Approximating and Analyzing Fitness Landscape for Evolutionary Search Enhancement

Pei, Yan
Graduate School of Design, Kyushu University

Takagi, Hideyuki
Faculty of Design, Kyushu University

出版情報：Foundation of Genetic Algorithms (FOGA). 2013, 2013-01
バージョン：
権利関係：

Approximating and Analyzing Fitness Landscape for Evolutionary Search Enhancement

Yan PEI
Graduate School of Design, Kyushu University
4-9-1 Shiobaru, Minami-ku
Fukuoka, 815-8540 Japan
peiyan@kyudai.jp

Hideyuki TAKAGI
Faculty of Design, Kyushu University
4-9-1 Shiobaru, Minami-ku
Fukuoka, 815-8540 Japan
http://www.design.kyushu-u.ac.jp/~takagi/

ABSTRACT

We introduce two techniques to approximate and analyze fitness landscape for evolutionary search enhancement. One involves dimensionality reduction method used for fitness landscape approximation to reduce the computational complexity of the fitting. The other uses Fourier transform to obtain the frequency information of fitness landscape for search acceleration and multi-modal optimization. We briefly describe the inspirations, principles and results of the two techniques.

Keywords

Fitness Landscape, Dimensionality Reduction, Fourier Analysis, Search Acceleration, Multi-modal Optimization

1. INTRODUCTION

There are three promising research directions in evolutionary computation (EC) community for obtaining an enhanced EC convergence. First is to approximate fitness landscape, second is to develop some novel strategies or mechanisms in the extent EC algorithms, and third is to develop some new EC algorithm inspired from biological, mathematical or physical phenomena and schemes.

This paper introduces two techniques on fitness landscape approximations for EC search enhancement in the first aspects. They involve the dimensionality reduction method and the Fourier transform method.

2. FITNESS LANDSCAPE APPROXIMATION

2.1 Approximating Fitness Landscape in an Original Space

Our original proposal is to approximate a n-D fitness landscape using a n-D unimodal function, use the peak point of the unimodal function as an elite individual, and replace the worst individual with the elite [1]. This idea was inspired by scale-space filtering [7]; the roughest approximation of signal is a single peak function, and roughly speaking, its peak location is near the global peak of the original signal.

The scale-space filtering was developed to detect several small peaks in signal originally and has been used to detect edges in image processing nowadays. As the amplitudes of small signal peaks in the time duration of low amplitude are smaller than noise added to signal in the time duration of big amplitude, we cannot distinguish the small signal peaks and noise on bigger signal using an amplitude threshold. However, human can distinguish them visually. The idea of the scale-space filtering is to convolve Gaussian functions with gradually changed variances with the signal. Scale-space is the collection of the convolved signals; imagine x-y axes are a time axis and a variance axis, respectively. Convolved time signals become dull according to increasing variances, and the dullest signal has only single peak. This method allows us to trance the nearest peaks from the final single peak to the direction of smaller variances gradually and finally reach several original signal peaks. Noise peaks on signal of bigger amplitude are not detected due to further bigger signal peaks, while small signal peaks without noise are detected.

The biggest feature of our original proposal [1] is *Low Risk, and High Return*. Even if the fitness of an elite individual is poor, just the worst individual among many individuals is replaced with the poor elite individual that is still better than the worst individual, while the elite individual becomes a powerful parent when its fitness is high.

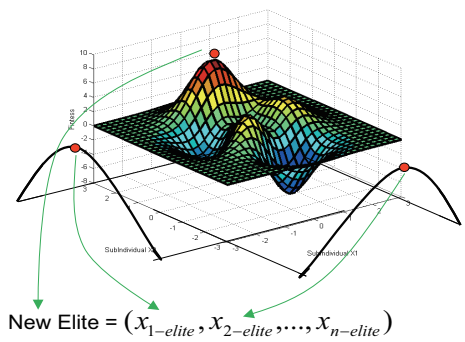


Figure 1: Dimensionality reduction, obtain elite in each projected dimensional landscape and synthesizing elite methods.

2.2 Approximating Fitness Landscape in a Dimensionality Reduced Space

It is not easy for conventional approximation methods to find an accurate landscape corresponding to the original multi-dimensional space. An alternative method is to reduce the dimensionality of the original search space and find approximation curve expressions in the lower dimensional space [2]. In general, this proposed method represents a novel local search method for accelerating EC convergence and reduce the computational complexity significantly [3], and it is this method that represents its original contribution.

2.2.1 Dimensionality Reduction

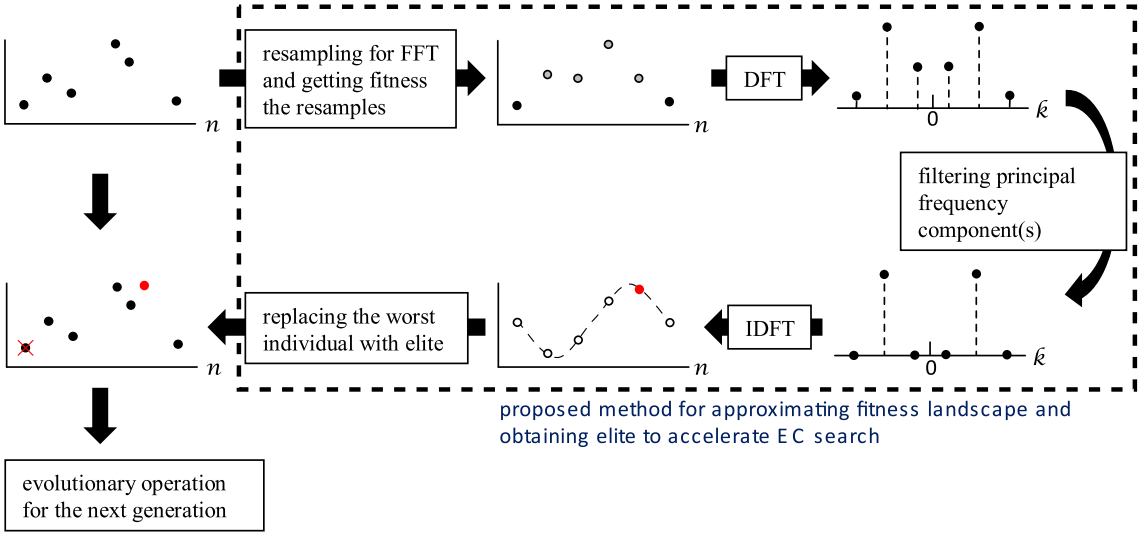


Figure 2: Flow diagram of our proposed method. The search space is re-sampled at uniform intervals. Fitness values are calculated and their frequency characteristics are calculated by the DFT. The primary frequency component is filtered, and an original fitness landscape is approximated with a trigonometric function by applying the inverse DFT to the filtered frequency components. The elite is obtained from the approximated landscape and used to accelerate EC search by substituting it for the worst individual.

Our method for reducing the dimensionality of the searching space uses only one of the n parameter axes at a time instead of all n parameter axes, and projects individuals onto each 1-D regression space. The landscape of the n-D parameter space is given by a fitness function, $y = f(x_1, x_2, ..., x_n)$, and the fitness value of the m-th individual are given by Equation (1):

$$y_m = f(x_{1m}, x_{2m}, ..., x_{nm}) \ (m = 1, 2, ..., M) \qquad (1)$$

There are M individuals with n-D parameter variables. We project the individuals onto the $i-th$ 1-D spaces as follows:

$$(x_{i1}, y_1) \ (x_{i2}, y_2) \ ... \ (x_{im}, y_m)$$

Each of the n 1-D regression spaces has M projected individuals.

2.2.2 Landscape Approximation

We interpolate or approximate the landscape of each 1-D regression space using the projected M individuals and select elite from the n approximated 1-D landscape shapes. The method selects n elite points in n 1-D regression spaces, respectively:

$$x_{1-elite}, \ x_{2-elite}, ..., \text{ and } x_{n-elite}.$$

2.2.3 Elite Synthesization

The n-D elite used for accelerating EC convergence in the next generation is obtained as follows.(Fig. 1)

$$NewElite = (x_{1-elite}, x_{2-elite}, .., x_{n-elite})$$

The proposed methods replace the worst individual in each generation with the selected elite. Although we cannot deny the small possibility that the global optimum is located near the worst individual, the possibility that the worst individual will become a parent in the next generation is also low; removing the worst individual therefore presents the least risk and is a reasonable choice.

3. FOURIER ANALYSIS ON FITNESS LANDSCAPE

3.1 Fitness Landscape approximated by Fourier Transform

We introduced to use discrete Fourier transform (DFT) for approximating fitness landscape and accelerating EC search [4]. There are five steps for implementing this objective. They include re-sampling in the original search space, conducting 1 dimensional Fourier transform, filtering principal frequency component, conducting 1 dimensional inverse Fourier transform and obtaining elite into next generation search.

3.1.1 Concept of the Proposal

Fig. 2 shows the flow diagram of our proposed method. Frequency characteristics of a fitness landscape are obtained by resampling a search space at regular intervals and applying the DFT to a sequence of fitness values for the resampled points. We can approximate the original fitness landscape with a trigonometric function by filtering a primary frequency component and applying inverse DFT to the filtered frequency components. Our proposed method that aims to accelerate EC search using information of an approximated function obtained by Fourier transform can be considered as a regression model for the trigonometric functions (Eq. 2).

$$EC(X) = \sum_{i=0}^{N} a_i \cos(2\pi\omega_i X + B_i) \qquad (2)$$

3.1.2 Principal Frequency Component

The trigonometric functions that determine the main structure of a fitness landscape are the most important in the regression model of Eq. 2, and they are determined by the frequency, amplitude and phase information at the peaks in the power spectrum. Let us define them as *principal frequen-*

cy components (PFC) and call them the PFC, the second, and so on according to their maximum power ranking.

Although the original fitness landscape is not always periodical with $1/\omega$, we can universally approximate the local landscape with arbitrary accuracy if we use an arbitrary number of trigonometric functions. In practice, however, too many base functions introduce too much computational complexity, so in this paper we will approximate the original fitness landscape with just use one trigonometric function, i.e. we use only the first PFC.

The PFC describes the main structure of a fitness landscape and its shape, and depends in turn on the choice of sampling range and sampling frequency.

3.1.3 Evolution Control Method For Acceleration

After obtaining a concrete regression model with a PFC, we obtain the local or global landscape characteristics of EC search space in 1-D dimension and n-D dimension. From this landscape characteristics, the more EC landscape information can be obtained than that little number of individuals supporting.

To accelerate EC, we analyze an EC fitness landscape with this regression model and direct EC search or make new elite into the next generation. Peak point must locate around the points of $k(1/\omega) + b$ and $k(1/2\omega) + b$ for the maximum problem and the minimum problem, respectively. So if we can find out the respect individuals with this characteristic in original search space and put it into the next generation evolution, EC can be accelerated.

Reference [5] investigated using n-D Fourier transform to obtain the PFC to accelerate EC search, from its results, it can enhance the benchmark problems that were not accelerated by the 1-D Fourier method.

3.2 Fourier Analysis for Multi-modal Optimization

In this section, we introduce a Fourier Niching Method by obtaining fitness landscape frequency, which is one kind of information to show the potential multi-modal region and their count. After we obtain the frequency, phase and amplitude information in the search space by fast Fourier transform (FFT), the exact multi-modal region and count information are calculated, we can use those information to implement a novel niching method - Fourier Niching Method [6].

3.2.1 Obtaining Frequency Landscape

In order to obtain the frequency information, we should re-sample the data in the the search space and apply FFT. Some data sampling method were proposed in Reference [4].

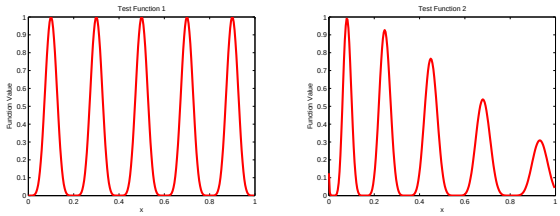
Table 1: Example functions

No.	Test function	Range	n
F1	$f(x) = sin^6(5\pi x)$	[0,1]	1
F2	$f(x) = e^{-2(ln2)(\frac{x-0.01}{0.8})^2} sin^6(5\pi(x^{0.75} - 0.05))$	[0,1]	1

Fig.3 shows two multi-modal benchmark functions, F1 and F2, shown in Table 1. F1 is with equal frequency and amplitude, and F2 is with unequal frequency and amplitude. After conducting FFT in the search space, we obtain the frequency and amplitude shown in Fig.4.

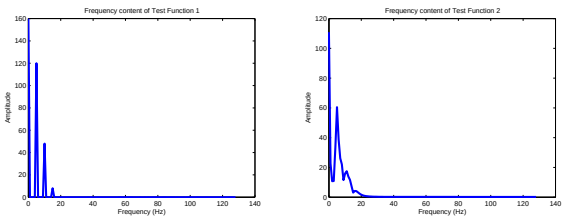
3.2.2 Computing Peak Range and Count

After we obtain the frequency (ω_i), phase (b_i) and amplitude (A_i) information in each dimensional, the PFC with



(a) (b)

Figure 3: Landscape of F1 and F2



(a) (b)

Figure 4: Frequency content of F1 and F2 by FFT

the related bigger amplitude are filtered, and we use those PFC to compute the peak regions and count [4].

$$f(x) = a_0 + \sum_{i=1}^{n}(a_i cos(\omega_i x) + c_i sin(\omega_i x) + b_i) \quad (3)$$

According to the inverse FFT, in the original space, the fitness landscape can be expressed by Eq.3. In our proposed approach, we only use the powerful amplitude as the approximation model, so it can be expressed as Eq.(4).

$$a_p cos(\omega_p x) + c_p sin(\omega_p x) = \sqrt{a_p^2 + c_p^2} cos(\omega_p x + b_p) \quad (4)$$

, and $b_p = arctan(\frac{a_p}{c_p})$. For the cos($x$) function, the minimum should local at $(\frac{T_{pi}}{2}) + b_{pi}$, $T_{pi} = \frac{2\pi}{\omega_{pi}}$, so in each dimension, the peak appears in the location within the search range around points $k_i(\frac{T_{pi}}{2*}) + b_{pi}$.

k_i are the count of peak in ith dimension, it can be calculated by Eq.(5).

$$k_i = Interity(\frac{Range_i}{T_i}) \quad (5)$$

The total count of peak in the search space is computed by Eq.(6), which is a combination of each dimensional peak number.

$$max(k_1, k_2, ..., k_n) \leq N \leq \prod_{i=1}^{n} k_i \quad (6)$$

3.2.3 Inserting Elite and Setting Search Radius

With the peak region and count information, we can roughly obtain the elite in the search space, where is the peak or is near the peak. We insert those elite into the population and replace with the individual with the related worse fitness value.

However, inserting those elite cannot ensure obtaining the final peak. Because in the some conventional EC, the parent is replaced by new offspring with high fitness value by greedy

criterion, which locates in anywhere. Based on this consideration, we should restrict the search range of the elite, so the elite search radius concept is proposed.

Search radius restricts the elite changed range in a small value to ensure its searching within the peak region. In this work, we set the search radius shown in Eq.(7), m is a tuning parameter.

$$SearchRadius_i = \frac{Range_i}{m * k_i} \tag{7}$$

4. CONCLUSION

In this paper, we briefly introduce our recent works on approximating and analyzing the fitness landscape to enhance EC search. Dimensionality reduction and Fourier transform are the main techniques involved in our studies. We will continue to investigate those methods' benefits, weaknesses, and limitations.

5. REFERENCES

[1] Ingu, T. and Takagi, H. "Accelerating a GA Convergence by Fitting a Single-Peak Function," IEEE Int. Conf. on Fuzzy Systems (FUZZ-IEEE'99), pp.1415-1420, Seoul, Korea (Aug. 22-25, 1999).

[2] Pei, Y. and Takagi, H., "Accelerating Evolutionary Computation with Elite Obtained in Projected One-dimensional Spaces", 5th Int. Conf. on Genetic and Evolutionary Computing (ICGEC2011), pp.89-92, Kinmen/Taiwan and Xiamen/China (Aug./Sept., 2011).

[3] Pei, Y. and Takagi, H., "Comparative Evaluations of Evolutionary Computation with Elite Obtained in Reduced Dimensional Spaces", 3rd Int. Conf. on Intelligent Networking and Collaborative Systems (INCoS2011), pp.35-40, Fukuoka, Japan (Nov./Dec., 2011).

[4] Pei,Y. and Takagi, H., "Fourier Analysis of the Fitness Landscape for Evolutionary Search Acceleration", 2012 IEEE Congress on Evolutionary Computation (IEEE CEC 2012), pp.2934-2940, Brisbane, Australia (June, 2012).

[5] Pei,Y. and Takagi, H., "Comparative Study on Fitness Landscape Approximation with Fourier TransformĄh", Sixth Int. Conf. on Genetic and Evolutionary Computation (ICGEC2012), pp.400-403, Kitakyushu, Japan (25-28, Aug, 2012).

[6] Pei,Y. and Takagi, H., "Fourier Niching Approach for Multi-modal Optimization", Joint Meeting of 2nd Evolutionary Computation Meeting and 6th Evolutionary Computation Frontier Meeting, pp.189-195, Toyonaka, Japan (March 9-10,2012) (in Japanese).

[7] Witkin, A. P., "Scale-space Filtering: a New Approach to Multi-scale Description", IEEE Int. Conf. on Acoustics, Speech, and Signal Processing (ICASSP1984), pp.150-153, San Diego, CA, USA (Mar., 1984).