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# The Popular Condensation Problem under Matroid Constraints 

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#### Abstract

The popular matching problem introduced by Abraham, Irving, Kavitha, and Mehlhorn is one of assignment problems in strategic situations. It is known that a given instance of this problem may admit no popular matching. For coping with such instances, Wu, Lin, Wang, and Chao introduced the popular condensation problem whose goal is to transform a given instance so that it has a popular matching by deleting a minimum number of agents. In this paper, we consider a matroid generalization of the popular condensation problem, and give a polynomial-time algorithm for this problem.


## 1 Introduction

In this paper, we consider a problem of assigning applicants to posts in strategic situations. Such problems occur, e.g., when a school assigns students to lectures or a firm assigns workers to tasks. For such strategic assignment problems, several solution concepts have been introduced. For example, Gärdenfors [5] introduced the concept of popularity. Intuitively speaking, if a matching $M$ is popular, then there exists no other matching $N$ such that more applicants prefer $N$ to $M$ than prefer $M$ to $N$. Using the concept of popularity, Abraham, Irving, Kavitha, and Mehlhorn [1] introduced the popular matching problem, and presented a linear-time algorithm for this problem. Several extensions of the popular matching problem have been investigated. For example, Manlove and Sng [10] considered a many-to-one variant of the popular matching problem, Mestre [11] considered a weighted variant, and Sng and Manlove [14] considered a weighted many-to-one variant. Furthermore, Kamiyama [6] introduced a matroid generalization of the popular matching problem, and gave a polynomial-time algorithm for this problem. This matroid generalization can represent a many-to-one variant of the popular matching problem presented by Manlove and Sng [10] and the popular matching with laminar capacity constraints (see [6] for details).

Unfortunately, it is known [1] that a given instance of the popular matching problem may admit no popular matching. For coping with such instances, several alternative solutions were presented. For example, Kavitha and Nasre [7] considered the problem of copying several posts so that a given instance admits a popular matching. Furthermore, Kavitha, Nasre, and Nimbhorkar [8] considered the problem of augmenting several posts with minimum costs. These problems have been shown to be hard in general. Wu, Lin, Wang, and Chao [15] considered the problem of transforming the set of agents so that a given instance admits a popular matching. More precisely, they introduced the popular condensation problem whose goal is to transform a given instance so that it has a popular matching by deleting a minimum number of agents,

[^0]and gave a polynomial-time algorithm for this problem. In this paper, we consider a matroid generalization of the popular condensation problem (i.e., the popular condensation problem in the matroid setting presented by Kamiyama [6]), and give a polynomial-time algorithm for this problem. Our algorithm can be regarded as a matroid generalization of the algorithm presented by Wu, Lin, Wang, and Chao [15].

The rest of this paper is organized as follows. In Section 2, we give the formal definition of our problem. In Section 3, we review a characterization of the existence of a popular matching under matroid constraints presented by Kamiyama [6]. In Section 4, we give our algorithm, and prove its correctness.

## 2 Preliminaries

We denote by $\mathbb{Z}_{+}$the set of non-negative integers. For each subset $X$ and each element $x$, we define $X+x:=X \cup\{x\}$ and $X-x:=X \backslash\{x\}$.

An ordered pair $\mathcal{M}=(U, \mathcal{I})$ is called a matroid, if $U$ is a finite set and $\mathcal{I}$ is a nonempty family of subsets of $U$ satisfying the following conditions.
(I1) If $I \in \mathcal{I}$ and $J \subseteq I$, then $J \in \mathcal{I}$.
(I2) If $I, J \in \mathcal{I}$ and $|I|<|J|$, then there exists an element $u$ in $J \backslash I$ with $I+u \in \mathcal{I}$

### 2.1 Problem formulation

Throughout this paper, we are given a finite simple bipartite graph $G=(V, E)$ in which $V$ is partitioned into two subsets $A$ and $P$, and each edge in $E$ connects a vertex in $A$ and a vertex in $P$. We call a vertex in $A$ an applicant, and a vertex in $P$ a post. We denote by $(a, p)$ the edge in $E$ between an applicant $a$ in $A$ and a post $p$ in $P$. For each vertex $v$ in $V$ and each subset $M$ of $E$, we define $M(v)$ as the set of edges in $M$ incident to $v$. Furthermore, for each subset $X$ of $A$ and each subset $M$ of $E$, we write $M(X)$ instead of $\cup_{a \in X} M(a)$.

In addition, we are given an injective function $\pi: E \rightarrow \mathbb{Z}_{+}$. That is, $\pi(e) \neq \pi\left(e^{\prime}\right)$ for every distinct edges $e, e^{\prime}$ in $E$. Intuitively speaking, $\pi$ represents preference lists of applicants. For each applicant $a$ in $A$ and each edges $e, e^{\prime}$ in $E(a)$, if $\pi(e)>\pi\left(e^{\prime}\right)$, then $a$ prefers $e$ to $e^{\prime}$. Since $\pi$ is injective, it represents "strict" preference lists of applicants. Without loss of generality, we assume that for each applicant $a$ in $A$, there exists a post $p_{a}$ in $P$ such that $E\left(p_{a}\right)=\left\{\left(a, p_{a}\right)\right\}$ and

$$
\forall e \in E(a)-\left(a, p_{a}\right): \pi(e)>\pi\left(\left(a, p_{a}\right)\right)
$$

Furthermore, for each post $p$ in $P$, we are given a matroid $\mathcal{M}_{p}=\left(E(p), \mathcal{I}_{p}\right)$. We assume that for each applicant $a$ in $A,\left\{\left(a, p_{a}\right)\right\} \in \mathcal{I}_{p_{a}}$. Furthermore, we assume that for each applicant $a$ in $A$, there exists a post $p$ in $P-p_{a}$ such that $(a, p) \in E$ and $\{(a, p)\} \in \mathcal{I}_{p}$.

For each subset $X$ of $A$, a subset $M$ of $E$ is called a matching with respect to $X$, if it satisfies the following two conditions.

- For every applicant $a$ in $A$,

$$
|M(a)|= \begin{cases}1 & \text { if } a \in X \\ 0 & \text { if } a \notin X\end{cases}
$$

- For every post $p$ in $P$, we have $M(p) \in \mathcal{I}_{p}$.

For each subset $X$ of $A$, each matching $M$ with respect to $X$, and each applicant $a$ in $X$, we denote by $\mu_{M}(a)$ the unique edge in $M(a)$. Let $M, N$ be matchings with respect to some subset $X$ of $A$. We denote by pre $_{M}(N)$ the number of applicants $a$ in $X$ with

$$
\pi\left(\mu_{N}(a)\right)>\pi\left(\mu_{M}(a)\right)
$$

i.e., $\operatorname{pre}_{M}(N)$ represents the number of applicants that prefer $N$ to $M$.

A matching $M$ with respect to a subset $X$ of $A$ is called a popular matching with respect to $X$, if

$$
\operatorname{pre}_{N}(M) \geq \operatorname{pre}_{M}(N)
$$

for every matching $N$ with respect to $X$. That is, there exists no other matching $N$ with respect to $X$ such that more applicants in $A$ prefer $N$ to $M$ than prefer $M$ to $N$. A subset $X$ of $A$ is called a popular condensation, if there exists a popular matching with respect to $X$. Notice that for every applicant $a$ in $A,\{a\}$ is a popular condensation. The goal of the popular condensation problem under matroid constraints is to find a maximum-size popular condensation.

### 2.2 Matroids

In this subsection, we give properties of matroids that will be used in the sequel.
Let $\mathcal{M}=(U, \mathcal{I})$ be a matroid. A subset $I$ in $\mathcal{I}$ is called an independent set in $\mathcal{M}$. For each subset $X$ of $U$, a subset $B$ of $X$ is called a base of $X$ in $\mathcal{M}$, if $B$ is an inclusion-wise maximal subset of $X$ that is an independent set in $\mathcal{M}$. We call a base of $U$ in $\mathcal{M}$ a base in $\mathcal{M}$. It follows from the condition (I2) that for each subset $X$ of $U$, every two bases of $X$ in $\mathcal{M}$ have the same size, which is called the rank of $X$ in $\mathcal{M}$ and denoted by $r_{\mathcal{M}}(X)$. It is known [12, Lemma 1.3.1] that

$$
\begin{equation*}
\forall X, Y \subseteq U: r_{\mathcal{M}}(X)+r_{\mathcal{M}}(Y) \geq r_{\mathcal{M}}(X \cup Y)+r_{\mathcal{M}}(X \cap Y) \tag{1}
\end{equation*}
$$

It follows from (1) that for every subsets $X, Y, Z$ of $U$ such that $X \subseteq Y$ and $Z \cap Y=\emptyset$,

$$
\begin{equation*}
r_{\mathcal{M}}(Y \cup Z)-r_{\mathcal{M}}(Y) \leq r_{\mathcal{M}}(X \cup Z)-r_{\mathcal{M}}(X) \tag{2}
\end{equation*}
$$

Furthermore, it follows from (1) and the non-negativity of $r_{\mathcal{M}}(\cdot)$ that

$$
\begin{equation*}
\forall X, Y \subseteq U: r_{\mathcal{M}}(X \cup Y) \leq r_{\mathcal{M}}(X)+r_{\mathcal{M}}(Y) \tag{3}
\end{equation*}
$$

Let $S$ be a subset of $U$. Define

$$
\begin{aligned}
\mathcal{I} \mid S & :=\{X \subseteq S \mid X \in \mathcal{I}\} \\
\mathcal{M} \mid S & :=(S, \mathcal{I} \mid S)
\end{aligned}
$$

It is not difficult to see that $\mathcal{M} \mid S$ is a matroid and $r_{\mathcal{M} \mid S}(X)=r_{\mathcal{M}}(X)$ for every subset $X$ of $S$. Define a function $r^{\prime}: 2^{U \backslash S} \rightarrow \mathbb{Z}_{+}$by

$$
r^{\prime}(X):=r_{\mathcal{M}}(X \cup S)-r_{\mathcal{M}}(S)
$$

In addition, we define

$$
\begin{aligned}
\mathcal{I} / S & :=\left\{X \subseteq U \backslash S\left|r^{\prime}(X)=|X|\right\}\right. \\
\mathcal{M} / S & :=(U \backslash S, \mathcal{I} / S)
\end{aligned}
$$

It is known [12, Proposition 3.1.6] that $\mathcal{M} / S$ is a matroid and $r_{\mathcal{M} / S}(X)=r^{\prime}(X)$ for each subset $X$ of $U \backslash S$.

Let $\mathcal{M}_{1}=\left(U_{1}, \mathcal{I}_{1}\right)$ and $\mathcal{M}_{2}=\left(U_{2}, \mathcal{I}_{2}\right)$ be matroids with $U_{1} \cap U_{2}=\emptyset$. Define

$$
\begin{aligned}
\mathcal{I}_{1} \oplus \mathcal{I}_{2} & :=\left\{X \subseteq U_{1} \cup U_{2} \mid X \cap U_{1} \in \mathcal{I}_{1}, X \cap U_{2} \in \mathcal{I}_{2}\right\} \\
\mathcal{M}_{1} \oplus \mathcal{M}_{2} & :=\left(U_{1} \cup U_{2}, \mathcal{I}_{1} \oplus \mathcal{I}_{2}\right)
\end{aligned}
$$

It is known [12, Proposition 4.2.12] that $\mathcal{M}_{1} \oplus \mathcal{M}_{2}$ is a matroid and

$$
\begin{equation*}
\forall X \subseteq U_{1} \cup U_{2}: r_{\mathcal{M}_{1} \oplus \mathcal{M}_{2}}(X)=r_{\mathcal{M}_{1}}\left(X \cap U_{1}\right)+r_{\mathcal{M}_{2}}\left(X \cap U_{2}\right) \tag{4}
\end{equation*}
$$

Let $\mathcal{M}_{1}=\left(U, \mathcal{I}_{1}\right)$ and $\mathcal{M}_{2}=\left(U, \mathcal{I}_{2}\right)$ be matroids on the same ground set $U$. A subset $I$ of $U$ is called a common independent set of $\mathcal{M}_{1}$ and $\mathcal{M}_{2}$, if $I \in \mathcal{I}_{1} \cap \mathcal{I}_{2}$. It is known [2, 9] that we can find a maximum-size common independent set of $\mathcal{M}_{1}$ and $\mathcal{M}_{2}$ in $O\left(n^{3} \gamma\right)$ time, where $n:=|U|$ and $\gamma$ is the time required to check whether $I-u+u^{\prime}$ and $I+u^{\prime}$ belong to $\mathcal{I}_{1}$ (or $\mathcal{I}_{2}$ ) for each independent set $I$ in $\mathcal{M}_{1}\left(\right.$ or $\left.\mathcal{M}_{2}\right)$ and each elements $u$ in $I$ and $u^{\prime}$ in $U \backslash I$ (see also [13, Section 41.2]). Furthermore, the following characterization of the size of a maximum-size common independent set is known.

Theorem 1 (Edmonds [3]). For each matroids $\mathcal{M}_{1}=\left(U, \mathcal{I}_{1}\right)$ and $\mathcal{M}_{2}=\left(U, \mathcal{I}_{2}\right)$, the size of a maximum-size common independent set of $\mathcal{M}_{1}$ and $\mathcal{M}_{2}$ is equal to

$$
\min _{X \subseteq U}\left(r_{\mathcal{M}_{1}}(X)+r_{\mathcal{M}_{2}}(U \backslash X)\right)
$$

The following lemmas will be used in the sequel.
Lemma 2 (see, e.g., [6]). Let $\mathcal{M}=(U, \mathcal{I}), S$, and $B$ be a matroid, a subset of $U$, and a base in $\mathcal{M} \mid S$, respectively. Then, for every subset $X$ of $U \backslash S, X$ is an independent set in $\mathcal{M} / S$ if and only if $X \cup B$ is an independent set in $\mathcal{M}$.

Lemma 3. Let $\mathcal{M}=(U, \mathcal{I})$ be a matroid. For each subsets $X, Y, Z$ of $U$ such that $X \subseteq Y$ and $Z \cap Y=\emptyset$, we have

$$
r_{\mathcal{M} / X}(Z)-r_{\mathcal{M} / Y}(Z) \leq r_{\mathcal{M}}(Y)-r_{\mathcal{M}}(X)
$$

Proof. It follows from the definition of $\mathcal{M} / X$ and $\mathcal{M} / Y$ that

$$
\begin{aligned}
& r_{\mathcal{M} / X}(Z)=r_{\mathcal{M}}(Z \cup X)-r_{\mathcal{M}}(X), \\
& r_{\mathcal{M} / Y}(Z)=r_{\mathcal{M}}(Z \cup Y)-r_{\mathcal{M}}(Y)
\end{aligned}
$$

This lemma follows from this and the monotonicity of $r_{\mathcal{M}}(\cdot)$.

## 3 Characterization

For each applicant $a$ in $A$, we define the $f$-edge $f(a)$ of $a$ as the unique element in

$$
\arg \max \left\{\pi((a, p)) \mid(a, p) \in E(a),\{(a, p)\} \in \mathcal{I}_{p}\right\}
$$

For each subset $X$ of $A$ and each post $p$ in $P$, we denote by $\Gamma_{X, p}$ the set of edges $(a, p)$ in $E(p)$ such that $a \in X$ and $(a, p)=f(a)$. For each subset $X$ of $A$ and each applicant $a$ in $X$, we define the $s$-edge $s_{X}(a)$ of $a$ as the unique edge in

$$
\arg \max \left\{\pi((a, p)) \mid(a, p) \in E(a)-f(a),\{(a, p)\} \in \mathcal{I}_{p} / \Gamma_{X, p}\right\}
$$

Notice that $s_{X}(a)$ is well-defined because there exists the post $p_{a}$. For each subset $X$ of $A$, we define the reduced edge set $\Pi_{X}$ by

$$
\Pi_{X}:=\left\{f(a), s_{X}(a) \mid a \in X\right\} .
$$

For each subset $X$ of $A$, we define a matroid $\mathcal{A}_{X}=\left(\Pi_{X}, \mathcal{I}_{X}\right)$ by

$$
\mathcal{I}_{X}:=\left\{M \subseteq \Pi_{X}|\forall a \in X:|M(a)| \leq 1\}\right.
$$

For each subset $X$ of $A$ and each post $p$ in $P$, we define

$$
\mathcal{P}_{X, p}:=\left(\mathcal{M}_{p} \mid \Gamma_{X, p} \oplus \mathcal{M}_{p} / \Gamma_{X, p}\right) \mid \Pi_{X}(p)
$$

Furthermore, for each subset $X$ of $A$, we define

$$
\mathcal{P}_{X}:=\bigoplus_{p \in P} \mathcal{P}_{X, p}
$$

For simplicity, we define $s(\cdot):=s_{A}(\cdot), \Gamma_{p}:=\Gamma_{A, p}, \Pi:=\Pi_{A}, \mathcal{A}:=\mathcal{A}_{A}$, and $\mathcal{P}:=\mathcal{P}_{A}$.
The following characterization of a popular condensation plays an important role. Precisely speaking, Kamiyama [6] proved Theorem 4 in the case of $X=A$, but Theorem 4 for a general subset $X$ of $A$ can be proved in the same way.

Theorem 4 (Kamiyama [6]). For each subset $X$ of $A, X$ is a popular condensation if and only if there exists a common independent set $M$ of $\mathcal{A}_{X}$ and $\mathcal{P}_{X}$ with $|M|=|X|$.

The following lemmas will be used in the sequel.
Lemma 5. There exists a subset $D$ of $A$ such that

$$
\begin{equation*}
|D|+r_{\mathcal{P}}(\Pi(A \backslash D))=\min _{F \subseteq \Pi}\left(r_{\mathcal{A}}(F)+r_{\mathcal{P}}(\Pi \backslash F)\right) \tag{5}
\end{equation*}
$$

Proof. Let $F$ be a minimizer of the right-hand side of (5). If $\Pi(X)=F$ for some subset $X$ of $A$, then the proof is done. Let $X$ be the set of applicants $a$ in $A$ such that there exists an edge $e$ in $F$ with $e \in \Pi(a)$, and assume that there exists an edge $e^{\prime}$ in $\Pi(X)$ with $e^{\prime} \notin F$. Clearly, we have

$$
\begin{gathered}
r_{\mathcal{A}}\left(F+e^{\prime}\right)=r_{\mathcal{A}}(F)(=|X|) \\
r_{\mathcal{P}}\left(\Pi \backslash\left(F+e^{\prime}\right)\right) \leq r_{\mathcal{P}}(\Pi \backslash F)
\end{gathered}
$$

This implies that there exists a subset $D$ of $A$ satisfying (5).
Lemma 6. For each subset $X$ of $A$, if there exists a common independent set $M$ of $\mathcal{A}_{X}$ and $\mathcal{P}_{X}$ with $|M|=|X|$, then

$$
\forall Y \subseteq X:|Y| \leq r_{\mathcal{P}_{X}}\left(\Pi_{X}(Y)\right)
$$

Proof. It follows from Theorem 1 that

$$
\begin{equation*}
\forall F \subseteq \Pi_{X}:|X| \leq r_{\mathcal{A}_{X}}(F)+r_{\mathcal{P}_{X}}\left(\Pi_{X} \backslash F\right) \tag{6}
\end{equation*}
$$

Let $Y$ be a subset of $X$. It follows from (6) with $F=\Pi_{X}(X \backslash Y)$ that

$$
\begin{aligned}
|X| & \leq r_{\mathcal{A}_{X}}\left(\Pi_{X}(X \backslash Y)\right)+r_{\mathcal{P}_{X}}\left(\Pi_{X} \backslash \Pi_{X}(X \backslash Y)\right) \\
& =|X \backslash Y|+r_{\mathcal{P}_{X}}\left(\Pi_{X}(Y)\right)
\end{aligned}
$$

which completes the proof.

## 4 Algorithm

Our algorithm PCuMC for the popular condensation problem under matroid constraints can be described as follows.

## Algorithm PCuMC

Step 1. Find a maximum-size common independent set $M$ of $\mathcal{A}$ and $\mathcal{P}$.
Step 2. Output the set $\Delta$ of applicants $a$ in $A$ with $M(a) \neq \emptyset$.

## End of Algorithm

From now on, we prove the correctness of the algorithm PCuMC. Let $M$ be the maximumsize common independent set found in Step 1 of the algorithm PCuMC, and we denote by $\Delta$ the output of the algorithm of PCuMC.

We first prove that $\Delta$ is a popular condensation.
Lemma 7. $\Pi_{\Delta}=\Pi(\Delta)$.
Proof. It suffices to prove that for every applicant $a$ in $\Delta$, we have $s_{\Delta}(a)=s(a)$. For proving this, we prove that for every post $p$, there exists a base $B$ in $\mathcal{M}_{p} \mid \Gamma_{p}$ with $B \subseteq \Gamma_{\Delta, p}$. If there exists such a base $B$, then $B$ is also a base in $\mathcal{M}_{p} \mid \Gamma_{\Delta, p}$. Thus, the above statement follows from Lemma 2.

Let $p$ be a post in $P$, and assume that any base in $\mathcal{M}_{p} \mid \Gamma_{p}$ is not a subset of $\Gamma_{\Delta, p}$. Since $M \cap \Gamma_{p}$ is an independent set in $\mathcal{M}_{p} \mid \Gamma_{p}$, there exists a base $B$ in $\mathcal{M}_{p} \mid \Gamma_{p}$ with $M \cap \Gamma_{p} \subseteq B$. The above assumption implies that there exists an edge $(a, p) \in B$ and $a \notin \Delta$. It follows from the definition of Step 2 that $M(a)=\emptyset$, which implies that $M+(a, p)$ is an independent set of $\mathcal{A}$. Furthermore, it follows from the condition (I1) that $\left(M \cap \Gamma_{p}\right)+(a, p)$ is an independent set in $\mathcal{M}_{p} \mid \Gamma_{p}$, and thus $M+(a, p)$ is an independent set in $\mathcal{P}$. Since $(a, p) \notin M$, these facts contradict the fact that $M$ is a maximum-size common independent set of $\mathcal{A}$ and $\mathcal{P}$. This completes the proof.

Lemma 8. $\Delta$ is a popular condensation.
Proof. It follows from Theorem 4 that if $M$ is a common independent set of $\mathcal{A}_{\Delta}$ and $\mathcal{P}_{\Delta}$, then the proof is done. It follows from Lemma 7 that $M$ is a subset of $\Pi_{\Delta}$. Furthermore, $M$ is clearly an independent set of $\mathcal{A}_{\Delta}$. What remains is to prove that $M$ is an independent set of $\mathcal{P}_{\Delta}$.

Let $p$ be a post of $P$. For proving that $M(p)$ is an independent set in $\mathcal{P}_{\Delta, p}$, we first prove that $M \cap \Gamma_{\Delta, p}$ is an independent set in $\mathcal{M}_{p} \mid \Gamma_{\Delta, p}$. It follows from the definition of Step 2 that

$$
\begin{equation*}
M \cap \Gamma_{\Delta, p}=M \cap \Gamma_{p} . \tag{7}
\end{equation*}
$$

Furthermore, $M \cap \Gamma_{p}$ is an independent set in $\mathcal{M}_{p}$. These facts implies that $M \cap \Gamma_{\Delta, p}$ is an independent set in $\mathcal{M}_{p} \mid \Gamma_{\Delta, p}$.

Next we prove that $M(p) \backslash \Gamma_{\Delta, p}$ is an independent set in $\mathcal{M}_{p} / \Gamma_{\Delta, p}$. It follows from (7) that $M(p) \backslash \Gamma_{\Delta, p}=M(p) \backslash \Gamma_{p}$. Moreover, in the same way as in the proof of Lemma 7, we can prove that there exists a base $B$ in $\mathcal{M}_{p} \mid \Gamma_{p}$ with $B \subseteq \Gamma_{\Delta, p}$. Since $\Gamma_{\Delta, p}$ is a subset of $\Gamma_{p}, B$ is also a base in $\mathcal{M}_{p} \mid \Gamma_{\Delta, p}$. Thus, since $M(p) \backslash \Gamma_{p}$ is an independent set in $\mathcal{M}_{p} / \Gamma_{p}$, it follows from these facts and Lemma 2 that $M(p) \backslash \Gamma_{\Delta, p}$ is an independent set in $\mathcal{M}_{p} / \Gamma_{\Delta, p}$.

Next we prove that $\Delta$ is a maximum-size popular condensation. Let $D$ be a subset of $A$ satisfying (5) in Lemma 5, and define $Q:=A \backslash D$.

Lemma 9. $|A \backslash \Delta|=|Q|-r_{\mathcal{P}}(\Pi(Q))$.
Proof. It follows from $|\Delta|=|M|$ and Theorem 1 that

$$
|\Delta|=|M|=|D|+r_{\mathcal{P}}(\Pi(A \backslash D))=|A|-|Q|+r_{\mathcal{P}}(\Pi(Q)),
$$

which completes the proof.
Lemma 10. For every popular condensation $\Omega$, we have $|A \backslash \Omega| \geq|Q|-r_{\mathcal{P}}(\Pi(Q))$.
Proof. Define $\Omega_{0}:=A \backslash \Omega, \Omega_{1}:=Q \cap \Omega_{0}$, and $\Omega_{2}:=\Omega_{0} \backslash \Omega_{1}$. Notice that $Q \backslash \Omega_{1}$ is a subset of $\Omega$. Thus, it follows from Theorem 4 and Lemma 6 that

$$
\left|Q \backslash \Omega_{1}\right| \leq r_{\mathcal{P}_{\Omega}}\left(\Pi_{\Omega}\left(Q \backslash \Omega_{1}\right)\right) .
$$

It follows from this that if

$$
\begin{equation*}
r_{\mathcal{P}_{\Omega}}\left(\Pi_{\Omega}\left(Q \backslash \Omega_{1}\right)\right) \leq r_{\mathcal{P}}(\Pi(Q))+\left|\Omega_{2}\right|, \tag{8}
\end{equation*}
$$

then the proof is done because $|A \backslash \Omega|=\left|\Omega_{1}\right|+\left|\Omega_{2}\right|$.
Let $p$ be a post in $P$. Define $F_{1}$ as the set of edges $(a, p)$ in $\Gamma_{p}$ such that $a \in Q$ or $a \in \Omega_{2}$. In addition, define $F_{2}$ as the set of edges $(a, p)$ in $\Pi(p)$ such that $a \in Q$ and $(a, p)=s(a)$. Notice that

$$
F_{1} \cup F_{2}=(\Pi(Q) \cap \Pi(p)) \cup\left\{(a, p) \in \Gamma_{p} \mid a \in \Omega_{2}\right\} .
$$

It follows from this and (3) that

$$
\begin{align*}
r_{\mathcal{P}_{p}}\left(F_{1} \cup F_{2}\right) & =r_{\mathcal{P}_{p}}\left((\Pi(Q) \cap \Pi(p)) \cup\left\{(a, p) \in \Gamma_{p} \mid a \in \Omega_{2}\right\}\right) \\
& \leq r_{\mathcal{P}_{p}}(\Pi(Q) \cap \Pi(p))+r_{\mathcal{P}_{p}}\left(\left\{(a, p) \in \Gamma_{p} \mid a \in \Omega_{2}\right\}\right) \\
& \leq r_{\mathcal{P}_{p}}(\Pi(Q) \cap \Pi(p))+\left|\left\{(a, p) \in \Gamma_{p} \mid a \in \Omega_{2}\right\}\right|  \tag{9}\\
& =r_{\mathcal{P}_{p}}(\Pi(Q) \cap \Pi(p))+\left|\left\{a \in \Omega_{2} \mid f(a) \in \Pi(p)\right\}\right| .
\end{align*}
$$

Define $F_{1}^{\prime}$ as the set of edges $(a, p)$ in $\Gamma_{\Omega, p}$ with $a \in Q \backslash \Omega_{1}$. In addition, define $F_{2}^{\prime}$ as the set of edges $(a, p)$ in $\Pi_{\Omega}(p)$ such that $a \in Q \backslash \Omega_{1}$ and $(a, p)=s_{\Omega}(a)$. Notice that

$$
\begin{equation*}
F_{1}^{\prime} \cup F_{2}^{\prime}=\Pi_{\Omega}\left(Q \backslash \Omega_{1}\right) \cap \Pi_{\Omega}(p) . \tag{10}
\end{equation*}
$$

It follows from (4), (9), and (10) that if

$$
\begin{equation*}
r_{\mathcal{P}_{\Omega, p}}\left(F_{1}^{\prime} \cup F_{2}^{\prime}\right) \leq r_{\mathcal{P}_{p}}\left(F_{1} \cup F_{2}\right), \tag{11}
\end{equation*}
$$

then (8) follows and the proof is done because

$$
\begin{aligned}
r_{\mathcal{P}_{\Omega}}\left(\Pi_{\Omega}\left(Q \backslash \Omega_{1}\right)\right) & =\sum_{p \in P} r_{\mathcal{P}_{\Omega, p}}\left(\Pi_{\Omega}\left(Q \backslash \Omega_{1}\right) \cap \Pi_{\Omega}(p)\right) \\
& =\sum_{p \in P} r_{\mathcal{P}_{\Omega, p}}\left(F_{1}^{\prime} \cup F_{2}^{\prime}\right) \\
& \leq \sum_{p \in P} r_{\mathcal{P}_{p}}\left(F_{1} \cup F_{2}\right) \\
& \leq \sum_{p \in P} r_{\mathcal{P}_{p}}(\Pi(Q) \cap \Pi(p))+\sum_{p \in P}\left|\left\{a \in \Omega_{2} \mid f(a) \in \Pi(p)\right\}\right| \\
& =r_{\mathcal{P}}(\Pi(Q))+\left|\Omega_{2}\right| .
\end{aligned}
$$

It follows from (4) that

$$
\begin{align*}
r_{\mathcal{P}_{p}}\left(F_{1} \cup F_{2}\right) & =r_{\mathcal{M}_{p}}\left(F_{1}\right)+r_{\mathcal{M}_{p} / \Gamma_{p}}\left(F_{2}\right), \\
r_{\mathcal{P}_{\Omega, p}}\left(F_{1}^{\prime} \cup F_{2}^{\prime}\right) & =r_{\mathcal{M}_{p}}\left(F_{1}^{\prime}\right)+r_{\mathcal{M}_{p} / \Gamma_{\Omega, p}}\left(F_{2}^{\prime}\right) \tag{12}
\end{align*}
$$

For every edge $(a, p)$ in $F_{2}^{\prime} \backslash F_{2}$, we have

$$
\pi(f(a))>\pi((a, p))>\pi(s(a))
$$

This implies that $(a, p) \notin \mathcal{I}_{p} / \Gamma_{p}$ for every edge $(a, p)$ in $F_{2}^{\prime} \backslash F_{2}$. Thus, we have

$$
\begin{equation*}
r_{\mathcal{M}_{p} / \Gamma_{p}}\left(F_{2}\right)=r_{\mathcal{M}_{p} / \Gamma_{p}}\left(F_{2} \cup F_{2}^{\prime}\right) \tag{13}
\end{equation*}
$$

Furthermore, it follows from the monotonicity of $r_{\mathcal{M}_{p} / \Gamma_{\Omega, p}}(\cdot)$ that

$$
\begin{equation*}
r_{\mathcal{M}_{p} / \Gamma_{\Omega, p}}\left(F_{2}^{\prime}\right) \leq r_{\mathcal{M}_{p} / \Gamma_{\Omega, p}}\left(F_{2} \cup F_{2}^{\prime}\right) \tag{14}
\end{equation*}
$$

It follows from $\Gamma_{\Omega, p} \subseteq \Gamma_{p}$ and Lemma 3 that

$$
\begin{equation*}
r_{\mathcal{M}_{p} / \Gamma_{\Omega, p}}\left(F_{2} \cup F_{2}^{\prime}\right)-r_{\mathcal{M}_{p} / \Gamma_{p}}\left(F_{2} \cup F_{2}^{\prime}\right) \leq r_{\mathcal{M}_{p}}\left(\Gamma_{p}\right)-r_{\mathcal{M}_{p}}\left(\Gamma_{\Omega, p}\right) \tag{15}
\end{equation*}
$$

Since $\Gamma_{p} \backslash \Gamma_{\Omega, p}=F_{1} \backslash F_{1}^{\prime}$ and $F_{1}^{\prime} \subseteq \Gamma_{\Omega, p}$, it follows from (2) that

$$
\begin{equation*}
r_{\mathcal{M}_{p}}\left(\Gamma_{p}\right)-r_{\mathcal{M}_{p}}\left(\Gamma_{\Omega, p}\right) \leq r_{\mathcal{M}_{p}}\left(F_{1}\right)-r_{\mathcal{M}_{p}}\left(F_{1}^{\prime}\right) \tag{16}
\end{equation*}
$$

It follows from (13), (14), (15), and (16) that

$$
\begin{align*}
r_{\mathcal{M}_{p} / \Gamma_{\Omega, p}}\left(F_{2}^{\prime}\right)-r_{\mathcal{M}_{p} / \Gamma_{p}}\left(F_{2}\right) & \leq r_{\mathcal{M}_{p} / \Gamma_{\Omega, p}}\left(F_{2} \cup F_{2}^{\prime}\right)-r_{\mathcal{M}_{p} / \Gamma_{p}}\left(F_{2} \cup F_{2}^{\prime}\right) \\
& \leq r_{\mathcal{M}_{p}}\left(\Gamma_{p}\right)-r_{\mathcal{M}_{p}}\left(\Gamma_{\Omega, p}\right)  \tag{17}\\
& \leq r_{\mathcal{M}_{p}}\left(F_{1}\right)-r_{\mathcal{M}_{p}}\left(F_{1}^{\prime}\right)
\end{align*}
$$

It follows from (12) and (17) that

$$
\begin{aligned}
r_{\mathcal{P}_{\Omega, p}}\left(F_{1}^{\prime} \cup F_{2}^{\prime}\right)-r_{\mathcal{P}_{p}}\left(F_{1} \cup F_{2}\right) & =r_{\mathcal{M}_{p}}\left(F_{1}^{\prime}\right)-r_{\mathcal{M}_{p}}\left(F_{1}\right)+r_{\mathcal{M}_{p} / \Gamma_{\Omega, p}}\left(F_{2}^{\prime}\right)-r_{\mathcal{M}_{p} / \Gamma_{p}}\left(F_{2}\right) \\
& \leq r_{\mathcal{M}_{p}}\left(F_{1}^{\prime}\right)-r_{\mathcal{M}_{p}}\left(F_{1}\right)+r_{\mathcal{M}_{p}}\left(F_{1}\right)-r_{\mathcal{M}_{p}}\left(F_{1}^{\prime}\right) \\
& =0 .
\end{aligned}
$$

This implies (11), which completes the proof.
Theorem 11. The algorithm PCuMC correctly solves the popular condensation problem under matroid constraints.

Proof. This theorem follows from Lemmas 8, 9 and 10.
Here we analyze the time complexity of the algorithm PMuMC. Define $m:=|E|$, and we assume that for each post $p$ in $P$, each independent set $I$ in $\mathcal{M}_{p}$, and each edges $e$ in $I$ and $e^{\prime}$ in $E(p) \backslash I$, we can check in $O(\gamma)$ time whether $I-e+e^{\prime}$ and $I+e^{\prime}$ belong to $\mathcal{I}_{p}$. It follows from Lemma 2 that once we find a base in $\mathcal{M}_{p} \mid \Gamma_{p}$, for each independent set $I$ in $\mathcal{M}_{p} / \Gamma_{p}$ and each edges $e$ in $I$ and $e^{\prime}$ in $E(p) \backslash\left(I \cup \Gamma_{p}\right)$, we can check in $O(\gamma)$ time whether $I-e+e^{\prime}$ and $I+e^{\prime}$ belong to $\mathcal{I}_{p} / \Gamma_{p}$. Thus, for each independent set $I$ in $\mathcal{P}$ and each edges $e$ in $I$ and $e^{\prime}$ in $E \backslash I$, we can check in $O(\gamma)$ time whether $I-e+e^{\prime}$ and $I+e^{\prime}$ are independent sets in $\mathcal{P}$. This implies that the time complexity of the algorithm PCuMC is $O\left(m^{3} \gamma\right)$.

Next we consider a weighted variant of the popular condensation problem under matroid constraints. More precisely, in this problem, we are given a weight function $w: A \rightarrow \mathbb{Z}_{+}$. The goal is to find a maximum-size popular condensation $X$ maximizing $\sum_{a \in X} w(a)$. This problem can be solved as follows. For each edge $(a, p)$ in $\Pi$, we define the weight of $(a, p)$ as $w(a)$. This weighted variant can be solved by finding a maximum-size common independent set of $\mathcal{A}$ and $\mathcal{P}$ with maximum-weight at Step 1 of the algorithm PCuMC. It is known [4] that this problem can be solved in $O\left(m^{3} \gamma\right)$ time (see also [13, Section 41.3]). Thus, this weighted variant can be solved in $O\left(m^{3} \gamma\right)$ time.

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