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Hydrodynamics in shallow closed water bodies such as shallow lakes and reservoirs, which is primarily caused by wind shear acting on the water surface, is one of the key issues in the field of water environment. It is relevant to water quality and sustainable management of closed water bodies through influencing the distribution of dissolved substances, nutrients, microorganisms and plankton. However, the circulation can be influenced by the excessive growth of floating aquatic plants in summer, which may make water quality of closed water bodies become worse. Therefore, knowledge of water movements in closed water bodies in response to winds blowing over the water surface is great of significance in examining and maintaining their water quality in good condition. Aiming at that objective, this research has been done to build a two–dimensional, unsteady, laterally averaged model for simulating the wind–induced hydrodynamics in closed water bodies. This paper will present primary results regarding to development of a two–dimensional numerical model for simulating the wind–induced flow in closed water bodies. On the basis of this research, the model can be extended to simulate water quality in these water bodies.

Key words: finite volume method, numerical simulation, water environment, wind–induced flow

INTRODUCTION

Closed water bodies, such as lakes and reservoirs, are well–known as major surface water sources for life. Human life depends on them for a multitude of uses including drinking, power generation, agricultural irrigation, etc., but they may also be subject to pollution caused by these and other activities, which may degrade their water quality. Therefore, lakes and reservoirs are still the subject of great environmental concern.

One of the key issues, which has drawn attention from researchers in the field of water environment so far, is hydrodynamics in lakes and reservoirs because it is closely related to questions of environmental protection, water supply and ecology (Sündermann, 1979). Hydrodynamics in closed water bodies is primarily caused by wind shear stress acting on the water surface, affected partly by density gradients, aquatic plants and other factors. Therefore, it is complicated, and we are far from having solved all problems. For shallow closed water bodies, unlike deeper closed water bodies which are often subject to thermal stratification during certain times of the year, vertical density gradients are usually very small and can be neglect in calculation. Therefore, in this research, wind is mentioned as a principal source of mechanical energy for the hydrodynamics in closed water bodies because the wind–induced flow significantly affects water quality in the closed–water area (Mori *et al.*, 2001). In addition, floating aquatic plants are expected as one of the main factors

greatly impacting on the hydrodynamics in the shallow closed water bodies. In an effort to contribute to the more understanding of the hydrodynamics in the shallow closed water bodies, this research has been done on the basis of the incompressible Navier–Stokes equations with the simplifying assumption that the flow in the closed water bodies is well–mixed laterally in order to develop a two–dimensional, unsteady, laterally averaged model for simulating the circulation.

The Navier–Stokes equations can not be solved analytically in an original form. Therefore, they must be treated by approximation with the help of the discretization methods. In this research, the finite volume method (FVM) was applied for the discretization of the equations. To solve the discretized equations, the SIMPLE algorithm (Patankar, 1980), which has been well–known in the field of fluid dynamics, was applied and programmed in Fortran 90 (Nyhoff and Leestma, 1997; 1999) with the use of the Tri–Diagonal Matrix Algorithm (TDMA).

The developed model was applied for the simulation of circulation in an assumed lake with two cases: without and with partly existences of aquatic plants on the water surface of the lake. The preliminary results of the simulation will be presented below.

MATERIALS AND METHODS

Governing equations

In this research, the ultimate objective is to develop a κ–ε model for simulation of wind–induced flow and water quality in closed water bodies. The numerical simulations are dealt with an unsteady–state, two–dimensional, laterally averaged flow, which is governed by a

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set of partial differential equations, including the momentum and continuity equations in their primitive form as shown below:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho w)}{\partial z} = 0 \tag{1}$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho w)}{\partial x} + \frac{\partial(\rho u w)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}$$

$$+ (\mu_x \frac{\partial u}{\partial x}) + \frac{\partial}{\partial z}(\mu_z \frac{\partial u}{\partial z}) \tag{2}$$

$$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho w u)}{\partial x} - \frac{\partial}{\partial x}(\mu_x \frac{\partial w}{\partial x}) + \frac{\partial(\rho w w)}{\partial z}$$

$$- \frac{\partial}{\partial z}(\mu_z \frac{\partial w}{\partial z}) = \rho g - \frac{\partial p}{\partial z} \tag{3}$$

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho u k)}{\partial x} + \frac{\partial(\rho w k)}{\partial z} = \frac{\partial}{\partial x}(\frac{\mu_t}{\sigma_k} \frac{\partial k}{\partial x})$$

$$+ \frac{\partial}{\partial z}(\frac{\mu_t}{\sigma_k} \frac{\partial k}{\partial z}) + \rho G - \rho \varepsilon \tag{4}$$

$$\frac{\partial(\rho \varepsilon)}{\partial t} + \frac{\partial(\rho u \varepsilon)}{\partial x} + \frac{\partial(\rho w \varepsilon)}{\partial z} = \frac{\partial}{\partial x}(\frac{\mu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x})$$

$$+ \frac{\partial}{\partial z}(\frac{\mu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial z}) + C_1 \frac{\rho \varepsilon}{k} G - C_2 \frac{\rho \varepsilon^2}{k} \tag{5}$$

$$\frac{\partial C}{\partial t} + \frac{\partial(uC)}{\partial x} + \frac{\partial(wC)}{\partial z} = \frac{\partial}{\partial x}(K_h \frac{\partial C}{\partial x})$$

$$+ \frac{\partial}{\partial z}(K_v \frac{\partial C}{\partial z}) + S \tag{6}$$

$$\mu_t = \rho C_\mu \frac{k^2}{\varepsilon} \tag{7}$$

Where u and w are the velocities in the x–horizontal and z–vertical directions, respectively, ρ is water density, p is pressure, g is gravity acceleration, k and ε are the turbulent kinetic energy and kinetic dissipation rate, respectively; μ_t is kinetic eddy viscosity coefficient, $\sigma_k(=1.0)$ and $\sigma_\varepsilon(=1.3)$ are the Prandtl numbers of the kinetic energy and kinetic dissipation rate, respectively; $C_\mu(=0.09)$, $C_1(=1.44)$ and $C_2(=1.9)$ are numerical constants; C is the concentration of the constituent, K_h is horizontal turbulent diffusivity, K_v is vertical turbulent diffusivity, and S represents the external sources or sinks of the constituent.

Discretization

Equations (1) – (6) will be discretized by the finite volume method (FVM) where velocity components (u, v) are set at the cell faces while water quality variables, k, ε and pressure (p) are set at the center of the control vol-
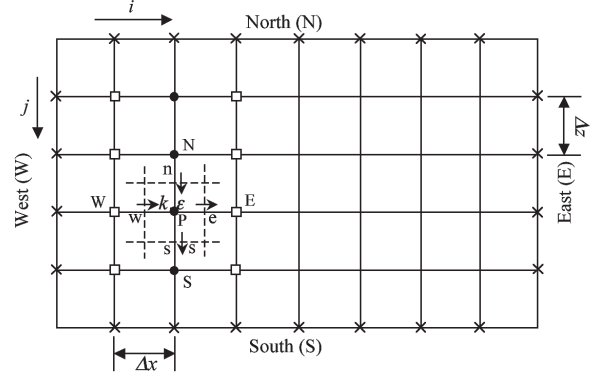

Fig. 1. Staggered grid describing control volume with flow variables for a two–dimensional situation.

ume (P) so that the system creates a staggered grid as shown in Fig. 1.

In the staggered grid, the calculated domain is divided into control volumes defined by the dashed lines. The pressure and k, are ε stored at the intersections of two unbroken grid lines. These are indicated by the capital letters P, W, E, N, and S. The u–velocity components are stored at the east and the west cell faces of the control volume and are indicated by the lower case letters e and w. The w–velocity components are located at the north and south cell faces of the control volume, which are indicated by the lower case letters n and s. Also, $\varDelta x$ and $\varDelta z$ are the sub–intervals of the calculated length and depth, respectively.

Numerical algorithm

After discretization, a system of the dicretized equations (1) – (6) will be solved numerically by using an algorithm as shown in Fig. 2.

Within this paper, as a first step, the research focused on solving the system of equations (1) to (3) in which μ_x and are μ_z temporarily calculated by using an empirical formula with assumption that they are horizontally and vertically–integrated.

In order to numerically solve equations (1) – (3), these equations were discretized by the finite–volume method shown in Fig. 1. After discretization, the discretized continuity equation becomes :

$$[(\rho u)_e - (\rho u)_w]\varDelta z + [(\rho w)_s - (\rho w)_n]\varDelta x = 0 \tag{8}$$

and the discretized u–momentum equation becomes

$$a_e^{(u)} u_e = \sum a_{nb}^{(u)} u_{nb} + b^{(u)} + (p_P - p_E)\varDelta z \tag{9}$$

with : $a_e^{(u)} = \frac{\rho_e^o \varDelta x \varDelta z}{\varDelta t} + a_E^{(u)} + a_W^{(u)} + a_S^{(u)} + a_N^{(u)}; b^{(u)}$
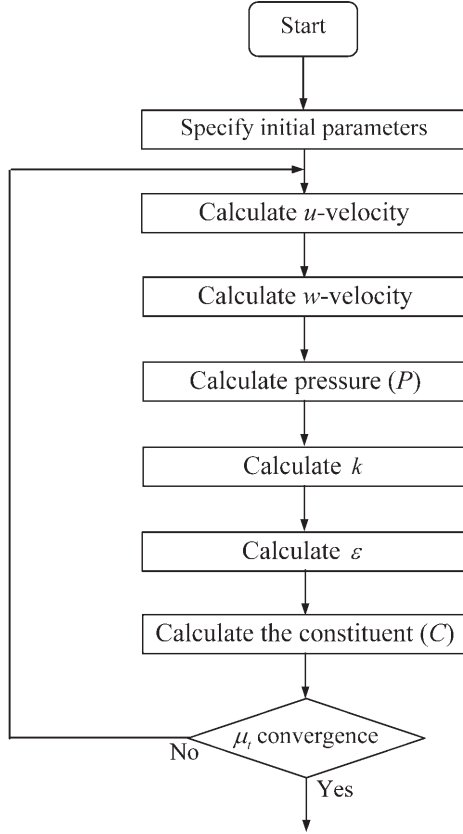
Fig. 2. The flow chart describing the algorithm for solving hydrodynamic and water quality equations.

$$= u_e^{\,o}\ \frac{\rho_e^{\,o}\varDelta x\varDelta z}{\varDelta t}$$

and the discretized w–momentum equation can be written as:

$$a_n^{(w)}w_n = \sum a_{nb}^{(w)}w_{nb} + b^{(w)} + (p_N - p_P)\varDelta x \qquad (10)$$

with : $a_n^{(w)} = \dfrac{\rho_n^{\,o}\varDelta x\varDelta z}{\varDelta t} + a_E^{(w)} + a_W^{(w)} + a_S^{(w)} + a_N^{(w)}$; $b^{(w)}$

$$= -\ \rho_n g\ \varDelta x\varDelta z + w_e^{\,o}\ \frac{\rho_e^{\,o}\varDelta x\varDelta z}{\varDelta t}$$

where $\rho_e^{\,o}$, $\rho_n^{\,o}$, $u_e^{\,o}$ and $w_n^{\,o}$ refer to the known values at time t, while all other values are the unknown values at time $t + \varDelta t$. The coefficients with superscripts (u) and (w) are the coefficients corresponding to u and w. Also, $a_{nb}^{(u)}$ and $a_{nb}^{(w)}$ refer to the neighbour coefficients $a_E^{(u)}$, $a_W^{(u)}$, $a_N^{(u)}$, $a_S^{(u)}$, $a_E^{(w)}$, $a_W^{(w)}$, $a_N^{(w)}$, and $a_S^{(w)}$, which account for the combined convection–diffusion influence at the control–volume faces of the u–cell and w–cell, respectively. The values of these coefficients are obtained on the basis of the power–law scheme (Patankar, 1980). The velocity components u_{nb} and w_{nb} are those at the neighbouring nodes outside the control volume. p_E, p_W, p_N, and p_S refer to the pressure at the east, the west, the north and the south faces of the control volume, respectively.

To solve equations (8) – (10), the SIMPLE algorithm (Patankar, 1980), which is essentially a guess–and–correct procedure for the calculation of pressure on the staggered grid introduced above, is applied. To initiate the SIMPLE calculation process, a pressure field p^* is estimated. The discretized momentum equations (9) and (10) are solved using the estimated pressure field to yield velocity components u^* and w^* as follows :

$$a_e^{(u)}u_e^* = \sum a_{nb}^{(u)}u_{nb}^* + b^{(u)} + (p_P^* - p_E^*)\varDelta z \qquad (11)$$

$$a_n^{(w)}w_n^* = \sum a_{nb}^{(w)}w_{nb}^* + b^{(w)} + (p_N^* - p_P^*)\varDelta x \qquad (12)$$

Defining the correction p' as the difference between the correct pressure field p and the estimated pressure field p^*, so that :

$$p = p^* + p' \qquad (13)$$

Similarly defining the velocity correction u' and w' to relate the correct velocities u and w to the estimated velocities u^* and w^*:

$$u = u^* + u' \qquad (14)$$

$$w = w^* + w' \qquad (15)$$

By subtracting equations (11) and (12) from (9) and (10) respectively, we obtain

$$a_e^{(u)}(u_e - u_e^*) = \sum a_{nb}^{(u)}(u_{nb} - u_{nb}^*)$$
$$+ [(p_P - p_P^*) - (p_E - p_E^*)]\varDelta z \qquad (16)$$

$$a_n^{(w)}(w_n - w_n^*) = \sum a_{nb}^{(w)}(w_{nb} - w_{nb}^*)$$
$$+ [(p_N - p_N^*) - (p_P - p_P^*)]\varDelta x \qquad (17)$$

Using the correction formulae (13) – (15), the equations (16) and (17) can be written as follows :

$$a_e^{(u)}u_e' = \sum a_{nb}^{(u)}u_{nb}' + (p_P' - p_E')\varDelta z \qquad (18)$$

$$a_n^{(w)}w_n' = \sum a_{nb}^{(w)}w_{nb}' + (p_N' - p_P')\varDelta x \qquad (19)$$

In the SIMPLE algorithm, the terms $\sum a_{nb}^{(u)}u_{nb}'$ and $\sum a_{nb}^{(w)}w_{nb}'$ are dropped to simplify equations (18) and (19) for velocity corrections. Therefore, we obtain :

$$u_e' = \frac{\varDelta z}{a_e^{(u)}}\ (p_P' - p_E') \qquad (20)$$

$$w_n' = \frac{\varDelta x}{a_n^{(w)}}\ (p_N' - p_P') \qquad (21)$$

Substituting equations (20) and (21) into (14) and (15) gives :

$$u_e = u^* + \frac{\varDelta z}{a_e^{(u)}}\ (p_P' - p_E') \qquad (22)$$

$$w_n = w^* + \frac{\varDelta x}{a_n^{(w)}}\ (p_N' - p_P') \qquad (23)$$

Similarly we have :

$$u_w = u_w^* + \frac{\Delta z}{a_w^{(u)}} \ (p_W' - p_P') \tag{24}$$

$$w_s = w_s^* + \frac{\Delta x}{a_s^{(w)}} \ (p_P' - p_S') \tag{25}$$

Substituting equations (22) – (25) into the discretized continuity equation (8), we draw the pressure–correction equation which plays an important part in the SIMPLE algorithm as follows :

$$a_P p_P' = a_E p_E' + a_W p_W' + a_N p_N' + a_S p_S'$$
$$+ [(\rho u^*)_w - (\rho u^*)_e] \Delta z + [(\rho u^*)_n - (\rho u^*)_s] \Delta x \tag{26}$$

where : $a_P = a_E + a_W + a_N + a_S$

$$a_E = \rho_e + \frac{(\Delta z)^2}{a_e} \ ; \qquad a_W = \rho_w + \frac{(\Delta z)^2}{a_w}$$

$$a_N = \rho_n + \frac{(\Delta x)^2}{a_n} \ ; \qquad a_S = \rho_S + \frac{(\Delta x)^2}{a_s}$$

The procedure of the SIMPLE algorithm, which is applied at each time step, is summarized in a flow chart as shown in Fig. 3.

In Fig. 3, α_p is the pressure under–relaxation factor. The other symbols have already been explained above.

The SIMPLE algorithm is extended to transient calculations to cover the desired time period. A flow chart for the calculation of unsteady–state flows will be shown in Fig. 4.

In Fig. 4, t is time, Δt is the calculated time step, and t_{max} is the desired time period. Other symbols were already explained above.

In performing the SIMPLE algorithm to solve the flow variables, the TDMA (Tri–Diagonal Matrix Algorithm) or Thomas' algorithm, which has become almost standard for the treatment of tridiagonal systems of equations (Anderson, 1995), is employed by the line–by–line method.

To solve a general two–dimensional discretized equation for water variables (ϕ) with a form such as

$$a_P \phi_P = a_W \phi_W + a_E \phi_E + a_S \phi_S + a_N \phi_N + b \tag{27}$$

The equation can be re–arranged in the form

$$-a_S \phi_S + a_P \phi_P - a_N \phi_N = a_W \phi_W + a_E \phi_E + b \tag{28}$$

Considering the discretized equations for the grid points along a chosen line, we can see that they contain the variables at the grid points (shown by the symbol □ in Fig. 1) along the two neighboring lines (ϕ_W and ϕ_E). If the right–hand side of the equation (28) is assumed to be temporarily known, the equations along the chosen line would look like one–dimensional equations and could be solved by the TDMA. Subsequently the calcu-
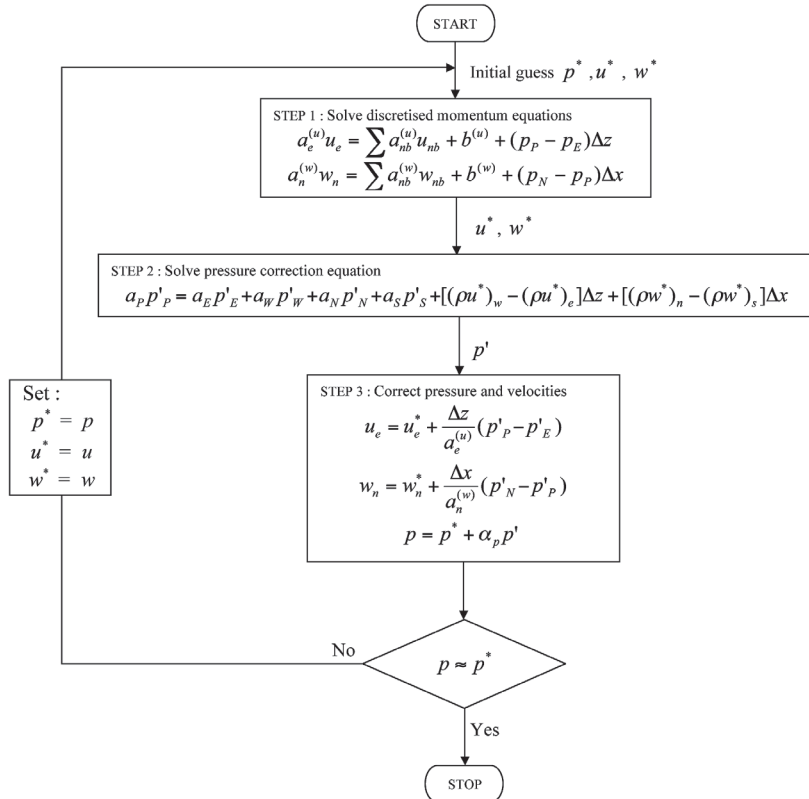

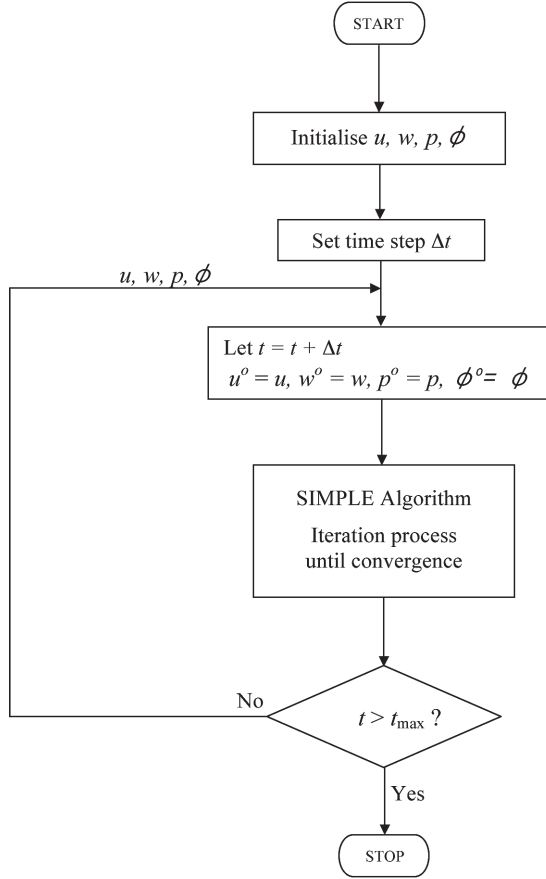**Fig. 3.** The flow chart of the SIMPLE algorithm at each time step.

Fig. 4. The flow chart for the unsteady flow calculation with application of the SIMPLE algorithm.

Fig. 5. The grid arrangement describing velocity components at boundaries.

lation is moved to the next line. If we sweep from the west to the east, the values of ϕ_W to the west of point P are known from the calculation performed on the previous line. Values of ϕ_E to its east, however, are unknown, so the solution process must be iterative. The line–by–line calculation procedure is repeated several times until a converged solution is obtained. The method is illustrated in Fig. 1 above (Versteeg and Malalasekera, 1995) where the points designated by the symbol (•) are the points at which the values are calculated, the points designated by the symbol (□) are considered to be temporarily known, and the points designated by the symbol (✕) are known boundary values.

Boundary Conditions

Boundary conditions at the bottom, the walls and the water surface area covered by floating aquatic plants of the reservoir are taken as the no–slip boundary condition ($u = w = 0$).

On the free water surface, w is set to zero, and u is calculated from the following relationship (Cole and Buchak, 1995) :

$$\tau_S = C_D \rho_a U^2 = C_D \rho_w U_S^2 \qquad (29)$$

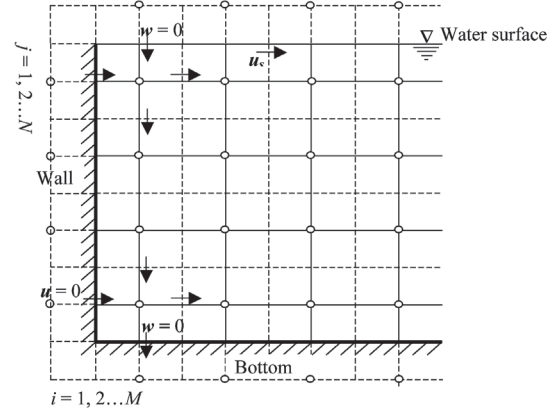where τ_S is surface shear stress at water surface, C_D is drag coefficient, ρ_a is air density, U is the wind speed at 10 m above the water surface, ρ_w is water density, U_S is surface velocity in water. From equation (29), U_S is calculated as follows :

$$u_S = \sqrt{\frac{\rho_a}{\rho_w}} \cdot U \approx 0.03U \qquad (30)$$

Equation (30) is known as the " 3% rule " (Cole and Buchak, 1995). The arrangement of velocity components at boundaries is illustrated in Fig. 5.

RESULTS AND DISCUSSION

Data requirement for the model

To calculate the velocities of flow field induced by wind, the data of wind speed must be collected. In addition to wind speed data, the data on reservoir size such as the average depth of reservoir (H), the average length of reservoir (L) must be also collected.

In this model, the vertical eddy viscosity (μ_z) is calculated from the following formula (Bengtsson, 1973) :

$$\mu_z = \rho c h U \qquad (31)$$

where h is average depth of the reservoir, c is constant $= 2 \times 10^{-5}$, the other symbols have already been explained above.

The horizontal eddy viscosity (μ_x) was defined according to the vertical eddy viscosity : $\mu_x = E\mu_z$, with E is an constant. In this research, E was found equal to 100.

Model application and initial results

In this research, assuming that there is a small natural lake (no exchange with external waters) with the average water depth of 2.2 m and the average width of 80 m. Assuming that wind velocity at the height of 10 m above the lake water surface is 2 m/s in the calculated period. In the vertical plane of calculation, the lake width was divided into intervals of 2 m, and the depth of lake was divided into intervals of 0.2 m to create cells of (2 m ✕ 0.2 m). The calculated domain was arranged

in the staggered grid illustrated in Fig. 1 above. The time step for the calculation was selected as 0.5 hour. In closed water bodies, the problem of the excessive growth of floating aquatic plants usually occurs in summer. Therefore, the simulations were performed under both cases including without and with floating aquatic plants on the water surface. In the case with floating aquatic plants, it is assumed that the floating aquatic plants would grow gradually from the sides toward the center of the lake, and the assumed covering percentage of aquatic plants is 50 % of water surface area. The results of simulation for these cases were illustrated in graphics by Stanford Software as described in Figs. 6–8 below

Fig. 6 & 7 visually characterize the wind–induced flows in the lake under the cases without and with 50 % of floating aquatic plant coverage on the water surface. They indicated that wind can help the lake circulate by mixing waters in upper layers into the lower layers. In closed water bodies, the problems such as thermal stratification in summer, lack of dissolved oxygen in the bottom layer easily occur. Therefore, this physical process (the circulation) is very important because it can help narrow the difference in concentration of water quality
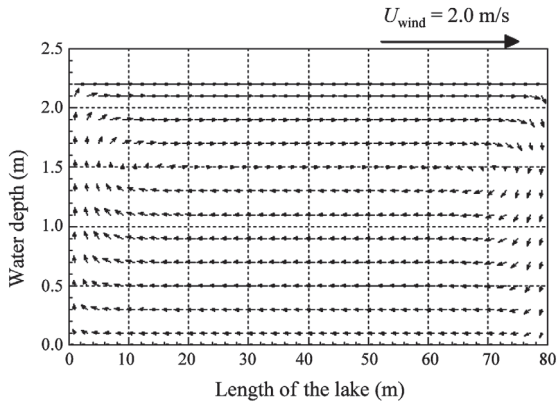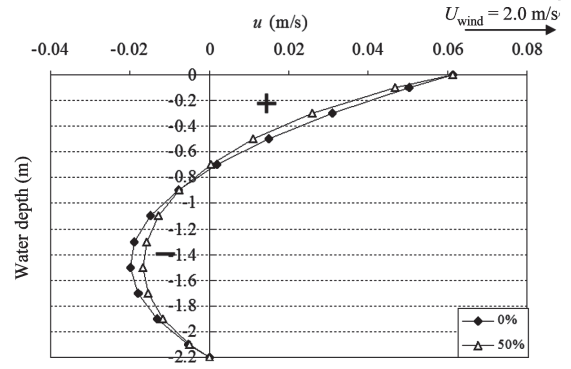

Fig. 8. The profile of u velocity distribution corresponding to cases with and without aquatic plants covering on the water surface.

variables between upper layers and lower layers. In other words, it positively influences the distribution of water temperature, dissolved oxygen and other water quality variables in the closed water bodies. It can be clearly seen that floating aquatic plants significantly affected the wind–induced flow pattern in the lake. Fig. 6 indicates that the circulation in the lake covers almost the cross section of the lake, corresponding to the case without the coverage of aquatic plants. As the aquatic plants grow from the sides toward the center of the lake, covering 50% of water surface area as described in Fig. 7, the region of the circulation would be narrowed from the sides to the center of the lake, creating water regions under the floating aquatic plant area without participating in the circulation. Because these water regions were not involved in the circulation, water quality problems could occur in these regions such as the lack of dissolved oxygen (DO) which in turn can cause other problems of water quality.

Fig. 8 indicates the change in u–velocity distribution in the center of the lake, corresponding to cases without and with the aquatic plant covering on the water surface. It can be easily seen that in the center of the lake, the flow current in the layers in the top one–third of the lake has the same direction as the wind, while the flow current in the bottom two–thirds of the lake has a direction inverse to that of the wind. The flow rate of the circulation can be calculated by following formula:

$$q_r = \int_0^h u\,\mathrm{d}z \tag{32}$$

where q_r is the flow rate in the circulation per unit width (cm^2/s), h is the depth of the wind–induced flow (cm), u is water velocity in the circulation.

It can be seen from Fig. 8 and formula (32) that in the case with 50% of the aquatic plants' coverage, the flow rate of the circulation is smaller than that of the circulation in the case without the coverage of the aquatic plants. That is because: The total wind force (F) acting on the free water surface area of the lake is defined : $F = \tau_s L$, (where τ_s is surface shear stress) caused by wind at water surface, which is defined by equation

Fig. 6. Wind–induced flow in the lake without the covering of aquatic plants under wind speed equal to 2 m/s.
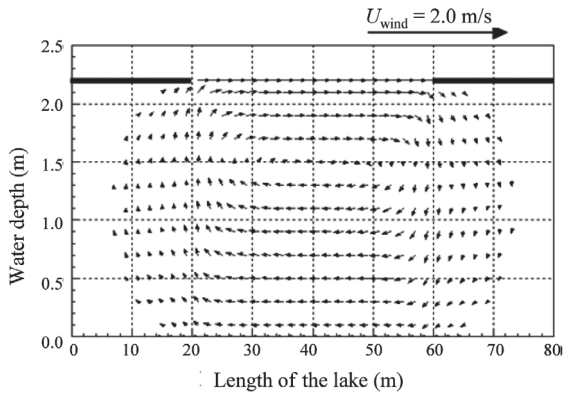

Fig. 7. Wind–induced flow in the lake with 50% of the water surface area assumed to be covered by aquatic plants under wind speed equal to 2 m/s.

(29), and L is the length of the free water surface. When the floating aquatic plant area increases, the total wind force (F) acting on the remaining free water surface decreases due to the decrease in the free water surface length (L). As a result, the rate of the wind–induced flow decreases as the area of the floating aquatic plants increases. In other words, it is the wind force (F) which is the reason causing the wind–induced flow in closed water bodies while floating aquatic plants could decrease this force by narrowing the free water surface.

CONCLUSIONS

From the results of simulation presented above, some conclusions can be drawn as follows :

1. In closed water bodies, such as lakes and reservoirs, where the exchange with external waters is usually small, wind plays an important part in the circulation of the closed water bodies by mixing the upper waters down through the water column. The higher wind speed is, the better the mixing capability of a closed water body is.

2. Floating aquatic plants can significantly affect on both the pattern and the rate of the wind–induced flow in the closed water bodies through narrowing the free water surface to decrease the wind force (F). The wind–induced flow, in turn, can affect water quality in the closed water bodies by influencing the distribution of their water quality variables.

3. As floating aquatic plants grow from the sides toward the center of a closed water body, the region of the circulation tends to be narrowed from the sides to the center of the closed water body, creating water regions of no–circulation under floating aquatic plant area, which may cause water quality problems such as the lack of dissolved oxygen in the bottom layer of these regions.

4. This research presented the initial results in the first step of development of k–ε model in which eddy viscosity coefficients (μ_x and μ_z) are averaged for the whole lake. In order to gain more realistic results of calculation, in the next step, this model will be extended to incorporate k and ε variables for simulation of wind–induced flow and water quality in lakes.

ACKNOWLEDGMENTS

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