_2}{3(2\pi)^3} k_F^3 = \frac{8\pi}{3(2\pi)^3} k_F^3, \tag{4.30}$$

where $g(=2)$ is a number of spin degrees of freedom of the baryon, S_2 is a volume of two-dimensional surface of a three-dimensional-sphere. In order to exploit this relation and our result $\mu_B = \mu_B(n_B)$, we introduce an effective baryon mass $m(k)$, which depends on the momentum k. This effective mass is interpreted as a reflection of the complicated interaction among nucleons. A similar concept is seen in the ordinary field theory approach [40]. Then it is possible to express μ_B as

$$\mu_B(k_F) = \sqrt{k_F^2 + m^2(k_F)}. \tag{4.31}$$

In this context, by extending the formula for the free fermion gas, we get the energy density ρ and pressure p of the nuclear matter as,

$$\rho\left(k_F\right) = \rho_c + \int_{k_c}^{k_F} dk k^2 \sqrt{k^2 + m^2\left(k\right)} \tag{4.32}$$

$$p\left(k_F\right) = p_c + \frac{1}{3}\int_{k_c}^{k_F} dk \frac{k^4}{\sqrt{k^2 + m^2\left(k\right)}} \tag{4.33}$$

where ρ_c and p_c denote the critical density and pressure respectively. Here we introduce the lower bound of the momentum, k_c, as

$$n_c = \frac{8\pi}{3\left(2\pi\right)^3} k_c^3,$$

since the nuclear matter considered here is defined for $n \geq n_c = 0.7\mathrm{fm}^{-3}$. This is our proposal to obtain the energy momentum tensor of the nuclear matter based on the holographic approach. Then we obtain the EoS, the relation between ρ and p, at any value of the density to solve the TOV equations.

4.2.2 Numerical Data of $\mu_B\left(n_B\right)$ and $m^2\left(k\right)$ and EoS

Now we obtain the momentum dependence of an effective nucleon mass $m(k)$ in nuclear matter phase from our (μ_B, n_B) data by using the Eqs. (4.30) and (4.31).

First we assume that the form of the function $m^2\left(n_B\right)$ as

$$m^2\left(n_B\right) = a n_B^{\alpha}. \tag{4.34}$$

We can use this function to fit the parameters a and α from the data of the relation between μ_B and n_B shown in Fig. 4.1a. Then we have $a = 8.54 \times 10^3$ and $\alpha = 1.44$ in the natural unit (mass unit is set to GeV). The fitted function Eq. (4.34) and the data points which used to fit the function is shown in Fig. 4.2. Because $n_B$depends on k_F via the relation Eq. (4.30), The mass Eq. (4.34) is also dependent on k_F as

$$m^2\left(k_F\right) = a\left(\frac{8\pi}{3\left(2\pi\right)^3} k_F^3\right)^{\alpha}$$

The effective nucleon mass $m(k_F)$ reflects the interaction of nucleons in the nuclear matter. Then the values of the Fermi momentum k_c and the mass m_0 at phase transition point are obtained as $k_c \sim 0.55\mathrm{GeV}$ and $m_0 \equiv m\left(k_c\right) \sim 2.23\mathrm{GeV}$.

Next, we calculate the Fermi momentum dependence of the energy density and the pressure from Eqs. (4.32) and (4.33). From the above relation of $m^2(k_F)$, we assume that

$$m^2\left(k\right) = a\left(\frac{8\pi}{3\left(2\pi\right)^3} k^3\right)^{\alpha}$$

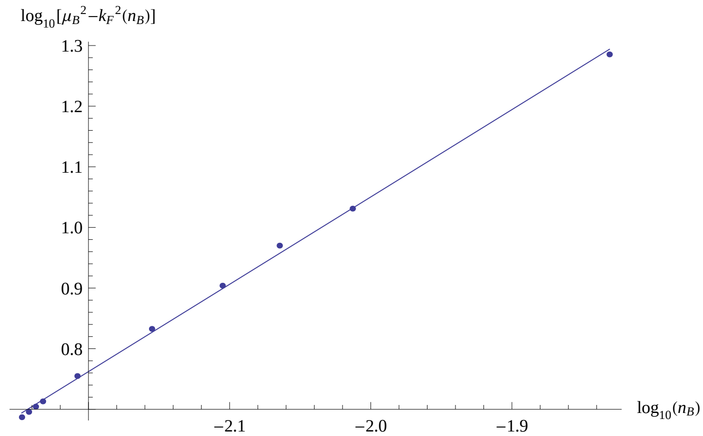


Figure 4.2: $\log\left(\mu_B^2 - k_F^2\right)$ v.s. $\log n_B$. Ten data points in the region $2.3\text{GeV} < \mu_B < 6.91\text{GeV}$ are used to fit the function $m^2\left(k_F\right) = \mu^2 - k_F^2$.

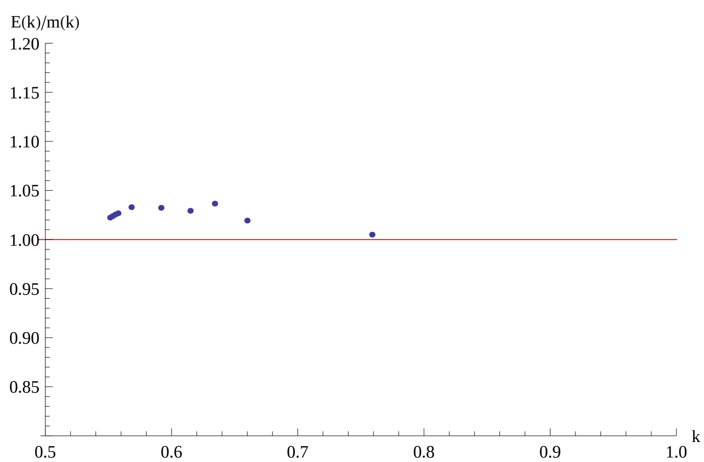


Figure 4.3: $\mu_B\left(n_B\left(k_F\right)\right)/m\left(k_F\right)$ v.s. k_F. The horizontal line shows unity. In $k > k_c \sim 0.55$, $E\left(k\right) = \sqrt{k^2 + m^2\left(k\right)}$ can be approximately replaced by $m\left(k\right)$.

in the energy function $\sqrt{k^2+m^2(k)}$. Moreover, since $2-3\alpha<0$ and

$$\begin{aligned}\sqrt{k^2+m^2(k)} &= m(k)\left(1+\mathcal{O}\left(\frac{k^2}{m^2(k)}\right)\right)\\ &= m(k)\left(1+\mathcal{O}\left(k^{2-3\alpha}\right)\right)\end{aligned}$$

and $\sqrt{k_c^2+m_0^2}\sim m_0$, we can approximate $\sqrt{k^2+m^2(k)}$ as $m(k)$ in the whole region of $k_c<k$. The ratio $\sqrt{k^2+m^2(k)}/m(k)$ is shown in Fig. 4.3. Then we can calculate the integrals in Eqs. (4.32) and (4.33), and have

$$\rho(k_F) = \rho_c + b_1\left(k_F^{3\alpha/2+3} - k_c^{3\alpha/2+3}\right), \tag{4.35}$$

$$p(k_F) = \rho_c + b_2\left(k_F^{5-3\alpha/2} - k_c^{5-3\alpha/2}\right), \tag{4.36}$$

where

$$b_1 = \frac{\sqrt{a\left(\frac{1}{3\pi^2}\right)^\alpha}}{\pi^2(3\alpha/2+3)}, \quad b_2 = \frac{1}{3\pi^2(5-3\alpha/2)\sqrt{a\left(\frac{1}{3\pi^2}\right)^\alpha}}.$$

From Eqs. (4.35) and (4.36) we obtain the relation between ρ and p, the EoS, as

$$\rho(p) = \rho_c + b_1\left(\left(\frac{1}{b_2}(p-p_c)+k_c^{5-3\alpha/2}\right)^{\frac{3\alpha/2+3}{5-3\alpha/2}} - k_c^{3\alpha/2+3}\right). \tag{4.37}$$

Parameters $a(=8.54\times10^3)$ and $\alpha(=1.44)$ lead to $b_1=0.158$ and $b_2=1.48\times10^{-3}$. Fig. 4.4 shows the relationship between ρ and p.

In the region of $p-p_c \gg b_2 k_c^{5-3\alpha/2}\sim 2.72\times10^{-4}$, the EoS Eq. (4.37) can be approximated as

$$p-p_c = \frac{b_2}{b_1^\gamma}(\rho-\rho_c)^\gamma.$$

The value of the index $\gamma(=\frac{5-3\alpha/2}{3\alpha/2+3}\sim 0.55<1)$ is in contrast to the value of that for many other kind of matters, which satisfy $\gamma>1$. For example, normal stars satisfy $\gamma=4/3$ and the ideal fermion gas satisfies $\gamma=5/3$. The origin of $\alpha\sim1.44$ is the effective nucleon mass $m(k_F)$. As the nucleon density increases, the effective nucleon mass increase in our holographic model.

4.2.3 Application to Neutron Star and solution of TOV Equations

Since we have the EoS as in Eq. (4.37), we may eliminate a unknown function, $\rho(r)$, in TOV equations, Eq. (4.29), and can solve them. Then TOV equations are

$$\begin{aligned}\frac{dm}{dr} &= 4\pi r^2\rho(p(r)),\\ \frac{dp}{dr} &= -\frac{G\rho(p(r))\,m(r)}{r^2}\left[1+\frac{p(r)}{\rho(p(r))}\right]\left[1+\frac{4\pi r^3 p(r)}{m(r)}\right]\left[1-\frac{2Gm(r)}{r}\right]^{-1},\end{aligned}$$

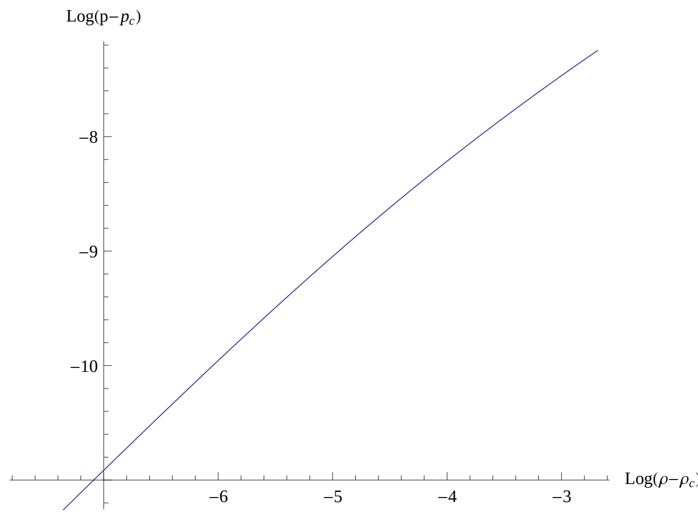


Figure 4.4: The relationship between ρ and p. It is approximated as $p - p_c = \text{const.} \times (\rho - \rho_c)^\gamma$ with $\gamma = 0.55$

where $\rho\,(p\,(r))$ is the function in Eq. (4.37). Then, we can estimate the radius and mass of neutron stars by specifying parameters ρ_c and p_c.

The most natural choice of the parameters ρ_c and p_c is

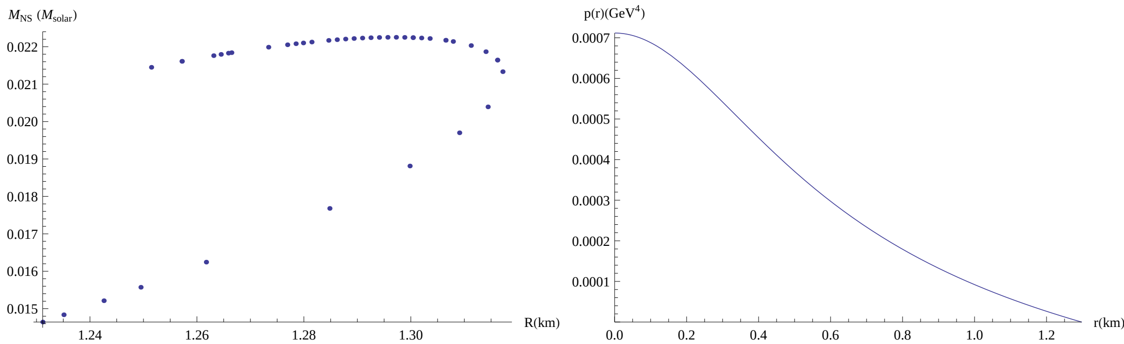
$$\rho_c = m_B n_B\,(k_c)\,, \quad p_c = 0, \tag{4.38}$$

where $m_B = m\,(k_c) \sim 2.3\text{GeV}$ and $n_B\,(k_c)$ are the effective mass and the baryon number density at the transition point $k = k_c$, respectively. This is natural because ρ_c and p_c represent energy density and a pressure at the transition point, respectively. We require $p_c = 0$ for simplicity. If we choose $p_c > 0$, we cannot solve the TOV equations by only using our EoS and we would need another assumption about the EoS in the region of $0 < p < p_c$. Such an additional assumption makes the point of the discussion obscure.Setting the parameters as in Eq. (4.38), we get the following values of our neutron star,

$$M \sim 2.22 \times 10^{-2} M_s, \quad R \sim 1.30\text{km}, \tag{4.39}$$

where M is the maximum mass of the neutron star, M_s is the solar mass and R is the radius of the neutron star when the mass of the neutron star is maximum. The relation between total mass M and the radius of the neutron star R, and the pressure $p\,(r)$ at the distance r from the center of the star with maximum mass are shown in Fig.4.5. These values of the results are very smaller than the real values of the maximum mass $M \sim \mathcal{O}\,(1) \times M_s$ and the radius $R \sim \mathcal{O}\,(10\text{km})$ of the neutron star.

The considerably different values in Eq. (4.39) about our neutron star imply that there are some contributions that we omitted. Since our neutron stars are too small (or too soft), we may infer that the missing contribution is some repulsive forces. In fact, we have dropped the contributions of repulsive force which exist in the case of two-baryon system when we



(a) the total mass M v.s. the radius R

(b) the pressure $p(r)$ v.s. the distance r from the center of the star.

Figure 4.5: The relationship between physical quantities of the neutron star.

apply the dilute gas approximation to our baryonic system in Sec.4.1.3. As a result, the baryon number density becomes rather large and the neutron stars become very small. This point will be improved by taking into account the repulsive force in Eqs. (4.15), (4.16).

4.3 Summary:Holographic cold nuclear matter

In this section, we consider the finite baryon density system in the Sakai-Sugimoto model and applying it to the neutron star. In the model, baryons are given by the corresponding solitons of $SU(N_f)$ gauge field on D8-branes.

First we construct the system with finite baryon number density in Sec. 4.1. We review one-baryon and two-baryon system and confirm that two-baryon system can be regarded as the sum of the two one-baryon system in the limit such that each baryon size is much smaller than the separation between baryons. Then we construct the finite baryon density system by using dilute gas approximation of baryons and mean field approximation of the $U(1)$ gauge field $A_0(x^\mu, z)$. The z dependent $U(1)$ gauge field $A_0(z)$ is dynamically solved and the boundary value of the field defines the quark chemical potential $\mu_q = \mu_B/N_c$. By comparing the energy density of the finite baryon density system and that of the zero baryon density system with fixed μ_B, we obtain the relation between the chemical potential and the baryon density which shows the first-order phase transition of the baryon number density at $\mu_B \simeq 2.3$GeV. At the critical point, the baryon number density and size of the baryon are calculated as $n_B \simeq 0.70\text{fm}^{-3}$ and $\rho \simeq 0.079$fm, respectively. The stable finite baryon system implies that the existence of the attractive force which have not been seen in two-baryon system[26]. Moreover we retain the some expected features of the n_B-μ_B diagram, that is, "silver blaze problem" and "first-order phase transition". On the other hand, our baryon number density is slightly larger than the normal nuclear density and the baryon size is very smaller than the real size of the baryons ~ 1fm, though it is slightly improved from the result of Ref. [24]. These values imply that our finite baryon system is too soft and the repulsive force is needed to improve these result.

Secondly, we apply our baryonic system to the construction of the neutron star in order to check the validity in Sec. 4.2. We regard our baryonic matter as the matter of the neutron since they have no electric charges. We introduce the Fermi momentum and the momentum dependent mass $m(k)$ to reconstruct the energy density ρ and the pressure p of our neutron system. Fitting $m(k)$ and using some stochastic relations, we find the relation between ρ and p, namely EoS, and one of the unknown functions in TOV equations is eliminated. Since we have first-order phase transition, we must introduce two free parameters, p_c and ρ_c, which are the pressure and the energy density at the critical point. We set these values as $p_c = 0,\ \rho_c = m(k_c)\, n_B(k_c)$ for our purpose. Then we solve the TOV equations and physical quantities of neutron stars are studied. The resultant mass and the radius, however, are very small compare to the familiar values of the quantities. Thus this results also imply the absence of repulsive force. In our setup, baryons have been introduced as dilute gas of solitons and the weak attractive force which is needed to form the stable nuclear matter is realized via the Dirac-Born-Infeld action in the curved background. However, the repulsive interaction which has been seen in the two baryon system [26] is absent, because we have neglected the overlap among solitons in Eqs. (4.18) and (4.19). Because of the absence of repulsive forces, the mass and radius of neutron stars we obtained here are much smaller than the empirical values.

Chapter 5

Discussions

In this thesis, we discuss two awkward topics of field theory by using gauge/gravity duality.

Holographic Unruh effect We consider a quark accelerated with a uniform acceleration a in the non-conformal gauge field theory in Chap. 3. We apply the ERT to the dAdS model to move to the Rindler frame in which the quark is static. Applying the gauge/gravity duality to the Rindler-dAdS model, we find the temperature which agree with the Unruh temperature and an anisotropy of the effects of the temperature.

Our analyses are based on two assumptions. First, we assume that we may apply the gauge/gravity duality to the coordinate transformed system. Second, we assume that the usual four dimensional Rindler transformation, Eq. (E.3), corresponds the ERT, Eq. (3.7), in terms of the gauge/gravity correspondence. The former assumption cannot be avoided when we use the gauge/gravity duality as a tool of analyses. The latter assumption may be improved, perhaps.

The ERT is introduced by Xiao in Ref. [14] to analyze an accelerated quark in the context of the AdS/CFT correspondence. We employ the ERT in the dAdS model in Chap. 3 as it is. One think that it is inattentive and the ERT should be changed. Possibly it may be true. However I think that using the ERT as it is is plausible. Since the Unruh temperature is independent of the detail of the field theory (See Appendix E.1 and Refs. [41], [42], [2]), it is expected that the procedure of calculating the Unruh temperature by using the gauge/gravity duality is also independent of the detail of the model. So if the ERT corresponds to the usual four dimensional Rindler transformation in the AdS/CFT correspondence, it may be true in other gauge/gravity duality models. Of course, there might be other ways we obtain the correct Unruh temperature. It is interesting to look for such methods.

The anisotropy of the effect of the temperature might be found even if we use any other transformation instead of the ERT, since the transformation must agree with the usual four dimensional Rindler transformation, which is anisotropic, on the boundary. Thus, in terms of the gauge/gravity duality, it is natural that the Unruh effect is anisotropic. However the behavior of the effect will be changed if we choose any other transformation instead of the ERT.

It is interesting that looking for the correct transformation which corresponds the usual four dimensional Rindler transformation in terms of the gauge/gravity duality. One of ways to accomplish the work is tuning the effect of the coordinate transformation in terms of the gauge/gravity duality to the Unruh effect of the corresponding theory.

Holographic cold nuclear matter We consider the system with finite baryon density by using the Sakai-Sugimoto model in Chap. 4. Applying the dilute gas approximation to the $SU(2)$ gauge field and the mean field approximation to the $U(1)$ gauge field, we find the relation between the baryon chemical potential μ_B and the baryon number density n_B; a first-order phase transition from $n_B = 0$ phase to finite n_B phase. The critical value of the baryon number density is considerably larger than the value of the normal nuclear density. We also consider neutron stars and calculate the upper bound of the mass and the radius of neutron stars. However these values are much smaller than the empirical values.

These results imply that the absence of the repulsive force which is omitted in the dilute gas approximation. The consideration of the contribution of the repulsive force must be needed to improve our finite baryon density system. However the tractability of our system relies on the simple form of the $SU(N_f)$ soliton solution by our approximation. So it is very difficult to add the contribution of the repulsive force even if we restrict our consideration only to the contributions from two-baryon interaction Eqs. (4.13) and (4.14). One of the ways of including these contributions into our model is simply adding these potentials to the energy density Eq. (4.27) as

$$\tilde{\epsilon} \equiv \epsilon(\mu, n_B, \rho) + \frac{Z n_B}{2}\left(H_{pot}^{(SU(2))}(|r|, \rho) + H_{pot}^{(U(1))}(|r|, \rho)\right),$$

where

$$\begin{aligned} H_{pot}^{(SU(2))}(|r(n_B)|, \rho) &= \frac{4\pi^2 a N_c}{3}\rho^4 \frac{c_1}{|r(n_B)|^2}, \\ H_{pot}^{(U(1))}(|r(n_B)|, \rho) &= \frac{N_c}{8\pi^2 a}\frac{1}{|r(n_B)|^2}\left[\frac{1}{2} + 2\frac{2c_2 - 1}{5}\right], \end{aligned}$$

with $|r(n_B)| = \left(1/\left(\frac{4}{3}\pi n_B\right)\right)^{1/3}$ and c_1, c_2 is the averaged values of Eq. (4.17) and Z is the number of the nearest neighbor baryons. We leave the analysis of this direction as the future work and do not discuss anymore.

Finally I want to discuss a way of deepening the understanding of gauge/gravity duality. Currently, gauge/gravity duality is used as a tool for analyzing theories and for giving predictions for the physical quantities which are difficult to access in other methods. However, in fact, we have a trouble when we use the tool for investigating new physics. Gauge/gravity duality is surely useful for the calculations in strongly interacting QFT, but we do not know why the duality actually works, how we generalize the duality, and how we identify a quantity of a gauge theory with a quantity of the dual gravitational theory. Moreover we do not know how far we can extend the AdS/CFT correspondence. Thus we should study gauge/gravity duality itself, while the application is also important. To do this, it is useful that the applications of gauge/gravity duality to well-known physics in order to make clear how it works. The Unruh effect is one of such topics to apply the duality for the purpose of advancing our understanding of duality, because the phenomenon is very universal among various field theories [2] and it contains the non-trivial coordinate transformation. It is very interesting for us to investigate the relation between the 4-dimensional ordinary Rindler transformation and 5-dimensional extended Rindler transformation not restricted to Eq. (3.7) in terms of gauge/gravity duality. I believe that deeper understanding of such interesting and special applications must become a clue to advance our understanding of general features of gauge/gravity duality.

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Appendix A

Review:Superstring theory

Since the gauge/gravity duality is based on the superstring theory, analyses with the duality need the knowledge of the superstring theory.

String theory was originally constructed as the theory of hadrons in early 1970s. The theory can explain some stringy feature of hadrons like Regge trajectory and linear rising potential between a quark and an anti-quark. But, the theory is well-defined only in 26 dimensional spacetime and contains tachyons which unstabilize the theory. In addition, it also has massless spin 1 and 2 particles which do not exist in actuality. So this theory is not appropriate for describing hadrons. Once the massless spin 1 particles are regarded as gauge particles and the massless spin 2 particles as gravitons, the string theory with supersymmetry, the superstring theory, has become a good candidate of the ultimate theory of Nature.

The fundamental objects of string theories are fundamental superstring (F-string) and D-branes. The superstring is a 2-dimensional object. A string may have one of two different forms:an open string or a closed string. An open string contains gauge particles, while a closed string has gravitons. Open strings must be ended on D-branes. D-branes are $p+1$-dimensional objects and they contains a gauge theory on their action. The characters in the superstring theory is displayed in Fig. A.1.

A.1 Superstring

A.1.1 Action

The string is a one-dimensional generalization of the relativistic particle to a two-dimensional object. The action is given by Nambu and Goto as

$$S_{NG} = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-\det\left(\frac{\partial X^{\mu}}{\partial \sigma^{a}}\frac{\partial X_{\mu}}{\partial \sigma^{b}}\right)}\ , \tag{A.1}$$

where $\sigma^0 = \tau,\ \sigma^1 = \sigma$ and $\mu = 0, \cdots, d = D-1,\ a, b = 0, 1$. A point on the string is specified by a point on the world sheet (τ, σ), and $X^{\mu}(\tau, \sigma)$ is the function which maps the point

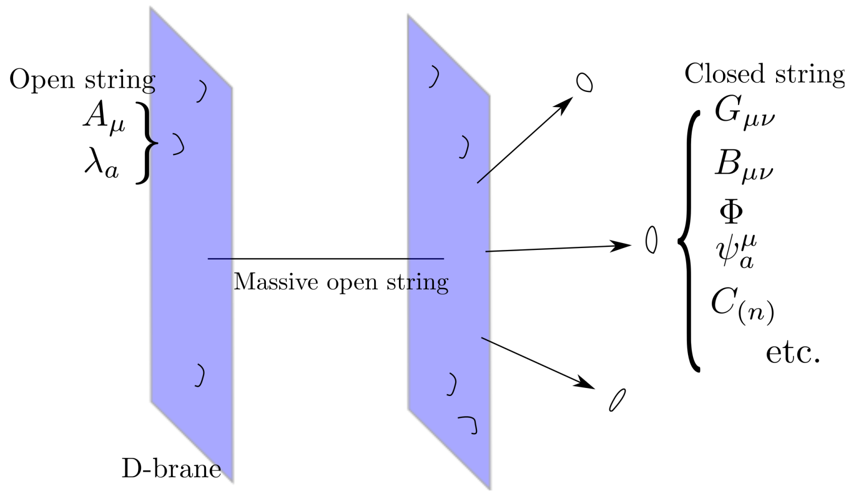


Figure A.1: Characters in superstring theory. Open strings whose ends end on the same position have massless states, namely the state corresponds to gauge field A_μ and the state which corresponds to the gaugino λ_a, while open strings whose ends end on different positions become massive. The states of open strings are localized on the D-branes. On the other hand, closed strings can propagate in the whole region of spacetime.

on τ-σ space to the point on $d+1$-dimensional spacetime. The action is a straightforward generalization of that for a relativistic particle because both of them are represented as the integration of invariant volume. But it is difficult to treat the Nambu-Goto (NG) action because fields X^μ are in the square root. So the Polyakov action S_P is often used instead of the NG action,

$$S_P = -\frac{1}{4\pi\alpha'}\int d\tau d\sigma \sqrt{-\gamma}\gamma^{ab}\partial_a X^\mu \partial_b X_\mu \ , \tag{A.2}$$

where $\gamma_{ab}(\tau,\sigma)$ is a metric on the world-sheet, independent of X^μ. The equivalence of two actions can be checked by eliminating γ_{ab} from Eq. (A.2) by using the equations of motion.

The Polyakov action is invariant under the following three transformations;

- Poincaré transformations:

$$\delta X^\mu = a^\mu{}_\nu X^\nu + b^\mu, \quad \delta\gamma^{ab} = 0.$$

- Reparametrizations:

$$\sigma^a \to f^a(\tau,\sigma) = \sigma'^a, \quad \gamma_{ab}(\tau,\sigma) = \frac{\partial f^c}{\partial \sigma^a}\frac{\partial f^d}{\partial \sigma^b}\gamma'_{cd}(\tau',\sigma')$$

- Weyl transformations:

$$\gamma_{ab} \to \gamma'_{ab} = e^{2\omega(\tau,\sigma)}\gamma_{ab}, \quad \delta X = 0$$

Here reparametrizations and Weyl transformations are local symmetries. These local symmetries must be fixed by choosing a gauge-fixing condition. Two of the three components of γ_{ab} are fixed by the reparametrization invariance, while the remaining one is fixed by the Weyl invariance. Then γ_{ab} has only one component, but this remaining component also can be gauged away by using the Weyl transformations. So we can choose γ_{ab} as

$$\gamma_{ab} = \eta_{ab} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{A.3}$$

Even after choosing this gauge, there is a residual gauge symmetry. In the light-cone coordinate

$$\begin{aligned} \sigma^{\pm} &= \tau \pm \sigma,\ \partial_{\pm} \equiv \frac{\partial}{\partial \sigma^{\pm}} = \frac{1}{2}\partial_\tau \pm \frac{1}{2}\partial_\sigma,\ \eta^{LC}_{ab} = \begin{pmatrix} \eta^{LC}_{--} & \eta^{LC}_{-+} \\ \eta^{LC}_{+-} & \eta^{LC}_{++} \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix}, \\ ds^2 &= \eta_{ab} d\sigma^a d\sigma^b = 2\eta^{LC}_{+-} d\sigma^+ d\sigma^- \end{aligned} \tag{A.4}$$

we can easily see from (A.4) that successive transformations,

$$\begin{cases} \sigma^{\pm} \to \sigma'^{\pm}(\sigma^{\pm}) & \text{(reparametrization)} \\ \eta^{LC}_{ab} \to \frac{\partial\sigma^+}{\partial\sigma'^+}\frac{\partial\sigma^-}{\partial\sigma'^-}\eta^{LC}_{ab} & \text{(Weyl transformation)} \end{cases}, \tag{A.5}$$

make the metric η^{LC} invariant.

Once we fix the gauge as in Eq. (A.3), we can write the bosonic string action as

$$S = -\frac{1}{4\pi\alpha'} \int d\tau d\sigma \partial_a X_\mu \partial^a X^\mu \tag{A.6}$$

Adding the fermionic action to the gauge fixed bosonic action Eq. (A.6), we get a supersymmetric string action

$$S = -\frac{1}{4\pi\alpha'} \int d\tau d\sigma \left(\partial_a X_\mu \partial^a X^\mu + \overline{\psi}^\mu \rho^a \partial_a \psi_\mu \right), \tag{A.7}$$

where ρ^a, with $a = 0, 1$, represent the two-dimensional Dirac matrices, which obey the Dirac algebra

$$\left\{\rho^a, \rho^b\right\} = 2\eta^{ab}.$$

We will use an explicit form of these matrices,

$$\rho^0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \rho^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

ψ^μ $(\mu = 0, 1, \ldots, D-1)$are D Majorana fermions which satisfy the Majorana condition

$$\psi^c \equiv C\overline{\psi}^T = \psi \equiv \begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix},$$

where $\overline{\psi}^{\mu} \equiv \psi^{\dagger} i\rho^0$ and $C = i\rho^0$ is a charge conjugation matrix which satisfies $\rho^{aT} = -C^{-1}\rho^a C$. In our notation, the Majorana condition means that the spinors are real,

$$\psi^{\mu *} = \psi^{\mu}. \tag{A.8}$$

We will sometimes suppress the Lorentz index μ from now on for the notational simplicity. Then the Majorana condition Eq. (A.8) is written as $\psi^* = \psi$.

The action is invariant up to total derivative term under the infinitesimal supersymmetric transformation

$$\begin{aligned}
\delta X^{\mu} &= \bar{\epsilon}\psi^{\mu} \\
\delta\psi^{\mu} &= \rho^a \partial_a X^{\mu} \epsilon
\end{aligned}$$

where ϵ is a infinitesimal constant Majorana spinor.

A.1.2 Equations of motion, boundary conditions and solutions

The equations of motion of the action (A.7) is written as

$$\partial_+ \partial_- X = 0, \tag{A.9}$$

$$\partial_+ \psi_- = 0, \quad \partial_- \psi_+ = 0, \tag{A.10}$$

where $\partial_{\pm} \equiv \frac{\partial}{\partial \sigma^{\pm}}$ and $\sigma^{\pm} \equiv \tau \pm \sigma$. Since the string has a finite interval of σ, we also have boundary conditions

$$\partial_\sigma X \delta X|_{\sigma=\pi} - \partial_\sigma X \delta X|_{\sigma=0} = 0, \tag{A.11}$$

$$(\psi_+ \delta\psi_+ - \psi_- \delta\psi_-)|_{\sigma=\pi} - (\psi_+ \delta\psi_+ - \psi_- \delta\psi_-)|_{\sigma=0} = 0. \tag{A.12}$$

So Eqs. (A.9) and (A.10) are just free wave equations, we can solve them with appropriate boundary conditions and the choice of the boundary conditions which satisfy Eqs. (A.11) and (A.12) gives some features, open or closed and NS or R, to the string.

Open strings One of the types of strings is the open string. In this case, $\sigma = \pi$ term and $\sigma = 0$ term in Eqs. (A.11) and (A.12) vanish respectively and then conditions reduce to

$$\begin{aligned}
\partial_\sigma X|_{\sigma=\pi} \text{ or } \delta X|_{\sigma=\pi} &= 0 \quad \text{and} \quad \partial_\sigma X|_{\sigma=0} \text{ or } \delta X|_{\sigma=\pi} = 0, \\
\psi_+|_{\sigma=0} &= \psi_-|_{\sigma=0}, \quad \psi_+|_{\sigma=\pi} = \pm\, \psi_-|_{\sigma=\pi}.
\end{aligned} \tag{A.13}$$

Since one relative sign can be absorbed in the definition of the field ψ, we can choose the definition such as Eq. (A.13). The remaining relative sign at $\sigma = \pi$ determines the spin-statics of the states of the string.

Boundary conditions for bosonic fields X at one end are of two types. One of these is the Neumann boundary condition,

$$\partial_\sigma X|_{\sigma=0 \text{ or } \pi} = 0,$$

and the other is the Dirichlet boundary condition,

$$X(\sigma=0) = X_0 \quad \text{or} \quad X(\sigma=\pi) = X_\pi.$$

Actually, these conditions are linked with the D-branes on which the string ends.The general solution of the equation of motion for an open string with the Neumann boundary conditions on both ends is given by

$$X^\mu(\tau,\sigma) = x^\mu + l_s^2 p^\mu \tau + i l_s \sum_{m\neq 0} \frac{1}{m}\alpha_m^\mu e^{-im\tau}\cos(m\sigma), \tag{A.14}$$

and the one with the Dirichlet boundary conditions on both ends is given by

$$X^\mu(\tau,\sigma) = \left(1-\frac{\sigma}{\pi}\right)X_0 + X_\pi\frac{\sigma}{\pi} + i l_s \sum_{m\neq 0}\frac{1}{m}\alpha_m^\mu e^{-im\tau}\sin(m\sigma). \tag{A.15}$$

We denote $\alpha_0^\mu = l_s p^\mu$ for the open string.

Next we consider boundary conditions for fermionic fields ψ. One of these is the Ramond condition (R)

$$\psi_+|_{\sigma=\pi} = \psi_-|_{\sigma=\pi}, \tag{A.16}$$

and the other condition is the Neveu-Schwarz boundary condition (NS)

$$\psi_+|_{\sigma=\pi} = -\,\psi_-|_{\sigma=\pi}. \tag{A.17}$$

The general solution in the R sector is

$$\begin{aligned}\psi_-^\mu(\tau,\sigma) &= \frac{1}{\sqrt{2}}\sum_{n\in\mathbb{Z}} d_{-,n}^\mu e^{-in(\tau-\sigma)},\\ \psi_+^\mu(\tau,\sigma) &= \frac{1}{\sqrt{2}}\sum_{n\in\mathbb{Z}} d_{+,n}^\mu e^{-in(\tau+\sigma)},\end{aligned}$$

and that in the NS sector is

$$\begin{aligned}\psi_-^\mu(\tau,\sigma) &= \frac{1}{\sqrt{2}}\sum_{r\in\mathbb{Z}+1/2} b_{-,r}^\mu e^{-ir(\tau-\sigma)},\\ \psi_+^\mu(\tau,\sigma) &= \frac{1}{\sqrt{2}}\sum_{r\in\mathbb{Z}+1/2} b_{+,r}^\mu e^{-ir(\tau+\sigma)}.\end{aligned}$$

Closed strings The other type of the string is the closed string. The boundary condition is written by

$$\begin{aligned} X(\sigma) &= X(\sigma+\pi), \\ \psi_{\pm}(\sigma) &= \psi_{\pm}(\sigma+\pi) \quad \text{or} \quad \psi_{\pm}(\sigma) = -\psi_{\pm}(\sigma). \end{aligned}$$

These conditions satisfy Eqs. (A.11) and (A.12). In this case, $X(\tau,\sigma) = X_{-}(\tau-\sigma) + X_{+}(\tau+\sigma)$ is written by

$$\begin{aligned} X^{\mu}_{-}(\tau-\sigma) &= \frac{1}{2}x^{\mu} + \frac{1}{2}l_s^2 p^{\mu}(\tau-\sigma) + \frac{i}{2}l_s \sum_{n\neq 0}\frac{1}{n}\alpha^{\mu}_{-,n}e^{-2in(\tau-\sigma)}, \\ X^{\mu}_{+}(\tau+\sigma) &= \frac{1}{2}x^{\mu} + \frac{1}{2}l_s^2 p^{\mu}(\tau+\sigma) + \frac{i}{2}l_s \sum_{n\neq 0}\frac{1}{n}\alpha^{\mu}_{+,n}e^{-2in(\tau+\sigma)}. \end{aligned}$$

We denote $\alpha^{\mu}_0 = \frac{1}{2}l_s p^{\mu}$ for the closed string.

Boundary conditions for fermionic fields are two types again, and we call the periodic condition $\psi_{\pm}(\sigma) = \psi_{\pm}(\sigma+\pi)$ "R" and call the anti-periodic condition $\psi_{\pm}(\sigma) = -\psi_{\pm}(\sigma+\pi)$ "NS". In the R (periodic) sector, the general solution of the fermionic field is written by

$$\begin{aligned} \psi_{-}(\tau,\sigma) &= \sum_{n\in\mathbb{Z}} d^{\mu}_{-,n}e^{-2in(\tau-\sigma)}, \\ \psi_{+}(\tau,\sigma) &= \sum_{n\in\mathbb{Z}} d^{\mu}_{+,n}e^{-2in(\tau+\sigma)}, \end{aligned}$$

and in the NS (anti-periodic) sector,

$$\begin{aligned} \psi_{-}(\tau,\sigma) &= \sum_{r\in\mathbb{Z}+1/2} b^{\mu}_{-,r}e^{-2ir(\tau-\sigma)}, \\ \psi_{+}(\tau,\sigma) &= \sum_{r\in\mathbb{Z}+1/2} b^{\mu}_{+,r}e^{-2ir(\tau+\sigma)}. \end{aligned}$$

A.1.3 Quantization

To perform canonical quantization, we impose the (anti) commutation relations on the modes in the Fourier expansion of the fields,

$$[\alpha^{\mu}_m, \alpha^{\nu}_n] = m\delta_{m+n,0}\eta^{\mu\nu}, \tag{A.18}$$

$$\{b^{\mu}_r, b^{\nu}_s\} = \eta^{\mu\nu}\delta_{r+s,0}, \quad \{d^{\mu}_m, b^{\nu}_n\} = \eta^{\mu\nu}\delta_{m+n,0}, \tag{A.19}$$

for the open string. For the closed string, there are two copies of these relations, which correspond to two different type of modes, α_{-}, b_{-}, d_{-} and α_{+}, b_{+}, d_{+}.

The Weyl invariance of the action, Eq. (A.2), requires that the trace of energy-momentum tensor is zero, $\gamma^{ab}T_{ab} \sim \frac{1}{\sqrt{-\gamma}}\gamma^{ab}\frac{\delta S}{\delta\gamma_{ab}} = 0$. On the other hand, the equation of motion for the metric requires that all components of the energy-momentum tensor are zero classically, $T_{ab} = 0$. So then two components of the energy-momentum tensor do not vanish and they are zero classically. In superstring theory, we can generalize this argument, and we have two non-vanishing components of the energy-momentum tensor T_{ab} and two non-vanishing components of the supercurrent J,

$$\begin{aligned} T_{++} &= \partial_+ X_\mu \partial_+ X^\mu + \frac{i}{2}\psi_+^\mu \partial_+ \psi_{+\mu}, \\ T_{--} &= \partial_- X_\mu \partial_- X^\mu + \frac{i}{2}\psi_-^\mu \partial_- \psi_{-\mu}, \\ J_+ &= \psi_+^\mu \partial_+ X_\mu, \quad J_- = \psi_-^\mu \partial_- X_\mu, \end{aligned}$$

and all of them are zero classically. Their Fourier modes are called the super-Virasoro generator,

$$L_m = \frac{1}{\pi}\int_{-\pi}^{\pi} d\sigma e^{im\sigma} T_{++} = \frac{1}{2}\sum_{n\in\mathbb{Z}} :\alpha_{-n}\cdot\alpha_{m+n}: + L_m^{(f)} \quad m \in \mathbb{Z},$$

and in R sector,

$$\begin{aligned} L_m^{(f)} &= \frac{1}{2}\sum_{n\in\mathbb{Z}}\left(n + \frac{m}{2}\right) : d_{-n}\cdot d_{m+n} : , \\ F_m &= \frac{\sqrt{2}}{\pi}\int_{-\pi}^{\pi} d\sigma e^{im\sigma} J_+ = \sum_{n\in\mathbb{Z}} \alpha_{-n}\cdot d_{m+n} \quad m \in \mathbb{Z}, \end{aligned}$$

or in NS sector,

$$\begin{aligned} L_m^{(f)} &= \frac{1}{2}\sum_{r\in\mathbb{Z}+1/2}\left(r + \frac{m}{2}\right) : b_{-r}\cdot b_{m+r} : , \\ G_r &= \frac{\sqrt{2}}{\pi}\int_{-\pi}^{\pi} d\sigma e^{ir\sigma} J_+ = \sum_{n\in\mathbb{Z}} \alpha_{-n}\cdot b_{r+n} \quad r \in \mathbb{Z} + \frac{1}{2}, \end{aligned}$$

for the open string. The physical state $|\phi\rangle$ must satisfy the condition $\langle\phi| T_{++} |\phi\rangle = 0$, and the condition may be reduced to the conditions, $(L_0 - a)|\phi\rangle = 0$ and $L_m|\phi\rangle = 0$ $(m > 0)$. The former of these conditions for physical state is regard as the mass shell condition. This condition can be rewritten as

$$\alpha' M^2 = N^{(b)} + N^{(f)} - a, \tag{A.20}$$

where $M^2 \equiv -p^2$ and

$$N^{(b)} = \sum_{n=1}^{\infty}\left(-\alpha^-_{-n}\alpha^+_n - \alpha^+_{-n}\alpha^-_n + \sum_{i=1}^{8}\alpha^i_{-n}\alpha^i_n\right), \tag{A.21}$$

$$N^{(f)} = \begin{cases} \sum_{n=1}^{\infty} n\left(-d^-_{-n}d^+_n - d^+_{-n}d^-_n + \sum_{i=1}^{8} d^i_{-n}d^i_n\right) & \text{(R sector)} \\ \sum_{r=1/2}^{\infty} r\left(-b^-_{-r}b^+_r - b^+_{-r}b^-_r + \sum_{i=1}^{8} b^i_{-r}b^i_r\right) & \text{(NS sector)} \end{cases}, \tag{A.22}$$

$$a = \begin{cases} 0 & \text{(R sector)} \\ \frac{1}{2} & \text{(NS sector)} \end{cases}. \tag{A.23}$$

For the purpose of seeing mass spectrum, it is easy that we quantize the string in light cone gauge. The light cone coordinates for spacetime is defined as

$$X^{\pm} = \frac{1}{\sqrt{2}}\left(X^0 \pm X^9\right), \quad \psi^{\pm} = \frac{1}{\sqrt{2}}\left(\psi^0 \pm \psi^9\right).$$

Since there is the residual symmetry corresponds to Eq. (A.5), we can perform the gauge fixing as

$$\alpha^+_{n\neq 0} = 0,\ b^+_r = 0,\ d^+_{n\neq 0} = 0.$$

Then we can solve $T_{++} = T_{--} = J_+ = J_- = 0$ for minus modes $\alpha^-,\ d^-,\ b^-$ and the dynamics of string is described by only eight transverse modes $\alpha^i,\ d^i,\ b^i\ (i = 1, \cdots 8)$. In this gauge, Eqs. (A.21) and (A.22) are written as

$$N^{(b)} = \sum_{n=1}^{\infty}\sum_{i=1}^{8}\alpha^i_{-n}\alpha^i_n,$$

$$N^{(f)} = \begin{cases} \sum_{n=1}^{\infty}\sum_{i=1}^{8} n d^i_{-n}d^i_n & \text{(R sector)} \\ \sum_{r=1/2}^{\infty}\sum_{i=1}^{8} r b^i_{-r}b^i_r & \text{(NS sector)} \end{cases}.$$

The ground state is defined as

$$\alpha^i_n \left|0;k;\pm\right\rangle_R = d^i_n \left|0;k;\pm\right\rangle_R = 0 \quad \text{for} \quad n > 0$$

for R sector and

$$\alpha^i_n \left|0;k\right\rangle_{NS} = b^i_r \left|0;k\right\rangle_{NS} = 0 \quad \text{for} \quad n, r > 0$$

for NS sector, where k is a total momentum of the open string. Since $[M^2, d^i_0] = 0$, the ground state for R sector is degenerated. Eq. (A.19) implies that d^i_0 behaves as eight dimensional gamma matrices, $d^i_0 \sim \Gamma^i$. So the vacuum for R-sector should be represented as a (Euclidean) eight dimensional Majorana-Weyl spinor. So it has eight real components.

To make the quantum theory consistent, we must perform the GSO projection. The G-parity is defined as

$$G = \begin{cases} \Gamma\,(-)^{\sum_{n=1}^{\infty} d_{-n}\cdot d_n} & \text{(R sector)} \\ (-)^{\sum_{r=1/2}^{\infty} b_{-n}\cdot b_n + 1} & \text{(NS sector)} \end{cases},$$

where Γ is the ten-dimensional analog of the Dirac matrix γ_5 in four dimensions. It satisfies $\Gamma\,|0;k;\pm\rangle_R = \pm\,|0;k;\pm\rangle_R$. In the NS sector, we truncate the G-parity odd states. On the other hand, in the R sector we can project on states with negative or positive G-parity.

The lowest mass state After the GSO projection, the lowest mass states are massless states. For the open string, the states are given by

$$b^i_{-1/2}\,|0;k\rangle_{NS}\ \ (\text{NS sector})\,,\qquad |0;k;+\rangle_R\ \text{ or }\ |0;k;-\rangle_R\ \ (\text{R sector})\,.$$

In the NS sector there are eight states ($i = 1, \cdots, 8$) and in R sector there are also eight states. The states in NS sector represent a massless spacetime vector field, namely gauge field, because it has suffix i. On the other hand, the states in R sector represent a massless spacetime fermion because the states $|0;k;\pm\rangle_R$ is 8-dimensional spinor.

For the closed string, the states are represented as the direct product of two copies of open string states. However, since the closed string share the total momentum between the left-moving modes and the right-moving modes, the level matching condition $N^{(+)} = N^{(-)}$ are imposed on each state. In addition, there are two distinct theories depending on chiralities of the ground states of two R sectors. The theory which has two same gravitinos is called "type IIB theory", while the theory which has two distinct gravitinos is called "type IIA theory". Then massless states of closed string are as follows.

- Type IIB theory

$$\begin{array}{rcl}
|+\rangle_R & \otimes & |+\rangle_R \\
b^i_{-,-1/2}\,|0\rangle_{NS} & \otimes & |+\rangle_R \\
|+\rangle_R & \otimes & b^i_{+,-1/2}\,|0\rangle_{NS} \\
b^i_{-,-1/2}\,|0\rangle_{NS} & \otimes & b^j_{+,-1/2}\,|0\rangle_{NS}
\end{array}$$

  - NS-NS sector: This sector contains the dilaton Φ (1 state), the graviton G_{ij} (35 states) and the antisymmetric two-form gauge field B_{ij} (28 states).
  - NS-R and R-NS sector:Each of these sectors contains the dilationo $\tilde{\phi}_a$ (8 states) and the gravitino ψ^i_a (56 states). The two gravitinos have the same chirality.
  - R-R sector:This sector contains a zero-form C (1 state), a two-form gauge field C_{ij} (28 states), a four-form gauge field C_{ijkl} with a self dual five-form gauge strength $*F_5 = F_5$ (35 states)

- type IIA theory

$$\begin{array}{rcl}
|-\rangle_R & \otimes & |+\rangle_R \\
b^i_{-,-1/2}\,|0\rangle_{NS} & \otimes & |+\rangle_R \\
|-\rangle_R & \otimes & b^i_{+,-1/2}\,|0\rangle_{NS} \\
b^i_{-,-1/2}\,|0\rangle_{NS} & \otimes & b^j_{+,-1/2}\,|0\rangle_{NS}
\end{array}$$

- NS-NS sector: This sector contains the same states as IIB theory.
- NS-R and R-NS sector:This sector contains the same states as IIB theory. But the two gravitinos have the opposite chirality.
- R-R sector:This sector contains a one-form C_i (8 state), a three-form gauge field C_{ijk} (56 states).

We often write a n-form as C_n and introduce $C_5,\ C_6,\ C_7,\ C_8$ which are dual to $C_3,\ C_2,\ C_1,\ C_0$.

Since the massless states for two types of the superstring theories are field contents of SUGRA, in the low energy limit $l_s \to 0$, the superstring theory will be reduced to ten dimensional SUGRA. In this limit, the closed string action may be written as

$$S = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-G} R \left(1 + \mathcal{O}\left(l_s^2 R\right)\right), \tag{A.24}$$

where $G_{10} \sim l_s^8 \sim \alpha'^4$ is ten dimensional Newton constant, G is the determinant of the metric and R is the curvature of the ten dimensional spacetime. We have $\alpha' R$ correction in the low energy action. In $\alpha' R \ll 1$ case, the low energy action will coincide with the usual Einstein-Hilbert action.

A.1.4 Background fields

Massless states in NS-NS sector of closed string $G_{\mu\nu},\ B_{\mu\nu},\ \Phi$ have a special meaning. If there are background fields of $G_{\mu\nu},\ B_{\mu\nu},\ \Phi$, the (Euclidean) string action Eq. (A.2) should be generalize to

$$S = S_G + S_B + S_\Phi, \tag{A.25}$$

$$\begin{aligned} S_G &= \frac{1}{4\pi\alpha'} \int_M d^2\sigma \sqrt{\gamma} \gamma^{ab} G_{\mu\nu} \partial_a X^\mu \partial_b X^\nu, \\ S_B &= \frac{1}{4\pi\alpha'} \int_M d^2\sigma \sqrt{\gamma} \gamma^{ab} B_{\mu\nu} \partial_a X^\mu \partial_b X^\nu, \\ S_\Phi &= \frac{1}{4\pi} \int_M d^2\sigma \sqrt{\gamma} \Phi R^{(2)}(\gamma), \end{aligned} \tag{A.26}$$

where $R^{(2)}(\gamma)$ is the scalar curvature of the two-dimensional string world sheet computed from the world sheet metric γ_{ab} and M represents a geometry of world sheet. Since S_Φ has no X , S_Φ is one order higher than other terms in the α' expansion.

S_G is a simple generalization of the Polyakov action, Eq. (A.2), with non-trivial background gravitational field. We may obtain it by replacing $\eta_{\mu\nu}$ with $G_{\mu\nu}$ in Eq. (A.2). Thus $G_{\mu\nu}$ behaves as the gravitational field in the theory. S_B may exist when the string is oriented. In terms of the states, orientation is defined by the orientifold projection Ω for closed string. This projection works as $\Omega b^i_\pm \Omega^{-1} \to b^i_\mp$. When the string theory is invariant under this transformation, the states which are changed by this transformation such as $B_{ij} b^i_- b^j_+ \left|0\right\rangle$ are eliminated. S_B is a two-form analog of the coupling of a one-form Maxwell field to the world line of a charged particle, $q\int A_\mu dx^\mu = q\int A_\mu \frac{dx^\mu}{d\tau}d\tau$. In this sense an oriented string can be regarded as a charged object. The dilaton term S_Φ has a special feature when the dilaton field is constant $\Phi = \Phi_0$. Then the term in Eq. (A.26) is rewritten as

$$S_{\Phi_0} = \Phi_0 \chi\left(M\right), \quad \chi\left(M\right) = \frac{1}{4\pi}\int_M d^2\sigma\sqrt{\gamma}R^{(2)}\left(\gamma\right) = 2 - 2n_h - n_b - n_c,$$

where $\chi\left(M\right)$ is the Euler characteristic of M and it is determined by the number of handles n_h, the number of boundaries n_b, and the number of cross-caps n_c of the Euclidean world sheet M.

The path integral of the string world sheet with background fields is written by

$$Z\left[G, B, \Phi\right] \sim \int D\gamma_{ab}\int DX^\mu e^{-S[h,X;G,B,\Phi]}, \tag{A.27}$$

where $S\left[h, X; G, B, \Phi\right]$ is the same as in Eq. (A.25) and $\int D\gamma$ means sum over all Riemann surfaces (M, h). Since Riemann surfaces of different topology are not diffeomorphic, Eq. (A.27) should be regarded as

$$Z = \sum_{n_h, n_b, n_c} Z\left(n_h, n_b, n_c\right) + \tilde{Z}, \tag{A.28}$$

where $\tilde{Z}$ is a term of non-perturbative contributions. The first term of Eq. (A.28) is a perturbative expansion for a constant mode of the dilaton e^{Φ_0},

$$\sum_{n_h, n_b, n_c} Z\left(n_h, n_b, n_c\right) = \sum_{\chi} e^{-\chi\Phi_0} Z\left(\chi\right),$$

where $e^{-\chi\Phi_0}Z\left(\chi\right)$ represents the contribution of the world sheets with the Euler characteristic χ. Thus the coupling constant of the string is defined as

$$g_s \equiv e^{\Phi_0}.$$

Some examples of the string world sheet M is displayed in Fig. A.2a. We can read the relation between the closed string coupling g_s and the open string coupling g_o,

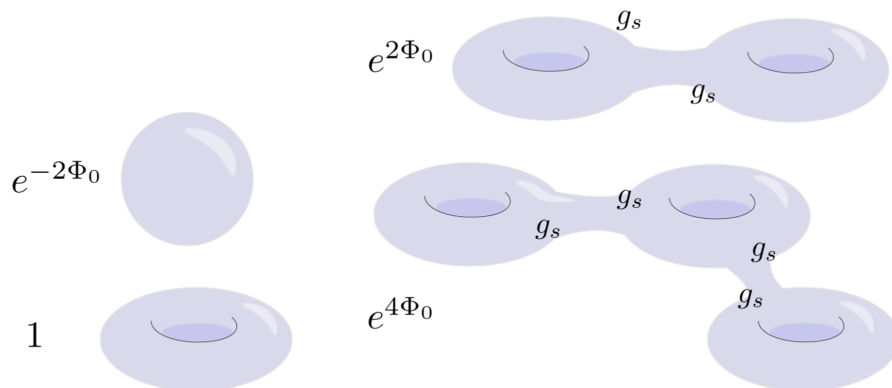
$$g_s = g_o^2,$$

from the Fig. A.2b.

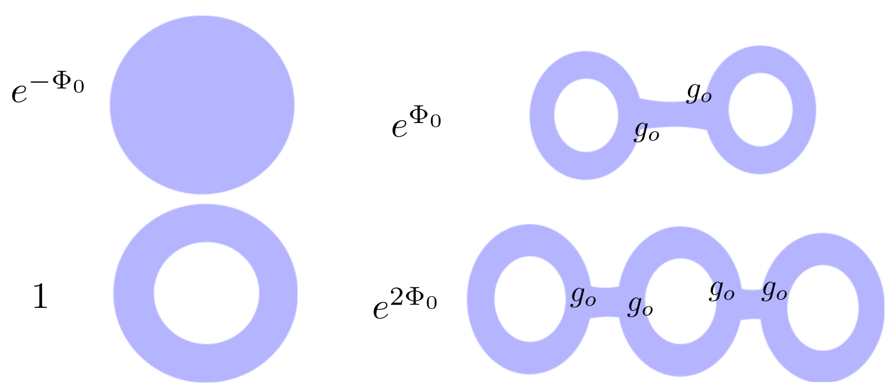
A.2 D-branes

Another important object of the string theory is Dp-brane which is a p-dimensional extended object. The bosonic part of its low energy effective action is written in terms of the massless fields of the string theory,

$$S_{Dp} = S_{DBI} + S_{CS},$$



(a) We display oriented closed string world sheets. The leading order contribution comes from the sphere ($\chi = 2$) diagrams. Next to leading order contribution comes from the torus diagrams which has the one handle, $\chi = 2 - 2 \times 1 = 0$ and $e^{\chi\Phi_0} = e^0$. Third contribution comes from the world sheets which has two handles, $\chi = 2 - 2 \times 2 = -2$ and $e^{-\chi\Phi_0} = e^{+2\Phi_0} = 1 \times g_s^2$. Fourth contribution comes from the world sheets which has three handles, $\chi = 2 - 2 \times 3 = -4$ and $e^{-\chi\Phi_0} = e^{+4\Phi_0} = 1 \times g_s^4$. Third and fourth contributions may be interpreted that the contribution of the torus type world sheet, 1, times g_s^n, where n is a number of the three point vertices of the world sheet.



(b) The oriented open string world sheet with $\chi = 1$(one boundary=disc), $\chi = 0$ (two boundaries=cylinder, $e^{\chi\Phi_0} = 1$) , $\chi = -1$ (three boundaries, $e^{-\chi\Phi_0} = e^{\Phi_0} = 1 \times g_o^2$, $\chi = -2$ (four boundaries, $e^{-\chi\Phi_0} = e^{2\Phi_0} = 1 \times g_o^4$) are displayed. Third and fourth contributions may be interpreted that the contribution of the cylinder type world sheet, 1, times g_o^n, where n is a number of the three point vertices of the world sheet.

Figure A.2: Some examples of the string world sheet.

$$
\begin{aligned}
S_{DBI} &= -T_{Dp}\int d^{p+1}\xi e^{-\Phi}\sqrt{-\det\left(G_{ab}+B_{ab}+2\pi\alpha' F_{ab}\right)}, \\
S_{CS} &= \mu_p \int \left(Ce^{B+2\pi\alpha' F}\right)_{p+1},
\end{aligned}
$$

where F_{ab} is the $U(1)$ gauge field strength on the single Dp-brane, $G_{ab} \equiv G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu$, $B_{ab} \equiv B_{\mu\nu}\partial_a X^\mu \partial_b X^\nu$ are pull-back of the background metric $G_{\mu\nu}$ and two-form field $B_{\mu\nu}$ to $p+1$-dimensional world volume and $a, b = 0, 1, \cdots, p$. Dp-brane tension is given as

$$
T_{Dp} = \frac{1}{(2\pi)^p (\alpha')^{(p+1)/2}}.
$$

In the term S_{CS},

$$
C \equiv \sum_{n=0}^{8} C_n,
$$

and $()_{p+1}$ means that one should extract the $p+1$-form piece from parenthesis. The R-R charge μ_p is

$$
\mu_p = \frac{1}{(2\pi)^p (\alpha')^{(p+1)/2}}.
$$

If N Dp-branes are put on the same spacetime region, then field F_{ab} will be promoted to $U(N)$ gauge fields and the action becomes

$$
\begin{aligned}
S_{DBI} &= -T_{Dp}\mathrm{Tr}\int d^{p+1}\xi e^{-\Phi}\sqrt{-\det\left(G_{ab}+B_{ab}+2\pi\alpha' F_{ab}\right)}, && \text{(A.29)} \\
S_{CS} &= \mu_p \mathrm{Tr}\int \left(Ce^{B+2\pi\alpha' F}\right)_{p+1}, && \text{(A.30)}
\end{aligned}
$$

where $\partial_a X$ in the pull-backs are replaced by $D_a X = \partial_a X + [A_a, X]$, so we treat Xs as adjoint representations of $U(N)$ gauge group.

It is interesting that the DBI action contains the Yang-Mills action. In the flat spacetime metric $G_{\mu\nu} = \eta_{\mu\nu}$ and without $B_{\mu\nu}$, choosing the gauge as $\xi^a = X^a$ for $a = 0, \cdots, p$, and denoting $X^i = 2\pi\alpha'\Phi^i$ for $i = p+1, \cdots, 9$, we can expand the action Eq. (A.29) as

$$
\begin{aligned}
S_{DBI} = & -\frac{T_{Dp}(2\pi\alpha')^2}{4}\int d^{p+1}\xi e^{-\Phi}\mathrm{Tr}\left[F_{ab}F^{ab} + 2D_a\Phi^i D_a\Phi^i + \left(\left[\Phi^i, \Phi^j\right]\right)^2\right] \\
& + \text{const.} + \text{higher order of } 2\pi\alpha' . \qquad \text{(A.31)}
\end{aligned}
$$

The leading terms of low energy expansion of the action contains the Yang-Mills action. Because of $\left\langle e^{-\Phi}\right\rangle = 1/g_s$, the Yang-Mills coupling is written as

$$
\begin{aligned}
g_{YM,p}^2 &= \frac{g_s}{T_{Dp}(2\pi\alpha')^2} \\
&= (2\pi)^{p-2}(\alpha')^{(p-3)/2} g_s.
\end{aligned}
$$

Appendix B

Hamilton formalism

In this section we explain the Hamilton formalism which we used in Sec. 3.3.3.

We start with the action which has the form

$$S = \int d\sigma \mathcal{L} \left[\left\{ q^i (\sigma), \partial_\sigma q^i (\sigma) \right\} \right] \tag{B.1}$$

with symmetry under the reparametrization $\sigma \to \sigma' = f(\sigma)$,

$$\int d\sigma \mathcal{L} \left[\left\{ q^i (\sigma), \partial_\sigma q^i (\sigma) \right\} \right] = \int d\sigma' \mathcal{L} \left[\left\{ q^{i\prime} (\sigma'), \partial_{\sigma'} q^{i\prime} (\sigma') \right\} \right] \quad \text{and} \quad q^i (\sigma) = q^{i\prime} (\sigma').$$

We define the conjugate momenta,

$$p_i \equiv \frac{\partial \mathcal{L}}{\partial \left(\partial_\sigma q_i \right)},$$

and the Hamiltonian,

$$\mathcal{H} \left[\left\{ q^i, p_i \right\} \right] \equiv \sum_i p_i \partial_\sigma q^i - \mathcal{L} \left[\left\{ q^i, \partial_\sigma q^i \right\} \right].$$

Then the equations of motion can be written as

$$\frac{\partial q^i}{\partial \sigma} = \frac{\partial \mathcal{H}}{\partial p_i}, \quad \frac{\partial p_i}{\partial \sigma} = -\frac{\partial \mathcal{H}}{\partial q^i}.$$

When this Hamiltonian satisfies a condition

$$\mathcal{H} = \Delta \left[\left\{ q^i, \partial_\sigma q^i, p_i \right\} \right] \tilde{\mathcal{H}} \left[\left\{ q^i, p_i \right\} \right], \quad \tilde{\mathcal{H}} \left[\left\{ q^i, p_i \right\} \right] = 0,$$

the equations of motion become

$$\begin{aligned} \frac{\partial q^i}{\partial \sigma} &= \frac{\partial \left(\Delta \tilde{\mathcal{H}} \right)}{\partial p_i} = \Delta \frac{\partial \tilde{\mathcal{H}}}{\partial p_i}, \\ \frac{\partial p_i}{\partial \sigma} &= -\frac{\partial \left(\Delta \tilde{\mathcal{H}} \right)}{\partial q^i} = -\Delta \frac{\partial \tilde{\mathcal{H}}}{\partial p_i}. \end{aligned} \tag{B.2}$$

On the other hand, the reparametrization invariance of the action, Eq. (B.1), implies that we may define the alternative conjugate momenta p'_i and Hamiltonian $\mathcal{H}'$ as

$$\begin{aligned} p'_i &\equiv \frac{\partial \mathcal{L}}{\partial\left(\partial_{\sigma'} q'_i\right)}, \\ \mathcal{H}'\left[\left\{q^{i\prime}, p'_i\right\}\right] &\equiv \sum_i \partial_{\sigma'} q^{i\prime} p'_i - \mathcal{L}\left[\left\{q^{i\prime}, \partial_\sigma q^{i\prime}\right\}\right] = \mathcal{H}\left[\left\{q^{i\prime}, p'_i\right\}\right] \end{aligned}$$

and the equations of motion are

$$\begin{aligned} \frac{\partial q^{i\prime}}{\partial \sigma'} &= \Delta \frac{\partial \tilde{\mathcal{H}}}{\partial p'_i} \rightarrow \frac{\partial \sigma}{\partial \sigma'} \frac{\partial q^{i\prime}\left(\sigma'\left(\sigma\right)\right)}{\partial \sigma} = \Delta\left[\left\{q^{i\prime}, \partial_{\sigma'} q^{i\prime}, p'_i\right\}\right] \frac{\partial \tilde{\mathcal{H}}\left[\left\{q^{i\prime}, p'_i\right\}\right]}{\partial p'_i}, \\ \frac{\partial p'_i}{\partial \sigma'} &= -\Delta \frac{\partial \tilde{\mathcal{H}}}{\partial q^{i\prime}} \rightarrow \frac{\partial \sigma}{\partial \sigma'} \frac{\partial p'_i\left(\sigma'\left(\sigma\right)\right)}{\partial \sigma} = -\Delta\left[\left\{q^{i\prime}, \partial_{\sigma'} q^{i\prime}, p'_i\right\}\right] \frac{\partial \tilde{\mathcal{H}}\left[\left\{q^{i\prime}, p'_i\right\}\right]}{\partial q^{i\prime}}. \end{aligned}$$

Setting $\frac{\partial \sigma}{\partial \sigma'} = \Delta$, we reduce these equations of motion to

$$\begin{aligned} \frac{\partial q^{i\prime}\left(\sigma'\left(\sigma\right)\right)}{\partial \sigma} &= \frac{\partial \tilde{\mathcal{H}}\left[\left\{q^{i\prime}, p'_i\right\}\right]}{\partial p'_i}, \\ \frac{\partial p'_i\left(s'\left(\sigma\right)\right)}{\partial \sigma} &= -\frac{\partial \tilde{\mathcal{H}}\left[\left\{q^{i\prime}, p'_i\right\}\right]}{\partial q^{i\prime}}. \end{aligned} \tag{B.3}$$

We notice that these equations are regard as the usual Hamilton's equations of motion for the Hamiltonian $\tilde{\mathcal{H}}$ rather than $\mathcal{H}$. Because two set of the equations of motion Eqs. (B.2) and (B.3) describe the same physics, we get the trajectory $q^i\left(\sigma\right) = q^{i\prime}\left(s'\left(\sigma\right)\right)$ as the solution of the equations Eq. (B.3).

Appendix C

Small q analysis in the dAdS model

The dAdS model is one of the simplest generalization of the AdS model. Its distinction from the AdS model is handled by the parameter q in dilaton Φ as

$$e^{\Phi} = 1 + \frac{q}{r^4}.$$

Thus the dAdS model with $q = 0$ is coincide with the original AdS model. Then we expect that the perturbative expansion in parameter q may work. However, the validity of this expectation depends on situations. In this section we consider the perturbative expansions in parameter q in two different situations, namely we consider the calculation of the vacuum expectation values of the Wilson loops and the calculations of the accelerated string solutions. In the former case the expansion works, while in the latter case it does not work.

C.1 Wilson loop

Wilson loops with bottom at $r_b \gg q^{1/4}$ can be estimated by expanding quantities in $\frac{q}{r_b^4}$.

In this expansion, the separation between quark and antiquark L, Eq. (2.24), and the potential between them V, Eq. (2.25), are expanded as

$$L = \sqrt{1 + \frac{q}{r_b^4}} L_0 \sim \left(1 + \frac{1}{2}\frac{q}{r_b^4}\right) L_0, \tag{C.1}$$

and

$$V = \frac{2r_b}{2\pi\alpha'}\int_1^\infty dy \frac{y^2}{\sqrt{y^4-1}}\left(1 + \frac{q}{r_b^4}\frac{1}{y^4}\right) - 2m_q \ , \tag{C.2}$$

where r_b is a position of the bottom of the string in r direction, $L_0 = \frac{2R^2}{r_b}\frac{\sqrt{2}\pi^{3/2}}{\Gamma\left(\frac{1}{4}\right)^2}$ and m_q is the quark mass introduced to regularize the potential energy. Thinking naively, m_q should be defined as

$$\begin{aligned} m_q &= \frac{1}{2\pi\alpha'}\int_0^\infty dr e^{\Phi/2} \\ &= \frac{1}{2\pi\alpha'}\int_0^\infty dr\sqrt{1+\frac{q}{r^4}}. \end{aligned}$$

However, this quantity will diverge in the lower limit of the integral. To avoid this problem, we have two different prescription.

1. We use $m_q = \frac{1}{2\pi\alpha'}\int_0^\infty dr$ which is the definition of a quark mass in the AdS/CFT correspondence
2. We use $m_q = \frac{1}{2\pi\alpha'}\int_{r_{min}}^\infty dr\sqrt{1+\frac{q}{r^4}}$ and set r_{min} finite.

Which prescription is adopted is not so important, since it just determines an normalization of the potential. Thus we will define the quark mass m_q as

$$m_q = m_{q,0} + m_{q,1}q,$$

where $m_{q,0},\ m_{q,1}$ is

$$\begin{aligned} m_{q,0} &= \frac{1}{2\pi\alpha'}\int_{r_{min}}^\infty dr \\ &= \frac{r_b}{2\pi\alpha'}\int_{y_{min}}^\infty dy, \end{aligned}$$

$$\begin{aligned} m_{q,1} &= \frac{1}{2\pi\alpha'}\int_{r_{min}}^\infty dr\left(\frac{s}{2}\frac{1}{r^4}\right) \\ &= \frac{1}{2\pi\alpha'}\frac{1}{r_b^3}\frac{s}{6}y_{min}^{-3}, \end{aligned}$$

and $s=0,\ r_{min}=0$ in the case of prescription 1, $s=1,\ r_{min}\neq 0$ in the case of prescription 2. Then V can be re written as

$$\begin{aligned} V(L) &= V_0(L) + V_1(L) + 2\frac{r_{min}}{2\pi\alpha'} - \frac{q}{2\pi\alpha'}\frac{1}{3}\frac{s}{r_{min}^3}, \\ V_0(L) &= -\frac{4\pi^2(2\lambda)^{1/2}}{\Gamma\left(\frac{1}{4}\right)^4}\frac{1}{L}, \\ V_1(L) &= \frac{\Gamma\left(\frac{1}{4}\right)^4}{2(2\pi)^4}\frac{q}{(2\lambda)^{3/2}\alpha'^4}L^3. \end{aligned}$$

This description is valid in the region $0 < L \lesssim L_{lim}$,

$$L_{lim} \equiv L\left(r_b = q^{1/4}\right) = \frac{4\pi^{3/2}}{\Gamma\left(\frac{1}{4}\right)^2}\frac{\alpha'}{q^{1/4}}\left(2\lambda\right)^{1/2}.$$

In this region, the first order of the potential V_1 is bounded in the region,

$$0 < V_1\left(L\right) \lesssim \frac{2\pi^{1/2}}{\Gamma\left(\frac{1}{4}\right)^2}\frac{q^{1/4}}{\alpha'}.$$

In the limit $q \to 0$, this description is valid in all region of L and $V_1\left(L\right)$ vanishes. Thus, in this limit, the situation will be coincide with the original AdS/CFT case as might be expected. $V_1\left(L\right)$ is proportional to L^3. However, since L is bounded in the region $L \lesssim L_{lim}$, we cannot discuss the confinement property in this approximation. In fact, in the region $L \gg L_{lim}$ $\left(r_b^4 \ll q\right)$, $V\left(L\right)$ is proportional to L as seen in Eq. (2.26).

C.2 Accelerated string solution

Now we try to consider Eq. (3.8) by expanding quantities in $\frac{q}{r^4}$. However it turn out that such a calculation is invalid immediately.

In q expansion, we assume that $f\left(r\right)$ can be written as

$$f\left(r\right) = f_0\left(r\right) + f_1\left(r\right)q + \mathcal{O}\left(q^2\right), \tag{C.3}$$

where $f_0\left(r\right) = \frac{1}{a^2} - \frac{R^4}{r^2}$ is the solution of the equation of motion for the accelerated string in AdS spacetime Eq. (3.5). Substituting Eq. (C.3) into Eq. (3.8) and keep the terms up to first order of q, the equation of motion for f_1 becomes

$$\frac{1}{2}a\partial_r\left(\frac{r^4}{R^4}f_1' - a^2r^2f_1'\right)q = 2a\frac{q}{r^4}. \tag{C.4}$$

This equation can be solved by

$$f_1\left(r\right) = \frac{4}{3}\frac{1}{a^6R^8}\left(\ln\frac{r}{\sqrt{r^2-a^2R^4}} - \frac{a^2R^4}{2}\frac{1}{r^2} - \frac{a^4R^8}{4}\frac{1}{r^4}\right) + \frac{C}{a^2}\left(\frac{1}{2aR^2}\ln\left(\frac{r-aR^2}{r+aR^2}\right) + \frac{1}{r}\right) + D,$$

where $C,\ D$ is integration constants. Because $f\left(r \to \infty\right) = \frac{1}{a^2}$, $D = 0$ is required. C will be determined by imposing the condition $\lim_{q\to 0} f_1\left(r_c\right) < \infty$. Denoting $r_c = r_{c,0} + r_{c,1}q,\ r_{c,0} = R^2a$, we have

$$\begin{aligned} f_1\left(r_c\right) &= \frac{4}{3}\frac{1}{a^6R^8}\ln\frac{R^2a + r_{c,1}q}{\sqrt{2R^2ar_{c,1}q}}\left(\frac{r_{c,1}q}{2R^2a + r_{c,1}q}\right)^{\frac{3a^3R^6C}{8}} \\ &\quad -\frac{1}{3}\frac{1}{a^4R^4}\left(\frac{2}{a^2R^4 + 2R^2ar_{c,1}q} + \frac{1}{a^2R^4 + 4aR^2r_{c,1}q}\right) + \frac{C}{a^2}\frac{1}{R^2a + r_{c,1}q}. \end{aligned}$$

Thus $C = \frac{4}{3a^3R^6}$ is required to make $\lim_{q\to 0} f_1(r_c)$ regular and $f_1(r)$ becomes

$$f_1(r) = \frac{4}{3a^6R^8}\left(\ln\frac{r}{r+aR^2} - \left(\frac{aR^2}{r} + \frac{a^2R^4}{2}\frac{1}{r^2} + \frac{a^4R^8}{4}\frac{1}{r^4}\right)\right). \tag{C.5}$$

Above discussion seems to be valid when the relation, $\min(r) \equiv r_b \gg q^{1/4}$, is satisfied. However it is wrong. When we expand Eq. (3.8), we drop the term of order $\mathcal{O}(q^2)$. Since q is the constant and r $(r_b < r < \infty)$is variable, there are regions where the contributions of $\mathcal{O}(q^2)$ exceeds that of $\mathcal{O}(q)$. In fact in the region $r \gg q^{1/4}$, the situation such that

$$\frac{q}{r^4} \ll \frac{q^2}{R^2r^2}$$

can arise, for example. Thus we must keep every term. It implies that our naive estimation Eq.(C.5) may be wrong. In fact, an asymptotic solution of Eq. (3.8) behaves

$$f(r) = f_0(r) + \mathcal{O}\left(r^{-3}\right), \quad r \sim \infty,$$

while Eq. (C.5) behaves

$$f_1(r) = \mathcal{O}\left(r^{-1}\right), \quad r \sim \infty.$$

Appendix D

Expanding $U(1)$ in Sakai-Sugimoto model

In Sec. 4.1.3, we expand the action only in terms of the $SU(2)$ field strength. This is crucial to keep the validity of the analysis in Sec. 4.1.3. In fact, the critical chemical potential in Eq. (4.28) is sufficiently large not to be valid the perturbative expansion in terms of the $U(1)$ gauge field strength. Since $\frac{1}{2}\frac{27\pi}{2\lambda}\mu_q$ is order one, $\frac{1}{2}\frac{27\pi}{2\lambda}\partial_z A_0$ in Eq. (4.21) is also expected as $\mathcal{O}(1)$. Thus first term and second term inside the square root in Eq. (4.21) become the same order.

We can confirm above statement concretely by expanding the action in Eq. (4.21) in terms of $U(1)$ gauge field. Then the action, Eq. (4.21), becomes

$$S/V_4 = -\kappa \int_0^\infty dz \left(12\bar{q}^2 n_B \left(k^{-1/3} + k\right) - \frac{1}{2}k(z)\left(\partial_z \hat{A}_0\right)^2 - \frac{324\pi}{\lambda} n_B \hat{A}_0 \bar{q}^2\right), \tag{D.1}$$

where we subtract $S_0/V_4 = -\kappa \int_0^\infty dz k^{-2/3}$ from the action. Then the equations of motion for A_0 is written as

$$\partial_z \left(k\partial_z \hat{A}_0\right) = \frac{324\pi}{\lambda} n_B \bar{q}^2.$$

Imposing the same boundary conditions as in Eq. (4.25), we get

$$\begin{aligned} A_0(z) &= \frac{54 n_B \pi^3}{\lambda\left(1-\rho^2\right)^{3/2}}\left\{\rho\sqrt{1-\rho^2}\left(1 - \frac{\rho}{\sqrt{z^2+\rho^2}}\right)\right. \\ &\quad \left. + \left(3\rho^2 - 2\right)\left(\tan^{-1}\frac{\rho}{\sqrt{1-\rho^2}} - \tan^{-1}\sqrt{\frac{z^2+\rho^2}{1-\rho^2}}\right)\right\}. \end{aligned} \tag{D.2}$$

Numerical plot of $A_0(z)$ and $\partial_z A_0$ are displayed in Fig. D.1. We assure that the value of the chemical potential becomes larger than that in Eq. (4.28) and $\frac{1}{2}\frac{27\pi}{2\lambda}\partial_z A_0$ becomes order one in a certain region. Thus the perturbative expansion of the action, Eq. (4.21), in terms of the $U(1)$ gauge field strength is invalid.

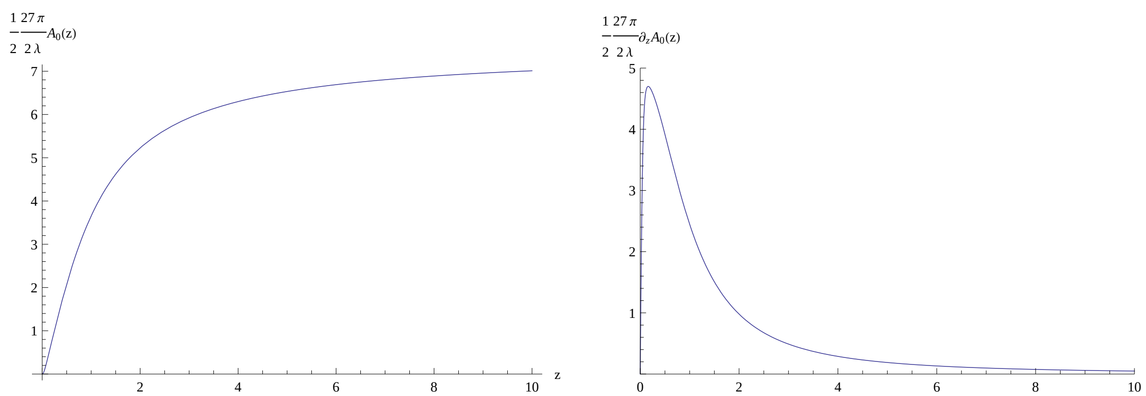


Figure D.1: Numerical plot of the function (D.2) and its derivative. These are evaluated at $\lambda = 16.6,\ \rho = 0.079,\ n_B = 0.019$, namely at critical values (4.28). The value of the chemical potential $A_0\left(z \to \infty\right)$ becomes larger than the value of Eq. (4.28). It comes from the invalidity of the expansion (D.1) because of the existence of the region of $\frac{1}{2}\frac{27\pi}{2\lambda}\partial_z A_0\left(z\right) > 1$.

Appendix E

Unruh effect

E.1 Field theoretical perspective

In this section, we will review the Unruh effect briefly. We argue along the Ref. [41] in this review. More detailed discussions for the Unruh effect are performed in Ref. [41] or Ref. [42].

Suppose a particle detector moves in Minkowski spacetime along the world line $x^{\mu}(\tau)$. The full Hamiltonian density of the system is written as

$$H = H_{\phi} + H_0 + H_{int}$$

$H_{\phi},\ H_0$ is non-interacting (free) Hamiltonian of the field ϕ and the detector, and $H_{int} = -L_{int} = -\int d\tau c_0 m(\tau)\,\phi[x(\tau)]$ is the detector-field interaction Hamiltonian. c_0 is a small coupling constant and $m(\tau)$ is the detector's monopole moment operator. When $\tau = -\infty$, the field ϕ is in the vacuum state $|0_M\rangle$, which is the usual oscillator vacuum ($a_k|0_M\rangle = 0$) in Minkowski spacetime. The states of the detector and the field will be changed to excited states $|\psi\rangle,\ |E\rangle\,(\mathcal{H}_0|E\rangle = E|E\rangle)$ from their ground state $|0_M\rangle,\ |E_0\rangle\,(H_0|E_0\rangle = E_0|E_0\rangle,\ E > E_0)$. For sufficient small c_0 the amplitude for transition can be written as

$$\langle E,\psi|\,(-iH_{int})\,|0_M,E_0\rangle = ic_0\int_{-\infty}^{\infty} d\tau\,\langle E,\psi|\,m(\tau)\,\phi[x(\tau)]\,|0_M,E_0\rangle\,.$$

The time evolution of $m(\tau)$ obey the Heisenberg equation $m(\tau) = e^{iH_0\tau}m(0)\,e^{-iH_0\tau}$, then the transition amplitude becomes

$$ic_0\,\langle E|\,m(0)\,|E_0\rangle\int_{-\infty}^{\infty} d\tau e^{i(E-E_0)\tau}\,\langle\psi|\,\phi[x(\tau)]\,|0_M\rangle\,. \tag{E.1}$$

Squaring Eq. (E.1) and summing over all possible state to transition, we get

$$c_0^2\sum_E\left|\langle E|\,m(0)\,|E_0\rangle^2\right|\int_{-\infty}^{\infty} d\tau d\tau'\mathcal{F}(E-E_0)\,,$$

where

$$\mathcal{F}\left(E-E_0\right)=e^{-i(E-E_0)(\tau-\tau')}G^{+}\left(x\left(\tau\right),x\left(\tau'\right)\right),$$

and

$$G^{+}\left(x\left(\tau\right),x\left(\tau'\right)\right)\equiv\left\langle 0_M\right|\phi\left[x\left(\tau\right)\right]\phi\left[x\left(\tau'\right)\right]\left|0_M\right\rangle.$$

Now we consider the transition probability per unit proper time. For simplicity, we restrict ϕ to be massless and restrict the trajectory $x\left(\tau\right)$ such that the massless propagator $D\left(x\left(\tau\right),x\left(\tau'\right)\right)$ only depend on the difference $\Delta\tau\equiv\tau-\tau'$, that is, $D\left(x\left(\tau\right),x\left(\tau'\right)\right)=D\left(\Delta\tau\right)$. Then the transition probability per unit proper time is given by

$$c_0^2\sum_E\left|\left\langle E\right|m\left(0\right)\left|E_0\right\rangle\right|^2\int_{-\infty}^{\infty}d\left(\Delta\tau\right)e^{-i(E-E_0)\Delta\tau}D^{+}\left(\Delta\tau\right),\tag{E.2}$$

where

$$D^{+}\left(\Delta\tau\right)=-\frac{1}{4\pi^2}\frac{1}{\left(t\left(\tau\right)-t\left(\tau'\right)-i\epsilon\right)^2-\left|\vec{x}\left(\tau\right)-\vec{x}\left(\tau'\right)\right|^2}.$$

If the motion of the detector is uniform motion $\tau=\sqrt{1-v^2}t,\ \vec{x}=\vec{x}_0+\vec{v}t$, the transition probability is

$$-c_0^2\sum_E\left|\left\langle E\right|m\left(0\right)\left|E_0\right\rangle\right|^2\frac{\sqrt{1-v^2}}{4\pi^2}\int_{-\infty}^{\infty}d\Delta t e^{-i(E-E_0)\sqrt{1-v^2}\Delta t}\frac{1}{\left(\Delta t-i\epsilon\right)^2}=0,$$

So the detector cannot detect any particles as expected. On the other hand, if we consider another complicated trajectories of the detector, the situation will be changed. Suppose the motion with an uniform acceleration a. In this case the world line is described as $x^2-t^2=\frac{1}{a^2},\quad y=z=0$, or

$$\begin{aligned}t&=\frac{1}{a}\sinh\left(a\tau\right),\\ x&=\frac{1}{a}\cosh\left(a\tau\right).\end{aligned}\tag{E.3}$$

Then we find

$$\begin{aligned}D^{+}\left(\Delta\tau\right)&=-\frac{a^2}{16\pi^2}\frac{1}{\sinh^2\left(\frac{a(\Delta\tau-i\epsilon)}{2}\right)}\\ &=-\frac{1}{4\pi^2}\sum_{k=-\infty}^{\infty}\frac{1}{\left(\Delta\tau-i\epsilon+2\pi\alpha k\right)^2}.\end{aligned}$$

Substituting this into Eq. (E.2), we get the final result of the transition probability as

$$\frac{c_0^2}{2\pi}\sum_E\frac{\left(E-E_0\right)\left|\left\langle E\right|m\left(0\right)\left|E_0\right\rangle\right|^2}{e^{\frac{2\pi}{a}(E-E_0)}-1}.$$

The factor $\left(e^{\frac{2\pi}{a}(E-E_0)}-1\right)^{-1}$ suggest that the accelerated detector sees a thermal bath with the temperature

$$T=\frac{a}{2\pi}.$$

In this section, though we demonstrate the emergence of the temperature by using a particle detector, the detector is not need for the demonstration in general. In the Ref. [2], the authors show the equation,

$$\langle 0_M|\left(\prod_i \phi\left(\vec{x_i},t_i\right)\right)_t|0_M\rangle=\frac{\mathrm{Tr}\left(e^{-\beta H^R}\left(\prod_i \phi\left(\vec{r_i},\eta_i\right)\right)_\eta\right)}{\mathrm{Tr}\left(e^{-\beta H^R}\right)}, \tag{E.4}$$

in terms of the partition function. Here $\beta=2\pi/a$, H^R is the Hamiltonian in Rindler coordinates, ϕ is a field. $()_t$, $()_\eta$ denote time and η ordering, $(\vec{r_i},\eta_i)$ is the same point as $(\vec{x_i},t_i)$ in Rindler coordinates. $|0_M\rangle$ represents the Minkowski vacuum state.

Note that Eq. (E.4) is shown about a large class of interacting field theories. It implies that the Unruh temperature $T_U=a/2\pi$ may be independent of the detail of the interaction or the sort of the fields. Thus we assume that the procedure to extract the Unruh temperature in terms of the gauge/gravity duality may not be affected from the detail of the theory. So it is natural that we employ the transformation Eq. (3.7) to get the temperature $T_U=a/2\pi$ in Chap. 3.

E.2 Comment on ERT

In Chap. 3, we perform ERT in order to move to co-moving frame for an accelerated quark and in order to obtain the correct Unruh temperature. However we may be able to consider other transformations which are appropriate for these purposes. Actually for the purpose of making the accelerated string Eq. (3.2) static, it is sufficient to transforming the coordinates as

$$\begin{aligned} x &= g\left(s,\beta\right)\cosh\left(\tilde{a}\tau\right), \\ t &= g\left(s,\beta\right)\sinh\left(\tilde{a}\tau\right), \\ r &= h\left(s,\beta\right). \end{aligned} \tag{E.5}$$

Since we want to regard $s\to\infty$ as the $r\to\infty$, we require

$$s\left(\beta,r=\infty\right)=\infty. \tag{E.6}$$

Then we may have various transformations which agree with the usual four-dimensional Rindler transformation on the boundary. It sounds strange. So it is a natural question whether the transformation Eq. (3.7) is the only transformation to describe the physics of the Unruh effect or not.

One of candidates for the transformation Eqs. (E.5) is

$$\begin{aligned} x &= \sqrt{\frac{1}{\tilde{a}^2} - \frac{1}{s^2}} e^{\tilde{a}\beta} \cosh\left(\tilde{a}\tau\right), \\ t &= \sqrt{\frac{1}{\tilde{a}^2} - \frac{1}{s^2}} e^{\tilde{a}\beta} \sinh\left(\tilde{a}\tau\right), \\ r &= sR^2 e^{-\tilde{a}\beta}, \end{aligned}$$

where $\tilde{a} < s < \infty$ and $\tilde{a} \neq a$. Under this transformation the solution with $q = 0$, Eq. (3.6), is mapped onto

$$\beta = \frac{1}{\tilde{a}} \ln \frac{\tilde{a}}{a}.$$

Then $aR^2 < r(s) < +\infty$. Note that if we choose $\tilde{a} \neq a$ only upper part $r > aR^2$ of the accelerated string solution is mapped again. In this case, however, we get the temperature

$$T_H = \frac{\tilde{a}}{2\pi}.$$

So imposing the conditions, "making string solution static" and "mapping only upper part of the string $r > r_c$", is not sufficient to determine the temperature. In this case an additional condition "setting the endpoint of the string on boundary as $\beta\left(s = s_{bdy}\right) = 0$" is needed. We use this condition through our analysis in Chap. 3.

Generally, the solution will be mapped by the transformation Eq. (E.5) as

$$x^2 - t^2 = f(r) \to g(s,\beta)^2 = f(h(s,\beta))$$

and the both sides of this equation is independent of τ. Then the metric after the transformation Eq. (E.5) becomes

$$\begin{aligned} ds^2 &= G_{xx}\left(-dt^2 + dx^2\right) + G_{rr}dr^2 \\ &= -G_{xx}\tilde{a}^2 g^2 d\tau^2 + \left[G_{xx}\left(\frac{\partial g}{\partial s}\right)^2 + \left(\frac{\partial h}{\partial s}\right)^2 G_{rr}\right] ds^2 \\ &\quad + \left[G_{xx}\left(\frac{\partial g}{\partial \beta}\right)^2 + \left(\frac{\partial h}{\partial \beta}\right)^2 G_{rr}\right] d\beta^2 + 2\left[G_{xx}\frac{\partial g}{\partial s}\frac{\partial g}{\partial \beta} + G_{rr}\frac{\partial h}{\partial s}\frac{\partial h}{\partial \beta}\right] ds d\beta \ . \end{aligned}$$

Since $f(r_c) = 0$,

$$g(s_c, \beta_c) = 0 \tag{E.7}$$

where β_c is the solution of the equation $h(s_c, \beta_c) = r_c$. To keep the metric diagonal and standard form of the finite temperature theory as in Eq. (2.10), we impose the condition

$$\begin{aligned} G_{s\beta} = G_{\beta s} &\equiv G_{xx}\frac{\partial g}{\partial s}\frac{\partial g}{\partial \beta} + G_{rr}\frac{\partial h}{\partial s}\frac{\partial h}{\partial \beta} = 0, \\ -G_{\tau\tau} &\equiv G_{xx}\tilde{a}^2 g^2 = A(\beta) B(s), \\ G_{ss} &\equiv \left[G_{xx}\left(\frac{\partial g}{\partial s}\right)^2 + \left(\frac{\partial h}{\partial s}\right)^2 G_{rr}\right] = A(\beta)\frac{1}{B(s)}, \end{aligned}$$

where $A(\beta)$, $B(s)$ are certain functions. Then $g(s,\beta)$, $h(s,\beta)$ should be satisfy the relations

$$
\begin{aligned}
A(\beta) &= G_{xx}(h(s,\beta))\sqrt{\frac{\partial g}{\partial s}\left[\frac{\partial g}{\partial s}\frac{\partial h}{\partial \beta}+\frac{\partial h}{\partial s}\frac{\partial g}{\partial \beta}\right]\left(\frac{\partial h}{\partial \beta}\right)^{-1}} \\
B(s) &= \frac{\tilde{a}g}{\sqrt{\frac{\partial g}{\partial s}\left[\frac{\partial g}{\partial s}\frac{\partial h}{\partial \beta}+\frac{\partial h}{\partial s}\frac{\partial g}{\partial \beta}\right]\left(\frac{\partial h}{\partial \beta}\right)^{-1}}}, \\
\frac{G_{xx}(h(s,\beta))}{G_{rr}(h(s,\beta))} &= \frac{\partial h}{\partial s}\frac{\partial h}{\partial \beta}\left(\frac{\partial g}{\partial s}\frac{\partial g}{\partial \beta}\right)^{-1}.
\end{aligned}
\tag{E.8}
$$

Wick rotating the time $\tau \to -i\tau_E$, and evaluating the metric near horizon $s = s_c + \epsilon^n$ by using $g = 0$, $\beta = \beta_c$ and $B(s = s_c + \epsilon^n) = C(s_c)\epsilon^l + o(\epsilon^l)$, we have

$$
ds^2 \sim \frac{A}{C(a)}n^2\left(d\rho^2 + \rho^2 d\theta^2\right), \quad \rho \equiv \frac{2}{n}\epsilon^{n/2}, \quad \theta \equiv \frac{C(s_c)}{2}\tau_E. \tag{E.9}
$$

It turn out that $l = n$ must be needed to avoid conical singularity. Thus $B(s)$ must be proportional to $s - s_c$ near horizon,

$$
B(s) \sim (s - s_c)C(s_c) \qquad (s \sim s_c). \tag{E.10}
$$

Then the temperature is determined as

$$
T_H = \frac{C(s_c)}{4\pi}. \tag{E.11}
$$

If the conditions Eqs. (E.7), (E.8), and (E.10) are satisfied, the metric has the temperature Eq. (E.11) and it may be different from $a/2\pi$. However these conditions are very complicated, so we do not find any other transformation which satisfy these conditions, except for the ERT, Eq. (3.7). Perhaps every transformation which satisfies these conditions represents a same physics. We may sophisticate the method of the gauge/gravity duality and deepen an understanding of the duality by investigating these circumstances. This subject is very attractive. But it is remained as open problem here.

We note that in the case of Eq. (2.22) and the transformation Eq. (E.5) takes the form as

$$
\begin{aligned}
x &= g(s)e^{a\beta}\cosh(a\tau), \\
t &= g(s)e^{a\beta}\sinh(a\tau), \\
r &= h(s)e^{-a\beta},
\end{aligned}
$$

the coefficient $g(s)$ and $h(s)$ can be determined uniquely by imposing the above conditions.

Performing this transformation, we have the Rindler coordinate,

$$\begin{aligned} ds^2 &= e^{\Phi/2}\Bigg\{ R^2 h^2 \left(\left(\frac{h'^4}{h^4} + g'^2 \right) ds^2 - g^2 a^2 d\tau^2 \right. \\ &\quad \left. + a^2 \left(g^2 + \frac{1}{h^2} \right) d\beta^2 + e^{-2a\beta} \left[dx_2^2 + dx_3^2 \right] \right) + R^2 d\Omega_5^2 \Bigg\}, \end{aligned}$$

where prime represents the derivative with respect to s, and we set as

$$\left(h^2\right)' = h^4 \left(g^2\right)'$$

to eliminate the non-diagonal part of the metric (it corresponds to the condition Eq. (E.8)). Then g is related to h as

$$g^2 = c_0^2 - \frac{1}{h^2}$$

where c_0 is an arbitrary constant. Imposing the condition Eq. (E.6), we must choose $c_0 = \frac{1}{a}$. Then we obtain ERT again and have the Rindler-dAdS metric,

$$\begin{aligned} x &= \sqrt{\frac{1}{a^2} - \frac{1}{h^2}} e^{a\beta} \cosh(a\tau), \\ t &= \sqrt{\frac{1}{a^2} - \frac{1}{h^2}} e^{a\beta} \sinh(a\tau), \\ r &= h e^{-a\beta}, \end{aligned}$$

$$\begin{aligned} ds^2 &= e^{\Phi/2} R^2 \Bigg\{ \frac{dh^2}{h^2 - a^2} - \left(h^2 - a^2\right) d\tau^2 \\ &\quad + h^2 \left(d\beta^2 + e^{-2a\beta} \left[dx_2^2 + dx_3^2 \right] \right) + d\Omega_5^2 \Bigg\}. \end{aligned}$$

Since

$$B(h) = C(h)(h - a), \; C(h) \equiv \frac{h(h+a)}{\sqrt{2a^2 - h^2}},$$

the condition Eq. (E.10) is satisfied and the temperature Eq. (E.11) becomes $a/2\pi$.

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