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Cuyt, Annie
Department of Mathematics and Computer Science University of Antwerp

Lee, Wen-shin
Department of Mathematics and Computer Science University of Antwerp

出版情報：COE Lecture Note. 49, pp.83-86, 2013-08-09. 九州大学マス・フォア・インダストリ研究所
バージョン：
権利関係：

チュートリアル 1

Tutorial 1

Sparse interpolation and signal processing

Annie Cuyt and Wen-shin Lee*

Department of Mathematics and Computer Science
University of Antwerp, Belgium

Conventional interpolation algorithms do not take sparsity into consideration and depend on the total degree or the maximum possible size of the function. Traditionally, polynomial interpolation of n values f_j at points x_j is a technique that determines the coefficients $a_i, i = 1, \ldots, n$ in the model $a_1\phi_1(x) + \cdots + a_n\phi_n(x)$ from the conditions

$$\sum_{i=1}^{n} a_i \phi_i(x_j) = f_j, \quad j = 1, \ldots, n,$$

where the functions $\phi_1(x), \ldots, \phi_n(x)$ satisfy the Haar condition. Several numerical techniques to determine the values a_i, for use with different $\phi_i(x)$, are well-known. The numerical conditioning of the problem and the stability of the algorithms have been analyzed in great detail.

On the other hand, sparse interpolation algorithms are sensitive to the number of nonzero terms in the underlying representation and thus account for the sparsity of the function. In computer algebra, the problem of interpolating a sparse polynomial has always been a major research focus. The purpose is to improve computational performance: sparse interpolation and representation algorithms are developed to control the intermediate swell encountered in symbolic computation.

In 1979, Zippel gave the first sparse polynomial interpolation algorithm [22]. Then in 1988, Ben-Or and Tiwari presented a different algorithm [2] that is based on the Berlekamp/Massey algorithm [15] from coding theory. The Ben-Or/Tiwari sparse interpolation algorithm can determine both the correct indices k_i and the coefficients a_i, for $i = 1, \ldots, m$, in the model $a_1 x^{k_1} + \ldots + a_m x^{k_m}$, with $k_1 < ... < k_m$, from the $2m$ conditions

$$\sum_{i=1}^{m} a_i x_j^{k_i} = f_j, \quad j = 1, \ldots, 2m.$$

Besides the monomial basis x^{i-1}, the problem of interpolating

$$\sum_{i=1}^{m} a_i \phi_{k_i}(x_j) = f_j, \quad j = 1, \ldots, 2m,$$

*wen-shin.lee@ua.ac.be

from $2m$ evaluations is also solved for certain sequences of functions $\phi_i(x)$, including the Chebyshev polynomials $T_{i-1}(x)$, the Pochhammer symbols $(x)_{i-1}$ [13] and some multivariate generalizations of these [2]. In addition, a probabilistic strategy called "early termination" is developed to detect the number of nonzero terms (being m) when it is not supplied in the input [12]. Sparse techniques solve the interpolation problem from a number of samples f_j proportional to the number of terms in the representation (being m) rather than the number of available generating elements (being k_m). In floating point arithmetic, the connection between Prony's method [18] and error-correcting codes has led to the development of symbolic-numeric sparse polynomial interpolation [9], which exploits a generalized eigenvalue reformulation [11, 10] and a link to Rutishauser's qd-algorithm [5]. This connection further enables a generalization of variants of Prony to other basis functions [8].

Closely related to Padé approximation, the classical method of Prony has found applications in the shape from moments problem [16], spectral analysis [14], and lately sparse sampling of signals with finite rate of innovation [21], etc. The modern least squares approaches [20, 19] of exponential modeling have evolved quite significantly from Prony's original version. Still, it is well-known that in general such inverse problem can be both ill-posed and ill-conditioned.

Interestingly, techniques from symbolic-numeric sparse interpolation can be used to tackle these numerical issues. New sparse interpolation algorithms are thus developed by drawing from various disciplines such as numerical linear algebra, computer algebra and numerical approximation theory. The new method is efficient, and the technique is generalized for functions $\phi_k(x)$ where the parameter k can vary continuously [6].

In signal processing, sparsity has recently emerged as an important concept [3, 7, 4]. Sparse signals admit a representation by a linear combination of only a few elementary waveforms or atoms. Currently, the acquisition and reconstruction of such signals receives a great deal of attention. The ultimate goal is to determine the underlying sparse representation directly from as few data samples as possible. In many applications, such technique offers a promising alternative to the standardized Fourier transform. Moreover, the fact that signals can be reconstructed from undersampled data opens up a whole new range of possibilities. In this talk, we discuss the use of interpolation methods in this setting. We depart from sparse polynomial interpolation in the field of computer algebra and explain an interesting connection to Prony's method, as well as some of its variants. We present some new signal processing methods and discuss the corresponding issues in linear algebra and approximation theory. Selected applications will be presented (e.g. [17, 1]).

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