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Microscopic Deformation Effects in Up-sampled Cloth Simulation

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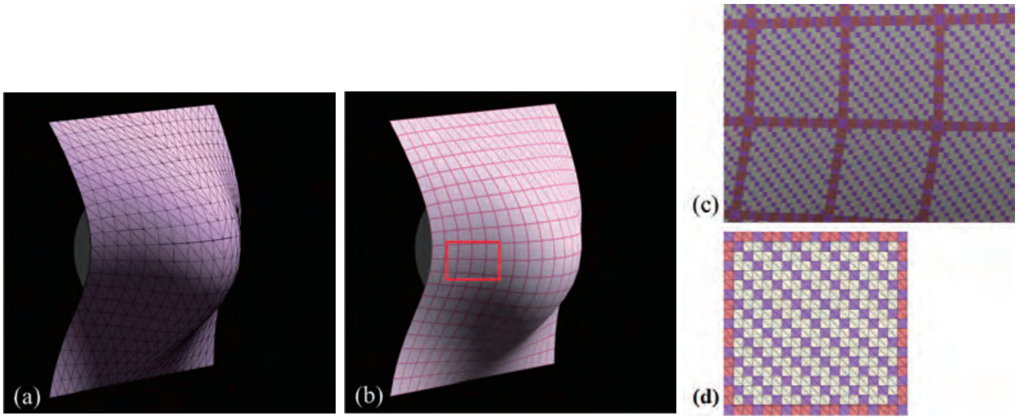

Figure 1. Up-sampled clothing simulation. (a)The simulated down-sampled mesh (800 triangles) that preserves macro characteristics unit-cells have such as anisotropy (b)Microscopic displacements attached to the down-sampled mesh (288k triangles). (c) Enlarged view of (b) showing up-sampled meshes with different stiffness that constitute a unit-cell shown in (d).

1. Introduction

Strong demands for simulating detailed but yet efficient cloth are pervasive in the CG animation industry especially for interactive applications such as games. However, cost of detailed cloth simulation is expensive and the use of highly detail cloth in real-time simulations is limited. Our goal is to accelerate high detail cloth simulation, more specifically made of anisotropic and inhomogeneous materials, by simulating only coarse meshes. In this paper, we propose an effective down-sampling method applicable for fine, heterogeneous and periodic structures by homogenization method. With our technique, we can simulate highly detailed cloth in real-time with a few coarse meshes

Advances in physics-based simulation see success in realistic clothing animation [1, 3, 4, 13]. Although these methods can produce highly detailed results that are also made of non-homogeneous or anisotropic elasticity, they make the simulation more costly, typically several minutes per frame. To accelerate cloth simulation, they simulate coarse meshes and then restore the fine detail [2, 7, 5, 14]. However, physically accurate results are not necessarily obtained because microscopic deformations such as anisotropy or inhomogeneity are not accounted in the material property of coarse mesh deformations.

Homogenization methods have been proposed [12, 6] in order to derive mechanical material property characteristics of macro deformations taking into account microscopic structures. However, these applications have been limited to deformation of solid materials and has not been applied to thin shells like cloth. Here we apply homogenization method to highly detailed cloth simulation.

As a pre-computation step, we homogenize periodic unit-cells in which anisotropic or non-homogeneous material property is embedded to obtain microscopic deformation called characteristic displacement which correlates coarse meshes deformation to fine mesh deformation. From characteristic displacement, we compute an effective elasticity tensor that approximates bulk material stiffness in the coarse simulation. At every simulation time step, we compute microscopic deformation from the macro strain on the coarse meshes, and then superimpose them in the deformed coarse meshes. Furthermore, we implement the linear elasticity to introduce homogenization method. The robust and efficient simulation of large deformation can be achieved by applying the warping stiffness method for thin shell.

Contributions of our method are listed as follows:

- Up-sampled technique which predicts microscopic in-plane deformation from 3D coarse mesh deformation.
- Efficient and robust cloth simulator for which warping stiffness method.

2. Method

2.1. **Homogenization.** As a pre-computation step, we obtain an effective elasticity tensor that approximate the same geometry like fine scale simulation using characteristic displacements which correspond to deformed coarse meshes. To approximate a fine simulation, we provide the following Eq. (1) that satisfies the conservation of the internal energy on both fine meshes and coarse meshes

$$W = \boldsymbol{\Sigma} : \boldsymbol{E} = \frac{1}{|Y|}\int_Y \boldsymbol{\sigma}^0 : \boldsymbol{\epsilon}^0 dy = \frac{1}{|Y|}\int_Y \boldsymbol{\epsilon}^0 : \boldsymbol{c}_y : \boldsymbol{\epsilon}^0 dy, \tag{1}$$

where Y is internal region of a unit-cell, $|Y|$ is its area. $\boldsymbol{\Sigma}$ and $\boldsymbol{E}$ are Cauchy stress tensor and linear strain on a unit-cell. while σ^0 and ϵ^0 are Cauchy stress tensor and linear strain tensor on fine meshes.

To homogenize the stretching and shearing stiffness of cloth, we first assume that unit-cells are placed on a two dimensional plane. The characteristic displacements $\boldsymbol{\chi}_{\alpha\beta} \in \mathbb{R}^{3n}$ of all n vertices of a unit-cell (with $1 \leq \alpha \leq \beta \leq 2$) can be computed by solving following problems.

$$\boldsymbol{K}\boldsymbol{\chi}_{\alpha\beta} = \int_Y \boldsymbol{B}^T\boldsymbol{C}\,dy : \boldsymbol{E}_{\alpha\beta} \quad (1 \leq \alpha \leq \beta \leq 2), \tag{2}$$

where $\boldsymbol{K} \in \mathbb{R}^{3n\times 3n}$ is a stiffness matrix, $\boldsymbol{B} \in \mathbb{R}^{2\times 3}$ is a deformed gradient tensor $\boldsymbol{C} \in \mathbb{R}^{3\times 3}$ is the elasticity tensor of each fine meshes on a unit-cell. However, $\boldsymbol{K}$ is still a singular matrix. To obtain homogenized stiffness matrix $\boldsymbol{C}^H \in \mathbb{R}^{3\times 3}$ that satisfies Eq. (2), we need to constraint that $\boldsymbol{\chi}_{\alpha\beta}$ varies periodically.

We assume that the deformations are small and that the stress on a unit-cell is equal to the total amount of stress on fine scales. Hence a macro stress on a unit-cell satisfies the following equality:

$$\boldsymbol{\Sigma} = \boldsymbol{C}^H : \boldsymbol{E} = \frac{1}{|Y|}\int_Y \boldsymbol{c}(\boldsymbol{y}) : (\mathbf{I} - \nabla_y \chi_{\alpha\beta}(\boldsymbol{y})dy : \boldsymbol{E}, \tag{3}$$

where $\boldsymbol{c}(\boldsymbol{y}) \in \mathbb{R}^{3\times3}$ is the elasticity tensor of each fine meshes on a unit-cell. By comparing the middle term of Eq. (3) with the right term, we obtain $\boldsymbol{C}^H$ as

$$\boldsymbol{C}^H = \frac{1}{|Y|}\int_Y \boldsymbol{c}(\boldsymbol{y}) : (\mathbf{I} - \nabla_y \chi_{\alpha\beta}(\boldsymbol{y})dy, \tag{4}$$

where $\boldsymbol{I} \in \mathbb{R}^{3\times3}$ is a unit tensor.

2.2. **Cloth Simulation Using Linear Elasticity.** So far, we illustrate homogenization technique for two dimensional linear elasticity. Our approach can be applied for linear elasticity because its computational cost is lower than the cost of non-linear elasticity and it is easy to implement. However, visual artifact often occur under large deformations of the simulation linear elasticity because its stiffness matrix is constant. In order to achieve robust and fast simulation of cloth, we applied the existing method of warping stiffness for thin shell.

To apply the linear strain tensor for large deformations, co-rotational methods has been proposed [9, 10]. However, these methods have not been introduced into membrane elements such as cloth because the rotation matrix can be unstable to compute from a plane in 3D space. We follow the approach of the *oriented particles* [8] to obtain a rotation matrix on a node. Moreover, in order to conform the normal vector $\boldsymbol{z} \in \mathbb{R}^3$ of the current deformed vertex with the initial normal vector $\boldsymbol{Z} \in \mathbb{R}^3$ rotated by the current rotation matrix, we add a revision matrix $\boldsymbol{z}\boldsymbol{Z}^T$ to a moment matrix $\boldsymbol{A} \in \mathbb{R}^{3\times3}$. Given rest position $\boldsymbol{x}_{0j}$, current position $\boldsymbol{x}_j$ and mass m_j for vertex j neighboring on vertex i and the current and rest mass centers $\boldsymbol{c}$ and $\boldsymbol{c}_0$ respectively, $\boldsymbol{A}$ is computed as

$$\boldsymbol{A} = \sum_{j\in\mathcal{N}_i}(m_j\boldsymbol{x}_j\boldsymbol{x}_{0j}^T + \boldsymbol{A}_j) - M\boldsymbol{c}\boldsymbol{c}_0^T + \boldsymbol{z}\boldsymbol{Z}^T, \tag{5}$$

where $\mathcal{N}_i$ is one-ling neighborhood of node i, $M = \sum_{j\in\mathcal{N}_i} m_j$ and the inertia moment $\boldsymbol{A}_j$ are defined as

$$\boldsymbol{A}_i = \frac{1}{5}m_i\begin{pmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & c^2 \end{pmatrix}\boldsymbol{R}_i, \tag{6}$$

where a,b and c are radii of an ellipsoid which has rotational inertia equivalent to triangles surrounding the node i. $\boldsymbol{R} \in \mathbb{R}^3$ is rotation matrix defined at each node. Then, $\boldsymbol{A}_i$ is decomposed by singular value decomposition method [9] as $\boldsymbol{USW^T}$. The rotation matrix $\boldsymbol{R}$ is computed as

$$\boldsymbol{R} = \boldsymbol{W}\boldsymbol{U}^T. \tag{7}$$

Using the rotation matrix $\boldsymbol{R}$, the elastic force $\boldsymbol{f}_i$ can be written as

$$\boldsymbol{f}_i = \boldsymbol{R}_i \sum_{j\in\mathcal{N}_i} \boldsymbol{k}_{ij}(\boldsymbol{R}_i^{-1}\boldsymbol{x}_j - \boldsymbol{x}_j^0), \tag{8}$$

where $\boldsymbol{k}_{ij}$ is a component of stiffness matrix associated with vertex i and vertex j.

2.3. **Localization.** With the machinery described above, we can obtain deformation of coarse mesh that is similar to the down-sampled high resolution simulation. As the final procedure of our runtime, we up-sample fine mesh vertex positions from coarse mesh deformation by interpolating pre-computed characteristic displacements.

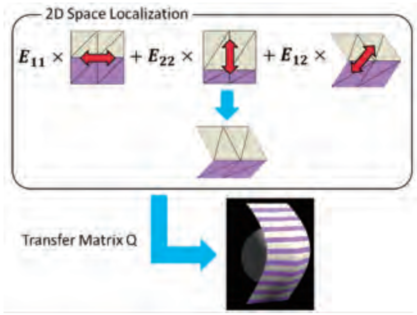

FIGURE 2. Localization into 3D space.

In two dimensional case, as shown in Fig. 2, microscopic displacements $\boldsymbol{w} \in \mathbb{R}^{3n}$ of all n vertices on a unit-cell are presented as linear combination of deformation modes as:

$$\boldsymbol{w} = -(\boldsymbol{E}_{11}\boldsymbol{\chi}_{11} + \boldsymbol{E}_{22}\boldsymbol{\chi}_{22}p + \boldsymbol{E}_{12}\boldsymbol{\chi}_{12}). \tag{9}$$

However, as cloth is deformed in 3D space, we cannot apply Eq. (9) directly for interpolation because those strain and displacements are defined on only a two dimensional plane. Therefore, at every time step, we translate the strains on current deformed coarse meshes into that on XY-plane and then compute microscopic displacements $\boldsymbol{w}$ on XY-plane by Eq. (9). Let $\boldsymbol{v_i} \in \mathbb{R}^3$ and $\boldsymbol{v_i^0} \in \mathbb{R}^3 \quad i \in \{1, 2, 3\}$, be the initial and deformed vertices of the triangle, respectively. Following the frame definition on a triangle in [11], we introduce a fourth initial vertex as

$$\boldsymbol{v}_3 = \boldsymbol{v}_0 + \frac{(\boldsymbol{v}_2 - \boldsymbol{v}_0) \times (\boldsymbol{v}_1 - \boldsymbol{v}_0)}{\|(\boldsymbol{v}_2 - \boldsymbol{v}_0) \times (\boldsymbol{v}_1 - \boldsymbol{v}_0)\|} \in \mathrm{R}^{3\times 3}. \tag{10}$$

Using an undeformed matrix $\boldsymbol{V^0} = \{\boldsymbol{v}_2^0 - \boldsymbol{v}_1^0, \boldsymbol{v}_3^0 - \boldsymbol{v}_1^0, \boldsymbol{v}_4^0 - \boldsymbol{v}_1^0\} \in \mathrm{R}^{3\times 3}$ and a deformed matrix $\boldsymbol{V} = \{\boldsymbol{v}_1 - \boldsymbol{v}_0, \boldsymbol{v}_2 - \boldsymbol{v}_0, \boldsymbol{v}_3 - \boldsymbol{v}_0\}$, an affine transformation matrix $\boldsymbol{Q} \in \mathbb{R}^{3\times 3}$ is given by

$$\boldsymbol{Q} = \boldsymbol{V}_0\boldsymbol{V}^{-1}. \tag{11}$$

Interpolated microscopic deformations on XY-plane are translated into current deformed coarse triangle in 3D space. The displacement for any vertex $\boldsymbol{x}_i$ inside this triangle can be computed as

$$\boldsymbol{x}_t = \boldsymbol{x}_0 + \boldsymbol{Q}(\boldsymbol{w}_i + \boldsymbol{x}_i^0 - \boldsymbol{x}_0^0), \tag{12}$$

where $\boldsymbol{x}_0$ is the present position of the vertex on a unit-cell that corresponds to a coarse mesh, $\boldsymbol{w}_i$ is displacement of fine meshes vertex i on a unit-cell, $\boldsymbol{x}_i^0$, and $\boldsymbol{x}_0^0$ are the initial positions of $\boldsymbol{x}_i$ and $\boldsymbol{x}_0$, respectively. We blend the displacements across adjacent unit-cells to avoid discontinuities without any artifacts.

3. Result

We apply our framework on a deformation of inhomogeneous shell in which soft and hard materials repeat line by line (Fig.6). As the results show, anisotropic homogenized coarse meshes behaved similar to the high-resolution simulation using non-homogeneous materials. Microscopic deformations also exhibit on coarse meshes due to localization so as to behave similar to the non-homogeneous one. Although the high-resolution simulation can take 1250 ms per time step, the simulation using our method can take 2.65 ms per time step. It is about 470 times faster than the naïve high-resolution simulation.

As shown in Fig.5, errors in strain of 1cm $\times$ 1cm unit-cells are small enough to be observed. And most of cloth simulators allow cloth to deform in up to 10 percent strain [4]. Therefore, localization can be practice to simulate cloth that has linear microscopic characteristics.

As an application of our method, we tested more subtle geometric detail made out of three different stiffness parameters (Fig.1). All the numbers are obtained on a laptop PC with Intel Core2 Duo$^{\text{TM}}$3.0 GHz.

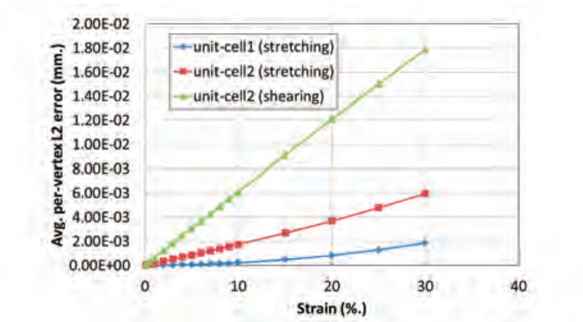

Figure 3. Error and performance statistics for the unit-cell shown in Fig. 6, and the uni-cell in Fig. 1. with respect to increase of strain. Error is computed as per-vertex L^2 error averaged over entire vertices.

4. Conclusion and Future Work

We proposed a fast simulation technique for non-homogenous elastic cloth. In addition, we presented a robust and fast simulation method by leveraging stiffness warping technique. As for future work, we plan to extend the current in-plane homogenization technique into bending so that we can handle more nonlinear elastic clothing deformation such as wrinkles.

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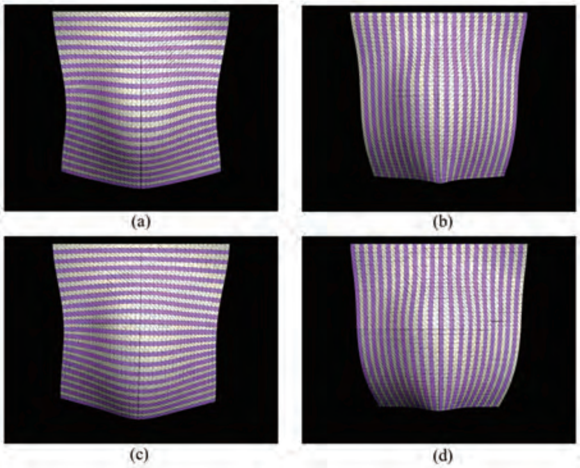

Figure 4. (a and b) have elastic properties of a layered object made out of a unit-cell (hard one is purple mesh and soft one is off-white mesh). (c and d) are the results of localization of coarse mesh (800 triangles) that the unit-cells are turned into anisotropic elastic properties on.

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