Efficient Image-Based Rendering Method using Spherical Gaussian

Iwasaki, Kei
Wakayama University | JST CREST

出版情報：MI lecture note series. 50, pp.52-55, 2013-10-21. 九州大学マス・フォア・インダストリ研究所

Efficient Image-based Rendering Method using Spherical Gaussian

Kei IWASAKI
Wakayama University, Wakayama, Japan / JST CREST

1. Introduction

Image-based rendering methods that capture the surrounding environment, store it as an environment map, and use it as the incident lighting (i.e. environment lighting) have been used to render realistic images. Real-time rendering of scenes illuminated by environmental lighting, is beneficial in many applications such as lighting/material design, animation, and games. For photorealistic rendering under environmental lighting, the triple product of the environmental lighting, the BRDF, and the visibility function is integrated. Integrating the triple product is computationally expensive and this prevents from real-time rendering under all-frequency environment lighting. To address this, we propose an efficient rendering method for scenes illuminated by all-frequency environmental lighting. Our method uses spherical Gaussian (SG) functions to efficiently integrate the triple product.

2. Spherical Gaussian

A spherical Gaussian (SG) is a function over the unit sphere and a type of spherical radial basis functions. A spherical Gaussian (SG) $G(\boldsymbol{\omega}_i; \boldsymbol{\xi}, \eta, \mu)$ is represented by the following equation:

$$G(\boldsymbol{\omega}_i; \boldsymbol{\xi}, \eta, \mu) = \mu e^{\eta(\boldsymbol{\omega}_i \cdot \boldsymbol{\xi} - 1)}, \tag{1}$$

where the unit vector $\boldsymbol{\xi}$ is the lobe axis, η is the lobe sharpness, and μ is the lobe amplitude that consists of RGB components.

SG has nice properties for rendering. Firstly, the product of two SGs is represented by another SG. That is, the product of two SGs is closed in SG basis.

$$G(\boldsymbol{\omega}_i; \boldsymbol{\xi}_1, \eta_1, \mu_1) \cdot G(\boldsymbol{\omega}_i; \boldsymbol{\xi}_2, \eta_2, \mu_2) = G(\boldsymbol{\omega}_i; \boldsymbol{\xi}_3, \eta_3, \mu_3), \tag{2}$$

where the lobe axis ξ_3, the lobe sharpness η_3, and the lobe amplitude μ_3 are calculated by the following equations.

$$\xi_3 = \frac{\eta_1 \boldsymbol{\xi}_1 + \eta_2 \boldsymbol{\xi}_2}{||\eta_1 \boldsymbol{\xi}_1 + \eta_2 \boldsymbol{\xi}_2||}, \tag{3}$$

$$\eta_3 = ||\eta_1 \boldsymbol{\xi}_1 + \eta_2 \boldsymbol{\xi}_2|| \tag{4}$$

$$\mu_3 = \mu_1 \mu_2 e^{||\eta_1 \boldsymbol{\xi}_1 + \eta_2 \boldsymbol{\xi}_2 - \eta_1 - \eta_2||} \tag{5}$$

Secondly, since SG is symmetric about the lobe axis, SG is easy to rotate just by rotating the lobe axis as the following equation.

$$\mathcal{R} G(\boldsymbol{\omega}_i; \boldsymbol{\xi}, \eta, \mu) = G(\boldsymbol{\omega}_i; \mathcal{R}\boldsymbol{\xi}, \eta, \mu), \tag{6}$$

where $\mathcal{R}$ is a rotation matrix.

Thirdly, the integral of SG over the unit sphere $\mathbb{S}^2$ is calculated analytically as the following equation.

$$\int_{\mathbb{S}^2} G(\boldsymbol{\omega}_i;\boldsymbol{\xi},\eta,\mu)d\boldsymbol{\omega} = \mu \int_0^{2\pi}\int_0^{\pi} e^{\eta(\cos\theta-1)}\sin\theta d\theta d\phi = \frac{2\pi}{\eta}(1-e^{-2\eta}). \tag{7}$$

Finally, the convolution of two SGs are calculated analytically as the following equation.

$$\int_{\mathbb{S}^2} G(\boldsymbol{\omega}_i;\boldsymbol{\xi}_1,\eta_1,\mu_1)\cdot G(\boldsymbol{\omega}_i;\boldsymbol{\xi}_2,\eta_2,\mu_2)d\boldsymbol{\omega}_i = \frac{4\pi\mu_1\mu_2}{e^{\eta_1+\eta_2}}\frac{\sinh(||\eta_1\boldsymbol{\xi}_1+\eta_2\boldsymbol{\xi}_2||)}{||\eta_1\boldsymbol{\xi}_1+\eta_2\boldsymbol{\xi}_2||}. \tag{8}$$

Although the convolution of two SGs is calculated analytically as the above equation, it is not closed in SG basis. To address this problem, we have derived the SG representation of the convolution of two SGs [2].

$$\begin{aligned}\int_{\mathbb{S}^2} G(\boldsymbol{\omega}_i;\boldsymbol{\xi}_1,\eta_1,\mu_1)\cdot G(\boldsymbol{\omega}_i;\boldsymbol{\xi}_2,\eta_2,\mu_2)d\boldsymbol{\omega} &= \frac{4\pi\mu_1\mu_2}{e^{\eta_1+\eta_2}}\frac{\sinh(||\eta_1\boldsymbol{\xi}_1+\eta_2\boldsymbol{\xi}_2||)}{||\eta_1\boldsymbol{\xi}_1+\eta_2\boldsymbol{\xi}_2||}\\ &= 2\pi\mu_1\mu_2\frac{e^{||\boldsymbol{\zeta}||-\eta_1-\eta_2}-e^{-||\boldsymbol{\zeta}||-\eta_1-\eta_2}}{||\boldsymbol{\zeta}||},\end{aligned} \tag{9}$$

where $\boldsymbol{\zeta}=\eta_1\boldsymbol{\xi}_1+\eta_2\boldsymbol{\xi}_2$. Here, we assume that both lobe sharpness values η_1 and η_2 are not too small. In practice, this assumption is valid for rendering applications. Then, we can assume that $e^{-||\zeta||-\eta_1-\eta_2}\approx 0$. Next, $||\boldsymbol{\zeta}||$ can be represented as follows:

$$\begin{aligned}||\boldsymbol{\zeta}|| &= \sqrt{\eta_1^2||\boldsymbol{\xi}_1||^2+2\eta_1\eta_2\boldsymbol{\xi}_1\cdot\boldsymbol{\xi}_2+\eta_2^2||\boldsymbol{\xi}_2||^2}\\ &= (\eta_1+\eta_2)\sqrt{1+\frac{2\eta_1\eta_2(\boldsymbol{\xi}_1\cdot\boldsymbol{\xi}_2-1)}{(\eta_1+\eta_2)^2}}.\end{aligned} \tag{10}$$

By using this representation, the numerator in Eq. (9) can be calculated as follows:

$$\begin{aligned}e^{||\boldsymbol{\zeta}||-\eta_1-\eta_2} &= e^{(\eta_1+\eta_2)\left(\sqrt{1+\frac{2\eta_1\eta_2(\boldsymbol{\xi}_1\cdot\boldsymbol{\xi}_2-1)}{(\eta_1+\eta_2)^2}}-1\right)}\\ &\approx e^{(\eta_1+\eta_2)\cdot\frac{\eta_1\eta_2(\boldsymbol{\xi}_1\cdot\boldsymbol{\xi}_2-1)}{(\eta_1+\eta_2)^2}}\\ &= e^{\frac{\eta_1\eta_2}{(\eta_1+\eta_2)}(\boldsymbol{\xi}_1\cdot\boldsymbol{\xi}_2-1)},\end{aligned} \tag{11}$$

where a linear approximation of the Taylor expansion $\sqrt{1+x}\approx 1+x/2$ is used. The denominator $||\boldsymbol{\zeta}||$ can be approximated with $\eta_1+\eta_2$, since $\frac{2\eta_1\eta_2(\boldsymbol{\xi}_1\cdot\boldsymbol{\xi}_2-1)}{(\eta_1+\eta_2)^2}$ can be considered as negligible. Finally, the convolution of two SGs can be represented by a single SG as:

$$\int_{S^2} G(\boldsymbol{\omega}_i;\boldsymbol{\xi}_1,\eta_1,\mu_1)\cdot G(\boldsymbol{\omega}_i;\boldsymbol{\xi}_2,\eta_2,\mu_2)d\boldsymbol{\omega} \approx G\left(\boldsymbol{\xi}_1,\boldsymbol{\xi}_2,\frac{\eta_1\eta_2}{\eta_1+\eta_2},\frac{2\pi\mu_1\mu_2}{\eta_1+\eta_2}\right). \tag{12}$$

3. Image-based Rendering using Spherical Gaussian

The outgoing radiance $L(\mathbf{x},\boldsymbol{\omega}_o)$ at point $\mathbf{x}$ in the outgoing direction $\boldsymbol{\omega}_o$ under environment lighting is calculated by the following equation.

$$L(\mathbf{x},\boldsymbol{\omega}_o)=\int_{\mathbb{S}^2} L(\boldsymbol{\omega}_i)V(\mathbf{x},\boldsymbol{\omega}_i)\rho(\mathbf{x},\boldsymbol{\omega}_i,\boldsymbol{\omega}_o)\max(0,\mathbf{n}(\mathbf{x})\cdot\boldsymbol{\omega}_i)d\boldsymbol{\omega}_i, \tag{13}$$

where $\mathbb{S}^2$ denotes the unit sphere in $\mathbb{R}^3$, $\boldsymbol{\omega}_i$ is the incident direction, $L(\boldsymbol{\omega}_i)$ is the distant lighting represented by the environment maps, $V(\mathbf{x},\boldsymbol{\omega}_i)$ is the visibility function at $\mathbf{x}$,

ρ is the BRDF, and $\mathbf{n}(\mathbf{x})$ is the normal at $\mathbf{x}$. To simplify the notation, we omit $\mathbf{x}$ in the following. Our method represents the BRDF ρ with $\rho(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = k_d + k_s\rho_s(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o)$ where k_d is a diffuse term and $k_s\rho_s(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o)$ is a specular term. By substituting this in Eq. (13), $L(\boldsymbol{\omega}_o)$ can be calculated from the sum of the diffuse component L_d and the specular component $L_s(\boldsymbol{\omega}_o)$ as follows:

$$L(\boldsymbol{\omega}_o) = k_d L_d + k_s L_s(\boldsymbol{\omega}_o),$$

$$L_d = \int_{\mathbb{S}^2} L(\boldsymbol{\omega}_i) V(\boldsymbol{\omega}_i) \max(0, \boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i, \tag{14}$$

$$L_s(\boldsymbol{\omega}_o) = \int_{\mathbb{S}^2} L(\boldsymbol{\omega}_i) V(\boldsymbol{\omega}_i) \rho_s(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) \max(0, \boldsymbol{\omega}_i \cdot \mathbf{n}) d\boldsymbol{\omega}_i. \tag{15}$$

To calculate L_d, our method approximates $L(\boldsymbol{\omega}_i)$ and the cosine term $\max(0, \mathbf{n} \cdot \boldsymbol{\omega}_i)$ with the sum of spherical Gaussians.

Our method represents the environmental lighting L as the sum of SGs; $L(\boldsymbol{\omega}_i) \approx \sum_{k=1}^{K} G(\boldsymbol{\omega}_i; \boldsymbol{\xi}_k, \eta_k, \mu_k)$ where K is the number of SG lobes. The cosine term is also approximated by a single SG as $G(\boldsymbol{\omega}_i; \mathbf{n}, \eta_c, \mu_c)$. By substituting these terms into Eq. (14), L_d is calculated by the following equation.

$$L_d \approx \sum_{k=1}^{K} \int_{\mathbb{S}^2} G(\boldsymbol{\omega}_i; \boldsymbol{\xi}_k, \eta_k, \mu_k) G(\boldsymbol{\omega}_i; \mathbf{n}, \eta_c, \mu_c) V(\boldsymbol{\omega}_i) d\boldsymbol{\omega}_i. \tag{16}$$

The product of two SGs can be represented with a single SG:

$$G(\boldsymbol{\omega}_i; \boldsymbol{\xi}, \eta, \mu) = G(\boldsymbol{\omega}_i; \boldsymbol{\xi}_k, \eta_k, \mu_k) G(\boldsymbol{\omega}_i; \mathbf{n}, \eta_c, \mu_c), \tag{17}$$

where the lobe sharpness η is $||\eta_k \boldsymbol{\xi}_k + \eta_c \mathbf{n}||$, the lobe axis $\boldsymbol{\xi}$ is $(\eta_k \boldsymbol{\xi}_k + \eta_c \mathbf{n})/\eta$, and the amplitude μ is $\mu_k \mu_c e^{\eta - \eta_k - \eta_c}$. The diffuse component L_d is calculated by integrating the product of the visibility function $V(\boldsymbol{\omega}_i)$ and the spherical Gaussian $G(\boldsymbol{\omega}_i; \boldsymbol{\xi}, \eta, \mu)$. The integral of the product of SG and the visibility function is efficiently computed by using spherical signed distance function [3] for static scenes and by using integral spherical Gaussian [1] for dynamic scenes.

To calculate the specular component L_s, our method represents the BRDF ρ and the cosine term as the sum of SGs. As [3], the BRDF $\rho(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o)$ can be calculated from the sum of the SGs as $\rho(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) \approx \sum_{j=1}^{J} G(\boldsymbol{\omega}_i; \boldsymbol{\xi}_j, \eta_j, \mu_j)$, where J is the number of SGs for ρ, and the lobe axis $\boldsymbol{\xi}_j$ depends on the outgoing direction $\boldsymbol{\omega}_o$. By substituting this into Eq. (15), $L_s(\boldsymbol{\omega}_o)$ can be rewritten as:

$$L_s(\boldsymbol{\omega}_o) \approx \sum_{j=1}^{J} \int_{\mathbb{S}^2} L(\boldsymbol{\omega}_i) G(\boldsymbol{\omega}_i; \boldsymbol{\xi}_j, \eta_j, \mu_j) G(\boldsymbol{\omega}_i; \mathbf{n}, \eta_c, \mu_c) V(\boldsymbol{\omega}_i) d\boldsymbol{\omega}_i. \tag{18}$$

Since the product of two SGs is also represented with a single SG $G_j(\boldsymbol{\omega}_i) = G(\boldsymbol{\omega}_i; \boldsymbol{\xi}'_j, \eta'_j, \mu'_j)$, $L_s(\boldsymbol{\omega}_o)$ can be rewritten as:

$$L_s(\boldsymbol{\omega}_o) \approx \sum_{j=1}^{J} \int_{\mathbb{S}^2} L(\boldsymbol{\omega}_i) G_j(\boldsymbol{\omega}_i) V(\boldsymbol{\omega}_i) d\boldsymbol{\omega}_i. \tag{19}$$

Our method approximates the integral of the triple product as:

$$L_s(\boldsymbol{\omega}_o) \approx \sum_{j=1}^{J} \int_{\mathbb{S}^2} L(\boldsymbol{\omega}_i) G_j(\boldsymbol{\omega}_i) d\boldsymbol{\omega}_i \frac{\int_{\mathbb{S}^2} G_j(\boldsymbol{\omega}_i) V(\boldsymbol{\omega}_i) d\boldsymbol{\omega}_i}{\int_{\mathbb{S}^2} G_j(\boldsymbol{\omega}_i) d\boldsymbol{\omega}_i}. \tag{20}$$

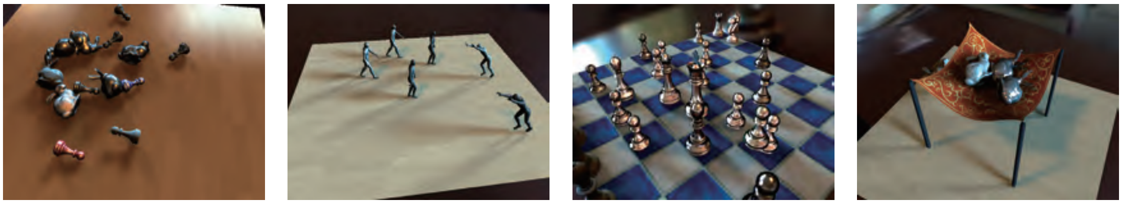

FIGURE 1. Rendering results of our method.

The numerator $\int_{\mathbb{S}^2} G_j(\boldsymbol{\omega}_i)V(\boldsymbol{\omega}_i)d\boldsymbol{\omega}_i$ can be calculated in the same way as L_d and the denominator $\int_{\mathbb{S}^2} G_j(\boldsymbol{\omega}_i)d\boldsymbol{\omega}_i$ can be calculated analytically as $\frac{2\pi}{\eta'_j}(1-e^{-2\eta'_j})$. The integral of the product of L and G is calculated as a 3D function $\Gamma_L(\boldsymbol{\xi},\eta)$:

$$\int_{\mathbb{S}^2} L(\boldsymbol{\omega}_i)G(\boldsymbol{\omega}_i;\boldsymbol{\xi},\eta,\mu)d\boldsymbol{\omega}_i = \mu\Gamma_L(\boldsymbol{\xi},\eta). \tag{21}$$

Our method precomputes $\Gamma_L(\boldsymbol{\xi},\eta)$ and stores the data as prefiltered environment maps for various lobe sharpness η as [3]. Therefore, our method can change the BRDFs (i.e. the SGs) at run-time. Since our method integrates the product of the BRDF approximated by the SGs and the lighting, our method can represent highly specular materials.

4. Results and Conclusion

Fig. 1 shows the rendering results of dynamic scenes illuminated by all-frequency environment lighting. We have implemented our rendering algorithm on a standard PC with an Intel Corei7 Extreme 965 CPU and a GeForce GTX 580 GPU. The rendering frame rates are 2.5-40 fps. The image resolutions are 640×480. We have proposed a real-time rendering method for fully dynamic scenes with highly specular BRDFs illuminated by all-frequency lighting by using spherical Gaussians.

References

[1] K. Iwasaki, W. Furuya, Y. Dobashi, and T. Nishita: Real-time Rendering of Dynamic Scenes under All-frequency Lighting using Integral Spherical Gaussian, Computer Graphics Forum(Eurographics 2012), Vol. 31, No. 2, pp. 727-734, 2012.

[2] K. Iwasaki, Y. Dobashi, and T. Nishita: Interactive Bi-scale Editing of Highly Glossy Materials, ACM Transactions on Graphics, Vol. 26, No.6, 144:1-144:7, 2012.

[3] J. Wang, P. Ren, M. Gong, J. Snyder, and B. Guo: All-frequency Rendering of Dynamic Spatially-Varying Reflectance, ACM Transactions on Graphics, Vol. 28, No.5, 133:1-133:10, 2009.