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Abstract

The popular matching problem introduced by Abraham, Irving, Kavitha, and Mehlhorn is one of assignment problems in strategic situations. It is known that a given instance of this problem may admit no popular matching. For coping with such instances, Wu, Lin, Wang, and Chao introduced the popular condensation problem whose goal is to transform a given instance so that it has a popular matching by deleting a minimum number of agents. In this paper, we consider a matroid generalization of the popular condensation problem, and give a polynomial-time algorithm for this problem.

1 Introduction

In this paper, we consider a problem of assigning applicants to posts in strategic situations. Such problems occur, e.g., when a school assigns students to lectures or a firm assigns workers to tasks. For such strategic assignment problems, several solution concepts have been introduced. For example, Gärdenfors [5] introduced the concept of popularity. Intuitively speaking, if a matching M is popular, then there exists no other matching N such that more applicants prefer N to M than prefer M to N. Using the concept of popularity, Abraham, Irving, Kavitha, and Mehlhorn [1] introduced the popular matching problem, and presented a linear-time algorithm for this problem. Several extensions of the popular matching problem have been investigated. For example, Manlove and Sng [10] considered a many-to-one variant of the popular matching problem, Mestre [11] considered a weighted variant, and Sng and Manlove [14] considered a weighted many-to-one variant. Furthermore, Kamiyama [6] introduced a matroid generalization of the popular matching problem, and gave a polynomial-time algorithm for this problem. This matroid generalization can represent a many-to-one variant of the popular matching problem presented by Manlove and Sng [10] and the popular matching with laminar capacity constraints (see [6] for details).

Unfortunately, it is known [1] that a given instance of the popular matching problem may admit no popular matching. For coping with such instances, several alternative solutions were presented. For example, Kavitha and Nasre [7] considered the problem of copying several posts so that a given instance admits a popular matching. Furthermore, Kavitha, Nasre, and Nimbhorkar [8] considered the problem of augmenting several posts with minimum costs. These problems have been shown to be hard in general. Wu, Lin, Wang, and Chao [15] considered the problem of transforming the set of agents so that a given instance admits a popular matching. More precisely, they introduced the popular condensation problem whose goal is to transform a given instance so that it has a popular matching by deleting a minimum number of agents,

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and gave a polynomial-time algorithm for this problem. In this paper, we consider a matroid generalization of the popular condensation problem (i.e., the popular condensation problem in the matroid setting presented by Kamiyama [6]), and give a polynomial-time algorithm for this problem. Our algorithm can be regarded as a matroid generalization of the algorithm presented by Wu, Lin, Wang, and Chao [15].

The rest of this paper is organized as follows. In Section 2, we give the formal definition of our problem. In Section 3, we review a characterization of the existence of a popular matching under matroid constraints presented by Kamiyama [6]. In Section 4, we give our algorithm, and prove its correctness.

2 Preliminaries

We denote by $\mathbb{Z}_+$ the set of non-negative integers. For each subset X and each element x, we define $X + x := X \cup \{x\}$ and $X - x := X \setminus \{x\}$.

An ordered pair $\mathcal{M} = (U, \mathcal{I})$ is called a *matroid*, if U is a finite set and $\mathcal{I}$ is a nonempty family of subsets of U satisfying the following conditions.

(I1) If $I \in \mathcal{I}$ and $J \subseteq I$, then $J \in \mathcal{I}$.

(I2) If $I, J \in \mathcal{I}$ and $|I| < |J|$, then there exists an element u in $J \setminus I$ with $I + u \in \mathcal{I}$

2.1 Problem formulation

Throughout this paper, we are given a finite simple bipartite graph $G = (V, E)$ in which V is partitioned into two subsets A and P, and each edge in E connects a vertex in A and a vertex in P. We call a vertex in A an *applicant*, and a vertex in P a *post*. We denote by (a, p) the edge in E between an applicant a in A and a post p in P. For each vertex v in V and each subset M of E, we define $M(v)$ as the set of edges in M incident to v. Furthermore, for each subset X of A and each subset M of E, we write $M(X)$ instead of $\cup_{a \in X} M(a)$.

In addition, we are given an injective function $\pi\colon E \to \mathbb{Z}_+$. That is, $\pi(e) \neq \pi(e')$ for every distinct edges e, e' in E. Intuitively speaking, π represents preference lists of applicants. For each applicant a in A and each edges e, e' in $E(a)$, if $\pi(e) > \pi(e')$, then a prefers e to e'. Since π is injective, it represents "strict" preference lists of applicants. Without loss of generality, we assume that for each applicant a in A, there exists a post p_a in P such that $E(p_a) = \{(a, p_a)\}$ and

$$\forall e \in E(a) - (a, p_a)\colon\ \pi(e) > \pi((a, p_a)).$$

Furthermore, for each post p in P, we are given a matroid $\mathcal{M}_p = (E(p), \mathcal{I}_p)$. We assume that for each applicant a in A, $\{(a, p_a)\} \in \mathcal{I}_{p_a}$. Furthermore, we assume that for each applicant a in A, there exists a post p in $P - p_a$ such that $(a, p) \in E$ and $\{(a, p)\} \in \mathcal{I}_p$.

For each subset X of A, a subset M of E is called a *matching with respect to* X, if it satisfies the following two conditions.

- For every applicant a in A,
$$|M(a)| = \begin{cases} 1 & \text{if } a \in X \\ 0 & \text{if } a \notin X. \end{cases}$$
- For every post p in P, we have $M(p) \in \mathcal{I}_p$.

For each subset X of A, each matching M with respect to X, and each applicant a in X, we denote by $\mu_M(a)$ the unique edge in $M(a)$. Let M, N be matchings with respect to some subset X of A. We denote by $\mathsf{pre}_M(N)$ the number of applicants a in X with

$$\pi(\mu_N(a)) > \pi(\mu_M(a)),$$

i.e., $\mathsf{pre}_M(N)$ represents the number of applicants that prefer N to M.

A matching M with respect to a subset X of A is called a *popular matching with respect to* X, if

$$\mathsf{pre}_N(M) \geq \mathsf{pre}_M(N)$$

for every matching N with respect to X. That is, there exists no other matching N with respect to X such that more applicants in A prefer N to M than prefer M to N. A subset X of A is called a *popular condensation*, if there exists a popular matching with respect to X. Notice that for every applicant a in A, $\{a\}$ is a popular condensation. The goal of the *popular condensation problem under matroid constraints* is to find a maximum-size popular condensation.

2.2 Matroids

In this subsection, we give properties of matroids that will be used in the sequel.

Let $\mathcal{M} = (U, \mathcal{I})$ be a matroid. A subset I in $\mathcal{I}$ is called an *independent set in* $\mathcal{M}$. For each subset X of U, a subset B of X is called a *base of* X *in* $\mathcal{M}$, if B is an inclusion-wise maximal subset of X that is an independent set in $\mathcal{M}$. We call a base of U in $\mathcal{M}$ a *base in* $\mathcal{M}$. It follows from the condition **(I2)** that for each subset X of U, every two bases of X in $\mathcal{M}$ have the same size, which is called the *rank of* X *in* $\mathcal{M}$ and denoted by $r_{\mathcal{M}}(X)$. It is known [12, Lemma 1.3.1] that

$$\forall X, Y \subseteq U\colon\ r_{\mathcal{M}}(X) + r_{\mathcal{M}}(Y) \geq r_{\mathcal{M}}(X \cup Y) + r_{\mathcal{M}}(X \cap Y). \tag{1}$$

It follows from (1) that for every subsets X, Y, Z of U such that $X \subseteq Y$ and $Z \cap Y = \emptyset$,

$$r_{\mathcal{M}}(Y \cup Z) - r_{\mathcal{M}}(Y) \leq r_{\mathcal{M}}(X \cup Z) - r_{\mathcal{M}}(X). \tag{2}$$

Furthermore, it follows from (1) and the non-negativity of $r_{\mathcal{M}}(\cdot)$ that

$$\forall X, Y \subseteq U\colon\ r_{\mathcal{M}}(X \cup Y) \leq r_{\mathcal{M}}(X) + r_{\mathcal{M}}(Y). \tag{3}$$

Let S be a subset of U. Define

$$\begin{aligned} \mathcal{I}|S &:= \{X \subseteq S \mid X \in \mathcal{I}\}, \\ \mathcal{M}|S &:= (S, \mathcal{I}|S). \end{aligned}$$

It is not difficult to see that $\mathcal{M}|S$ is a matroid and $r_{\mathcal{M}|S}(X) = r_{\mathcal{M}}(X)$ for every subset X of S. Define a function $r'\colon 2^{U \setminus S} \to \mathbb{Z}_+$ by

$$r'(X) := r_{\mathcal{M}}(X \cup S) - r_{\mathcal{M}}(S).$$

In addition, we define

$$\begin{aligned} \mathcal{I}/S &:= \{X \subseteq U \setminus S \mid r'(X) = |X|\}, \\ \mathcal{M}/S &:= (U \setminus S, \mathcal{I}/S). \end{aligned}$$

It is known [12, Proposition 3.1.6] that $\mathcal{M}/S$ is a matroid and $r_{\mathcal{M}/S}(X) = r'(X)$ for each subset X of $U \setminus S$.

Let $\mathcal{M}_1 = (U_1, \mathcal{I}_1)$ and $\mathcal{M}_2 = (U_2, \mathcal{I}_2)$ be matroids with $U_1 \cap U_2 = \emptyset$. Define

$$\begin{aligned}\mathcal{I}_1 \oplus \mathcal{I}_2 &:= \{X \subseteq U_1 \cup U_2 \mid X \cap U_1 \in \mathcal{I}_1,\ X \cap U_2 \in \mathcal{I}_2\},\\ \mathcal{M}_1 \oplus \mathcal{M}_2 &:= (U_1 \cup U_2, \mathcal{I}_1 \oplus \mathcal{I}_2).\end{aligned}$$

It is known [12, Proposition 4.2.12] that $\mathcal{M}_1 \oplus \mathcal{M}_2$ is a matroid and

$$\forall X \subseteq U_1 \cup U_2 \colon\ r_{\mathcal{M}_1 \oplus \mathcal{M}_2}(X) = r_{\mathcal{M}_1}(X \cap U_1) + r_{\mathcal{M}_2}(X \cap U_2). \tag{4}$$

Let $\mathcal{M}_1 = (U, \mathcal{I}_1)$ and $\mathcal{M}_2 = (U, \mathcal{I}_2)$ be matroids on the same ground set U. A subset I of U is called a *common independent set of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, if $I \in \mathcal{I}_1 \cap \mathcal{I}_2$. It is known [2, 9] that we can find a maximum-size common independent set of $\mathcal{M}_1$ and $\mathcal{M}_2$ in $O(n^3\gamma)$ time, where $n := |U|$ and γ is the time required to check whether $I - u + u'$ and $I + u'$ belong to $\mathcal{I}_1$ (or $\mathcal{I}_2$) for each independent set I in $\mathcal{M}_1$ (or $\mathcal{M}_2$) and each elements u in I and u' in $U \setminus I$ (see also [13, Section 41.2]). Furthermore, the following characterization of the size of a maximum-size common independent set is known.

Theorem 1 (Edmonds [3])**.** *For each matroids $\mathcal{M}_1 = (U, \mathcal{I}_1)$ and $\mathcal{M}_2 = (U, \mathcal{I}_2)$, the size of a maximum-size common independent set of $\mathcal{M}_1$ and $\mathcal{M}_2$ is equal to*

$$\min_{X \subseteq U} (r_{\mathcal{M}_1}(X) + r_{\mathcal{M}_2}(U \setminus X)).$$

The following lemmas will be used in the sequel.

Lemma 2 (see, e.g., [6])**.** *Let $\mathcal{M} = (U, \mathcal{I})$, S, and B be a matroid, a subset of U, and a base in $\mathcal{M}|S$, respectively. Then, for every subset X of $U \setminus S$, X is an independent set in $\mathcal{M}/S$ if and only if $X \cup B$ is an independent set in $\mathcal{M}$.*

Lemma 3. *Let $\mathcal{M} = (U, \mathcal{I})$ be a matroid. For each subsets X, Y, Z of U such that $X \subseteq Y$ and $Z \cap Y = \emptyset$, we have*

$$r_{\mathcal{M}/X}(Z) - r_{\mathcal{M}/Y}(Z) \le r_{\mathcal{M}}(Y) - r_{\mathcal{M}}(X).$$

Proof. It follows from the definition of $\mathcal{M}/X$ and $\mathcal{M}/Y$ that

$$\begin{aligned}r_{\mathcal{M}/X}(Z) &= r_{\mathcal{M}}(Z \cup X) - r_{\mathcal{M}}(X),\\ r_{\mathcal{M}/Y}(Z) &= r_{\mathcal{M}}(Z \cup Y) - r_{\mathcal{M}}(Y).\end{aligned}$$

This lemma follows from this and the monotonicity of $r_{\mathcal{M}}(\cdot)$. $\square$

3 Characterization

For each applicant a in A, we define the *f-edge $f(a)$ of a* as the unique element in

$$\arg\max\{\pi((a,p)) \mid (a,p) \in E(a),\ \{(a,p)\} \in \mathcal{I}_p\}.$$

For each subset X of A and each post p in P, we denote by $\Gamma_{X,p}$ the set of edges (a,p) in $E(p)$ such that $a \in X$ and $(a,p) = f(a)$. For each subset X of A and each applicant a in X, we define the *s-edge $s_X(a)$ of a* as the unique edge in

$$\arg\max\{\pi((a,p)) \mid (a,p) \in E(a) - f(a),\ \{(a,p)\} \in \mathcal{I}_p/\Gamma_{X,p}\}.$$

Notice that $s_X(a)$ is well-defined because there exists the post p_a. For each subset X of A, we define the *reduced edge set* Π_X by

$$\Pi_X := \{f(a), s_X(a) \mid a \in X\}.$$

For each subset X of A, we define a matroid $\mathcal{A}_X = (\Pi_X, \mathcal{I}_X)$ by

$$\mathcal{I}_X := \{M \subseteq \Pi_X \mid \forall a \in X\colon\ |M(a)| \leq 1\}.$$

For each subset X of A and each post p in P, we define

$$\mathcal{P}_{X,p} := (\mathcal{M}_p|\Gamma_{X,p} \oplus \mathcal{M}_p/\Gamma_{X,p})|\Pi_X(p).$$

Furthermore, for each subset X of A, we define

$$\mathcal{P}_X := \bigoplus_{p \in P} \mathcal{P}_{X,p}.$$

For simplicity, we define $s(\cdot) := s_A(\cdot)$, $\Gamma_p := \Gamma_{A,p}$, $\Pi := \Pi_A$, $\mathcal{A} := \mathcal{A}_A$, and $\mathcal{P} := \mathcal{P}_A$.

The following characterization of a popular condensation plays an important role. Precisely speaking, Kamiyama [6] proved Theorem 4 in the case of $X = A$, but Theorem 4 for a general subset X of A can be proved in the same way.

Theorem 4 (Kamiyama [6])**.** *For each subset X of A, X is a popular condensation if and only if there exists a common independent set M of $\mathcal{A}_X$ and $\mathcal{P}_X$ with $|M| = |X|$.*

The following lemmas will be used in the sequel.

Lemma 5. *There exists a subset D of A such that*

$$|D| + r_{\mathcal{P}}(\Pi(A \setminus D)) = \min_{F \subseteq \Pi}(r_{\mathcal{A}}(F) + r_{\mathcal{P}}(\Pi \setminus F)). \tag{5}$$

Proof. Let F be a minimizer of the right-hand side of (5). If $\Pi(X) = F$ for some subset X of A, then the proof is done. Let X be the set of applicants a in A such that there exists an edge e in F with $e \in \Pi(a)$, and assume that there exists an edge e' in $\Pi(X)$ with $e' \notin F$. Clearly, we have

$$\begin{aligned} r_{\mathcal{A}}(F + e') &= r_{\mathcal{A}}(F)\ (= |X|), \\ r_{\mathcal{P}}(\Pi \setminus (F + e')) &\leq r_{\mathcal{P}}(\Pi \setminus F). \end{aligned}$$

This implies that there exists a subset D of A satisfying (5). □

Lemma 6. *For each subset X of A, if there exists a common independent set M of $\mathcal{A}_X$ and $\mathcal{P}_X$ with $|M| = |X|$, then*

$$\forall Y \subseteq X\colon\ |Y| \leq r_{\mathcal{P}_X}(\Pi_X(Y)).$$

Proof. It follows from Theorem 1 that

$$\forall F \subseteq \Pi_X\colon\ |X| \leq r_{\mathcal{A}_X}(F) + r_{\mathcal{P}_X}(\Pi_X \setminus F). \tag{6}$$

Let Y be a subset of X. It follows from (6) with $F = \Pi_X(X \setminus Y)$ that

$$\begin{aligned} |X| &\leq r_{\mathcal{A}_X}(\Pi_X(X \setminus Y)) + r_{\mathcal{P}_X}(\Pi_X \setminus \Pi_X(X \setminus Y)) \\ &= |X \setminus Y| + r_{\mathcal{P}_X}(\Pi_X(Y)), \end{aligned}$$

which completes the proof. □

4 Algorithm

Our algorithm **PCuMC** for the popular condensation problem under matroid constraints can be described as follows.

Algorithm PCuMC

Step 1. Find a maximum-size common independent set M of $\mathcal{A}$ and $\mathcal{P}$.

Step 2. Output the set Δ of applicants a in A with $M(a) \neq \emptyset$.

End of Algorithm

From now on, we prove the correctness of the algorithm **PCuMC**. Let M be the maximum-size common independent set found in **Step 1** of the algorithm **PCuMC**, and we denote by Δ the output of the algorithm of **PCuMC**.

We first prove that Δ is a popular condensation.

Lemma 7. $\Pi_\Delta = \Pi(\Delta)$.

Proof. It suffices to prove that for every applicant a in Δ, we have $s_\Delta(a) = s(a)$. For proving this, we prove that for every post p, there exists a base B in $\mathcal{M}_p|\Gamma_p$ with $B \subseteq \Gamma_{\Delta,p}$. If there exists such a base B, then B is also a base in $\mathcal{M}_p|\Gamma_{\Delta,p}$. Thus, the above statement follows from Lemma 2.

Let p be a post in P, and assume that any base in $\mathcal{M}_p|\Gamma_p$ is not a subset of $\Gamma_{\Delta,p}$. Since $M \cap \Gamma_p$ is an independent set in $\mathcal{M}_p|\Gamma_p$, there exists a base B in $\mathcal{M}_p|\Gamma_p$ with $M \cap \Gamma_p \subseteq B$. The above assumption implies that there exists an edge $(a, p) \in B$ and $a \notin \Delta$. It follows from the definition of **Step 2** that $M(a) = \emptyset$, which implies that $M + (a, p)$ is an independent set of $\mathcal{A}$. Furthermore, it follows from the condition **(I1)** that $(M \cap \Gamma_p) + (a, p)$ is an independent set in $\mathcal{M}_p|\Gamma_p$, and thus $M + (a, p)$ is an independent set in $\mathcal{P}$. Since $(a, p) \notin M$, these facts contradict the fact that M is a maximum-size common independent set of $\mathcal{A}$ and $\mathcal{P}$. This completes the proof. □

Lemma 8. Δ *is a popular condensation.*

Proof. It follows from Theorem 4 that if M is a common independent set of $\mathcal{A}_\Delta$ and $\mathcal{P}_\Delta$, then the proof is done. It follows from Lemma 7 that M is a subset of Π_Δ. Furthermore, M is clearly an independent set of $\mathcal{A}_\Delta$. What remains is to prove that M is an independent set of $\mathcal{P}_\Delta$.

Let p be a post of P. For proving that $M(p)$ is an independent set in $\mathcal{P}_{\Delta,p}$, we first prove that $M \cap \Gamma_{\Delta,p}$ is an independent set in $\mathcal{M}_p|\Gamma_{\Delta,p}$. It follows from the definition of **Step 2** that

$$M \cap \Gamma_{\Delta,p} = M \cap \Gamma_p. \tag{7}$$

Furthermore, $M \cap \Gamma_p$ is an independent set in $\mathcal{M}_p$. These facts implies that $M \cap \Gamma_{\Delta,p}$ is an independent set in $\mathcal{M}_p|\Gamma_{\Delta,p}$.

Next we prove that $M(p) \setminus \Gamma_{\Delta,p}$ is an independent set in $\mathcal{M}_p/\Gamma_{\Delta,p}$. It follows from (7) that $M(p) \setminus \Gamma_{\Delta,p} = M(p) \setminus \Gamma_p$. Moreover, in the same way as in the proof of Lemma 7, we can prove that there exists a base B in $\mathcal{M}_p|\Gamma_p$ with $B \subseteq \Gamma_{\Delta,p}$. Since $\Gamma_{\Delta,p}$ is a subset of Γ_p, B is also a base in $\mathcal{M}_p|\Gamma_{\Delta,p}$. Thus, since $M(p) \setminus \Gamma_p$ is an independent set in $\mathcal{M}_p/\Gamma_p$, it follows from these facts and Lemma 2 that $M(p) \setminus \Gamma_{\Delta,p}$ is an independent set in $\mathcal{M}_p/\Gamma_{\Delta,p}$. □

Next we prove that Δ is a maximum-size popular condensation. Let D be a subset of A satisfying (5) in Lemma 5, and define $Q := A \setminus D$.

Lemma 9. $|A \setminus \Delta| = |Q| - r_{\mathcal{P}}(\Pi(Q))$.

Proof. It follows from $|\Delta| = |M|$ and Theorem 1 that

$$|\Delta| = |M| = |D| + r_{\mathcal{P}}(\Pi(A \setminus D)) = |A| - |Q| + r_{\mathcal{P}}(\Pi(Q)),$$

which completes the proof. $\square$

Lemma 10. *For every popular condensation Ω, we have $|A \setminus \Omega| \geq |Q| - r_{\mathcal{P}}(\Pi(Q))$.*

Proof. Define $\Omega_0 := A \setminus \Omega$, $\Omega_1 := Q \cap \Omega_0$, and $\Omega_2 := \Omega_0 \setminus \Omega_1$. Notice that $Q \setminus \Omega_1$ is a subset of Ω. Thus, it follows from Theorem 4 and Lemma 6 that

$$|Q \setminus \Omega_1| \leq r_{\mathcal{P}_\Omega}(\Pi_\Omega(Q \setminus \Omega_1)).$$

It follows from this that if

$$r_{\mathcal{P}_\Omega}(\Pi_\Omega(Q \setminus \Omega_1)) \leq r_{\mathcal{P}}(\Pi(Q)) + |\Omega_2|, \tag{8}$$

then the proof is done because $|A \setminus \Omega| = |\Omega_1| + |\Omega_2|$.

Let p be a post in P. Define F_1 as the set of edges (a,p) in Γ_p such that $a \in Q$ or $a \in \Omega_2$. In addition, define F_2 as the set of edges (a,p) in $\Pi(p)$ such that $a \in Q$ and $(a,p) = s(a)$. Notice that

$$F_1 \cup F_2 = (\Pi(Q) \cap \Pi(p)) \cup \{(a,p) \in \Gamma_p \mid a \in \Omega_2\}.$$

It follows from this and (3) that

$$\begin{aligned} r_{\mathcal{P}_p}(F_1 \cup F_2) &= r_{\mathcal{P}_p}((\Pi(Q) \cap \Pi(p)) \cup \{(a,p) \in \Gamma_p \mid a \in \Omega_2\}) \\ &\leq r_{\mathcal{P}_p}(\Pi(Q) \cap \Pi(p)) + r_{\mathcal{P}_p}(\{(a,p) \in \Gamma_p \mid a \in \Omega_2\}) \\ &\leq r_{\mathcal{P}_p}(\Pi(Q) \cap \Pi(p)) + |\{(a,p) \in \Gamma_p \mid a \in \Omega_2\}| \\ &= r_{\mathcal{P}_p}(\Pi(Q) \cap \Pi(p)) + |\{a \in \Omega_2 \mid f(a) \in \Pi(p)\}|. \end{aligned} \tag{9}$$

Define F_1' as the set of edges (a,p) in $\Gamma_{\Omega,p}$ with $a \in Q \setminus \Omega_1$. In addition, define F_2' as the set of edges (a,p) in $\Pi_\Omega(p)$ such that $a \in Q \setminus \Omega_1$ and $(a,p) = s_\Omega(a)$. Notice that

$$F_1' \cup F_2' = \Pi_\Omega(Q \setminus \Omega_1) \cap \Pi_\Omega(p). \tag{10}$$

It follows from (4), (9), and (10) that if

$$r_{\mathcal{P}_{\Omega,p}}(F_1' \cup F_2') \leq r_{\mathcal{P}_p}(F_1 \cup F_2), \tag{11}$$

then (8) follows and the proof is done because

$$\begin{aligned} r_{\mathcal{P}_\Omega}(\Pi_\Omega(Q \setminus \Omega_1)) &= \sum_{p \in P} r_{\mathcal{P}_{\Omega,p}}(\Pi_\Omega(Q \setminus \Omega_1) \cap \Pi_\Omega(p)) \\ &= \sum_{p \in P} r_{\mathcal{P}_{\Omega,p}}(F_1' \cup F_2') \\ &\leq \sum_{p \in P} r_{\mathcal{P}_p}(F_1 \cup F_2) \\ &\leq \sum_{p \in P} r_{\mathcal{P}_p}(\Pi(Q) \cap \Pi(p)) + \sum_{p \in P} |\{a \in \Omega_2 \mid f(a) \in \Pi(p)\}| \\ &= r_{\mathcal{P}}(\Pi(Q)) + |\Omega_2|. \end{aligned}$$

It follows from (4) that

$$\begin{aligned} r_{\mathcal{P}_p}(F_1 \cup F_2) &= r_{\mathcal{M}_p}(F_1) + r_{\mathcal{M}_p/\Gamma_p}(F_2), \\ r_{\mathcal{P}_{\Omega,p}}(F_1' \cup F_2') &= r_{\mathcal{M}_p}(F_1') + r_{\mathcal{M}_p/\Gamma_{\Omega,p}}(F_2'). \end{aligned} \tag{12}$$

For every edge (a, p) in $F_2' \setminus F_2$, we have

$$\pi(f(a)) > \pi((a, p)) > \pi(s(a)).$$

This implies that $(a, p) \notin \mathcal{I}_p/\Gamma_p$ for every edge (a, p) in $F_2' \setminus F_2$. Thus, we have

$$r_{\mathcal{M}_p/\Gamma_p}(F_2) = r_{\mathcal{M}_p/\Gamma_p}(F_2 \cup F_2'). \tag{13}$$

Furthermore, it follows from the monotonicity of $r_{\mathcal{M}_p/\Gamma_{\Omega,p}}(\cdot)$ that

$$r_{\mathcal{M}_p/\Gamma_{\Omega,p}}(F_2') \le r_{\mathcal{M}_p/\Gamma_{\Omega,p}}(F_2 \cup F_2'). \tag{14}$$

It follows from $\Gamma_{\Omega,p} \subseteq \Gamma_p$ and Lemma 3 that

$$r_{\mathcal{M}_p/\Gamma_{\Omega,p}}(F_2 \cup F_2') - r_{\mathcal{M}_p/\Gamma_p}(F_2 \cup F_2') \le r_{\mathcal{M}_p}(\Gamma_p) - r_{\mathcal{M}_p}(\Gamma_{\Omega,p}). \tag{15}$$

Since $\Gamma_p \setminus \Gamma_{\Omega,p} = F_1 \setminus F_1'$ and $F_1' \subseteq \Gamma_{\Omega,p}$, it follows from (2) that

$$r_{\mathcal{M}_p}(\Gamma_p) - r_{\mathcal{M}_p}(\Gamma_{\Omega,p}) \le r_{\mathcal{M}_p}(F_1) - r_{\mathcal{M}_p}(F_1'). \tag{16}$$

It follows from (13), (14), (15), and (16) that

$$\begin{aligned} r_{\mathcal{M}_p/\Gamma_{\Omega,p}}(F_2') - r_{\mathcal{M}_p/\Gamma_p}(F_2) &\le r_{\mathcal{M}_p/\Gamma_{\Omega,p}}(F_2 \cup F_2') - r_{\mathcal{M}_p/\Gamma_p}(F_2 \cup F_2') \\ &\le r_{\mathcal{M}_p}(\Gamma_p) - r_{\mathcal{M}_p}(\Gamma_{\Omega,p}) \\ &\le r_{\mathcal{M}_p}(F_1) - r_{\mathcal{M}_p}(F_1'). \end{aligned} \tag{17}$$

It follows from (12) and (17) that

$$\begin{aligned} r_{\mathcal{P}_{\Omega,p}}(F_1' \cup F_2') - r_{\mathcal{P}_p}(F_1 \cup F_2) &= r_{\mathcal{M}_p}(F_1') - r_{\mathcal{M}_p}(F_1) + r_{\mathcal{M}_p/\Gamma_{\Omega,p}}(F_2') - r_{\mathcal{M}_p/\Gamma_p}(F_2) \\ &\le r_{\mathcal{M}_p}(F_1') - r_{\mathcal{M}_p}(F_1) + r_{\mathcal{M}_p}(F_1) - r_{\mathcal{M}_p}(F_1') \\ &= 0. \end{aligned}$$

This implies (11), which completes the proof. □

Theorem 11. *The algorithm* **PCuMC** *correctly solves the popular condensation problem under matroid constraints.*

Proof. This theorem follows from Lemmas 8, 9 and 10. □

Here we analyze the time complexity of the algorithm **PMuMC**. Define $m := |E|$, and we assume that for each post p in P, each independent set I in $\mathcal{M}_p$, and each edges e in I and e' in $E(p) \setminus I$, we can check in $O(\gamma)$ time whether $I - e + e'$ and $I + e'$ belong to $\mathcal{I}_p$. It follows from Lemma 2 that once we find a base in $\mathcal{M}_p|\Gamma_p$, for each independent set I in $\mathcal{M}_p/\Gamma_p$ and each edges e in I and e' in $E(p) \setminus (I \cup \Gamma_p)$, we can check in $O(\gamma)$ time whether $I - e + e'$ and $I + e'$ belong to $\mathcal{I}_p/\Gamma_p$. Thus, for each independent set I in $\mathcal{P}$ and each edges e in I and e' in $E \setminus I$, we can check in $O(\gamma)$ time whether $I - e + e'$ and $I + e'$ are independent sets in $\mathcal{P}$. This implies that the time complexity of the algorithm **PCuMC** is $O(m^3\gamma)$.

Next we consider a weighted variant of the popular condensation problem under matroid constraints. More precisely, in this problem, we are given a weight function $w\colon A \to \mathbb{Z}_+$. The goal is to find a maximum-size popular condensation X maximizing $\sum_{a \in X} w(a)$. This problem can be solved as follows. For each edge (a, p) in Π, we define the weight of (a, p) as $w(a)$. This weighted variant can be solved by finding a maximum-size common independent set of $\mathcal{A}$ and $\mathcal{P}$ with maximum-weight at **Step 1** of the algorithm **PCuMC**. It is known [4] that this problem can be solved in $O(m^3\gamma)$ time (see also [13, Section 41.3]). Thus, this weighted variant can be solved in $O(m^3\gamma)$ time.

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