

# Mathematical Models of Visual Information Processing in the Human Brain and Applications to Image Processing

Arai, Hitoshi

Graduate School of Mathematical Sciences, The University of Tokyo | JST CREST

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# Mathematical Models of Visual Information Processing in the Human Brain and Applications to Image Processing

Hitoshi ARAI

Graduate School of Mathematical Sciences, The University of Tokyo, Japan /  
JST CREST

In this lecture I give a survey of joint works of Hitoshi Arai and Shinobu Arai. The main purpose of our study is to construct mathematical models of visual information processing in the brain, and to give applications to image processing.

On the past few decades, several studies have been made on mathematical models of visual information processing in the human brain. Our new models are constructed by using simple pinwheel framelets ([4]) and pinwheel framelets ([6]), which are a new class of the so-called framelets<sup>1</sup>. Before going to the main body of this lecture, we review (simple) pinwheel framelets.

## 1. PINWHEEL FRAMELETS

Some filters have been well known as models of simple cells in V1. For example, Gabor filters, DOG filters and ODOG filters. Recently Young pointed out that Gaussian derivatives are more appropriate than these filters. Adopting his theory, E. Escalante-Ramírez and Silván-Cárdenas<sup>2</sup> proposed a multiresolution model which consists of Gaussian derivatives. However Gaussian derivatives are not compactly supported: in other words, filters have infinite length in nature unless the filters are cut artificially. On the other hand, our simple pinwheel framelets ([4]) and pinwheel framelets ([6]) are produced by finite length filters (or compactly supported functions). The figure 1 is the maximal overlap version of our pinwheel framelets of degree 7 at the second level.

## 2. SIMULATIONS OF VISUAL ILLUSIONS

Based on the maximal overlap version of our pinwheel framelet we constructed a nonlinear model of visual information processing in the striate cortex. This model produces computer simulations of several lightness illusions in a unified way: the Hermann

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<sup>1</sup>General scheme of framelets was established by I. Daubechies, I. Han, B. Ron and Z. Shen, *Appl. Comput. Harm. Anal.*, 14 (2003), 1–46. Our (simple) pinwheel framelets are new framelets appropriate to study of vision.

<sup>2</sup>*Signal Processing: Image Communication*, 20 (2005), 801–812.

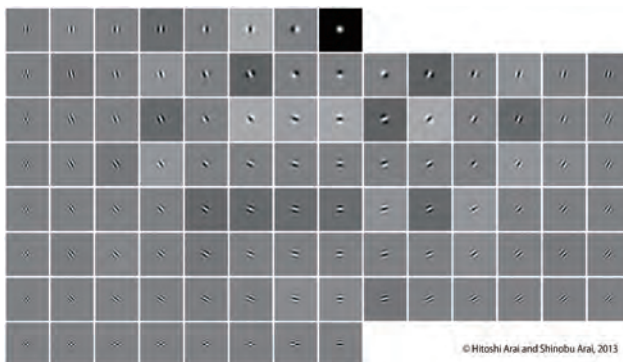


FIGURE 1. Maximal overlap version of the pinwheel framelet of degree 7 (level 2).

grid illusion, Mach band illusion, the Chevreul illusion etc. Here I show as an example simulations of the Hermann grid illusion and some variations.

The Hermann grid is shown in Fig.2. In this image, small gray spots are perceived at the intersections of white bands, however there are no such spots. (Note that the illusion disappear in foveal vision.) Our simulation of the illusion is Fig.3. So far it was thought that the Hermann grid illusion is caused by lateral inhibition in the retina (Baumgartner, 1960). However as reported in Spillmann and Levine<sup>3</sup> and Wolfe<sup>4</sup>, some evidences of a postretinal contribution to the illusion were found: the illusion is weakened when the grid is presented diagonally, and the strength of the illusion is effected by the number of intersections (see Fig.4). Our mathematical model can simulate these phenomena. For example I show the simulation of the former phenomenon in Fig.5.

### 3. APPLICATIONS TO IMAGE PROCESSING

Our nonlinear models of visual information processing in the brain have many applications not only to visual illusion, but also to image processing. In this section I show an example of applications to image processing of natural images. The original image (input image) is Fig.6(a), a photo of a natural scene of mountains in Japan. Fig.6(b) is the output image by our nonlinear model. This model has given us simulations of several lightness illusions, and some illusions related to color perception. Definitely the image is enhanced: for examples, thick foliage, ridgeline of mountains of a distant place, and dimly or vaguely clouds in Fig.6(a) are enhanced in Fig.6(b). Moreover, it

<sup>3</sup>*Perception* 23 (1994), 691–708.

<sup>4</sup>*Perception* 13 (1984), 33–40.

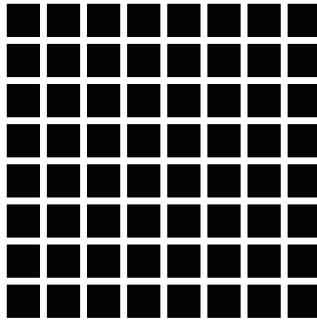


FIGURE 2. Hermann grid.



FIGURE 3. Simulation of the Hermann grid illusion by using our non-linear model with the pinwheel framelet of degree 5.

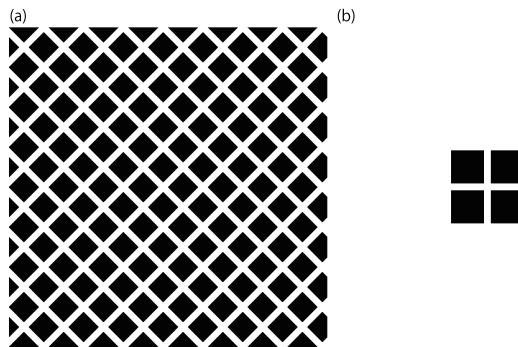


FIGURE 4. (a) Diagonally presented Hermann grid, (b) Hermann grid with one intersection of white roads.

does not distort unnecessarily the image. In this lecture, I will show superiority of our



FIGURE 5. Simulation of the diagonally presented Hermann grid illusion by using our nonlinear model with the pinwheel framelet of degree 5.

image processing based on our nonlinear model of vision in detail. This technique is patent pending (H. Arai and S. Arai, JST).



FIGURE 6. (a) Original image, (b) Processed image (output image).  
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#### 4. CREATION OF VISUAL ILLUSIONS

By one of our mathematical models of vision, we can transform usual images or designs to illusory images: some designs seem to be moving, although they are at rest in the images <sup>5</sup>. We call illusions produced by our method "fuyuu illusions". I show an example of fuyuu illusions. Fig.7 is an original image which contains no illusions.

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<sup>5</sup>Patent (H. Arai and S. Arai, JST).

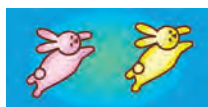


FIGURE 7. Original image: Two rabbits. ©Hitoshi Arai and Shinobu Arai, 2011.

By our program of making fuyuu illusions, we obtain Fig.8. Instruction of seeing this illusion is as follows: Please put the this book on a desk, and look at the small yellow disc at the center of Fig.8 . Then bring your face close to it, subsequently keep away your face from it, and repeat this action. You will feel that rabbits move circularly on the picture.



FIGURE 8. Jumping Rabbit illusion (H. Arai and S. Arai).

Why can our mathematical model create such illusions? An main idea is indicated in Fig.9.

## 5. OTHER RESULTS

We have other results related to mathematical models of visual information and applications to image processing. For example, mathematical algorithm of creating letter-row tilt illusions, noise reduction, edge detection, etc. Time permitting, I will show these other results.

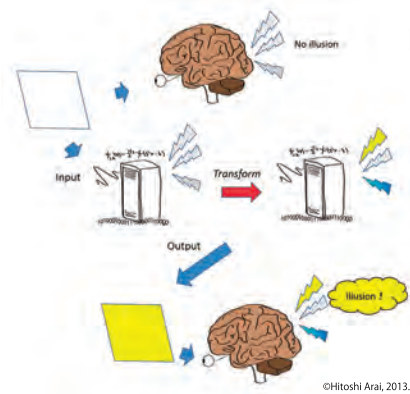


FIGURE 9. Idea of transforming to fuyuu illusions.

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