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Lagrangian approach to weakly nonlinear interaction of Kelvin waves and a symmetry-breaking bifurcation of a rotating flow

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Abstract. We develop a general framework of using the Lagrangian variables for calculating the energy of waves on a steady Euler flow and the mean flow induced by their nonlinear interaction. With the mean flow at hand, we can determine, without ambiguity, all the coefficients of the amplitude equations to third order in amplitude, for a rotating flow subject to a steady perturbation breaking the circular symmetry of the streamlines. Moreover, a resonant triad of waves is identified, which brings in the secondary instability of the Moore-Saffman-Tsai-Widnall instability, and, with the aid of the energetic viewpoint, resonant amplification of the waves without bound is numerically confirmed.

Keywords: Weakly nonlinear instability, Secondary instability, Lagrangian approach, Kelvin waves, Mean flow

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1. Introduction

Since the late 60s, the stability of vortices has attracted much attention in connection with the aircraft wake turbulence, because some instability mechanisms have been continually sought for promoting the destruction of trailing vortices. Moore and Saffman (1975) and Tsai and Widnall (1976) uncovered a ubiquitous feature of three-dimensional instability of a strained vortex tube, which is referred to as the Moore-Saffman-Tsai-Widnall (MSTW) instability. A rotating flow supports inertial waves or Kelvin waves. When rotational and/or translational symmetry is broken, a pair of Kelvin waves become amplifiable via symmetry-breaking perturbations (Guckenheimer and Mahalov 1992, Knobloch *et al.* 1994, Kerswell 2002). For the MSTW instability, the symmetry-breaking perturbation is a pure shear, induced by the companion vortex of the pair, which deforms circular streamlines into ellipses. Put another way, the S^1-symmetry of streamlines is reduced to $\mathbb{Z}_2$-symmetry, whereby two Kelvin waves with azimuthal wavenumbers m separated by two can be amplifiable (Eloy and Le Dizès 2001, Fukumoto 2003). The resonance instability between $m = 1$ and $m = -1$ modes were clearly detected for an anti-parallel vortex pair by Leweke and Williamson (1998) and for a rotating flow confined in a cylinder of elliptic cross-section by Malkus (1989) and Eloy *et al.* (2000). Instability of a resonant pair $(m, m+2) = (0, 2)$ was also reported (Kerswell 2002).

The next concern is a weekly nonlinear stage of the wave growth. The Hamiltonian bifurcation theory (Guckenheimer and Mahalov 1992, Knobloch, Mahalov and Marsden 1994) dictates the form of amplitude equations depending on the broken symmetry, and the remaining task is to calculate their coefficients based on the Euler equation (Waleffe 1989, Sipp 2000, Mason and Kerswell 1999, Rodrigues and Luca 2009, Lehner *et al.* 2010). On the way of proceeding to third-order in amplitude lies, at second order, the determination of the modification of the mean flow driven by interaction between Kelvin waves. In our previous papers (Mie and Fukumoto 2010, Fukumoto *et al.* 2010), we pointed out that postulation of the conservation of the kinetic energy, as usually employed so far, does not fully determine the mean flow, with an integration constant left undetermined, and showed that this difficulty is rescued by restricting the disturbances isovortical ones with use of the Lagrangian variables. As analyzed by Fukumoto and Mie (2013), our Lagrangian approach shares, though not the same, a common property with the generalized Lagrangian mean (GLM) theory (Andrew and McIntyre 1978, Bühler 2009). It is noteworthy that, in three dimensions, a Kelvin wave generates a second-order mean flow in not only the azimuthal but also the axial directions (Fukumoto and Hirota 2008). The latter had been overlooked by the previous investigations. The wave-induced mean flow is regarded as the pseudomomentum in the context of the GLM. With the knowledge of the mean flow, the program of determining the amplitude equations is completed (Mie and Fukumoto 2010). The instability mode of the stationary resonance of a pair $(m, m+1) = (-1, 1)$ of waves goes through the Hamiltonian pitchfork bifurcation and eventually reaches a limit cycle of finite amplitude. The resonance instability of a pair $(m, m+1) = (0, 2)$ falls into a chaotic state, through the Hamiltonian Hopf bifurcation, but the disturbance amplitude is bounded (Rodrigues and Luca 2009, Fukumoto and Mie 2013). In either event, the nonlinear terms arising via the self-interaction of a MSTW instability mode suppress the growth of the disturbance amplitude.

This does not account for experimental observation (Malkus 1989, Eloy *et al.* 2000). Rather, once a MSTW instability mode is invited, excitation of enumerable waves follows in a short period, resulting in transition to turbulence. Kerswell (1999) and Mason and Kerswell (1999) revealed a mechanism for wave excitation via the secondary instability of Kelvin waves which comes into play on the way of growth of the MSTW instability. For a rigidly rotating flow in a circular cylinder, Fukumoto *et al.* (2005) identified a resonance triad among waves of $m = \pm 1, 3$ and 4, with waves of $m = \pm 1$ being degenerate at $\omega_0 = 0$, and derived the amplitude equations for them. However, they were unable to show the secondary instability.

The above mentioned mean flow of second order in amplitude is brought as a by-product of calculating the excitation energy, of second order, of the Kelvin waves. The wave energy is a key ingredient of Krein's theory of Hamiltonian spectra. A necessary condition for linear instability is either that two degenerate waves have opposite signed energy or that the energy of the both is zero (Arnol'd 1986, Morrison 1998). The wave energy is systematically calculated by taking advantage of the Lagrangian variables (Hirota and Fukumoto 2008, Fukumoto and Hirota 2008, Fukumoto *et al.* 2011). As regards the Kelvin waves in three dimensions on the Rankine vortex embedded in an unbounded domain, their energy is calculated first by an *ad hoc* method of exploiting a derivative of the dispersion relation with respect to the frequency by Fukumoto (2003). Recently for the Kelvin waves confined in a circular cylinder, a direct link of the energy formula obtained by the Lagrangian approach is built with a derivative of the dispersion relation (Fukumoto *et al.* 2014). It turns out that the knowledge of the energy is indispensable for the nonlinear growth of waves via a three-wave resonance. In this paper, we find that the secondary instability of the stationary resonance mode is realizable by carefully choosing the total of the wave energy to be zero.

In §2, we give a brief sketch of the time evolution of the Lagrangian displacement and of its use for deriving of the mean flow. Sections 3 and 4 give an outline of the Kelvin waves, along with nonlinearly induced mean flow, and the linear stability theory of the MSTW instability for the stationary mode of the helical-helical wave resonance ($m = \pm 1$). Section 5 reproduces the weakly nonlinear theory for the self interaction of this stationary mode described in our previous paper (Mie and Fukumoto 2010). Section 6 describes, at some length, the derivation of the amplitude equations for a resonant triad of $m = \pm 1, 3$ and 4, with waves of $m = \pm 1$ being degenerate. Notably, by a careful choice of normalization of amplitudes, the amplitude equations are turned into canonical Hamiltonian equations. Numerical examples will be given which shows indefinite amplification of these four waves. Section 7 is devoted to a summary and conclusions.

2. Mean flow induced by nonlinear interaction of waves

The Lagrangian approach has been invoked in an effort for calculating the excitation energy, to second order in amplitude, of waves on a steady basic flow (Hirota and Fukumoto 2008, Fukumoto and Hirota 2008, Fukumoto *et al.* 2011), which is otherwise non-trivial. Behind this lies Kelvin-Arnold's theorem (Arnol'd 1966) that a steady state of the Euler flows is an extremal of the kinetic energy with respect to isovortical disturbance. The isovortical disturbances are constructed with ease in terms of the Lagrangian displacement, and with this criticality, the energy is expressible solely in terms of the first-order Lagrangian displacement field. The same is true of the second-order mean field, and these are referred to as the wave property (Andrew and McIntyre 1978, Bühler 2009). In a previous paper, we made asymptotic expansions of temporal evolution of the Lagrangian displacement field to arbitrary order in wave amplitude (Fukumoto *et al.* 2010).

We assume that the fluid is incompressible, with uniform mass density, as well as inviscid. The motion of an inviscid incompressible fluid is regarded as an orbit on $\mathrm{SDiff}(\mathcal{D})$, the group of the volume-preserving diffeomorphisms of the domain $\mathcal{D}$ (Arnol'd 1966). Its Lie algebra $\mathfrak{g}$ is the velocity field of the fluid. A one-parameter subgroup of $\varphi_t \in \mathrm{SDiff}(\mathcal{D})$ and its generator $u(t) \in \mathfrak{g}$ are linked by the definition

$$u(t_0) = \left.\frac{\partial}{\partial t}\right|_{t_0} \left(\varphi_t \circ \varphi_{t_0}^{-1}\right). \tag{1}$$

Suppose that an orbit displaced from φ_t is written at each instant t as $\varphi_{\alpha,t} \circ \varphi_t$ by means of a near-identity map $\varphi_{\alpha,t}$ labeled with a small parameter $\alpha(\in \mathbb{R})$. There exists a generator $\xi_\alpha(t) \in \mathfrak{g}$ for it, defined by $\varphi_{\alpha,t} = \exp \xi_\alpha(t)$. The disturbance velocity field $u_\alpha(t)$ is calculated from

$$u_\alpha(t_0) = \left.\frac{\partial}{\partial t}\right|_{t_0} \left(\varphi_{\alpha,t} \circ \varphi_t \circ \varphi_{t_0}^{-1} \circ \varphi_{\alpha,t_0}^{-1}\right). \tag{2}$$

Use of a geometric setting, combined with symbolic manipulation of the Lie algebra, facilitates perturbation expansions, in powers of α, of the Lagrangian field to a higher order, and a series representation u_α to arbitrary order in α was manipulated in the previous paper.

Translation into the language of the vector calculus is straightforward. Given a steady basic flow $\boldsymbol{U}_0(\boldsymbol{x})$, an orbit $\boldsymbol{x}(t)$ of a fluid particle constituting this basic flow is defined by $\mathrm{d}\boldsymbol{x}(t)/\mathrm{d}t = \boldsymbol{U}_0\left(\boldsymbol{x}(t)\right)$. Suppose that the particle position $\boldsymbol{x}$ is disturbed to $\exp(\boldsymbol{\xi}_\alpha(\boldsymbol{x},t))\boldsymbol{x}$. The exponent $\boldsymbol{\xi}_\alpha(\boldsymbol{x},t)$ turns out to be the kinematically accessible displacement field (Moffatt 1986). We expand it in a power series in α as

$$\boldsymbol{\xi}_\alpha(\boldsymbol{x},t) = \alpha\boldsymbol{\xi}_1(\boldsymbol{x},t) + \frac{\alpha^2}{2}\boldsymbol{\xi}_2(\boldsymbol{x},t) + \cdots, \tag{3}$$

and correspondingly $\boldsymbol{u}_\alpha(\boldsymbol{x},t)$ as

$$\boldsymbol{u}_\alpha(\boldsymbol{x},t) = \alpha\boldsymbol{u}_1(\boldsymbol{x},t) + \frac{\alpha^2}{2}\boldsymbol{u}_2(\boldsymbol{x},t) + \cdots. \tag{4}$$

The series development of (2) reads, to $O(\alpha^2)$,

$$\frac{\partial \boldsymbol{\xi}_1}{\partial t} + (\boldsymbol{U}_0 \cdot \nabla)\boldsymbol{\xi}_1 - (\boldsymbol{\xi}_1 \cdot \nabla)\boldsymbol{U}_0 = \boldsymbol{u}_1, \tag{5}$$

$$\frac{\partial \boldsymbol{\xi}_2}{\partial t} + (\boldsymbol{U}_0 \cdot \nabla)\boldsymbol{\xi}_2 - (\boldsymbol{\xi}_2 \cdot \nabla)\boldsymbol{U}_0 + (\boldsymbol{u}_1 \cdot \nabla)\boldsymbol{\xi}_1 - (\boldsymbol{\xi}_1 \cdot \nabla)\boldsymbol{u}_1 = \boldsymbol{u}_2. \tag{6}$$

It is noteworthy that the geometric approach developed by Fukumoto *et al.* (2010) is crucial to derive the second-order equation (6) and higher-order ones.

The right-hand side of (5) is determined by the requirement that the disturbance be *isovortical* or *kinematically accessible*; the vorticity flux across an arbitrary infinitesimal material surface as represented, in the local Cartesian coordinates, by $\omega_x \mathrm{d}y \wedge \mathrm{d}z + \omega_y \mathrm{d}z \wedge \mathrm{d}x + \omega_z \mathrm{d}x \wedge \mathrm{d}y$ is preserved. The isovortical disturbance is represented, order by order, with use of the Lagrangian displacement field, as

$$\boldsymbol{u}_1 = \mathcal{P}\left[\boldsymbol{\xi}_1 \times \boldsymbol{\omega}_0\right], \tag{7}$$

$$\boldsymbol{u}_2 = \mathcal{P}\left[\boldsymbol{\xi}_1 \times (\nabla \times (\boldsymbol{\xi}_1 \times \boldsymbol{\omega}_0)) + \boldsymbol{\xi}_2 \times \boldsymbol{\omega}_0\right], \tag{8}$$

where $\boldsymbol{\omega}_0 = \nabla \times \boldsymbol{U}_0$ is the vorticity of the basic field and $\mathcal{P}$ is an operator projecting to solenoidal vector field complying with the boundary condition. With the requirement of disturbance being isovortical, (5) and (6) are made closed for the Lagrangian variables $\boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$.

The local mean flow of $O(\alpha^2)$ induced by the self-interaction of a wave is obtained by taking an ensemble average of (8). Combined with (7), it takes the form

$$\overline{\boldsymbol{u}_2} = \overline{\mathcal{P}\left[\boldsymbol{\xi}_1 \times (\nabla \times \boldsymbol{u}_1) + \boldsymbol{\xi}_2 \times \boldsymbol{\omega}_0\right]}. \tag{9}$$

When the basic flow is steady, the last term identically vanishes in the same way as for the wave energy. Recently, Fukumoto and Mie (2013) verified that (9) coincides with the pseudomomentum

$$\mathsf{p} = \mathcal{P}\left(\frac{\alpha^2}{2}\overline{\boldsymbol{u}_2} + \bar{\boldsymbol{v}}^{\mathrm{S}}\right). \tag{10}$$

The Stokes drift, the second term, is absent $\bar{\boldsymbol{v}}^{\mathrm{S}} = \boldsymbol{0}$ for the Kelvin waves in confined geometry.

3. Kelvin wave

Kelvin waves are a family of neutrally stable oscillations, of infinitesimal amplitude α, on the core of a circular cylindrical vortex. We briefly recall the Kelvin waves in confined geometry (Mie and Fukumoto 2010). We take, as the basic flow, the rigid-body rotation of an inviscid incompressible fluid confined in a cylinder of circular cross-section of unit radius. This basic flow has both rotational symmetry about the cylinder axis and translational symmetry along it, featured by $\mathrm{SO}(2) \times \mathrm{O}(2)$.

Let us introduce cylindrical coordinates (r, θ, z) with the z-axis along the centerline. Let the r- and the θ-components of the two-dimensional basic velocity field $\boldsymbol{U}_0$ be U_0 and V_0, and the pressure be P_0. The suffix 0 signifies that these quantities pertain to the case of circular cross-section of the container. The basic flow is confined in $r \leq 1$, with the velocity field given by

$$U_0 = 0, \quad V_0 = r, \quad P_0 = r^2/2 - 1. \tag{11}$$

We may take, as the disturbance field $\tilde{\boldsymbol{u}} = \alpha \boldsymbol{u}_{01}$, a normal mode

$$\boldsymbol{u}_{01} = A_m(t)\boldsymbol{u}_{01}^{(m)}(r)\mathrm{e}^{im\theta}\mathrm{e}^{ik_0 z}, \quad A_m(t) \propto \mathrm{e}^{-i\omega_0 t}, \tag{12}$$

where A_m is a complex function of time t and ω_0 is the frequency. This velocity field represents a Kelvin wave with the azimuthal wavenumber $m(\in \mathbb{Z})$ and the axial wavenumber $k_0(\in \mathbb{R})$. The radial functions $\boldsymbol{u}_{01}^{(m)} = (u_{01}^{(m)}, v_{01}^{(m)}, w_{01}^{(m)})$ of the disturbance velocity and $p_{01}^{(m)}$ of the disturbance pressure are found from the equation of continuity and the linearized Euler equations. Here we write down the resulting functions only (Mie and Fukumoto 2010).

$$\begin{aligned}
p_{01}^{(m)} &= J_m(\eta_m r), \\
u_{01}^{(m)} &= \frac{i}{\omega_0 - m + 2}\left\{-\frac{m}{r}J_m(\eta_m r) + \frac{\omega_0 - m}{\omega_0 - m - 2}\eta_m J_{m+1}(\eta_m r)\right\}, \\
v_{01}^{(m)} &= \frac{1}{\omega_0 - m + 2}\left\{\frac{m}{r}J_m(\eta_m r) + \frac{2\eta_m}{\omega_0 - m - 2}J_{m+1}(\eta_m r)\right\}, \\
w_{01}^{(m)} &= \frac{k_0}{\omega_0 - m}J_m(\eta_m r),
\end{aligned} \tag{13}$$

where η_m is the radial wavenumber

$$\eta_m^2 = \left[\frac{4}{(\omega_0 - m)^2} - 1\right] k_0^2, \tag{14}$$

and J_m is the m-th Bessel function of the first kind. The boundary condition on the cylinder surface, $\boldsymbol{u}_{01} \cdot \boldsymbol{n} = u_{01}^{(m)} = 0$ at $r = 1$, provides the dispersion relation

$$J_{m+1}(\eta_m) = \frac{(\omega_0 - m - 2)m}{(\omega_0 - m)\eta_m}J_m(\eta_m). \tag{15}$$

The Lagrangian displacement field is found from (5) with identification of $\boldsymbol{u}_1 = \boldsymbol{u}_{01}$. For the rigid-body rotation (11), (5) yields simply

$$\boldsymbol{\xi}_1 = \frac{i}{\omega_0 - m}\boldsymbol{u}_{01}. \tag{16}$$

When calculating the nonlinear quantity, the complex conjugate terms, denoted with the symbol *c.c.*, of the linear field $\boldsymbol{u}_{01}$ and $\boldsymbol{\xi}_1$ is to be supplemented as $\boldsymbol{u}_{01} = A_m(t)\boldsymbol{u}_{01}^{(m)}(r)\mathrm{e}^{i(m\theta + k_0 z)} + c.c.$ For the rigid-body rotation $\boldsymbol{U}_0 = r\boldsymbol{e}_\theta$, it is advantageous to calculate the mean flow directly from the spatial average of (8) since simply $\nabla \times \boldsymbol{U}_0 = 2\boldsymbol{e}_z$. It turns out that the second-order field $\boldsymbol{\xi}_2$ has no contribution when spatially averaged, leaving

$$\begin{aligned}
\overline{\boldsymbol{u}_{02}} &= \overline{\mathcal{P}\left[\boldsymbol{\xi}_1 \times (\nabla \times (\boldsymbol{\xi}_1 \times \boldsymbol{e}_z))\right]} = \overline{\boldsymbol{\xi}_1 \times \partial \boldsymbol{\xi}_1/\partial z} \\
&= \frac{4ik}{(\omega_0 - m)^2}|A_m|^2(0, u_{01}^{(m)}w_{01}^{(m)}, -u_{01}^{(m)}v_{01}^{(m)}).
\end{aligned} \tag{17}$$

The Lagrangian approach is capable of calculating the mean flow at any points satisfied by the dispersion relation (k, ω_0). In view of the amplitude functions (13), the reality of (17) is guaranteed.

4. Moore-Saffman-Tsai-Widnall instability

We express elliptic cross-section with small eccentricity of the cylinder as $r = 1 + \epsilon \cos 2\theta/2 + O(\epsilon^2)$, using a small parameter ϵ. In conjunction with this distortion the basic flow is perturbed as

$$\boldsymbol{U} = \boldsymbol{U}_0 + \epsilon \boldsymbol{U}_1 + \cdots, \quad P = P_0 + \epsilon P_1 + \cdots; \\ U_1 = -r \sin 2\theta, \quad V_1 = -r\cos 2\theta, \qquad P_1 = 0. \tag{18}$$

The subscript designates order in elliptic parameter ϵ. The augmented term $\epsilon \boldsymbol{U}_1$ of $O(\epsilon)$ represents a steady two-dimensional pure shear whose stretching direction lies along $\theta = -\pi/4$ and whose direction of contraction is along $\theta = \pi/4$.

We superimpose disturbance field $\tilde{\boldsymbol{u}}$ to this two-dimensional basic flow. We consider asymptotic expansions of the velocity field in two small parameters ϵ and α as

$$\boldsymbol{u} = \boldsymbol{U} + \tilde{\boldsymbol{u}} = \boldsymbol{U}_0 + \epsilon \boldsymbol{U}_1 + \alpha \boldsymbol{u}_{01} + \epsilon\alpha \boldsymbol{u}_{11} + \alpha^2 \boldsymbol{u}_{02} + \alpha^3 \boldsymbol{u}_{03} + \cdots, \tag{19}$$

to $O(\alpha^3)$ in amplitude. Here, the velocity field $\boldsymbol{u}_{mn}$ occurs at $O(\epsilon^m \alpha^n)$. The boundary condition to be imposed at the rigid side wall is

$$\boldsymbol{u} \cdot \boldsymbol{n} = 0 \quad \text{at} \quad r = 1 + \epsilon \cos 2\theta/2, \tag{20}$$

where $\boldsymbol{n}$ is the unit outward normal vector to the cylinder boundary.

The elliptic strain $\epsilon \boldsymbol{U}_1$ mediates interaction of two Kelvin waves with $\mathrm{e}^{im\theta}$ and $\mathrm{e}^{i(m+2)\theta}$ at $O(\epsilon\alpha)$. Eloy and Le Dizès (2001) and Fukumoto (2003) made a thorough analysis of the MSTW instability for the Rankine vortex embedded in a plane shear flow, and showed that the parametric resonance instability occurs, at $O(\epsilon\alpha)$, at all the intersection points (k_0, ω_0) of the dispersion curves of the m and $m+2$ waves. The same is true of the rotating flow confined in a cylinder of elliptic cross-section (Vladimirov *et al.* 1983, Mie and Fukumoto 2010). The instability is dominated, among them, by the stationary mode $\omega_0 = 0$ of degenerate right- $(m = -1)$ and left- $(m = 1)$ handed helical waves, For later use, we reproduce the result of Mie and Fukumoto (2010) on this stationary resonance.

Under the restriction of $\omega_0 = 0$, the radial wavenumber is $\eta = \sqrt{3}k_0$. At $O(\alpha)$, we send a pair of waves of $m = \pm 1$

$$\boldsymbol{u}_{01} = A_- \boldsymbol{u}_{01}^{(-)} \mathrm{e}^{-i\theta} \mathrm{e}^{ik_0 z} + A_+ \boldsymbol{u}_{01}^{(+)} \mathrm{e}^{i\theta} \mathrm{e}^{ik_0 z} + c.c., \tag{21}$$

where the amplitudes $A_\pm = A_\pm(t)$ are complex-valued functions of time t. Being fueled by $\boldsymbol{U}_1$, excited at $O(\epsilon\alpha)$ is

$$\boldsymbol{u}_{11} = \left\{ B_- \boldsymbol{u}_{11}^{(-)} \mathrm{e}^{-i\theta} + B_+ \boldsymbol{u}_{11}^{(+)} \mathrm{e}^{i\theta} + B_{-3} \boldsymbol{u}_{11}^{(-3)} \mathrm{e}^{-3i\theta} + B_3 \boldsymbol{u}_{11}^{(3)} \mathrm{e}^{3i\theta} \right\} \mathrm{e}^{ik_0 z} \\ + c.c. \tag{22}$$

The radial functions $\boldsymbol{u}_{11}^{(\pm)}(r)$ are determined by solving the linearized Euler equations and the continuity equations, subject to the boundary condition (20) at $O(\epsilon\alpha)$,

$$u_{11} - u_{01} \cos 2\theta/2 + v_{01} \sin 2\theta = 0 \quad \text{at} \quad r = 1. \tag{23}$$

The boundary condition (23) provides inhomogeneous algebraic equations for $B_\pm$ and the solvability condition on them gives rise to, with the help of the dispersion relation (15),

$$\frac{1}{A_+}\frac{\partial A_-}{\partial t_{10}} = -\frac{1}{A_-}\frac{\partial A_+}{\partial t_{10}} = i\frac{3(3k_0^2+1)}{8(2k_0^2+1)} = ia, \tag{24}$$

where $t_{10} = \epsilon t$, the slow time scale, and k_0 is the solution of dispersion relation (15) $J_1(\eta) = -\eta J_0(\eta)$ with $\eta = \sqrt{3}k_0$. The degenerate modes with $\omega_0 = 0$ necessarily result in parametric resonance with growth rate $a = 3(3k_0^2+1)/[8(2k_0^2+1)]$ (Vladimirov *et al.* 1983) and with amplitude ratio of the eigen-function given by $A_-/A_+ = i$. Numerical values of the growth rate $\epsilon\sigma$ are, at a first few intersection points with $\omega_0 = 0$, $(k_0, \sigma) \approx$ (1.578, 0.5311), (3.286, 0.5542), $\cdots$.

5. Weakly nonlinear evolution of MSTW instability

The procedure for deriving weakly nonlinear amplitude equations, as expounded in Mie and Fukumoto (2010), begins with calculation of the mean flow or the drift current of $O(\alpha^2)$ induced by nonlinear interactions of linear waves. The superposed helical waves (21) drives, as the drift current (17),

$$4ik_0\left(0, \left(|A_-|^2+|A_+|^2\right)u_{01}^{(+)}w_{01}^{(+)}, \left(|A_-|^2-|A_+|^2\right)u_{01}^{(+)}v_{01}^{(+)}\right), \tag{25}$$

represented in the cylindrical coordinates. Generically, the radial component of mean flow is identically zero, but other components are both present as is seen from (17). Note that the existence of axial component had been overlooked before Fukumoto and Hirota (2008) derived it.

At $O(\alpha^3)$, the modes $\mathrm{e}^{\pm i\theta}\mathrm{e}^{ik_0z}$ again arise, which invites the compatibility conditions. The function $\boldsymbol{u}_{03}^{(m)}$ with $m = \pm 1$ is governed by $\mathcal{L}_{m,k_0}\boldsymbol{u}_{03}^{(m)} = \mathcal{N} - \partial\boldsymbol{u}_{01}^{(m)}/\partial t_{02}$, with $t_{02} = \alpha^2 t$. The calculation of $\mathcal{N}$ requires the precise form of the mean flow of $O(\alpha^2)$, and, given it, equation of $\boldsymbol{u}_{03}^{(m)}$ is somehow integrated for $m = \pm 1$ in terms of the Bessel functions. The boundary condition at $O(\alpha^3)$ yields inhomogeneous algebraic equations with a singular matrix. For solvability of these algebraic equations, $(\partial/\partial t_{02})\boldsymbol{u}_{01}^{(m)}$ must be properly adjusted. Implementing this procedure, we arrive at the amplitude equations, valid to $O(\alpha^3)$,

$$\frac{\mathrm{d}A_\pm}{\mathrm{d}t} = \mp i\left[\epsilon\left(a\overline{A_\mp} - p_1k_1A_\pm\right) + \alpha^2A_\pm\left(b|A_\pm|^2 + c|A_\mp|^2\right)\right], \tag{26}$$

where an overbar stands for the complex conjugate, a is defined by (24) and

$$p_1 = \frac{3(k_0^2+1)}{2k_0(2k_0^2+1)}, \tag{27}$$

$$b = \frac{-2k_0^4}{3(2k_0^2+1)}\left[\frac{4}{J_0(\eta)^2}\int_0^1 rJ_0(\eta r)^2J_1(\eta r)^2\mathrm{d}r - (11k_0^4+13k_0^2+5)J_0(\eta)^2\right],$$

$$c = \frac{k_0^2}{12(2k_0^2+1)}\left[\frac{64k_0^2}{J_0(\eta)^2}\int_0^1 rJ_0(\eta r)^2J_1(\eta r)^2\mathrm{d}r\right.$$

$$+ (20k_0^6 + 97k_0^4 + 14k_0^2 - 27)J_0(\eta)^2\Big]. \tag{28}$$

These coefficients are, with compact form at hand, readily calculated at all the intersection points on the k_0-axis ($\omega_0 = 0$). For the longest two wavelengths, we have $(k_0; a, b, c) \approx (1.579; 0.5312, -0.3976, 5.222), (3.286; 0.5542, -8.286, 53.39)$. The sign of coefficients (a, b, c) is unchanged, regardless of the choice of the intersection points: $a > 0$, $b < 0$ and $c > 0$.

It is remarkable that the normal form derived by Knobloch *et al.* (1994) automatically shows up with nonlinear terms $|A_\pm|^2 A_\pm, |A_\mp|^2 A_\pm$ fully incorporating the influence of mean flow (25). By contrast, in the Eulerian treatment, the amplitude of the mean flow in the azimuthal direction had to be separately introduced as a parameter to be determined indirectly to satisfy the conservation law of energy (Waleffe 1989, Sipp 2000, Lehner *et al.* 2010). The axial drift current was disregarded, perhaps because it has no contribution to the wave energy of $O(\alpha^2)$. Moreover, the imposition of the energy conservation law does not fully determine the wave-induced mean flow but leaves an undetermined constant whose *ad hoc* choice is liable to produce an unacceptable result (Mie and Fukumoto 2010).

As A_- and A_+ are complex functions of t, the amplitude equations (26) constitute a four-dimensional dynamical system. These coupled equations admit restriction of the phase space to a two-dimensional subspace with $A = A_- = -\overline{A_+}$ (Sipp 2000). The complex amplitude equations (26) collapses, by a choice of $\alpha^2 = \epsilon$, to

$$\frac{\mathrm{d}A}{\mathrm{d}t} = i\epsilon\left(-a\overline{A} + \beta|A|^2 A\right), \tag{29}$$

where $\beta = b + c$. Figure 1 draws the trajectory of the solution of (29) in the phase space $(\mathrm{Re}[A], \mathrm{Im}[A])$. The rigidly rotating state (the origin) is unstable, but the amplitude of the orbit necessarily saturates within the basin of the stable equilibria marked with the symbol 's'. Expressing the amplitude function as $A = |A|\mathrm{e}^{i\phi}$ ($\phi \in \mathbb{R}$), the modulus $|A|$ and the phase ϕ satisfy the following equations

$$\frac{\mathrm{d}|A|}{\mathrm{d}t} = -\epsilon a|A|\sin 2\phi, \quad \frac{\mathrm{d}\phi}{\mathrm{d}t} = -\epsilon a\cos 2\phi + \epsilon\beta|A|^2. \tag{30}$$

The linear effect predominates over the nonlinear effect for small disturbance amplitude $|A|(\ll 1)$. In case the equilibrium point $A = 0$ is unstable, the direction of disturbance vorticity ϕ is liable to be aligned at the stretching direction $\phi = -\pi/4$. The elliptic strain makes horizontal vortex lines continuously stretched, if they are oriented, on average, in the direction of $\phi = -\pi/4$. This is the mechanism for the MSTW instability at the linear stage. When the disturbance grows substantially, $|A| \approx 1$ say, the nonlinear effect is called into play. Because of $b + c > 0$, the nonlinear effect in (30) is to monotonically increase the phase angle ϕ if $\epsilon > 0$. The alignment of horizontal vorticity parallel to the direction $\phi = -\pi/4$ is distracted, and, as a consequence, the disturbance amplitudes saturates. It grows to no more than a finite value $|A|_{\mathrm{eq}} = \sqrt{a/\beta}$.

The aforementioned behavior does not coincide with the vigorous amplification of a number of waves and the ultimate disruption of a strained flow as observed in

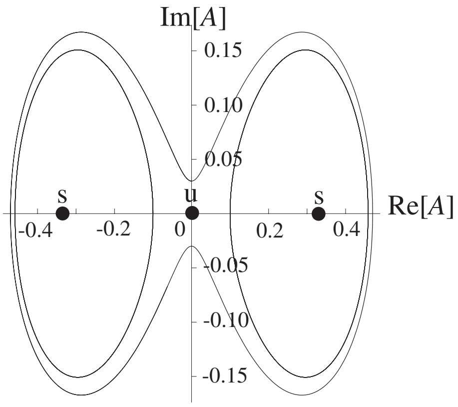

Figure 1. Trajectories in the phase space $(\mathrm{Re}[A], \mathrm{Im}[A])$ for $k = 1.579$. The dots designate equilibria (s: stable, u: unstable).

experiments of Malkus (1989) and Eloy *et al.* (2000). The nonlinear interaction of a single MSTW mode is of limited relevance to practical phenomena. As argued by Mason and Kerswell (1999), the secondary and the tertiary instability, which could be invited before reaching the stage of nonlinear saturation, will drastically alter the subsequent evolution. In the next section, we inquire into a mechanism for exciting waves.

6. Secondary instability of Kelvin waves

The secondary instability of the MSTW mode is possibly attributable to three-wave resonance as demonstrated numerically by Kerswell (1999) and Mason and Kerswell (1999). For a rigidly rotating flow confined in a circular cylinder, a candidate was pinpointed for resonantly interacting waves of $m = \pm 1$, 3 and 4 by Fukumoto *et al.* (2005) (figure 2). At $O(\alpha)$, we pose the following combination of Kelvin waves

$$\begin{aligned} \boldsymbol{u}_{01} &= \hat{A}_+ \boldsymbol{u}_{A_+}(r) \mathrm{e}^{i(\theta + k_0 z)} + \hat{A}_- \boldsymbol{u}_{A_-}(r) \mathrm{e}^{i(\theta - k_0 z)} + B_+ \boldsymbol{u}_{B_+}(r) \mathrm{e}^{i(3\theta + k_0 z/2)} \\ &\quad + B_- \boldsymbol{u}_{B_-}(r) \mathrm{e}^{i(3\theta + k_0 z/2)} + C_+ \boldsymbol{u}_{C_+}(r) \mathrm{e}^{i(4\theta + 3k_0 z/2)} + C_- \boldsymbol{u}_{C_-}(r) \mathrm{e}^{i(4\theta - 3k_0 z/2)} \\ &\quad + c.c. \end{aligned} \qquad (31)$$

where $k_0 \approx 3.286$, and the amplitudes $\hat{A}_\pm$, $B_\pm$ and $C_\pm$ are functions of t to be determined at higher orders. Notice by comparing (31) with (21) that $\hat{A}_+ = A_+$ but that $\hat{A}_- = \overline{A_-}$. In this section, we take the combination (31) since the resulting amplitude equations are made simpler in their signs. Figure 2 draws the dispersion relation $\omega_0 = \omega_0(k_0)$ for waves of $m = \pm 1$, 3 and 4. An infinite number of branches emanating from $(k_0, \omega_0) = (0, 1)$ correspond to the left-handed helical waves ($m = 1$), and the branches of the right-handed helical waves ($m = -1$) are reflection of those of $m = 1$ with respect to the k_0-axis. The dispersion curves of $m = 3$ emanate from $(k_0, \omega_0) = (0, 3)$ and those of $m = 4$ from $(k_0, \omega_0) = (0, 4)$. Fukumoto *et al.* (2005) sought a resonance triad, with placing one of the vertices at an intersection point of the $m = \pm 1$ curves on the k_0-axis, embedded in this infinite number of curves. Among non-rotating resonances, the second

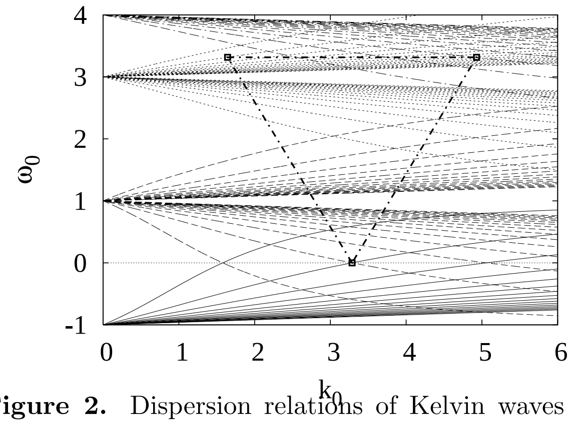

Figure 2. Dispersion relations of Kelvin waves of $m = \pm 1, 3, 4$ as picked out by Fukumoto *et al.* (2005)

radial mode, which is located at $(k_0, \omega_0) = (\beta, 0)$ with $\beta \approx 3.286$, is found to constitutes a near-resonance triad with companion vertices close to $(m, k_0, \omega_0) = (3, \beta/2, 3.315)$ and $(4, 3\beta/2, 3.315)$.

The perturbation equations derived from the Euler equations, with the straining field of $O(\epsilon)$, furnish the compatibility conditions at $O(\alpha^2)$. It is straightforward to fulfill these conditions, leaving

$$\frac{\mathrm{d}\hat{A}_\pm}{\mathrm{d}t} = i\left[\epsilon\left(-a\overline{\hat{A}_\mp} + p_1 k_1 \hat{A}_\pm\right) + \alpha q_1 \overline{B_\pm} C_\pm\right], \tag{32}$$

$$\frac{\mathrm{d}B_\pm}{\mathrm{d}t} = i\left[-\omega_0 B_\pm + \epsilon p_2 k_1 B_\pm + \alpha q_2 \overline{\hat{A}_\pm} C_\pm\right], \tag{33}$$

$$\frac{\mathrm{d}C_\pm}{\mathrm{d}t} = i\left[-\omega_0 C_\pm + \epsilon p_3 k_1 C_\pm + \alpha q_3 \hat{A}_\pm B_\pm\right]. \tag{34}$$

The coefficients a and p_1 are given respectively by (24) and (27), and are evaluated as $a \approx 0.5542$, $p_1 \approx 0.2383$. The rest are $p_2 \approx -0.1806$, $p_3 \approx 0.1274$, $q_1 \approx -10.23$, $q_2 \approx 3.150$ and $q_3 \approx -4.227$. We confirm that these values are the same as the one obtained by Fukumoto *et al.* (2005). A closer resonance is attained by choosing $\epsilon k_1 \approx -0.01268$.

The coupled system of ordinary differential equations (32)-(34) as they stand are not Hamiltonian. Fukumoto and Mie (2013) found, though numerically and therefore without mathematical justification, for the Hamiltonian Hopf bifurcation of the $(0, 2)$ waves, that normalizing the amplitudes so as to make the linear-wave part Hamiltonian transforms the whole amplitude equations as well into canonical Hamiltonian equations. We apply the same procedure to the three-wave interaction. Indeed setting

$$z_{1\pm} = \frac{\overline{\hat{A}_\pm}}{\sqrt{|q_1|}}, \quad z_{2\pm} = \frac{B_\pm \mathrm{e}^{i\omega_0 t}}{\sqrt{q_2}}, \quad z_{3\pm} = \frac{\overline{C_\pm}\mathrm{e}^{-i\omega_0 t}}{\sqrt{|q_3|}}, \tag{35}$$

renders (32)-(34) canonical Hamiltonian equations

$$\frac{\mathrm{d}z_{1\pm}}{\mathrm{d}t} = i\left[\epsilon\left(a\overline{z_{1\mp}} - p_1 k_1 z_{1\pm}\right) + \alpha\gamma z_{2\pm} z_{3\pm}\right], \tag{36}$$

$$\frac{\mathrm{d}z_{2\pm}}{\mathrm{d}t} = i\left[\epsilon p_2 k_1 z_{2\pm} + \alpha\gamma z_{1\pm}\overline{z_{3\pm}}\right], \tag{37}$$

$$\frac{\mathrm{d}z_{3\pm}}{\mathrm{d}t} = -i\left[\epsilon p_3 k_1 z_{3\pm} - \alpha\gamma z_{1\pm}\overline{z_{2\pm}}\right], \tag{38}$$

where

$$\gamma = \sqrt{|q_1 q_2 q_3|} \approx 11.67. \tag{39}$$

The coordinate transformation (35) acts to absorb the inertial waves from (32)-(34) as well as to normalize the amplitude. The Hamiltonian H associated with the above system is

$$H = H_+ + H_-, \tag{40}$$

where

$$H_\pm = \alpha\gamma \mathrm{Re}\left[z_{1\pm}\overline{z_{2\pm}z_{3\pm}}\right] + \frac{1}{2}\epsilon\Big(-a\mathrm{Re}\left[z_{1+}z_{1-}\right] + p_1 k_1 |z_{1\pm}|^2 - p_2 k_1 |z_{2\pm}|^2 + p_3 k_1 |z_{3\pm}|^2\Big). \tag{41}$$

The symbol Re[$\cdot$] designates the real part of a complex number. The term $\alpha\gamma\mathrm{Re}\left[z_{1\pm}\overline{z_{2\pm}z_{3\pm}}\right]$ is a Hamiltonian for a three-wave interaction (Alber *et al.* 1998). The Hamiltonian (40) itself is a conserved quantity of $O(\alpha^3)$. In addition to H, the second-order energies

$$\omega_0(|z_{2+}|^2 - |z_{3+}|^2), \quad \omega_0(|z_{2-}|^2 - |z_{3-}|^2) \tag{42}$$

are shown to be conserved independently.

In our previous investigation (Fukumoto *et al.* 2005), amplification of waves by this triad resonance did not occur. Here we fully exploit a knowledge of the energy of Kelvin waves to establish the triad-resonance instability. As a rule, on the side $\omega_0 > 0$ in figure 2, a cograde wave corresponding to an upgoing branch of the dispersion curve $\omega_0 = \omega_0(k_0)$ has positive energy, whereas a retrograde wave corresponding to a downgoing branch has negative energy. A stationary wave ($\omega_0 = 0$) has zero energy (Fukumoto 2003). As far as the triad resonance under question are concerned, the wave of $m = 3$ has positive energy and the wave of $m = 4$ has negative energy as is read off from figure 2, and this is reflected in the sign of (42). There is no contribution to the second-order energy from the amplitude $z_\pm$ of the stationary helical waves. It is probable that a superposition of waves with zero total energy can grow without bound, since they are capable of being excited at the expense of no energy supply. We have carried out numerical computation of (36)-(38) for a variety of initial conditions and, in fact, by carefully adjusting the total energy very close to zero, unlimited wave amplification through the triad resonance is established. Figure 3 illustrates a numerical example exhibiting the resonant growth with a choice of the initial condition $z_{1\pm}(0) = 0.5(1-i)$, $z_{2\pm}(0) = 0.05(1+i)$ and $z_{3\pm}(0) = -0.05\sqrt{2}i$. The solid line represents the evolution of $|z_{1\pm}(t)|$ with time t, and the dashed lines represent $|z_{2\pm}(t)|$ (blue) and $|z_{3\pm}(t)|$ (green), respectively. This choice of the initial conditions $z_{2\pm}(0)$ and $z_{3\pm}(0)$ cancels completely the second-order wave energy (42). We observe oscillating growth of these three waves.

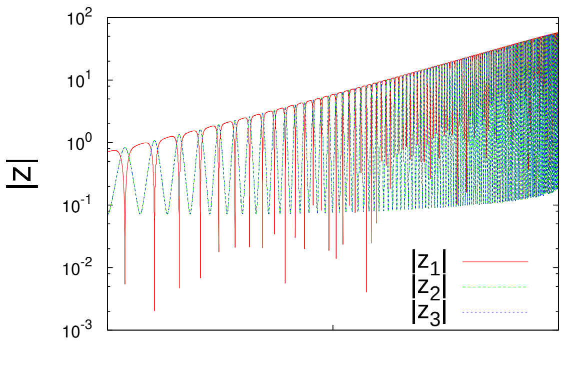

Figure 3. Evolution of the normalized amplitude $|z_{1\pm}(t)|$, $|z_{2\pm}(t)|$ and $|z_{3\pm}(t)|$ of Kelvin waves of $m = \pm1, 3$ and 4 constituting a resonance triad shown in figure 2. The initial condition is $z_{1\pm}(0) = 0.5(1 - i)$, $z_{2\pm}(0) = 0.05(1 + i)$, $z_{3\pm}(0) = -0.05\sqrt{2}i$.

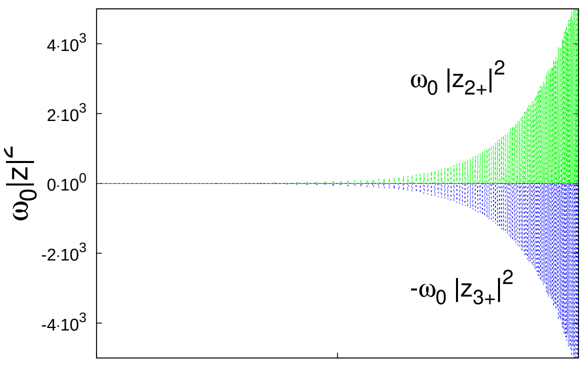

Figure 4. Evolution of the wave energy of $m = 3, 4$ corresponding to figure 3.

Figure 4 draws the time evolution of the wave energy $\omega_0|z_{2+}|^2$ of $m = 3$ and $-\omega_0|z_{3+}|^2$ of $m = 4$. We are convinced that the waves of $m = 3$ and 4 are amplified together without spending energy.

In order to gain an insight into the secondary instability via the three-wave resonance, we consider an initial stage when the disturbance amplitudes $z_{2\pm}$ and $z_{3\pm}$ are smaller than $z_{1\pm}$. In such a stage, (36)-(38) may be simplified to

$$\frac{\mathrm{d}z_{1\pm}}{\mathrm{d}t} = i\epsilon\left(a\overline{z_{1\mp}} - p_1k_1z_{1\pm}\right), \quad \frac{\mathrm{d}z_{2\pm}}{\mathrm{d}t} = i\alpha\gamma z_{1\pm}\overline{z_{3\pm}}, \quad \frac{\mathrm{d}z_{3\pm}}{\mathrm{d}t} = i\alpha\gamma z_{1\pm}\overline{z_{2\pm}}. \qquad (43)$$

The MSTW instability exclusively excites $z_{1\pm}$, being followed by the growth of components $z_{2\pm}$ and $z_{3\pm}$. A closer look at the amplitudes and the phases evolving according to (36)-(36) informs us of the following. The growth of $z_{2\pm}$ and $z_{3\pm}$ shifts the phase $\phi_{1\pm}$ of $z_{1\pm}$, accompanied by decrease of the disturbance amplitude $|z_{1\pm}|$. The decay of $z_{1\pm}$ further shifts the phase $\phi_{1\pm}$, and then in turn the disturbance amplitude $|z_{1\pm}|$ grows. This shift in the phase $\phi_{1\pm}$ also acts to shift the phases of $z_{2\pm}$ and $z_{3\pm}$. In this way, growth and decay of $|z_{1\pm}|$ occur alternately with those of $|z_{2\pm}|$ and $|z_{3\pm}|$, as observed in figure 3. The growth-and-decay cycles occur in a succession, with the peak becoming higher and with the period becoming shorter. The amplitude growth behaves almost exponentially in time. This ever accelerating amplification of Kelvin waves may be responsible for the wave excitations observed in Malkus' experiment (1989).

7. Conclusion

The energy holds a key to a deeper understanding of linear and weakly nonlinear stability of vortical flows, and the Lagrangian approach is instrumental in calculating the wave energy. For a flow of uniform vorticity, a rotating flow of an inviscid incompressible fluid shares, though the system is of infinite dimensional, common properties of spectra with a finite-dimensional Hamiltonian system. The instability and the resulting bifurcation are dictated by the prescription of Krein's theory of the Hamiltonian spectra. The instability is triggered by degeneracy of eigenvalues. A necessary condition for instability is either that the signs of the energies of the eigenmodes are opposite or that the energies are both zero. Here the energy means the excitation energy, a quantity of second order in amplitude that is necessary to excite the wave. In the context of three dimensions, the energy of the Kelvin waves was calculated, by trial and error, from a derivative of the dispersion relation by Fukumoto (2003), and continual effort follows to give a systematic derivation for it, by use of the Lagrangian variables (Fukumoto and Hirota 2008, Fukumoto *et al.* 2010, 2011). In the presence of a basic flow, calculation of the wave energy necessitates a nonlinear wave field. The use of the Lagrangian variables restricts disturbances to irrotational or isovortical ones, with which the steady flow is characterized as a state of the extremum of the kinetic energy. It is this criticality that makes feasible calculation of the wave energy in terms solely of linear displacement field. Recently a justification was given to the energy formula given in the form of a derivative of the dispersion relation (Fukumoto *et al.* 2014). The wave-induced mean flow of second order in amplitude was gained as a by-product (Fukumoto and Hirota 2008). The GLM theory reveals the close link between the mean flow and the wave energy (Fukumoto and Mie 2013). The wave-induced mean flow induced by the Kelvin wave coincides with a pseudomomentum. The wave energy of a steady basic flow is associated with the translational symmetry in time, and the azimuthal and axial components of wave-induced mean flow are associated with the rotational and translational symmetries of the basic flow.

The axial mean flow of $O(\alpha^2)$ is essentially of three dimensional; this disappears in the limit of the infinite axial wavelength. A precise specification of this component is, though ignored in the previous investigations, indispensable to go beyond the linear regime and to derive amplitude equations to $O(\alpha^3)$ (Mie and Hirota 2010). In §5, the amplitude equation has been unambiguously derived to $O(\alpha^3)$ using the mean flow obtained in §3. Realization of the secondary instability via three-wave resonance, pointed out by Kerswell (1999) and Mason and Kerswell (1999) as a plausible mechanism for excitation of waves, had gotten stuck in our previous investigation (Fukumoto *et al.* 2005). A quantitative knowledge of the wave energy has rescued this difficulty. In §6, the initial wave field comprising three waves $m = \pm 1, 3$ and 4 in resonance, with the frequency of $m = \pm 1$ being degenerate, is posed in such a way that the total wave energy is zero. With this choice, unbounded growth of the superposed waves of $m = \pm 1, 3$ and 4 is observed.

The determination of the mean flow is one of the crucial steps in common in the weakly nonlinear stability theories of fluid flows. We surmise that a similar difficulty is encountered for other problems, and the previous theories may well be reappraised. The use of the Lagrangian variables restricts the disturbance to isovortical ones, and this restriction has made possible the unique determination of the mean flow. However, there are cases where vorticity is imported externally and/or is created on the boundary. A wider framework of the weakly nonlinear stability theory is desired that can handle, by relaxing the restriction to isovortical disturbances, vortical disturbances. The simplicity of the analysis of the present investigation hinges upon the uniformity of vorticity field. This is special in the sense that continuous spectra are ruled out, leaving point spectra only. The non-uniformity in vorticity drastically alters the situation, by smearing out the point spectra into continuous ones. In such a case, the linear and weakly nonlinear stability theories call for entirely different treatment. Concerning parallel flows confined in slab geometry, Hirota and Fukumoto (2008) gave a formula for the wave energy associated with continuous spectra. The corresponding formulas for rotating flows of non-uniform vorticity with and without boundary have yet been unknown. A more drastic change arising from continuous spectra occurs for the secondary instability. There remain several stringent hurdles to overcome before we are able to deal with stability and bifurcations of practical flows.

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