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https://hdl.handle.net/2324/14230

出版情報：経済学研究院ディスカッション・ペーパー, 2009-03-24. 九州大学大学院経済学研究院
バージョン：
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SIMULATION STUDIES OF MULTIPLE PORTMANTEAU TESTS *

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Abstract. The portmanteau statistic based on the first m residual autocorrelations is used for diagnostic checks on the adequacy of fitting a model with varying m. Katayama (2008) proposed an approximation of the joint probability of multiple portmanteau tests with different degrees of freedom (DF). This distribution is easy to compute when all DF are even integers; its empirical behavior is clarified in terms of asymptotic theory. This paper presents detailed simulation study over a variety of ARMA(p, q) models and different m's to show the empirical sizes and powers of the proposed test.

Keywords. Autoregressive-moving average model; Portmanteau test; Residual autocorrelations; Goodness-of-fit; Multiple tests.

1 INTRODUCTION

This paper presents a detailed simulation study of multiple portmanteau tests by Katayama (2008). This paper details the results of a simulation study on a variety of ARMA(p, q) models and different m's which show the empirical sizes and powers of the proposed test. To save space, we use the notations given in Katayama (2008). We used S-PLUS to compute ten thousand runs of the simulation experiments.

2 Maximum value of DF

This section discusses how to decide the maximum value of Degree of Freedom (DF) for the individual portmanteau tests.

2.1 Variance/mean ratios for $p = q = 0$

First we review the results for the case of $p = q = 0$. Kwan and Sim (1996) and Kwan (2003) carried out simulation studies to check the empirical significance levels, means, variances and variance/mean ratios (VMR) of the modified portmanteau statistics, Q^*_m for $p + q = 0$. The study shows that when the empirical sizes increase, VMR increases, even though the empirical means are stable. Correspondingly, for $p = q = 0$, the theoretical VMR of Q^*_m, is $2 + 2(2m - 5)/n$ by Ljung and Box (1978, p.299). Combining these results, Tables 1 and 2 give results which show the discrepancies from the chi-squared approximation. For each cell in Table 1, the first number is the empirical significance level for the theoretical significance level of 5% given by Kwan and Sim (1996, Table 1) and Kwan (2003, Table 1). The number in parentheses is the empirical VMR given by Kwan and Sim (1996, Table 3) and Kwan (2003, Table 2). The number in brackets is the theoretical VMR $2 + 2(2m - 5)/n$ by Ljung and Box (1978). This table indicates that the theoretical and empirical VMR are very close, except for the case of DF$= 20$ and $n = 40$, and the empirical (theoretical) significance levels are proportional to VMR. For example, for $n = 50$ and DF$= 12$ and $n = 100$ and DF$= 20$, both empirical significance levels are 7.4%, while the empirical (theoretical) VMR are very similar.

*Discussion paper. Revision Date: March 24, 2009.

DF=	1	3	5	7	10	12	15	20	25
$n = 40$	5.3	5.1	5.7	6.3	7.1	7.7	8.7	9.4	
	(1.96)	(2.04)	(2.19)	(2.38)	(2.66)	(2.87)	(3.17)	(3.40)	
	[1.85]	[2.05]	[2.25]	[2.45]	[2.75]	[2.95]	[3.25]	[3.75]	
$n = 50$	4.9	5.0	5.3	5.9	6.8	7.4			
	(1.85)	(2.00)	(2.15)	(2.30)	(2.56)	(2.75)			
	[1.88]	[2.04]	[2.20]	[2.36]	[2.60]	[2.76]			
$n = 100$	5.0	5.2	5.4	5.8	5.9	6.2	6.7	7.4	8.2
	(1.95)	(2.00)	(2.07)	(2.13)	(2.24)	(2.31)	(2.47)	(2.71)	(2.93)
	[1.94]	[2.02]	[2.10]	[2.18]	[2.30]	[2.38]	[2.50]	[2.70]	[2.90]

Table 1: Empirical significance levels, empirical VMR and theoretical VMR of portmanteau statistics for $p = q = 0$.

DF=	6	8	10	12	14	16	18	20
$n = 50$	2.28	2.44	2.60	2.76	2.92	3.08	3.24	3.40
DF=	14	16	22	24	30	32	38	40
$n = 100$	2.46	2.54	2.78	2.86	3.10	3.18	3.42	3.50
DF=	28	30	44	46	60	62	76	78
$n = 200$	2.51	2.55	2.83	2.87	3.15	3.19	3.47	3.51

Table 2: Theoretical VMR of Q^*_m for $p = q = 0$.

2.2 Maximum value of DF and VMR for ARMA(p, q) models

To extend the simulation studies by Kwan and Sim (1996) and Kwan (2003) to general ARMA models, we propose the following as an approximation of the variance of the portmanteau statistics: Let $\boldsymbol{z}^*_m = \boldsymbol{T}_m \boldsymbol{r}_m$. Then we have:

$$\boldsymbol{z}^{*\prime}_m \boldsymbol{z}^*_m = \boldsymbol{z}^{*\prime}_m \boldsymbol{D}_m \boldsymbol{z}^*_m + \boldsymbol{z}^{*\prime}_m (\mathbb{I}_m - \boldsymbol{D}_m) \boldsymbol{z}^*_m \tag{1}$$

and $\boldsymbol{z}^*_m \xrightarrow{d} N_m(\boldsymbol{0}_m, \mathbb{I}_m)$ as $n \to \infty$. We note that: (i) $\boldsymbol{z}^{*\prime}_m \boldsymbol{z}^*_m$ is Q^*_m for $p = q = 0$, which is examined by Kwan and Sim (1996) and Kwan (2003) and this variance is $2m\{1+(2m-5)/n\}$ by Ljung and Box (1978, p.299). (ii) From Cochran's theorem, asymptotically, $\boldsymbol{z}^{*\prime}_m \boldsymbol{D}_m \boldsymbol{z}^*_m \sim \chi^2_{p+q}$, $\boldsymbol{z}^{*\prime}_m (\mathbb{I}_m - \boldsymbol{D}_m) \boldsymbol{z}^*_m \sim \chi^2_{m-p-q}$, and $\boldsymbol{z}^{*\prime}_m \boldsymbol{D}_m \boldsymbol{z}^*_m$ and $\boldsymbol{z}^{*\prime}_m (\mathbb{I}_m - \boldsymbol{D}_m) \boldsymbol{z}^*_m$ are independent. (iii) The portmanteau statistics, Q^*_m and Q^{**}_m, are approximately equal to $\boldsymbol{z}^{*\prime}_m (\mathbb{I}_m - \boldsymbol{D}_m) \boldsymbol{z}^*_m$. It follows that the approximate variance of the portmanteau statistics are:

$$\begin{aligned} \mathrm{Var}(\boldsymbol{z}^{*\prime}_m (\mathbb{I}_m - \boldsymbol{D}_m) \boldsymbol{z}^*_m) &= \mathrm{Var}(\boldsymbol{z}^{*\prime}_m \boldsymbol{z}^*_m) - \mathrm{Var}(\boldsymbol{z}^{*\prime}_m \boldsymbol{D}_m \boldsymbol{z}^*_m) \\ &= 2m + \frac{2m(2m-5)}{n} - 2(p+q) \\ &= 2(m-p-q) + \frac{2m(2m-5)}{n}. \end{aligned}$$

The approximate mean of the portmanteau statistics is $m - p - q$ from Ljung and Box (1978, p.300). Therefore, we can approximate the VMR of the portmanteau statistics by:

$$\mathrm{VMR}(m) = 2 + \frac{2m(2m-5)}{n(m-p-q)}. \tag{2}$$

Table 3 shows VMR(m) for $n = 50, 100, 200$, $p + q = 1, 2$, and various DF.

$p+q=1$				
DF=	8	12	16	20
$n=50$	2.58	2.91	3.23	3.55
DF=	16	24	32	40
$n=100$	2.62	2.94	3.26	3.58
DF=	30	46	62	78
$n=200$	2.59	2.91	3.23	3.55

$p+q=2$				
DF=	6	10	14	18
$n=50$	2.50	2.83	3.15	3.48
DF=	14	22	30	38
$n=100$	2.58	2.90	3.22	3.54
DF=	28	44	60	76
$n=200$	2.57	2.89	3.21	3.53

Table 3: Theoretical VMR of portmanteau statistics for $p+q=1,2$.

2.3 Simulation experiments for means, variances, and VMR

For simulation experiments, we consider the models $\{y_t\}_{t=1}^n$:

$$\begin{array}{lll} \text{AR}(1): & y_t - a y_{t-1} = \varepsilon_t & a = -0.9(0.3)0.9 \\ \text{MA}(1): & y_t = \varepsilon_t - a\varepsilon_{t-1} & a = -0.9(0.3)0.9 \\ \text{AR}(2)a: & y_t + 0.6 y_{t-1} - a y_{t-2} = \varepsilon_t & a = -0.3(0.1)0.3 \\ \text{AR}(2)b: & y_t - 0.6 y_{t-1} - a y_{t-2} = \varepsilon_t & a = -0.3(0.1)0.3 \\ \text{MA}(2)a: & y_t = \varepsilon_t + 0.6\varepsilon_{t-1} - a\varepsilon_{t-1} & a = -0.3(0.1)0.3 \\ \text{MA}(2)b: & y_t = \varepsilon_t - 0.6\varepsilon_{t-1} - a\varepsilon_{t-1} & a = -0.3(0.1)0.3 \\ \text{ARMA}(1,1)a: & y_t + 0.6 y_{t-1} = \varepsilon_t - a\varepsilon_{t-1} & a = -0.3(0.1)0.3 \\ \text{ARMA}(1,1)b: & y_t - 0.6 y_{t-1} = \varepsilon_t - a\varepsilon_{t-1} & a = -0.3(0.1)0.3 \end{array} \quad (3)$$

where $n = 50, 100, 200$ and $\{\varepsilon_t\} \sim NID(0,1)$. Note that the models in (3) are frequently used in this paper.

We first examine the empirical means, variances, and VMR of Q_m^* and Q_m^{**}. The results are given in Figures 1–24, where Q^* denotes Q_m^*, Q^{**} denotes Q_m^{**}, r denotes the theoretical VMR given by (2), R denotes the empirical VMR, M denotes the empirical mean divided by $m-p-q$ and V denotes the empirical variance divided by $m-p-q$.

These figures indicate:

1. Except for the case of Q_m^* and ϵ is close to one, the empirical means are stable around 1, which is consistent with the theoretical mean of the chi-squared random variable divided by its DF.

2. The V and R increases as m increases. Both of the theoretical values by the chi-squared random variable are two. However, the deviation from two is serious as m increases.

3. In almost all cases, r is larger than R and the difference between them increases as m increases. This result indicates the approximation of the theoretical VMR given by (2) is not a good approximation as m increases.

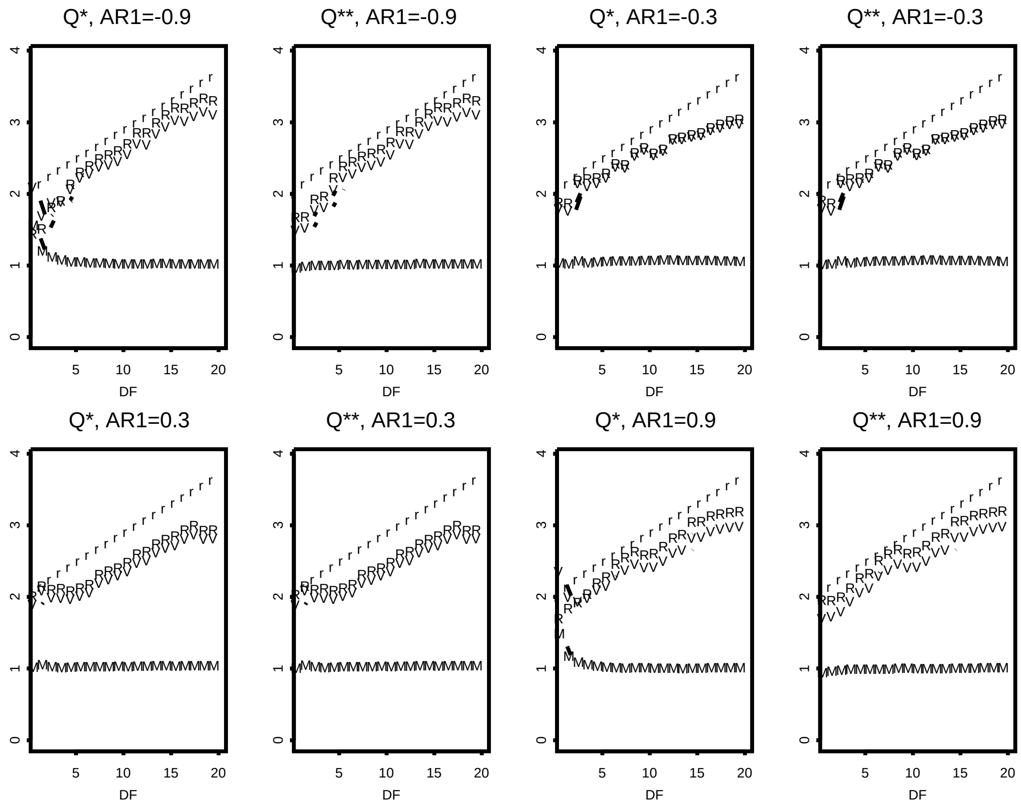

Figure 1: AR(1) models, $n = 50$

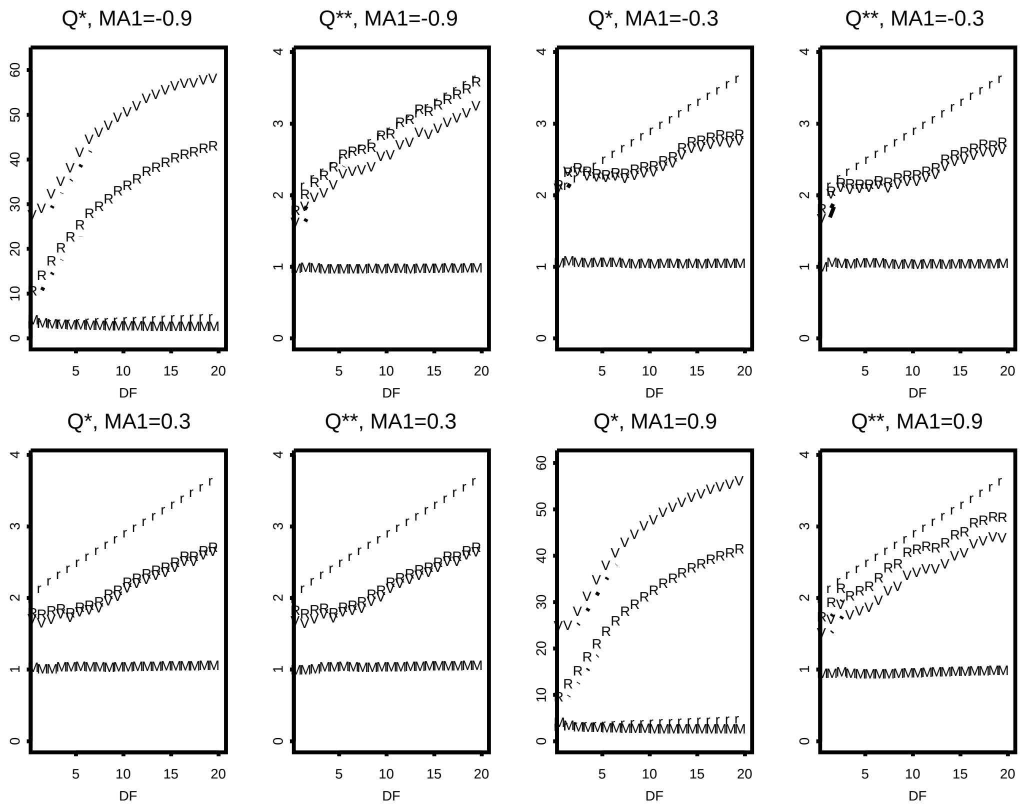

Figure 2: MA(1) models, $n = 50$

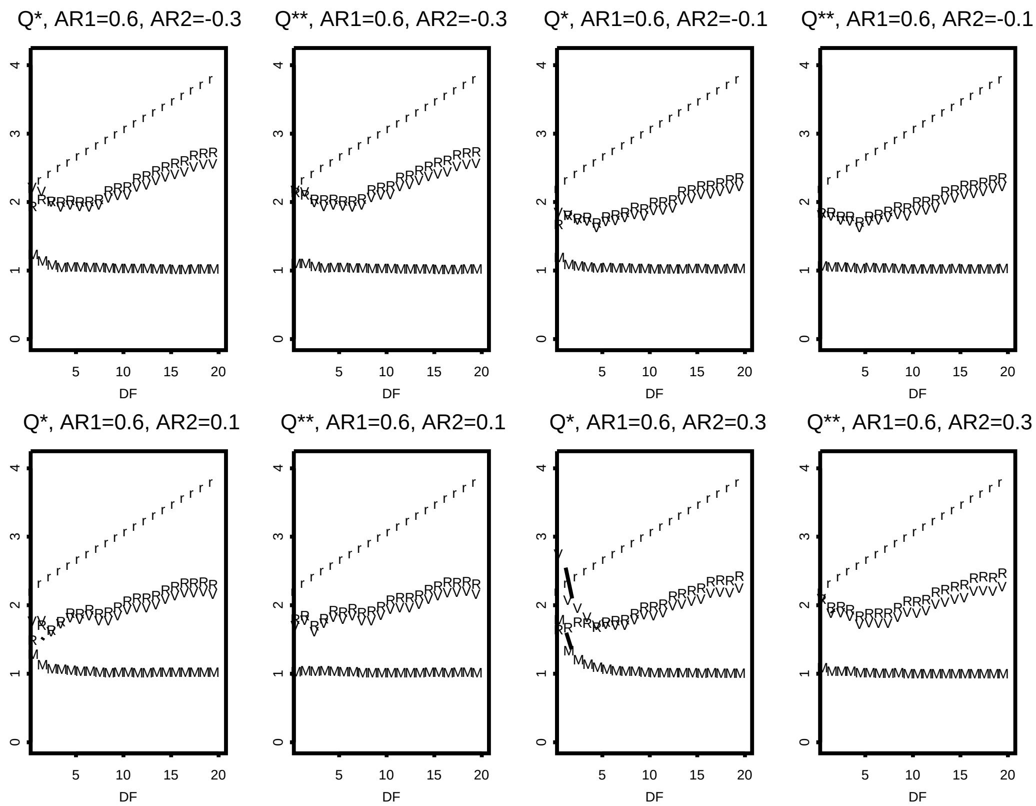

Figure 3: AR(2)models, $n = 50$

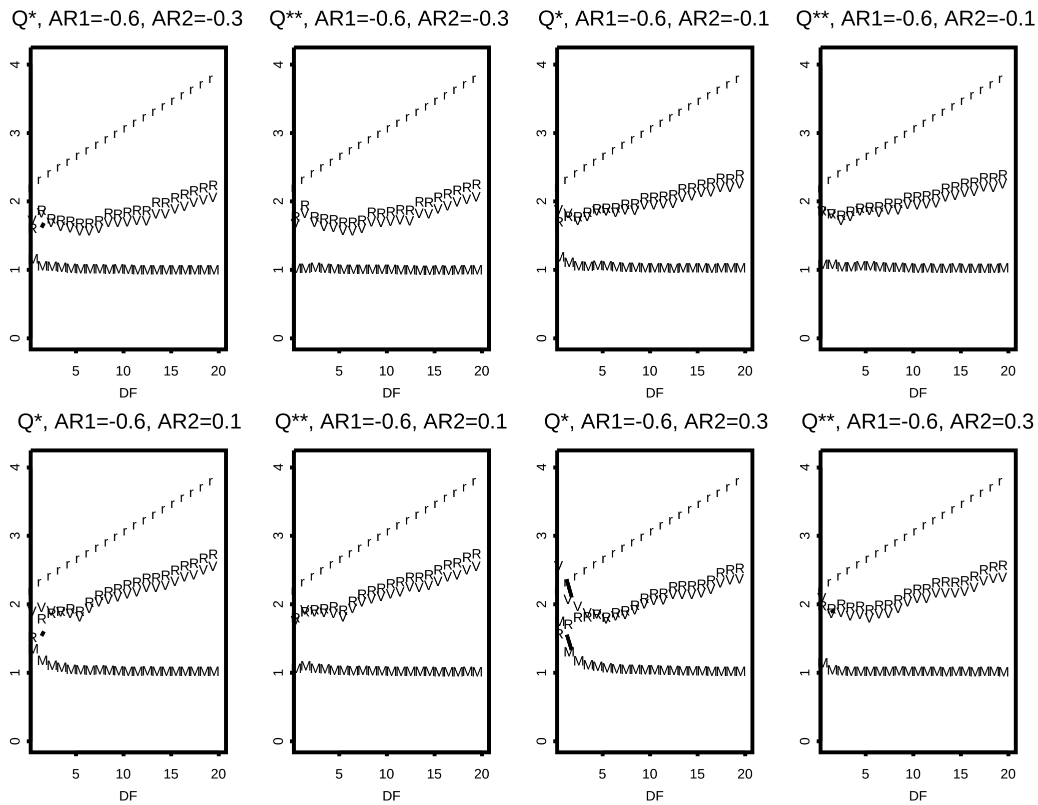

Figure 4: AR(2)models, $n = 50$

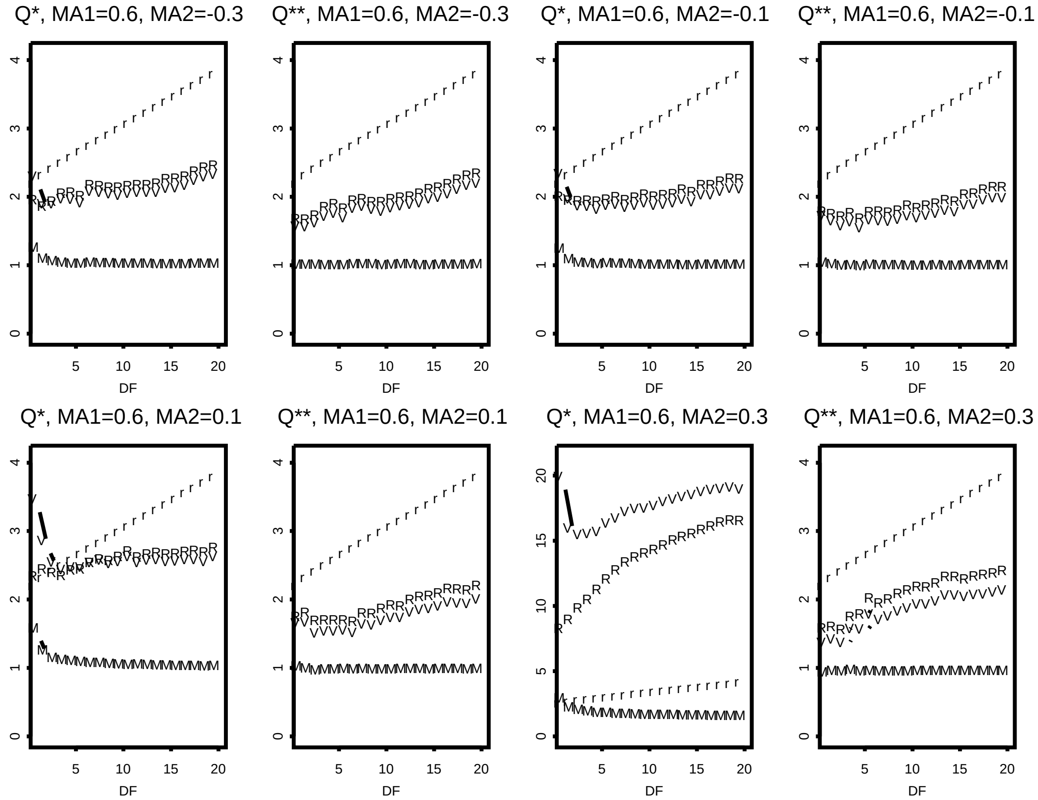

Figure 5: MA(2)models, $n = 50$

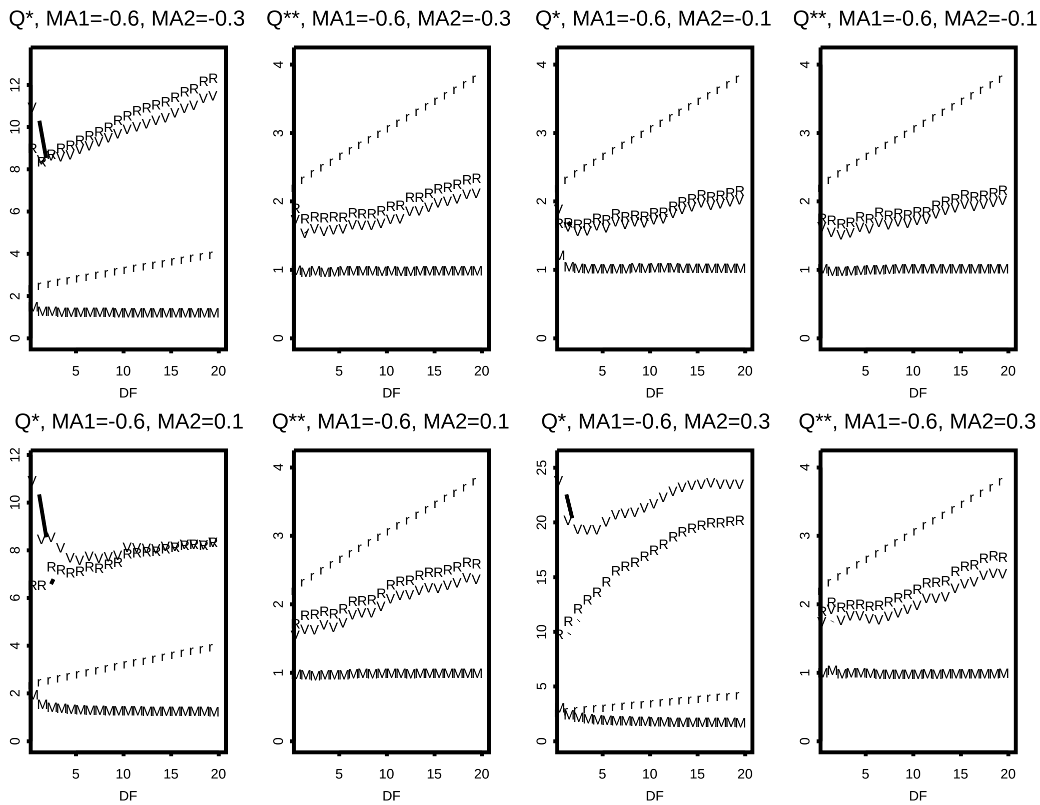

Figure 6: MA(2)models, $n = 50$

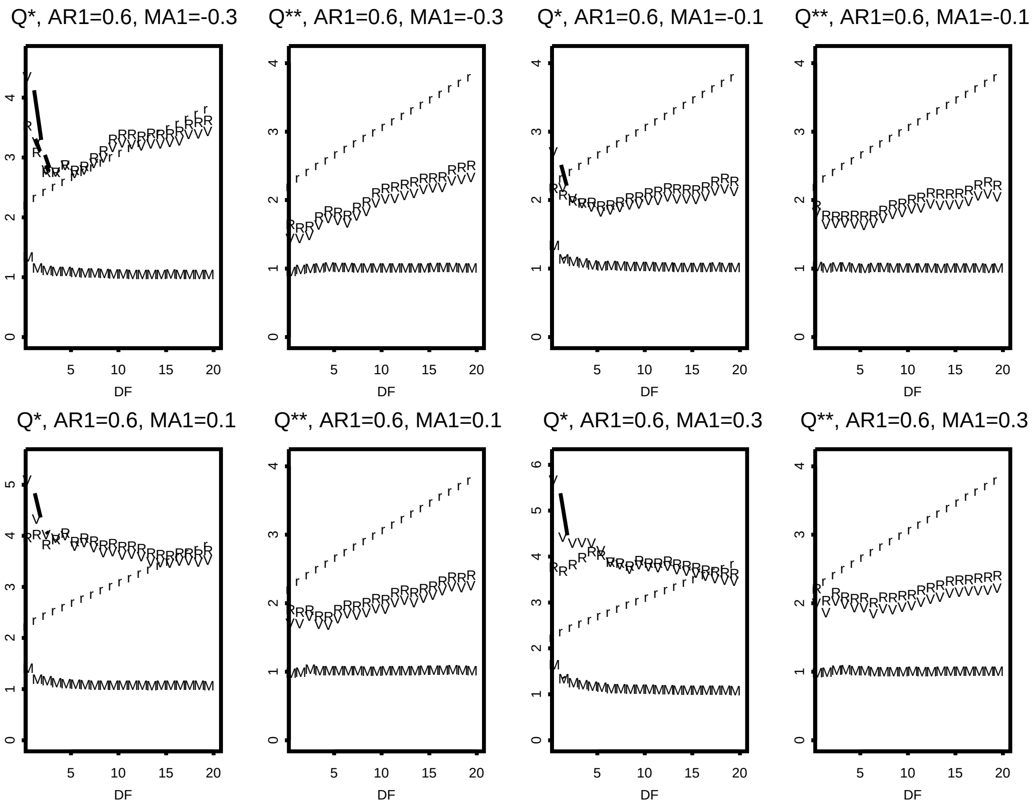

Figure 7: ARMA$(1,1)$models, $n = 50$

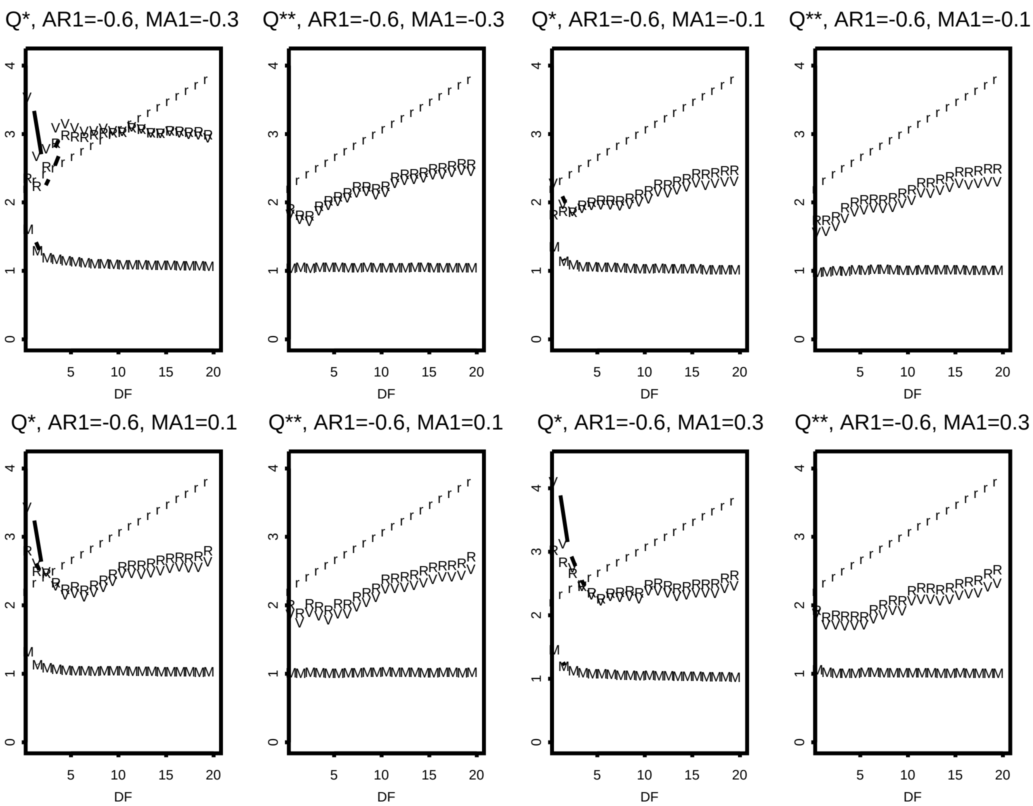

Figure 8: ARMA$(1,1)$models, $n = 50$

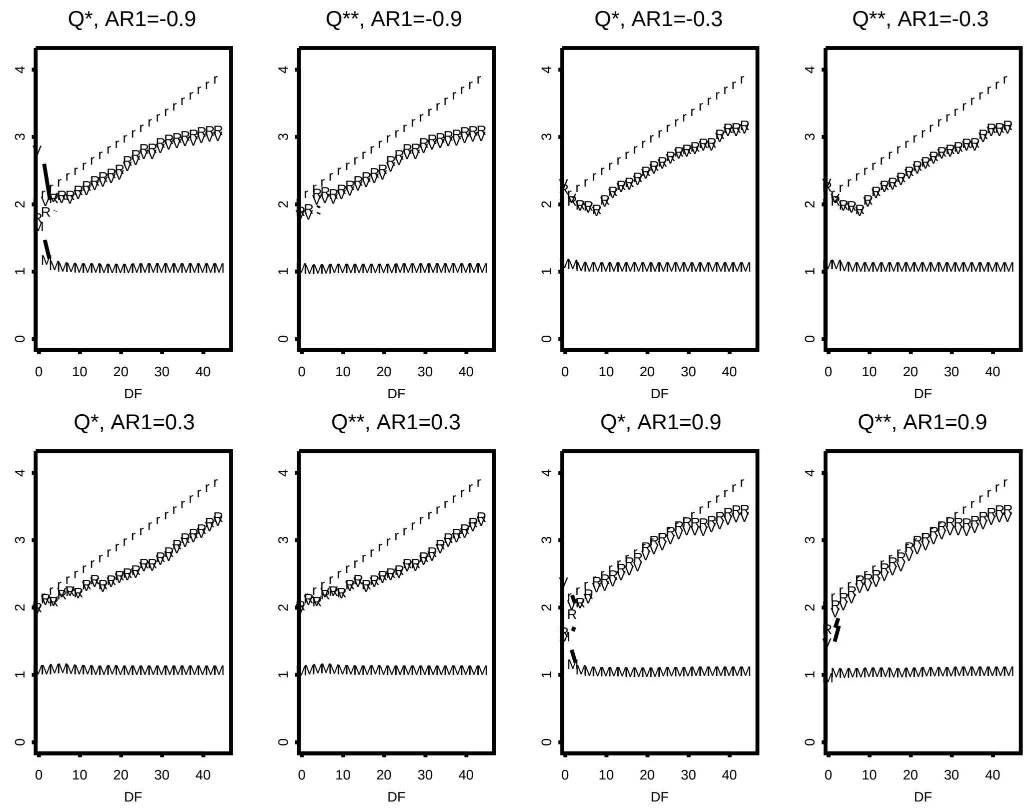

Figure 9: AR(1) models, $n = 100$

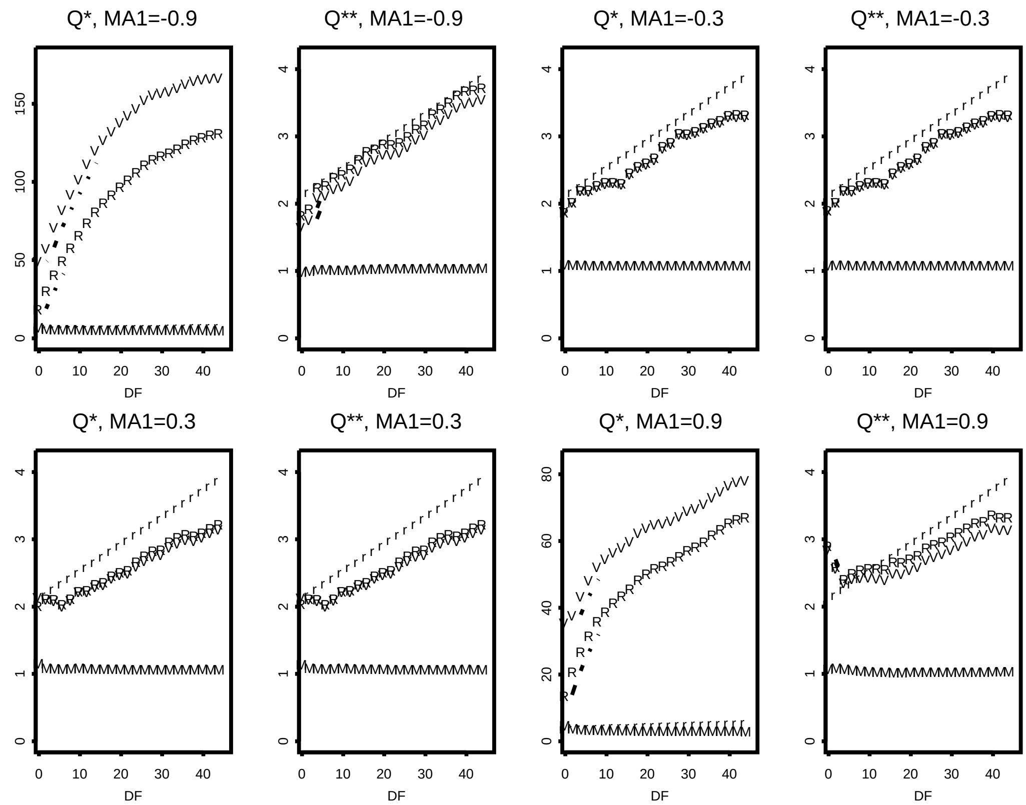

Figure 10: MA(1) models, $n = 100$

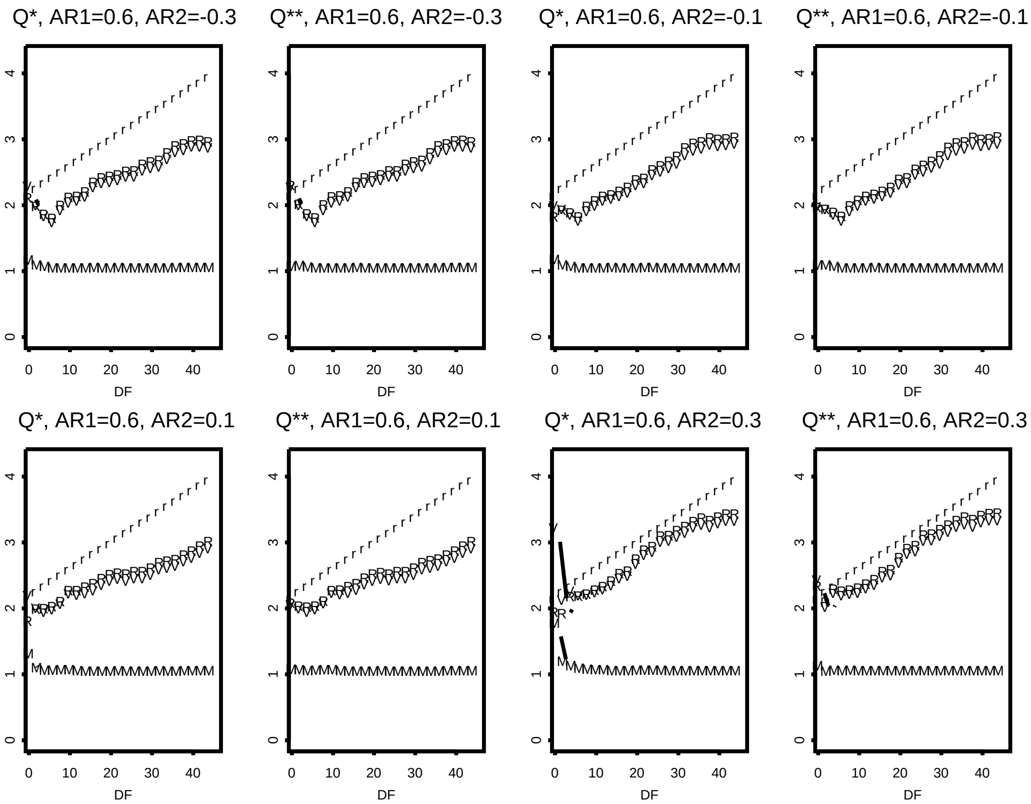

Figure 11: AR(2)models, $n = 100$

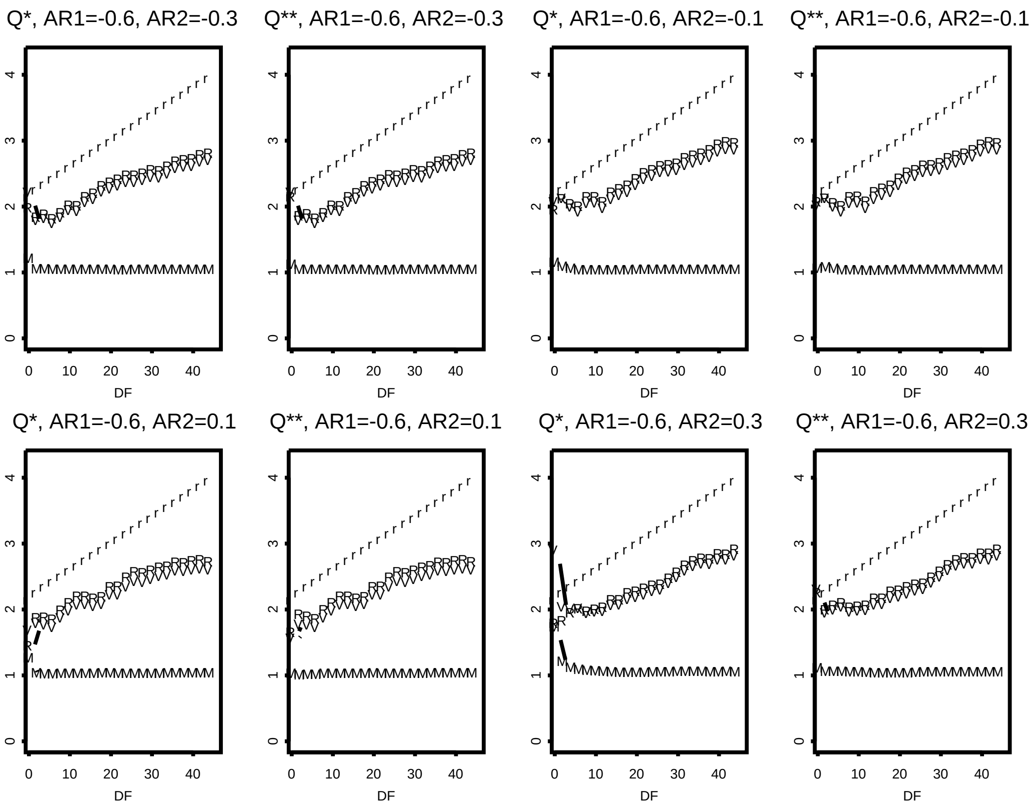

Figure 12: AR(2)models, $n = 100$

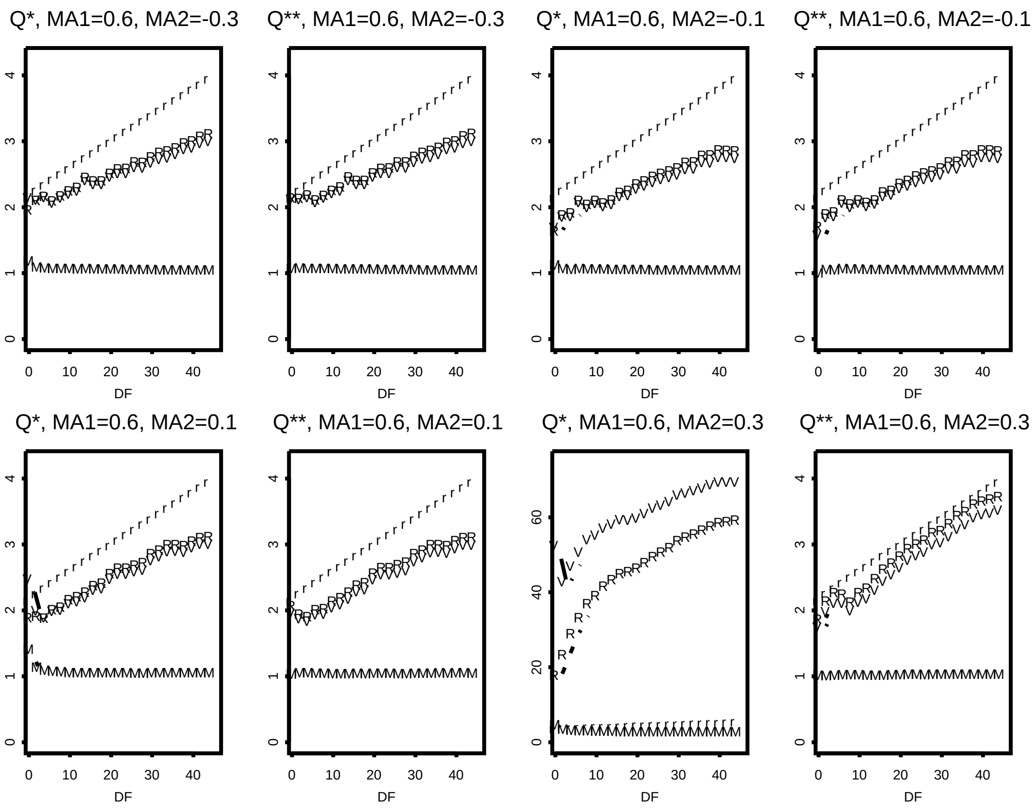

Figure 13: MA(2)models, $n = 100$

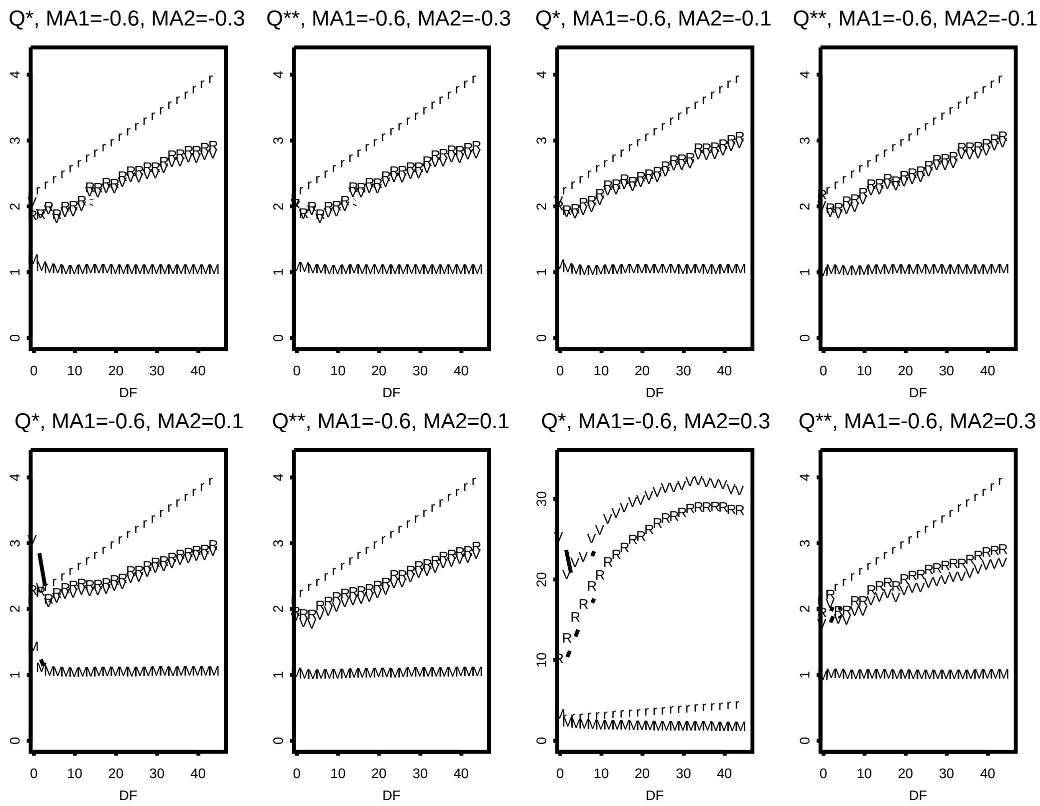

Figure 14: MA(2)models, $n = 100$

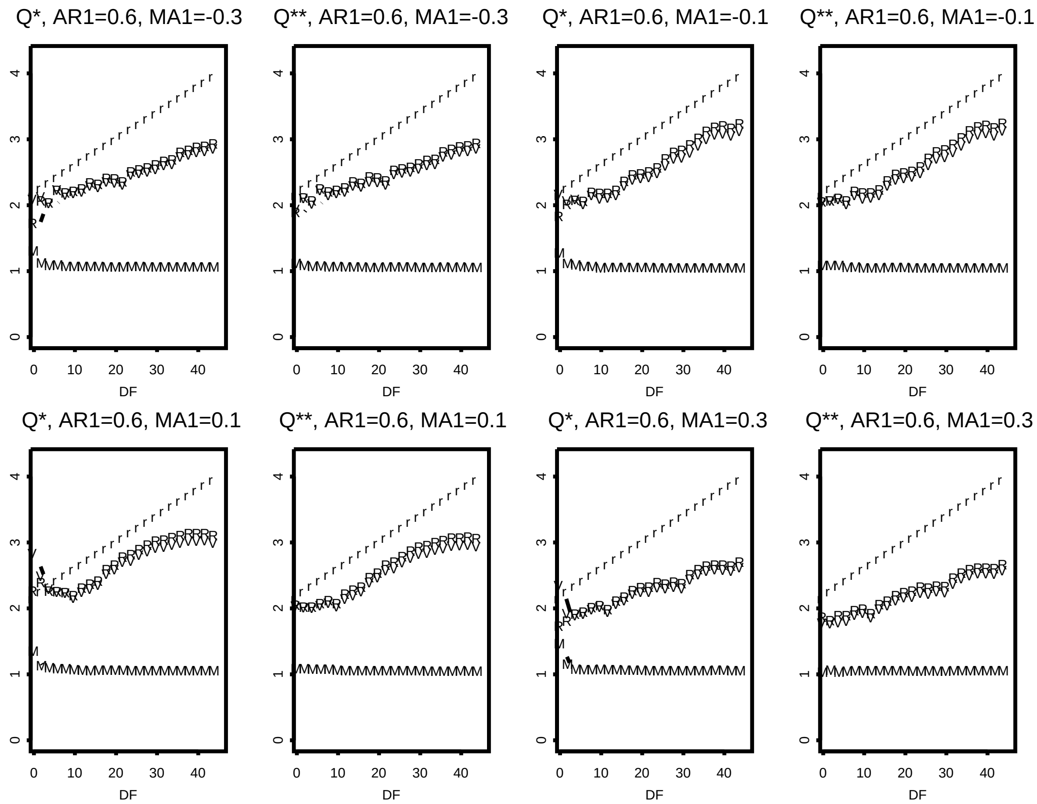

Figure 15: ARMA$(1,1)$models, $n = 100$

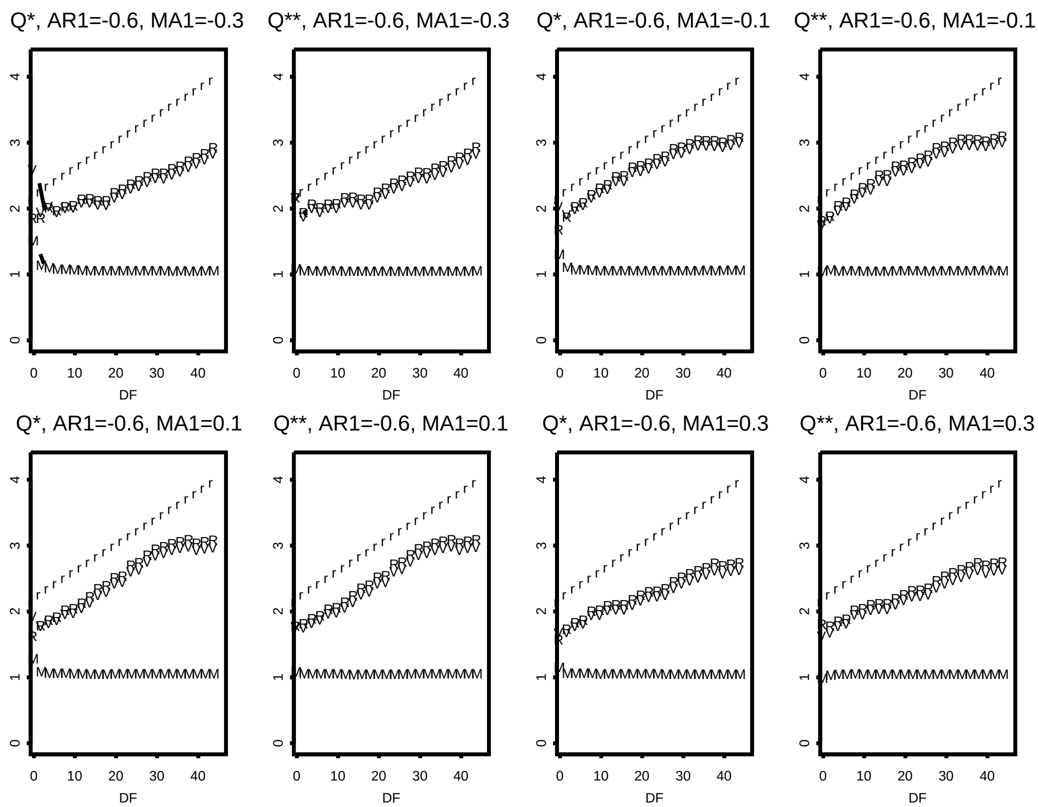

Figure 16: ARMA$(1,1)$models, $n = 100$

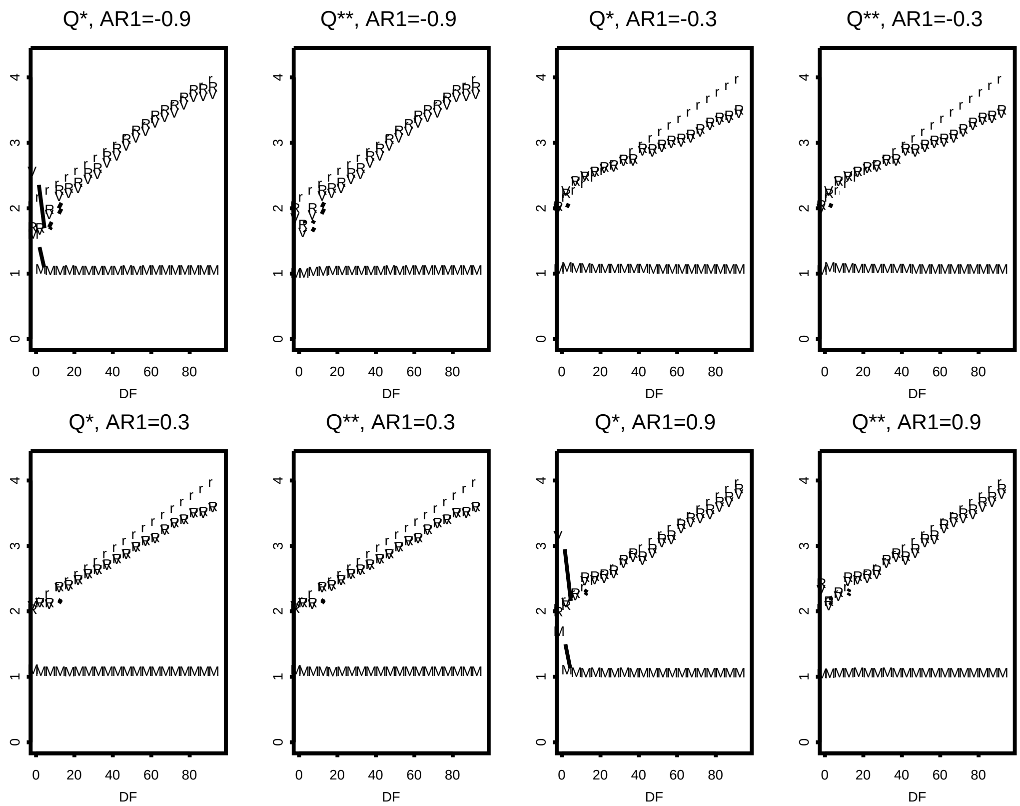

Figure 17: AR(1) models, $n = 200$

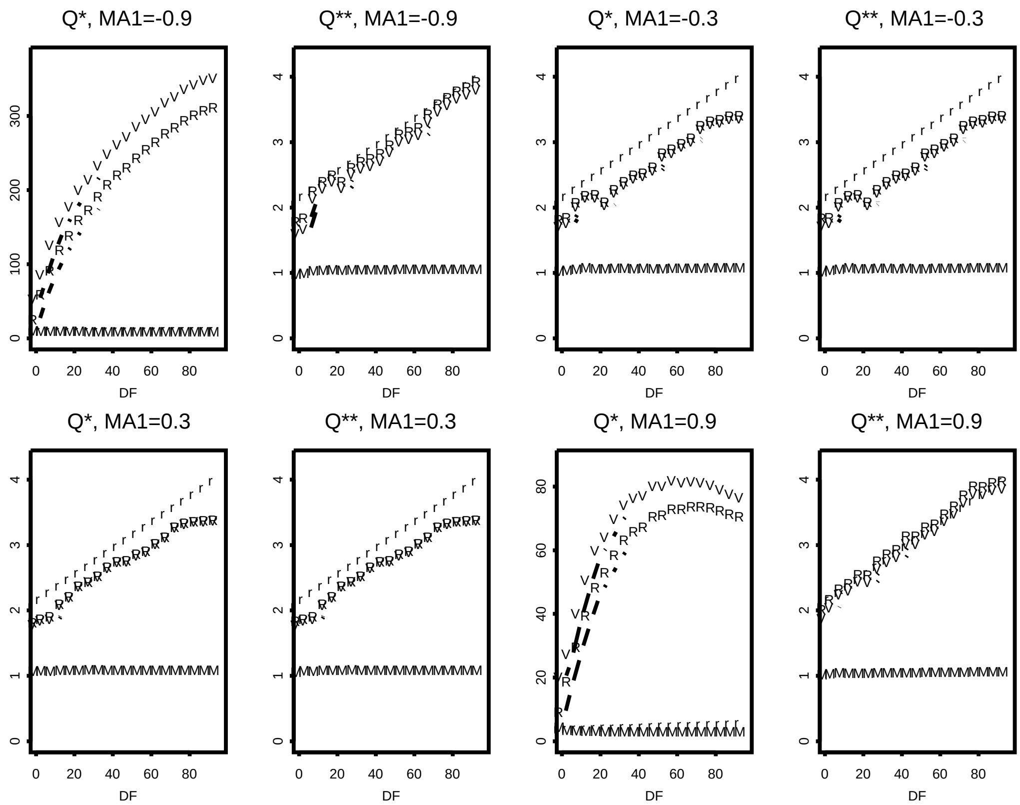

Figure 18: MA(1) models, $n = 200$

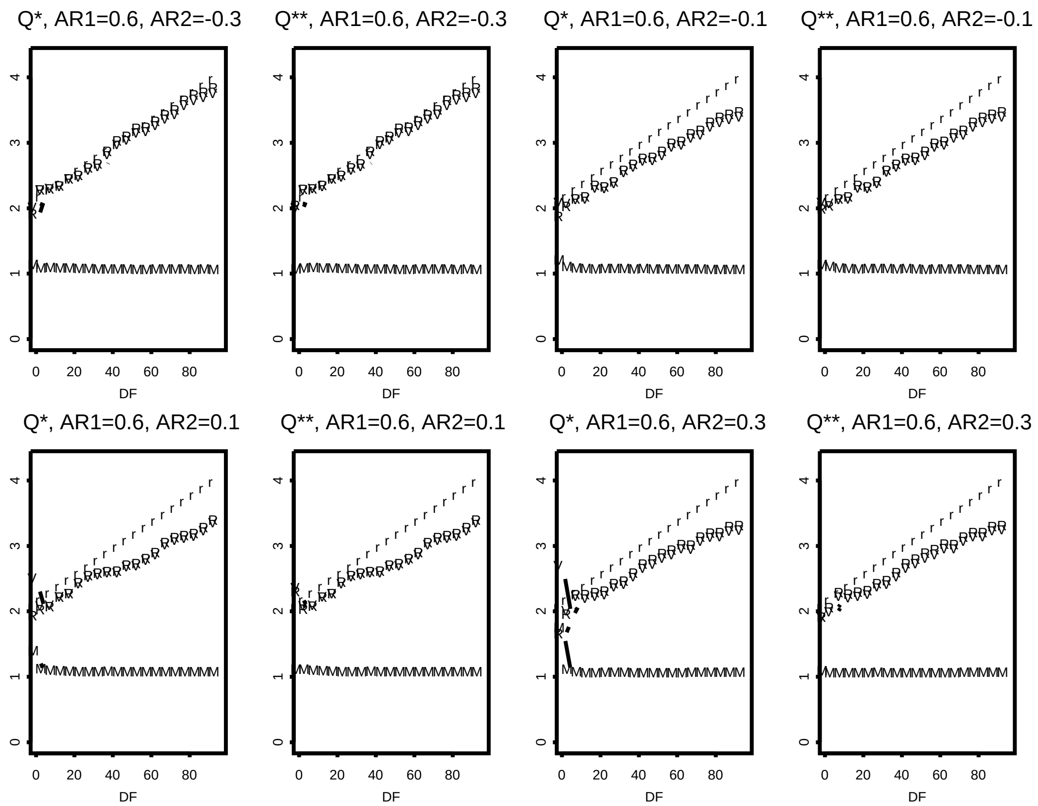

Figure 19: AR(2)models, $n = 200$

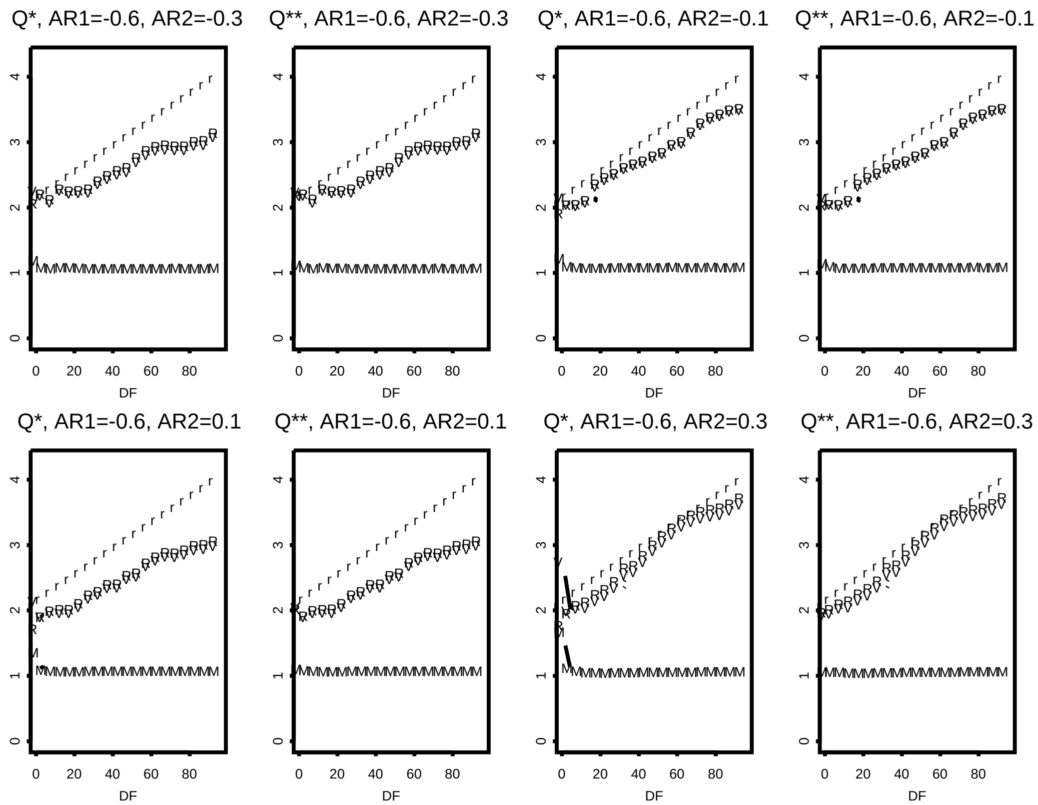

Figure 20: AR(2)models, $n = 200$

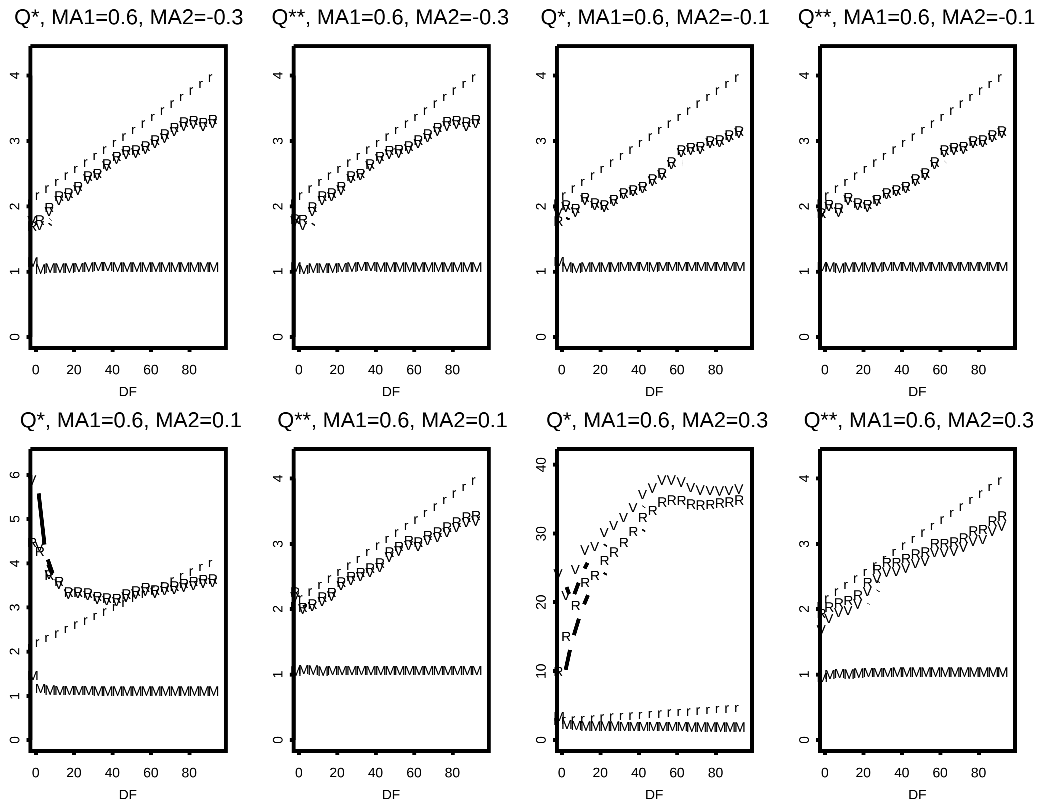

Figure 21: MA(2)models, $n = 200$

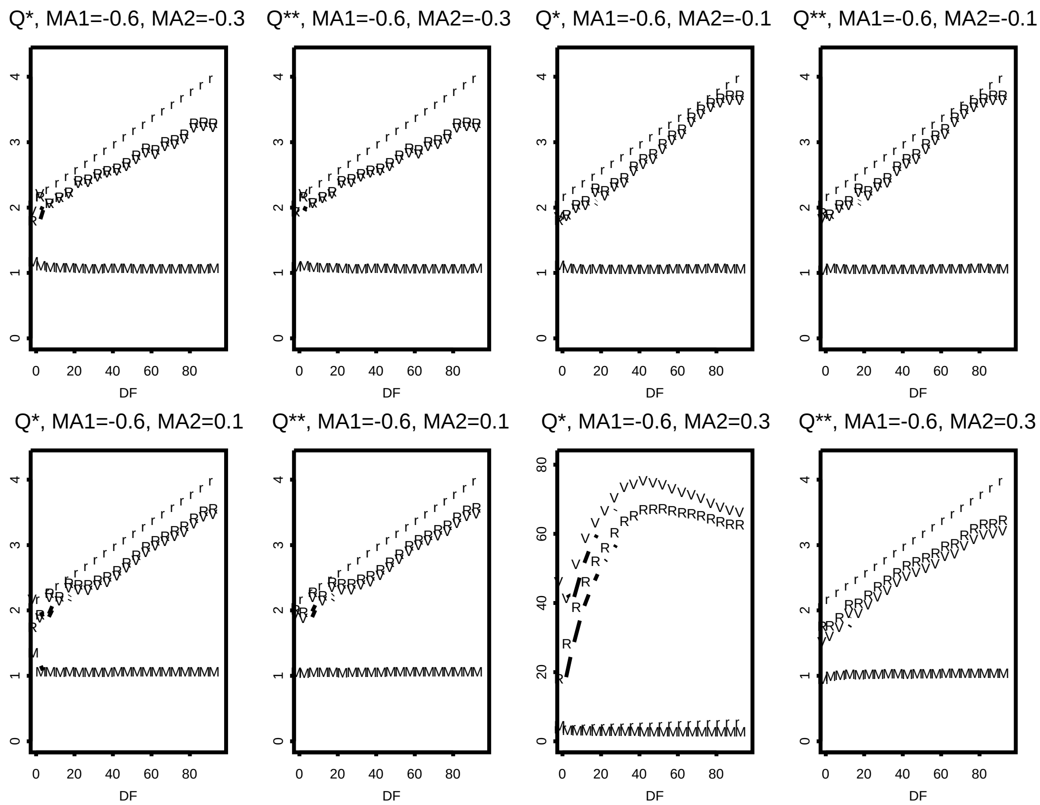

Figure 22: MA(2)models, $n = 200$

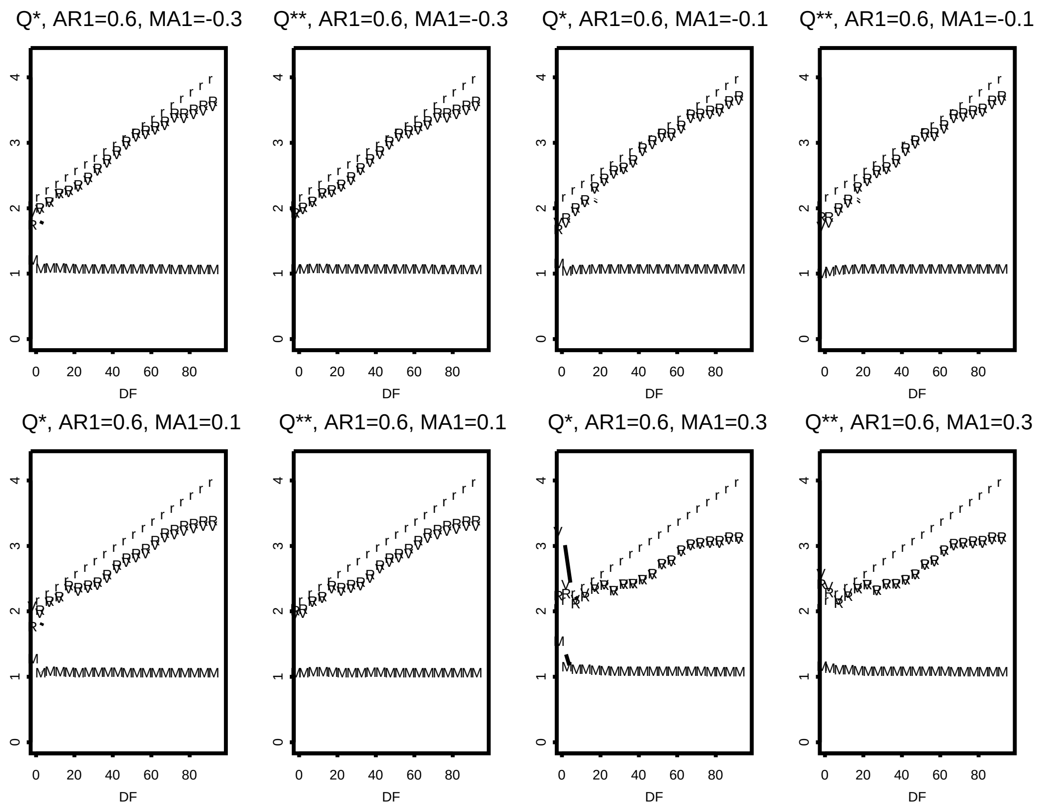

Figure 23: ARMA$(1, 1)$models, $n = 200$

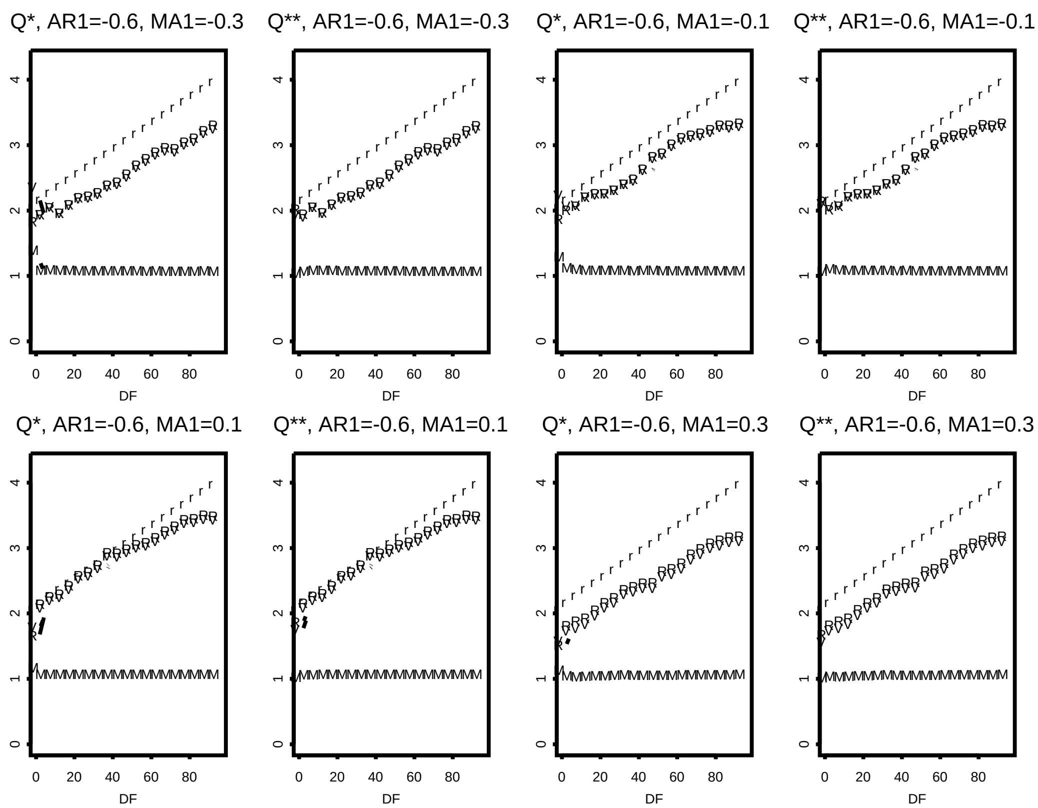

Figure 24: ARMA$(1, 1)$models, $n = 200$

2.4 Simulation experiments for empirical significance levels and VMR

We next conducted simulation studies to examine the relationship between the theoretical VMR given by (2) and the empirical significance levels by using the models in (3). The number of iterations is only 1,000 in these experiments. In each model, we computed the minimum DF of Q^*_m and Q^{**}_m when the percentages of the empirical significance level minus the theoretical significance level exceed 2 % for $\beta = 0.01, 0.05, 0.10$. Namely, the minimum DF, $m_{\min} - p - q$, is obtained from:

$$m_{\min} = \min_m \{m \,|\, 100(p^*_m - \beta) > 2.0, m - p - q = 1, 2, \ldots, n/2 - 5\},$$

where p^*_m denotes the empirical significance levels of the portmanteau statistics with $m - p - q$ DF. Thereafter, in each parameter of the models, Figures 25–36 depict VMR($m_{\min}$) given by (2), where Q^* denotes Q^*_m, Q^{**} denotes Q^{**}_m, O denotes $100\beta = 1\%$, F denotes $100\beta = 5\%$ and T denotes $10\beta = 10\%$. Note that when $100(p^*_m - \beta) \leq 2.0$ for all DF, the maximum value of DF, $n/2-5$, are chosen as $m_{\min}$ and these figures depict VMR($n/2 - 5$). The aim of this investigation is to check the appropriate maximum value of m for the individual portmanteau tests. Therefore, our interest is in finding the minimum value of VMR($m_{\min}$) of the models.

These figures indicate:

1. VMR($m_{\min}$) by Q^*_m takes the value 2 frequently. This phenomenon occurs when ϵ of the model is close to 1.

2. In contrast, almost all VMR($m_{\min}$) by Q^{**}_m exceed 3.0 except for a few cases. Therefore, one of the feasible inequalities for the maximum value of m for the individual portmanteau tests is:

$$2 + \frac{2m(2m-5)}{n(m-p-q)} \leq 3.0 \tag{4}$$

   to conduct.

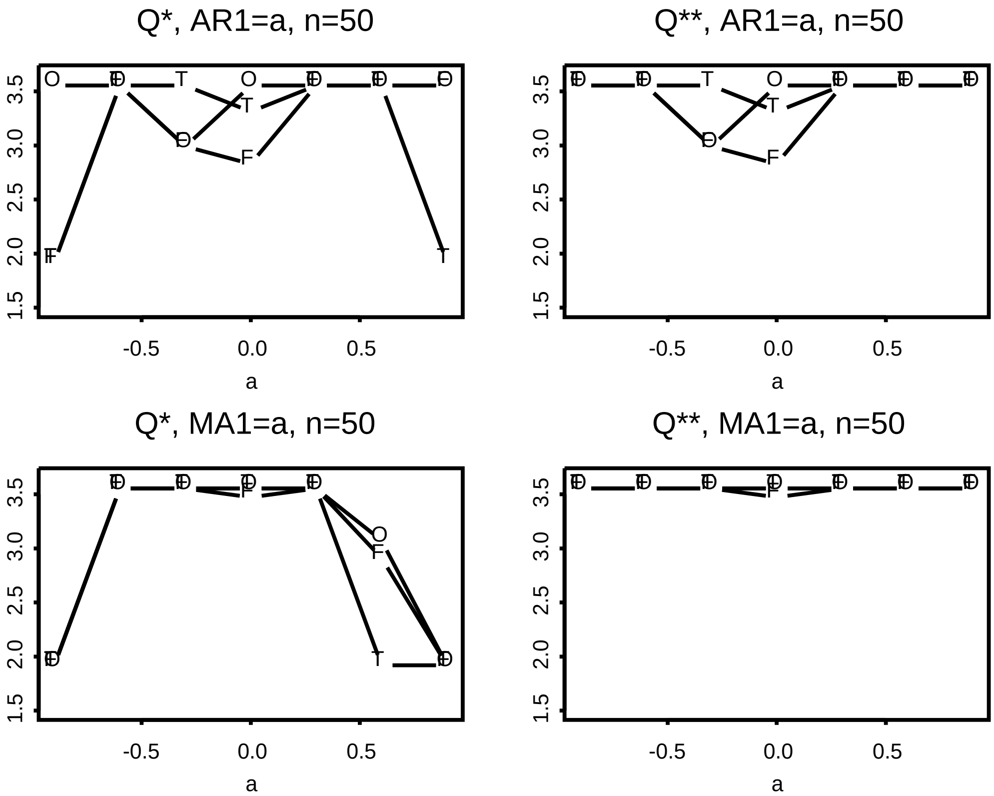

Figure 25: AR(1) models and MA(1) models, $n = 50$

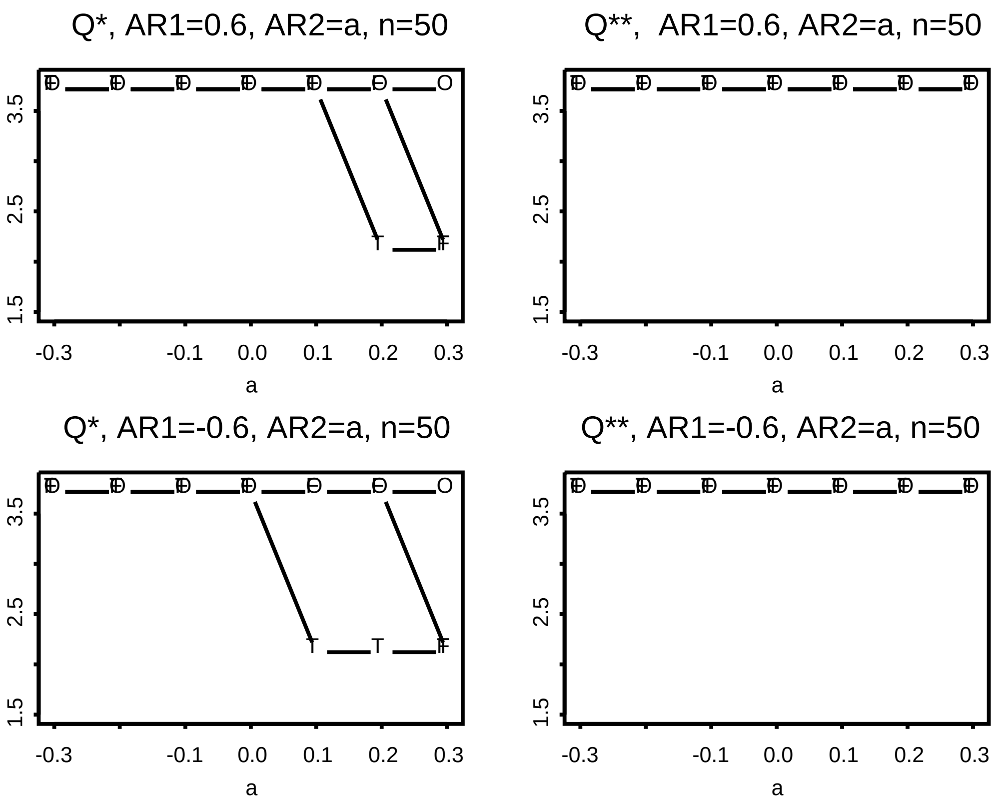

Figure 26: AR(2)models, $n = 50$

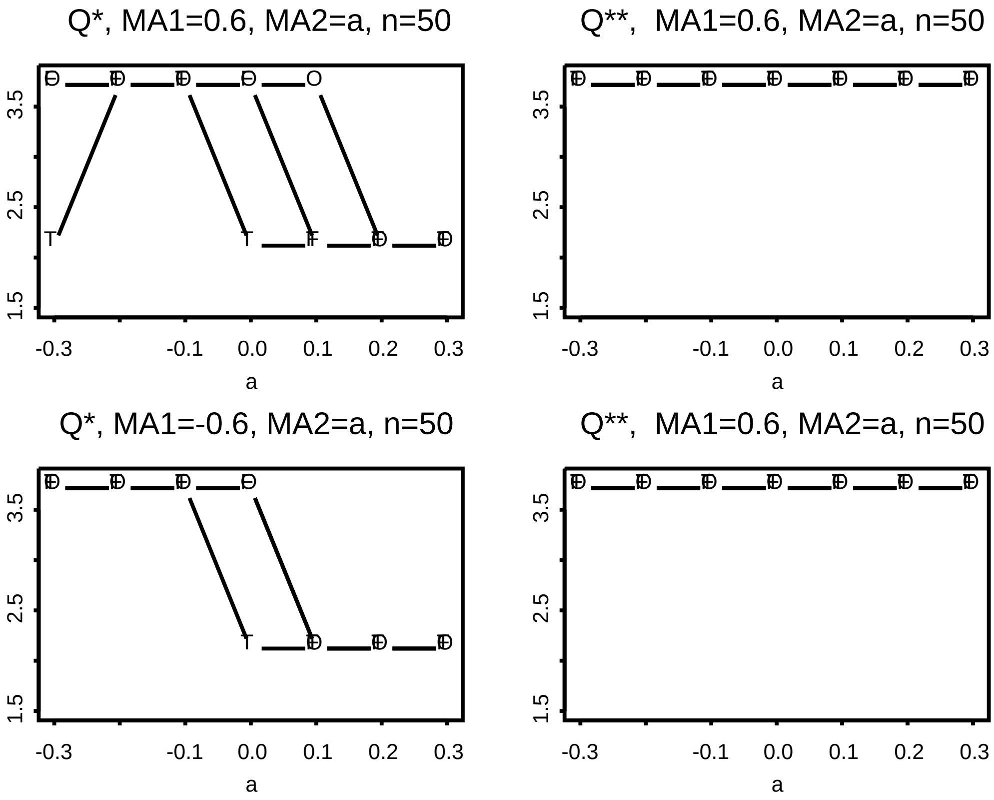

Figure 27: MA(2)models, $n = 50$

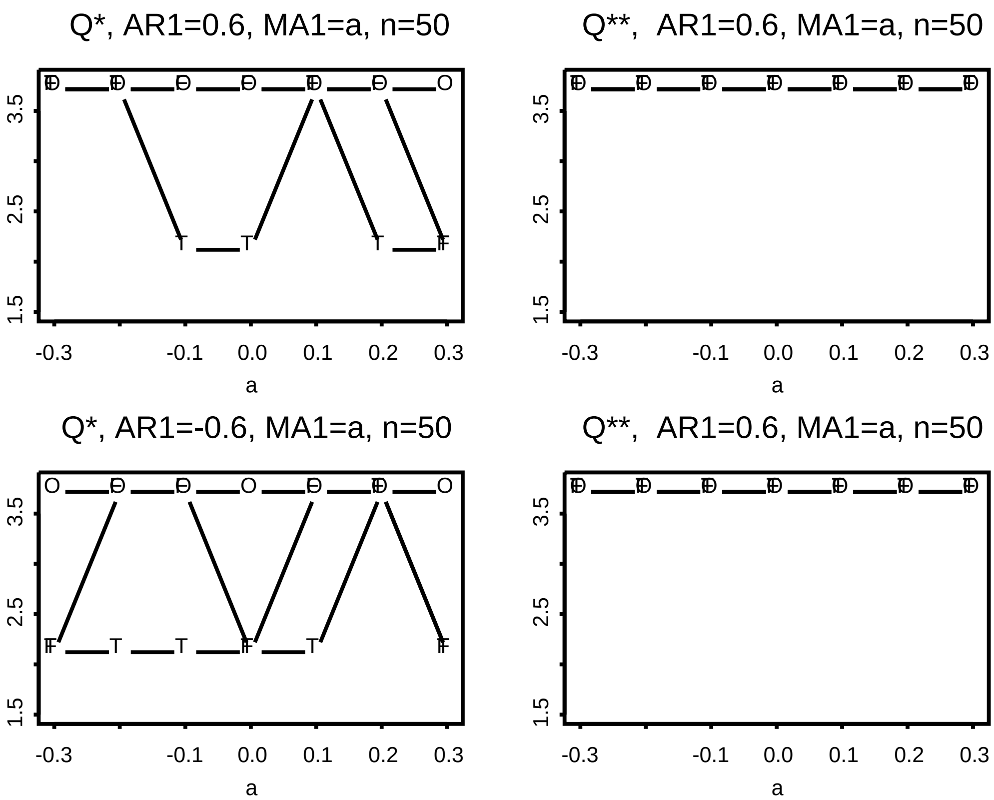

Figure 28: ARMA(1, 1)models, $n = 50$

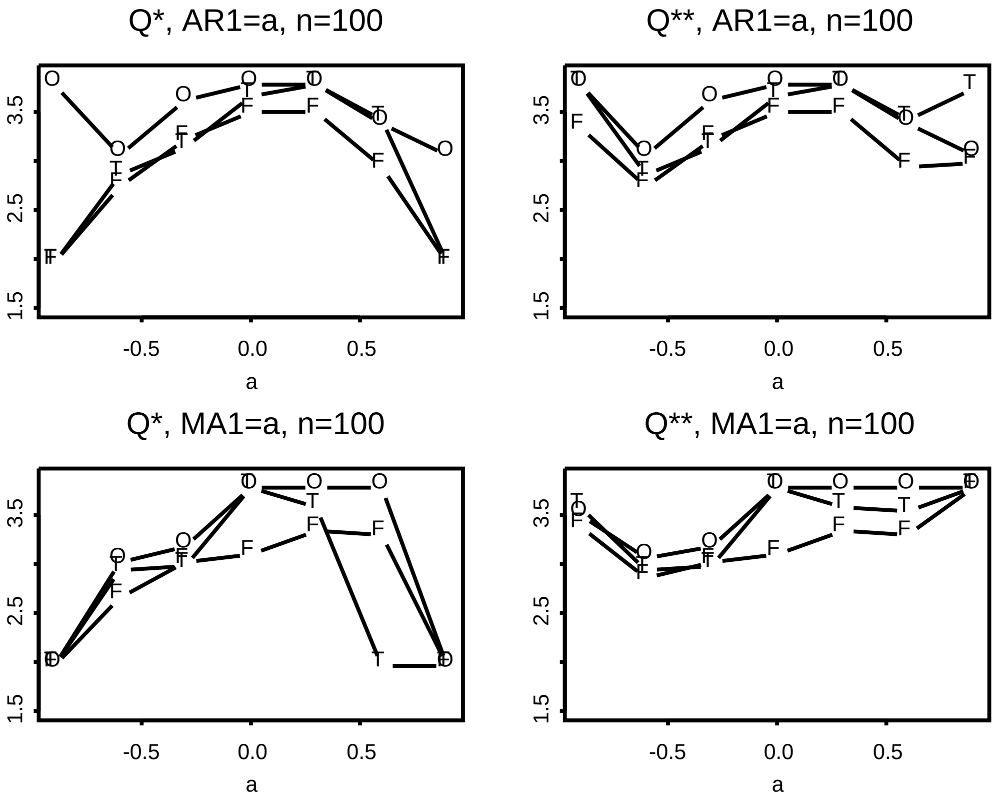

Figure 29: AR(1) models and MA(1) models, $n = 100$

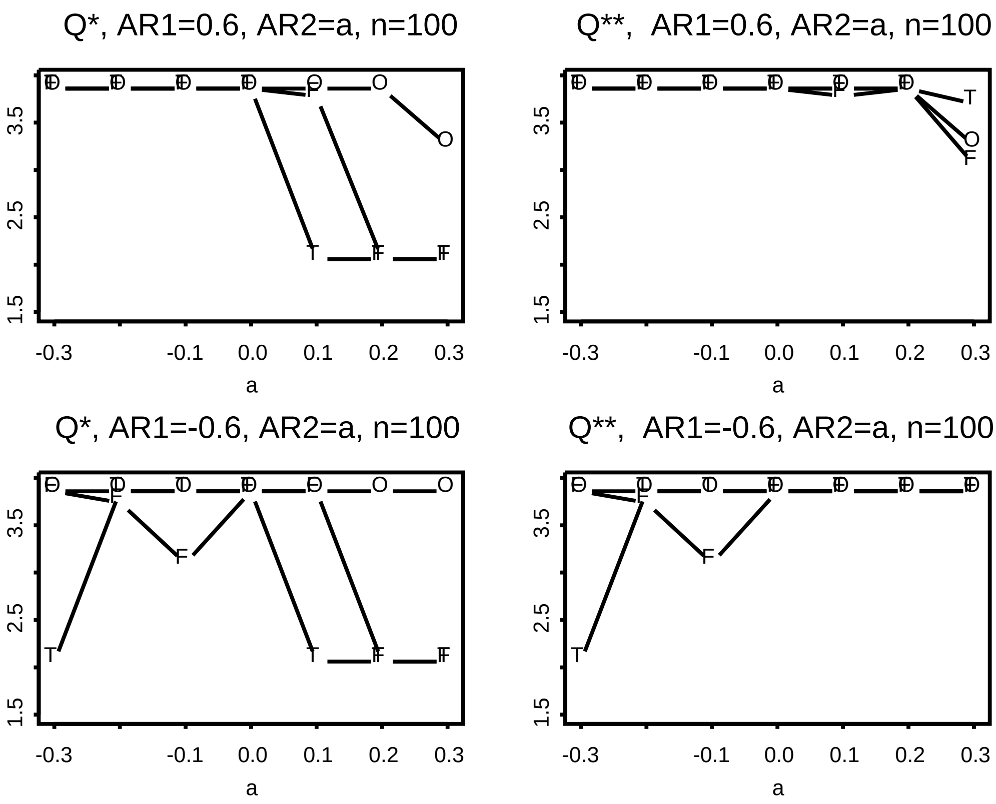

Figure 30: AR(2)models, $n = 100$

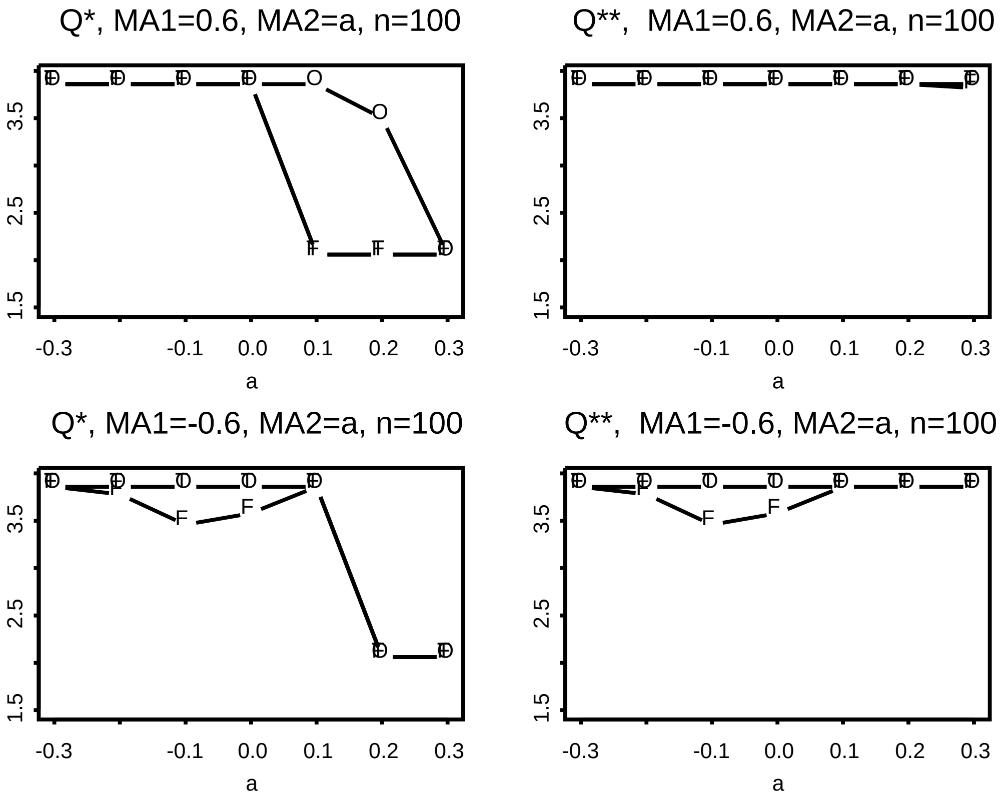

Figure 31: MA(2)models, $n = 100$

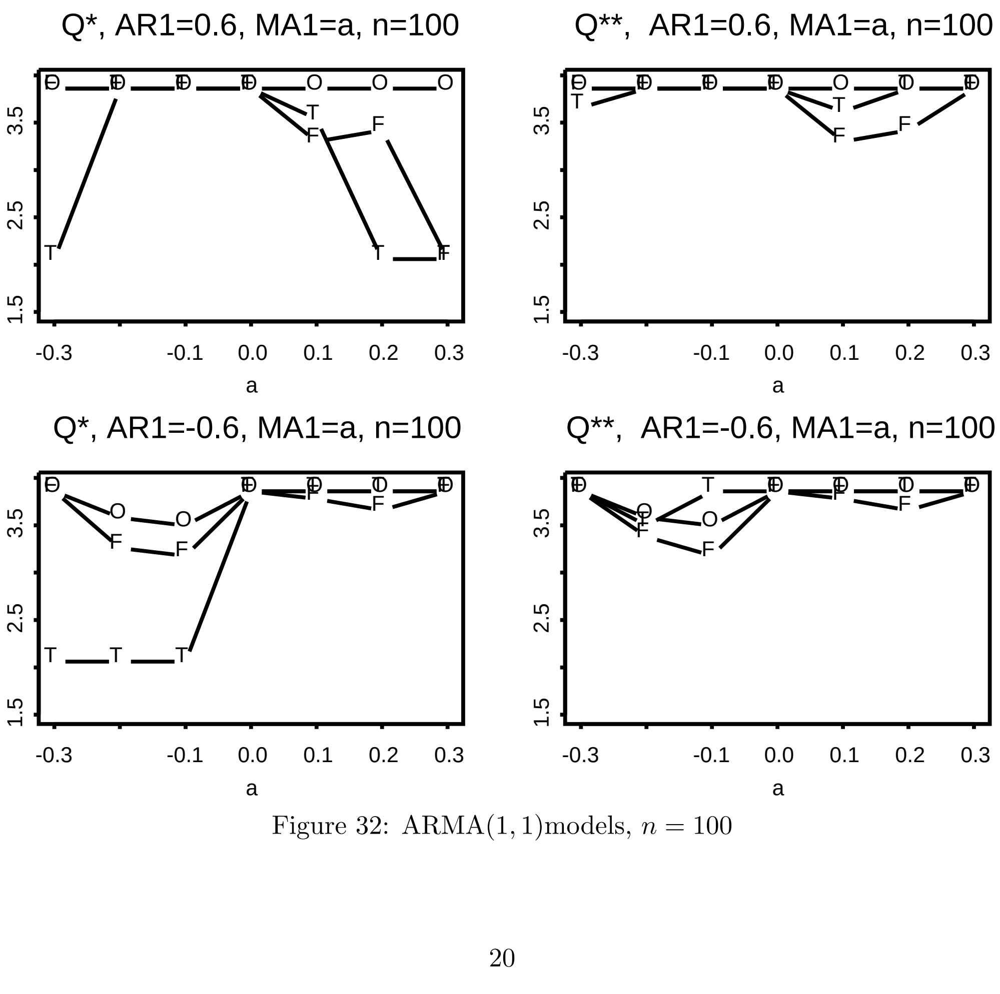

Figure 32: ARMA(1, 1)models, $n = 100$

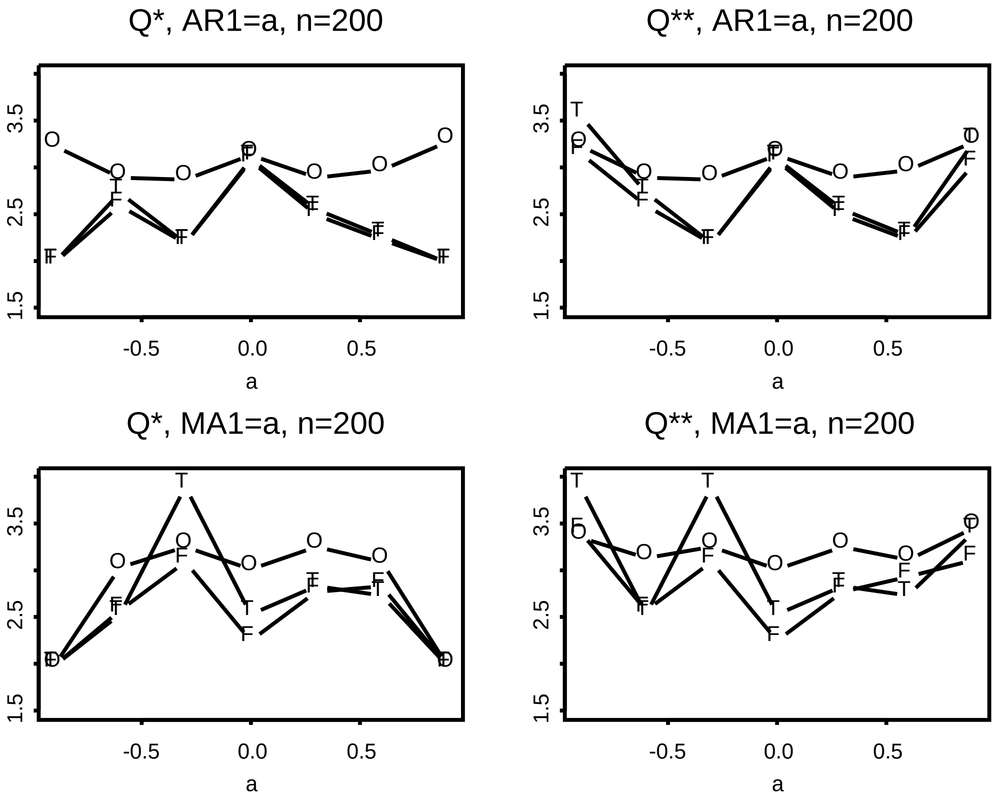

Figure 33: AR(1) models and MA(1) models, $n = 200$

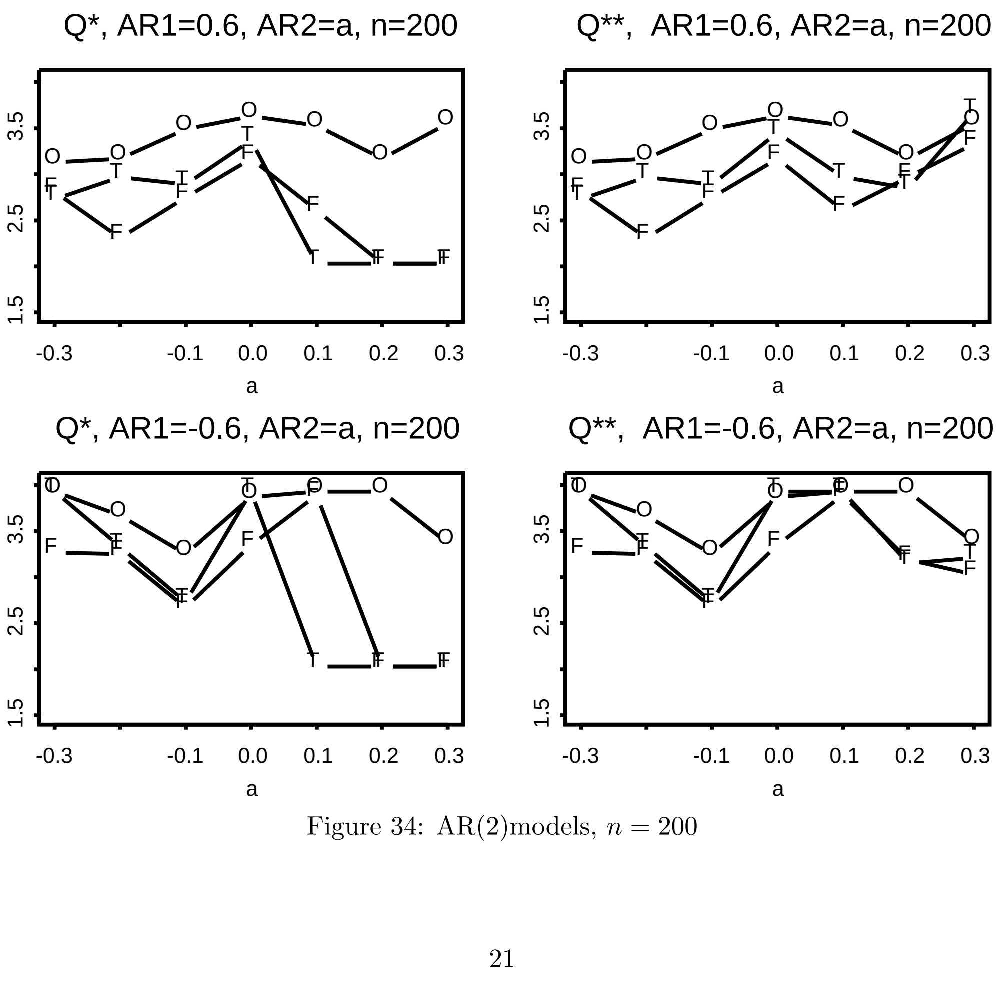

Figure 34: AR(2)models, $n = 200$

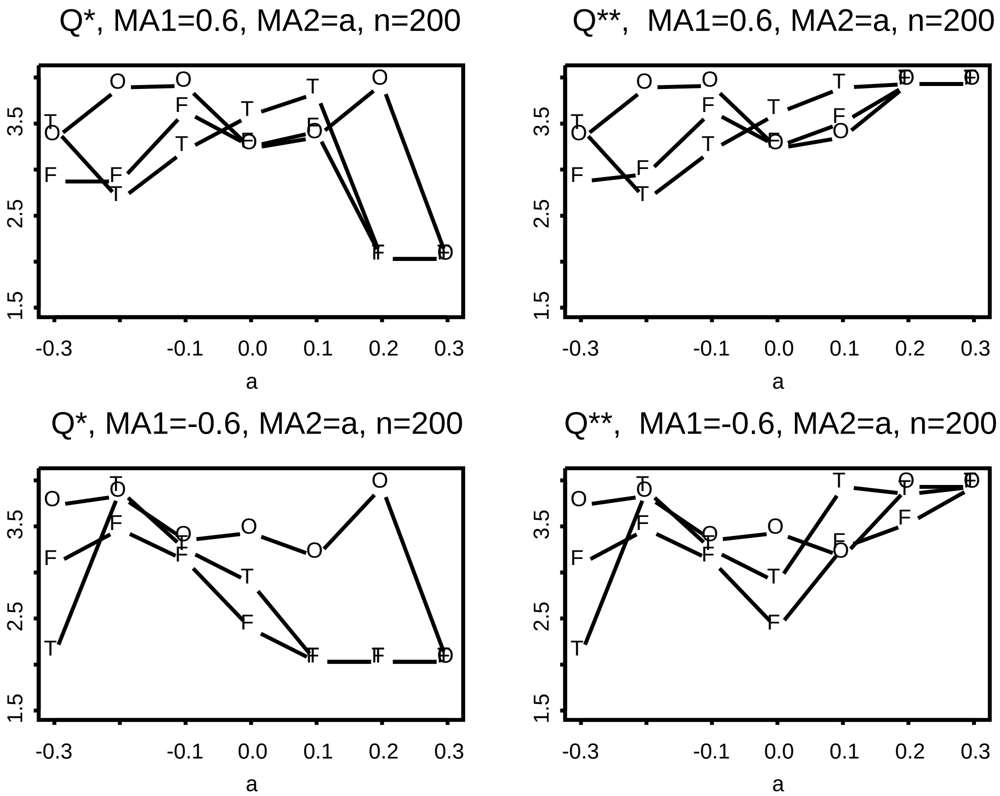

Figure 35: MA(2)models, $n = 200$

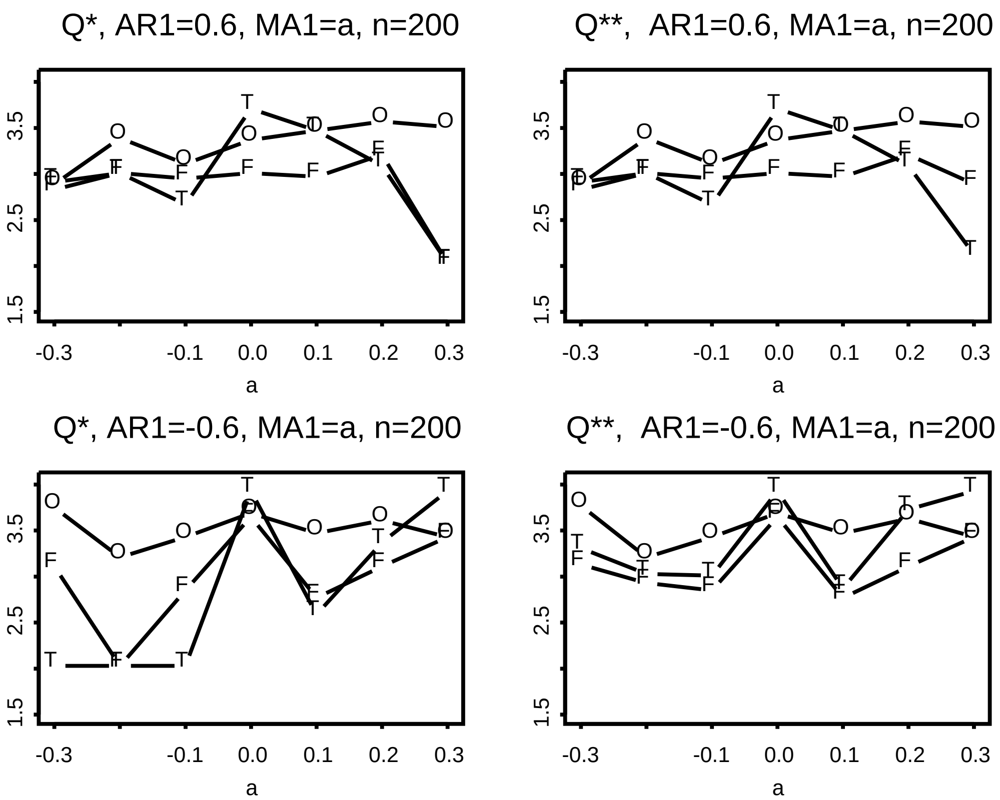

Figure 36: ARMA(1, 1)models, $n = 200$

3 EMPIRICAL SIZES AND POWERS OF MULTIPLE PORTMANTEAU TESTS

3.1 Empirical sizes

We carried out simulation studies of the models given by (3).

Figures 37–68 depict the joint empirical significance levels of these models with any possible even numbers of DF. In these figures, O denotes $100\alpha = 1\%$, F denotes $100\alpha = 5\%$ and T denotes $100\alpha = 10\%$, Q_m^* (solid line) and Q_m^{**} (dashed line) with the different sets of DF. To examine the maximum value of DF, we used the set of DF given in Table 3. Note that we denote consecutive even integers of the DF by using parentheses. For example, we denote the row vector of DF, $(2, 4, 6, 8)$ by 2(2)8. We reveal the following:

1. Q_m^* shows over-rejection behavior when the ϵ is close to 1.
2. The results by Q_m^{**} did not present serious over-rejection behavior even for large numbers of DF. Let the maximum value of m in Q_m^{**} be M. Then, in practice, with reference to the VMR in Table 3, we should define M by:

$$2 + \frac{2M(2M-5)}{n(M-p-q)} = a_M \tag{5}$$

   to retain the joint significance levels, where $\exists a_M \in [3.00, 3.25]$.

3. We also conducted the case of DF with discontinuous even integers. For example, see Figures 40, 44, 48 and so on. We came to similar conclusions as above.

To summarize the cases of the DF with discontinuous even integers, Figures 69–76 depict the joint empirical significance level with the different sets of the DF, where $n = 100$, Q^* denotes Q_m^* and Q^{**} denotes Q_m^{**}. Similarly to the figures above, O denotes $100\alpha = 1\%$, F denotes $100\alpha = 5\%$ and T denotes $100\alpha = 10\%$. We considered the nested sets of the DF, which have different numbers of partitions of the DF. For the AR(1) models and the MA(1) models in (3), solid lines denote $DF = 2(2)40$, dashed lines denote $DF = (2, 10, 20, 30, 40)$ and solid lines with dot denote $DF = (2, 20, 40)$. For the left models in (3), solid lines denote $DF = 2(2)38$, dashed lines denote $DF = (2, 10, 20, 30, 38)$ and solid lines with a dot denote $DF = (2, 20, 38)$. These figures reveal that the empirical sizes increase as the number of partitions of the DF decreases, which indicates that multiple portmanteau tests with any possible DF, $m(i) - p - q = 2i$, $i = 1, 2, \ldots, s$, are the most suitable sets of DF to retain the joint significance levels.

Figures 77 and 78 depict the joint empirical significance levels of these models with odd DF. In these figures, Q_m^* (solid line) and Q_m^{**} (dashed line) with $DF(3, 5, 9, 15, 21, 27)$, where O denotes $100\beta = 1\%$ $((\alpha_L, \alpha_U) = (0.02, 0.05))$, F denotes $100\beta = 5\%$ $((\alpha_L, \alpha_U) = (0.09, 0.21))$, and T denotes $100\beta = 10\%$ $((\alpha_L, \alpha_U) = (0.18, 0.34))$. For Q_m^*, Except for the case that ϵ close to 1, the joint empirical significance levels of the portmanteau statistics are within (α_L, α_U). For Q_m^{**}, the joint empirical significance levels of the portmanteau statistics are within (α_L, α_U) uniformly. However, there is a difference between α_Us and the empirical significance levels.

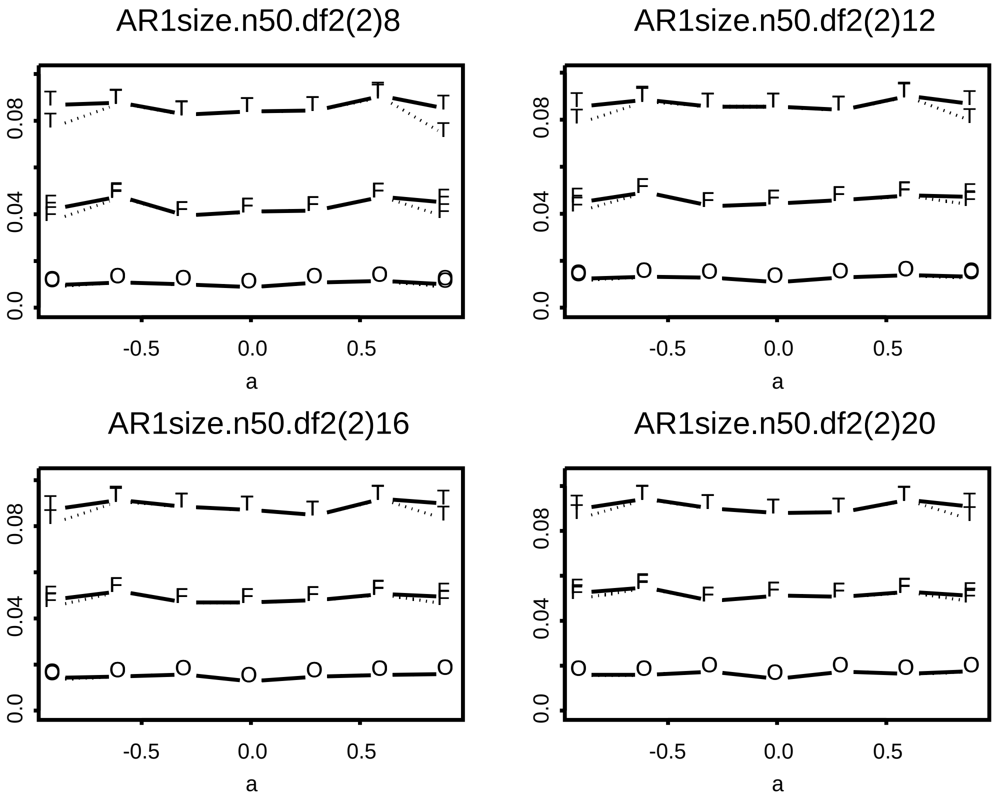

Figure 37: AR(1): $n = 50$, $DF = 2(2)8,\ 2(2)12,\ 2(2)16,\ 2(2)20$

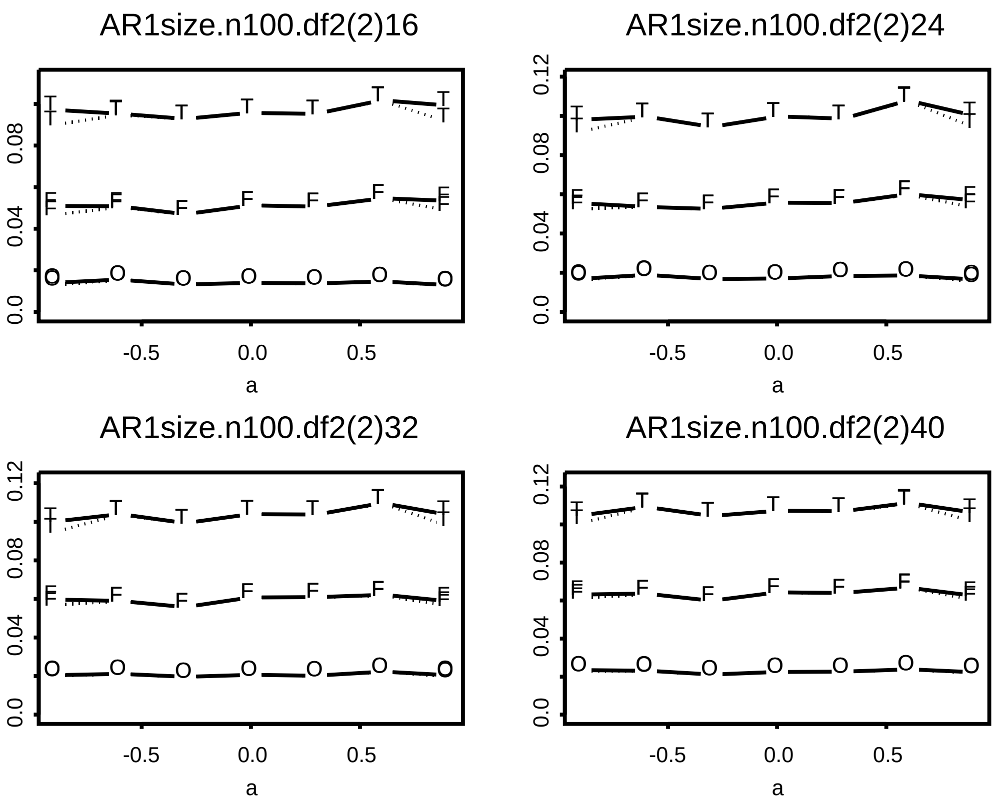

Figure 38: AR(1): $n = 100$, $DF = 2(2)16,\ 2(2)24,\ 2(2)32,\ 2(2)40$

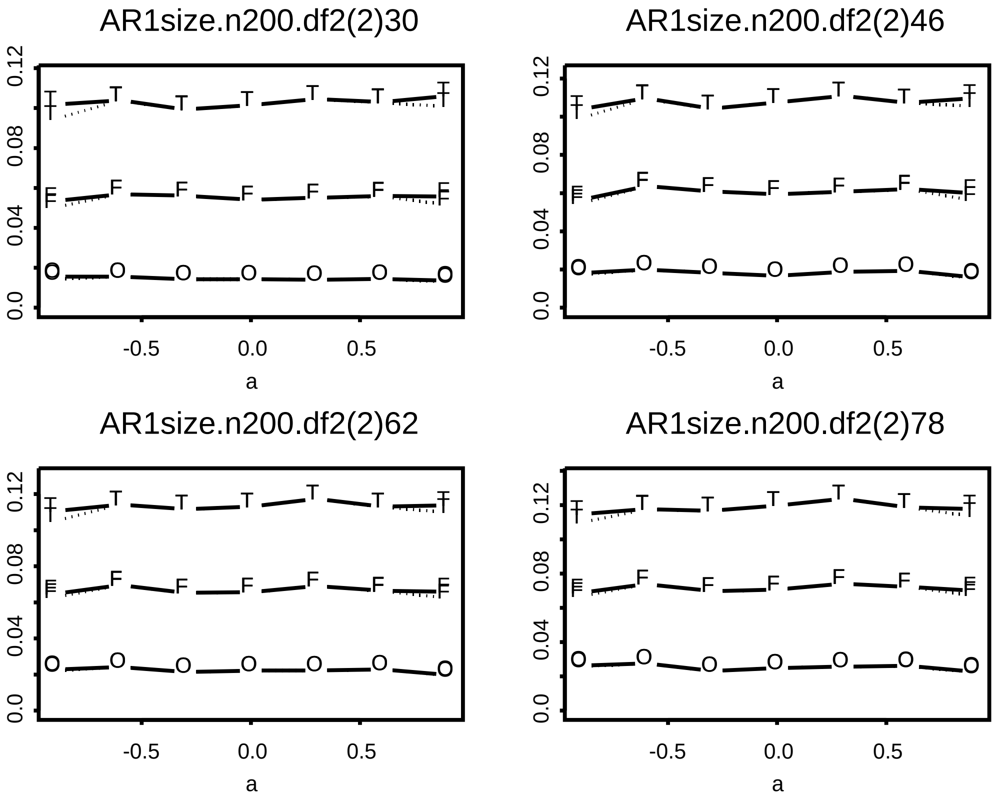

Figure 39: AR(1): $n = 200,\ DF = 2(2)30,\ 2(2)46,\ 2(2)62,\ 2(2)78$

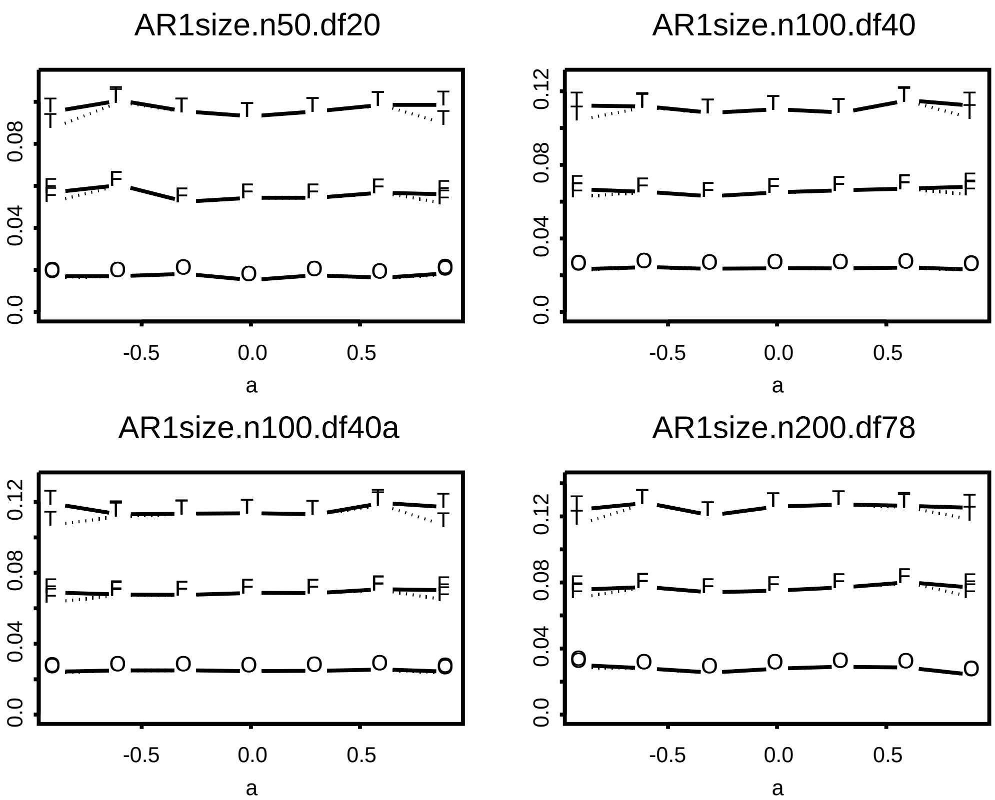

Figure 40: AR(1): $DF = (2, 10, 20),\ (2, 10, 20, 30, 40),\ (2, 20, 40),\ (2, 20, 40, 60, 78)$

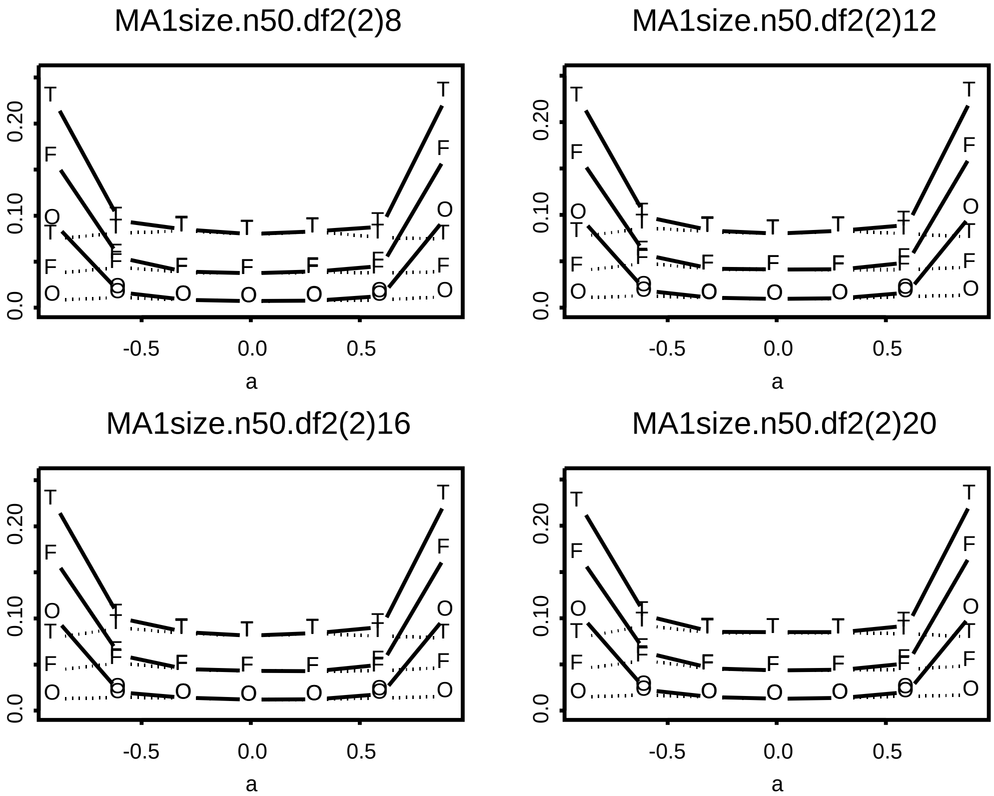

Figure 41: MA(1): $n = 50$, $DF = 2(2)8,\ 2(2)12,\ 2(2)16,\ 2(2)20$

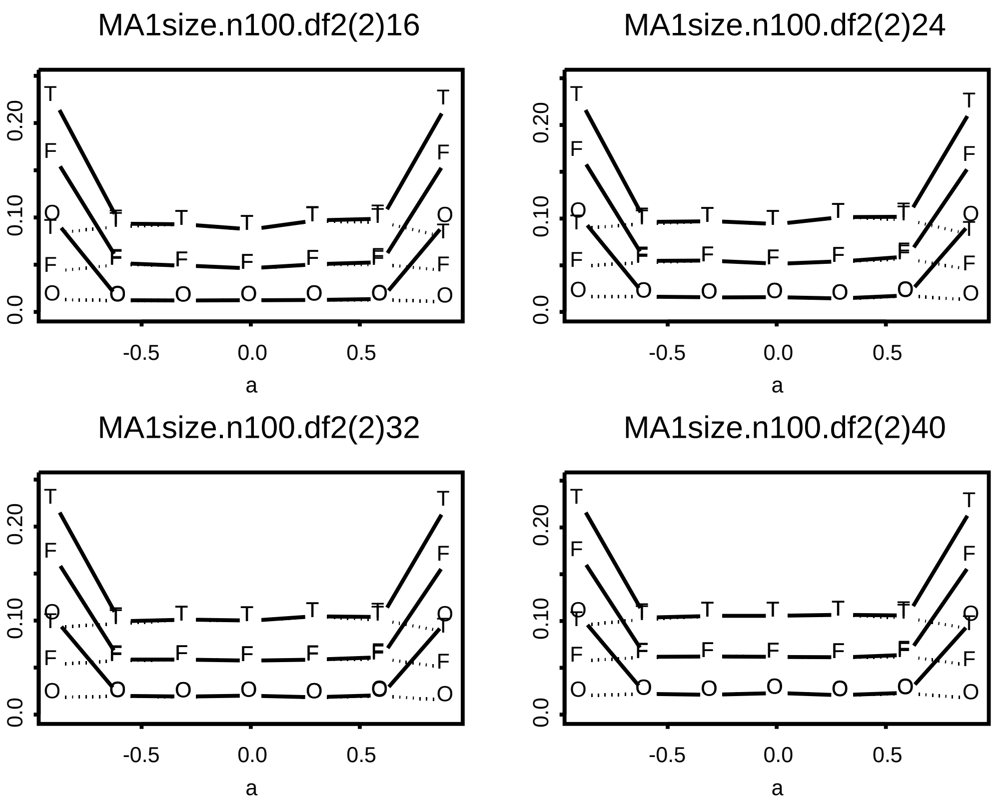

Figure 42: MA(1): $n = 100$, $DF = 2(2)16,\ 2(2)24,\ 2(2)32,\ 2(2)40$

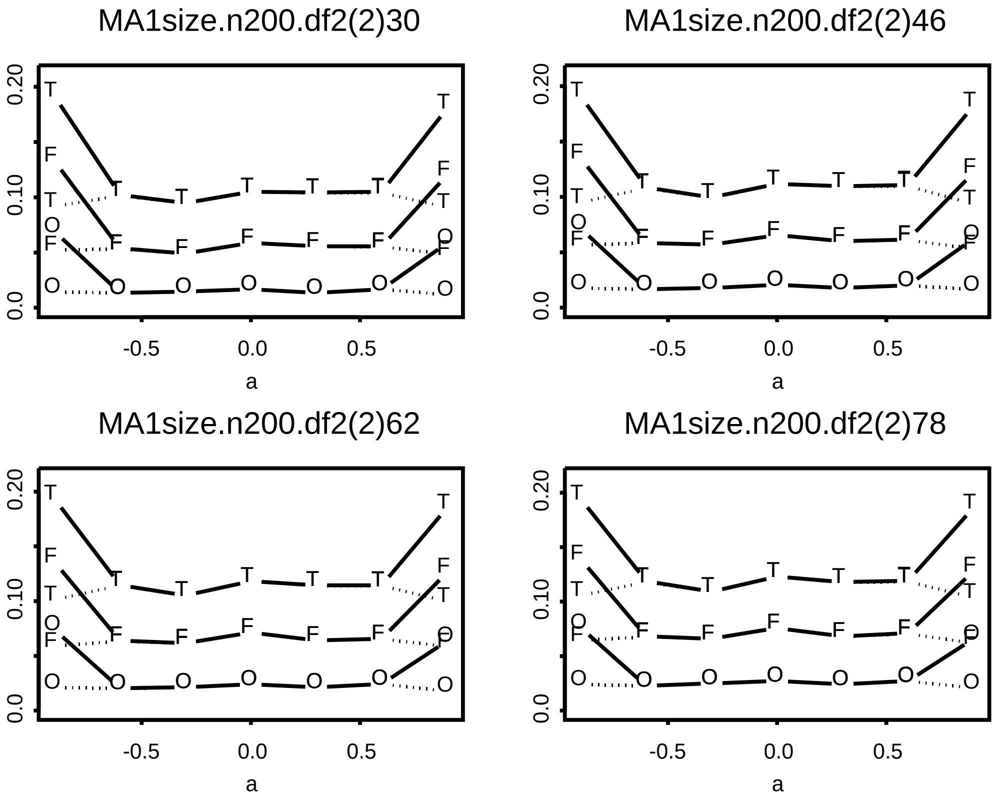

Figure 43: MA(1): $n = 200$, $DF = 2(2)30,\ 2(2)46,\ 2(2)62,\ 2(2)78$

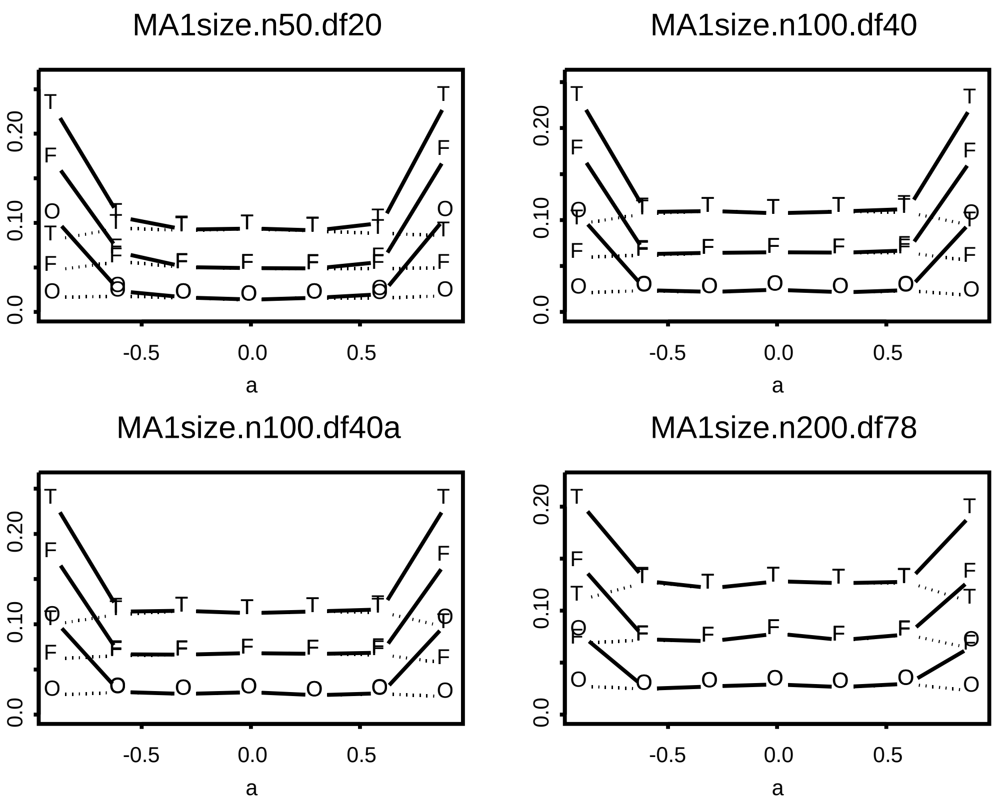

Figure 44: MA(1): $DF = (2, 10, 20),\ (2, 10, 20, 30, 40),\ (2, 20, 40),\ (2, 20, 40, 60, 78)$

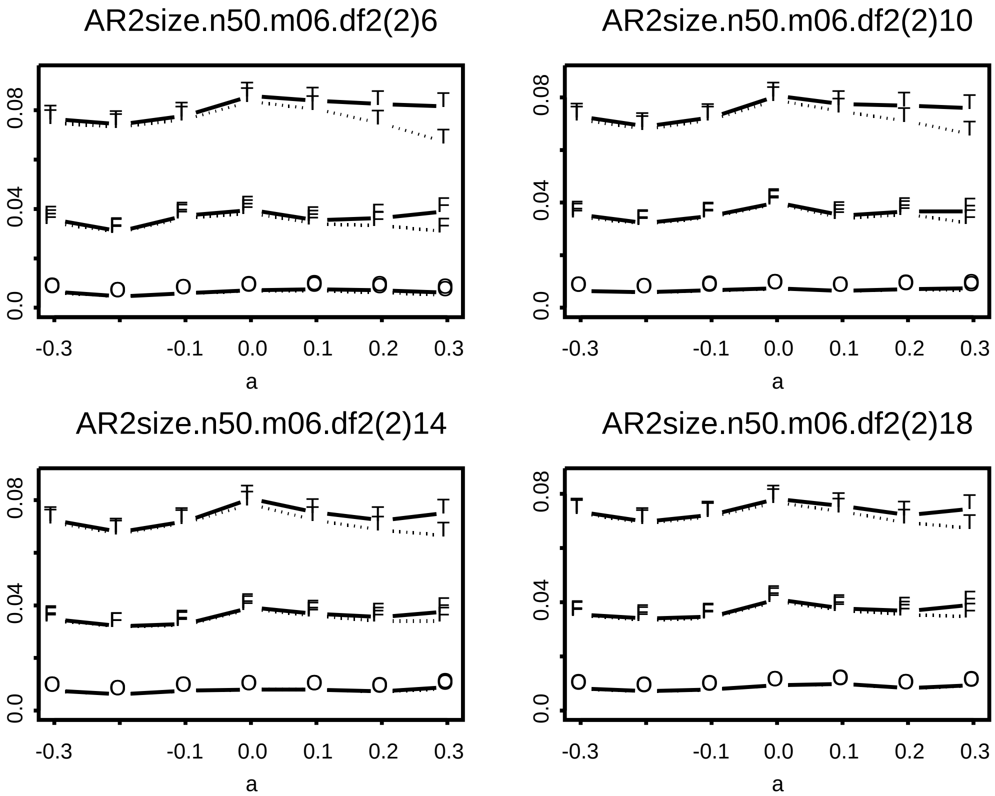

Figure 45: AR(2)a: $n = 50$, $DF = 2(2)6,\ 2(2)10,\ 2(2)14,\ 2(2)18$

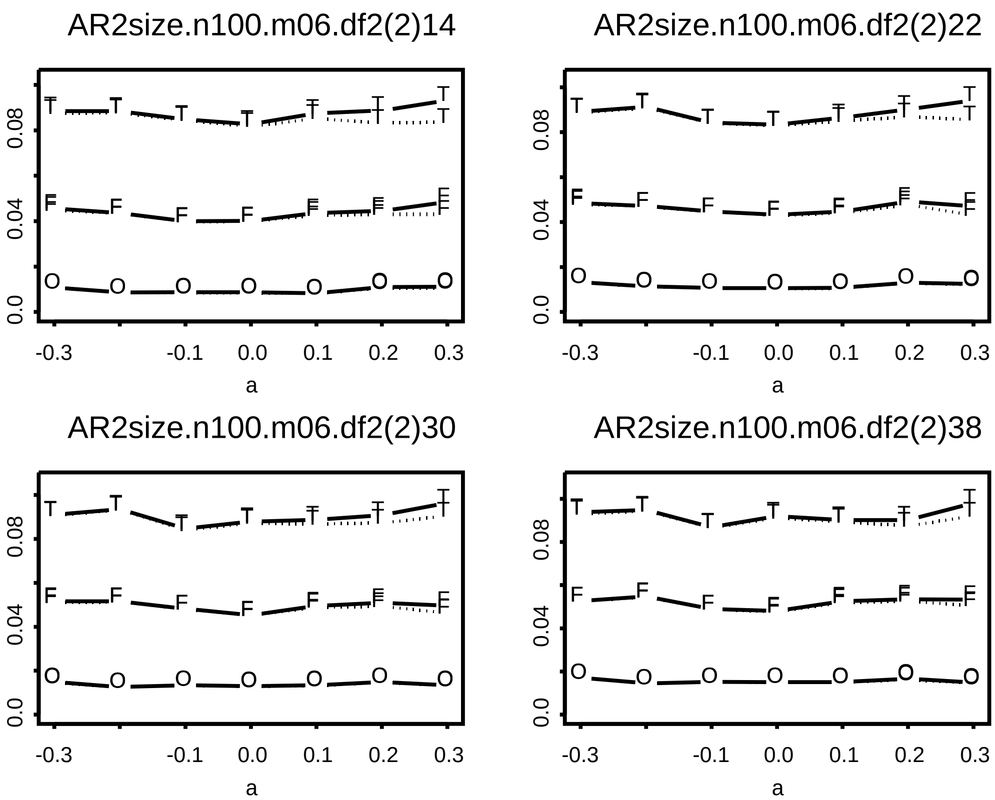

Figure 46: AR(2)a: $n = 100$, $DF = 2(2)14,\ 2(2)22,\ 2(2)30,\ 2(2)38$

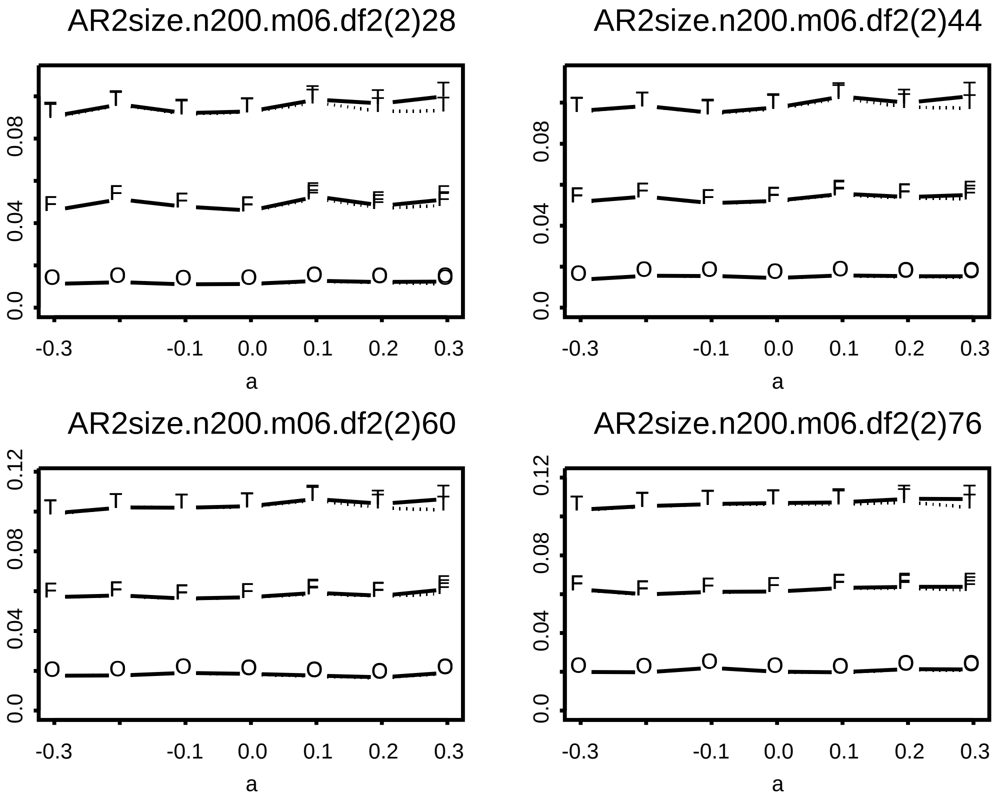

Figure 47: AR(2)a: $n = 200$, $DF = 2(2)28,\ 2(2)44,\ 2(2)60,\ 2(2)76$

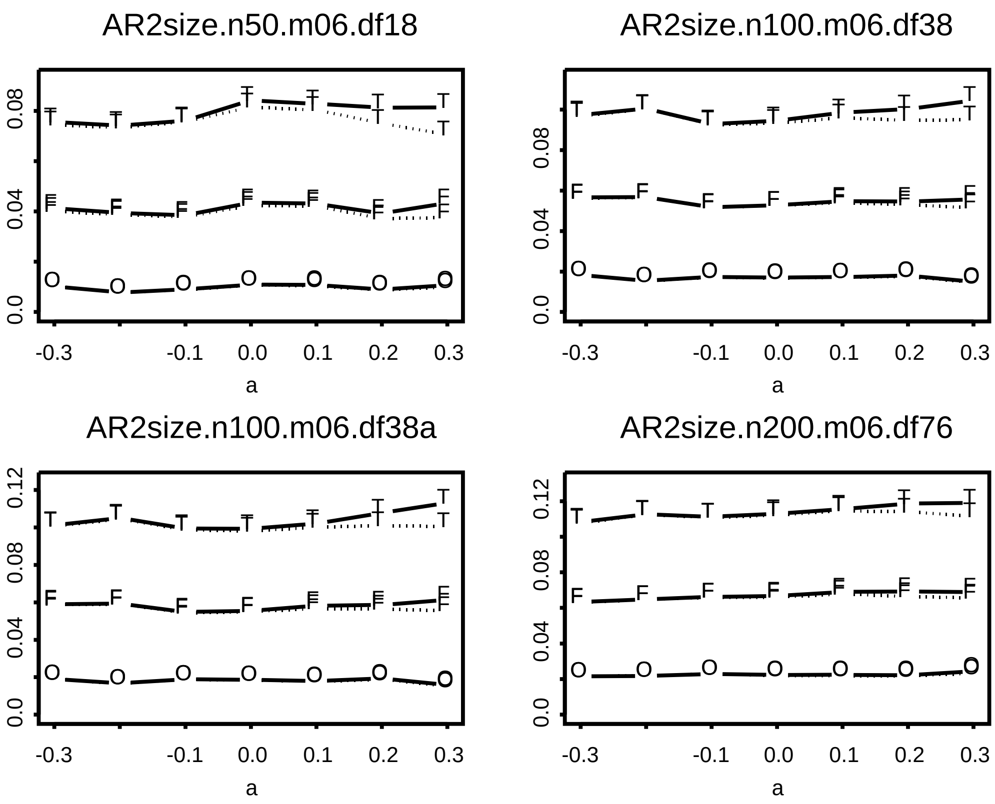

Figure 48: AR(2)a: $DF = (2, 10, 18),\ (2, 10, 20, 30, 38),\ (2, 20, 38),\ (2, 20, 40, 60, 76)$

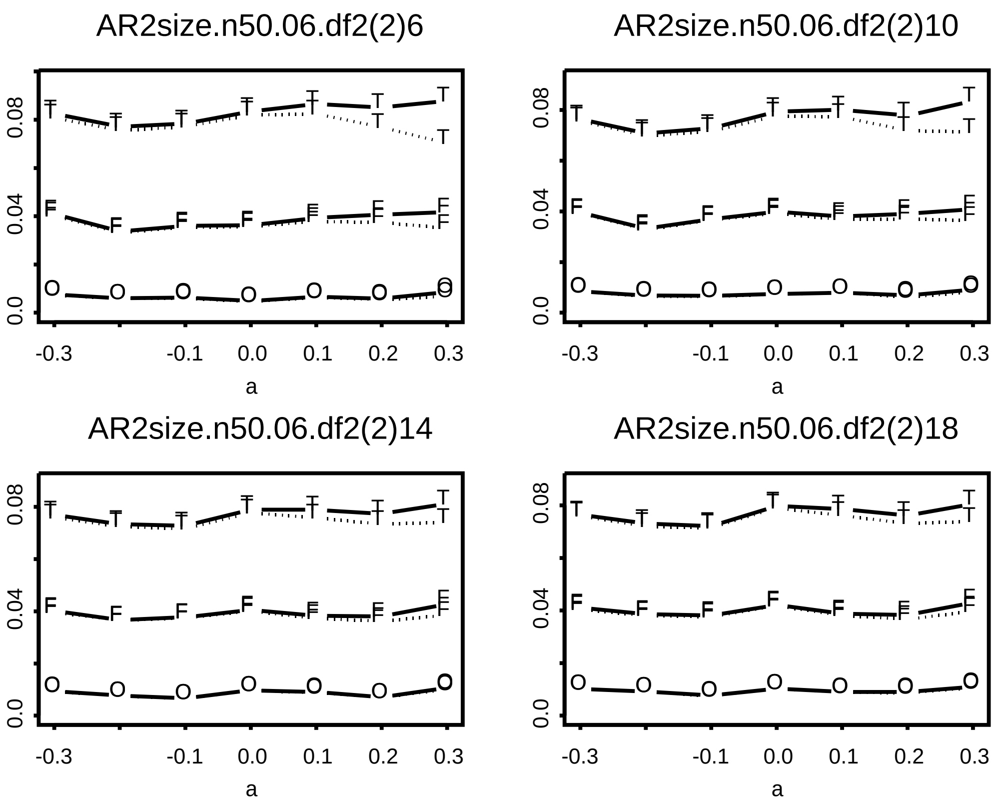

Figure 49: AR(2)b: $n = 50$, $DF = 2(2)6,\ 2(2)10,\ 2(2)14,\ 2(2)18$

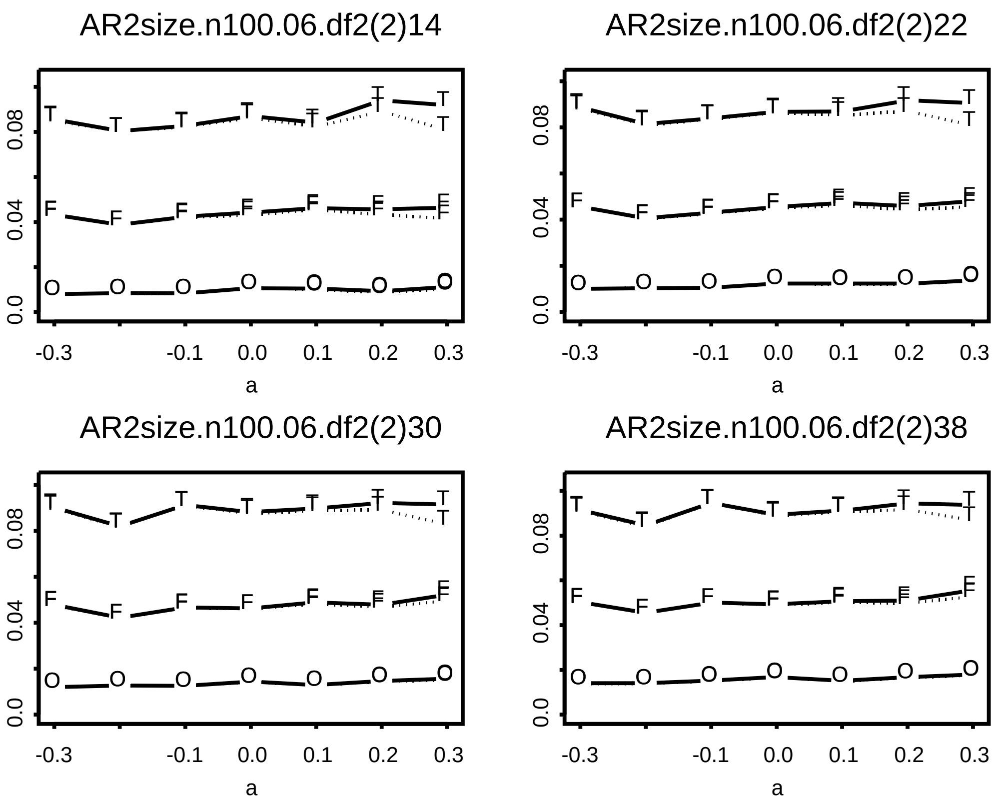

Figure 50: AR(2)b: $n = 100$, $DF = 2(2)14,\ 2(2)22,\ 2(2)30,\ 2(2)38$

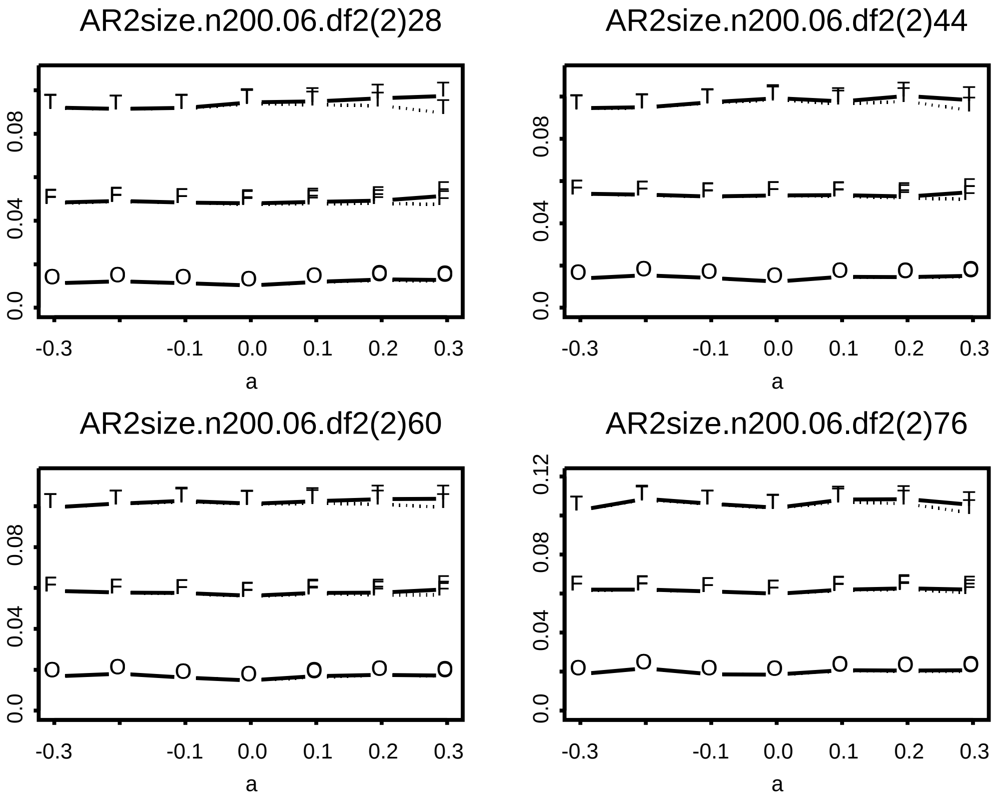

Figure 51: AR(2)b: $n = 200$, $DF = 2(2)28,\ 2(2)44,\ 2(2)60,\ 2(2)76$

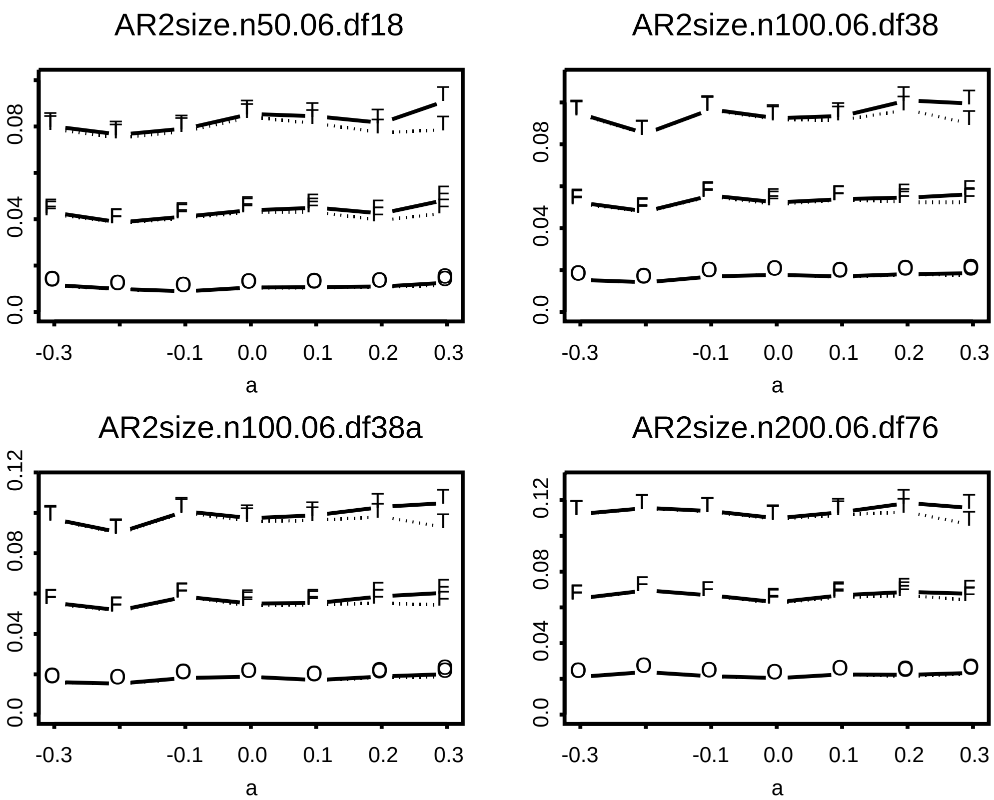

Figure 52: AR(2)b: $DF = (2, 10, 18),\ (2, 10, 20, 30, 38),\ (2, 20, 38),\ (2, 20, 40, 60, 76)$

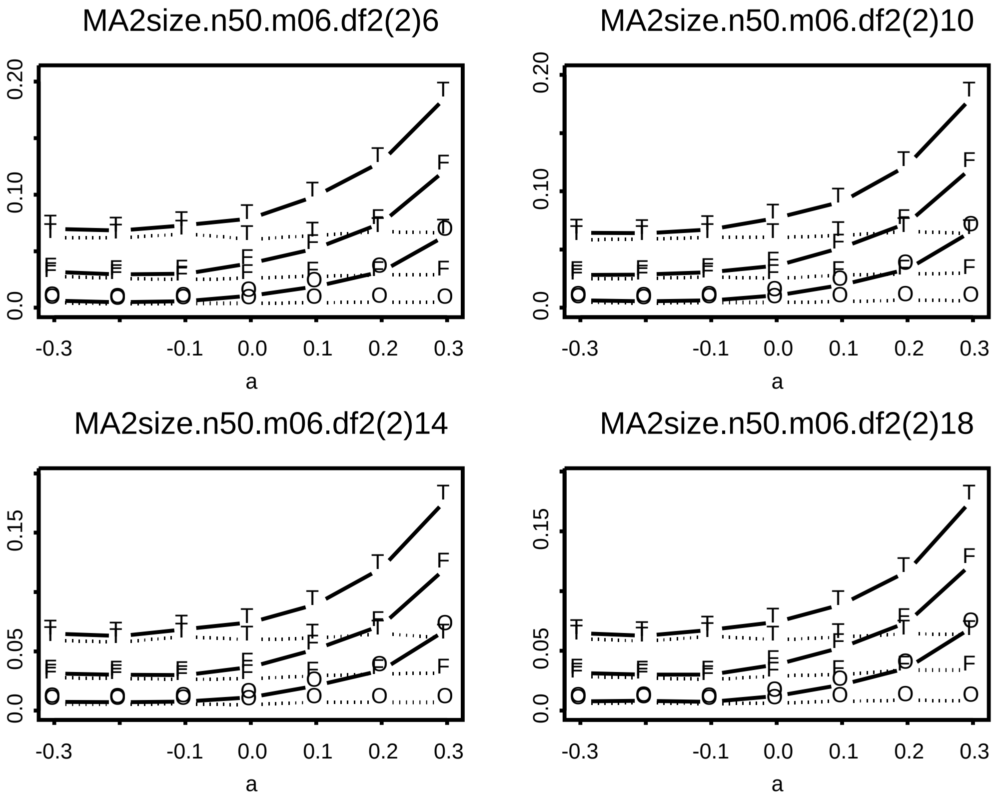

Figure 53: MA(2)a: $n = 50$, $DF = 2(2)6,\ 2(2)10,\ 2(2)14,\ 2(2)18$

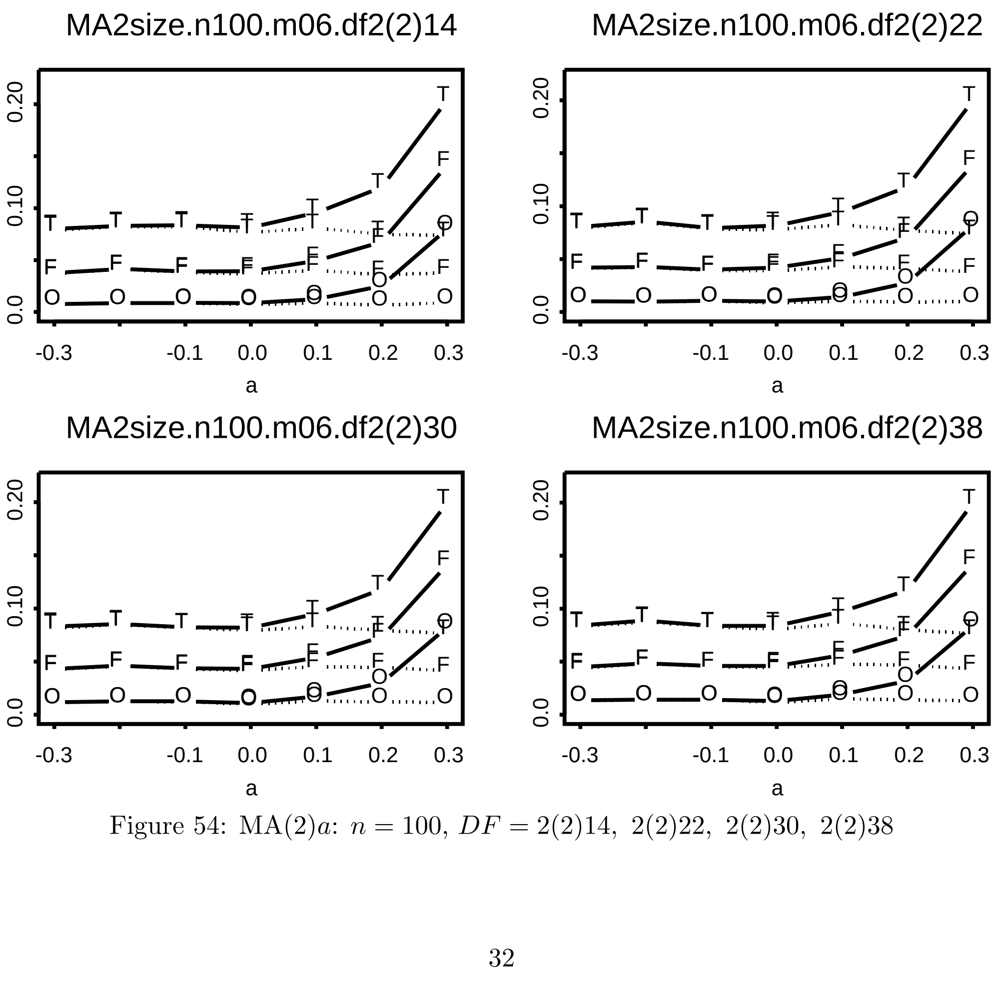

Figure 54: MA(2)a: $n = 100$, $DF = 2(2)14,\ 2(2)22,\ 2(2)30,\ 2(2)38$

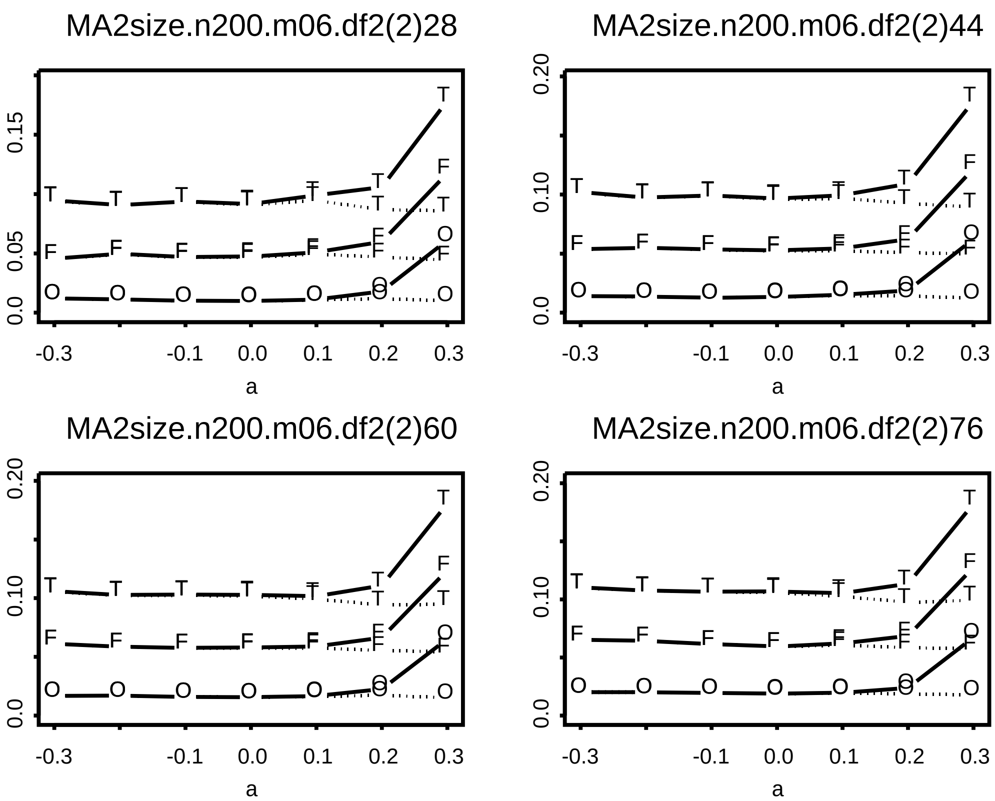

Figure 55: MA(2)a: $n = 200$, $DF = 2(2)28,\ 2(2)44,\ 2(2)60,\ 2(2)76$

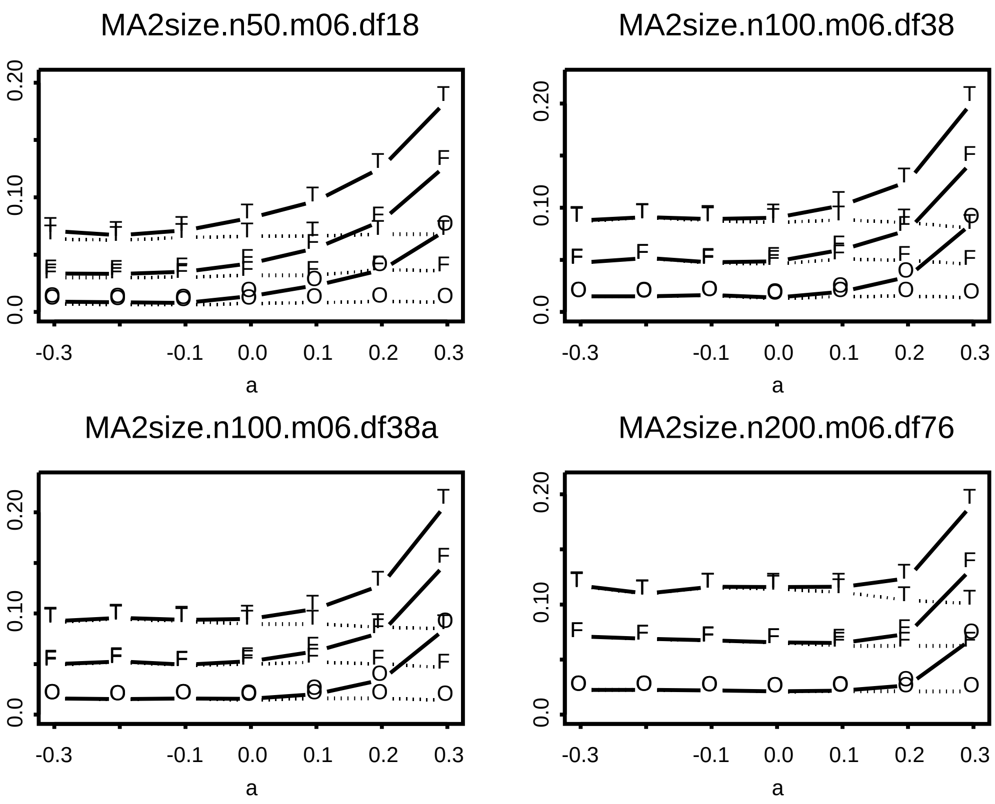

Figure 56: MA(2)a: $DF = (2, 10, 18),\ (2, 10, 20, 30, 38),\ (2, 20, 38),\ (2, 20, 40, 60, 76)$

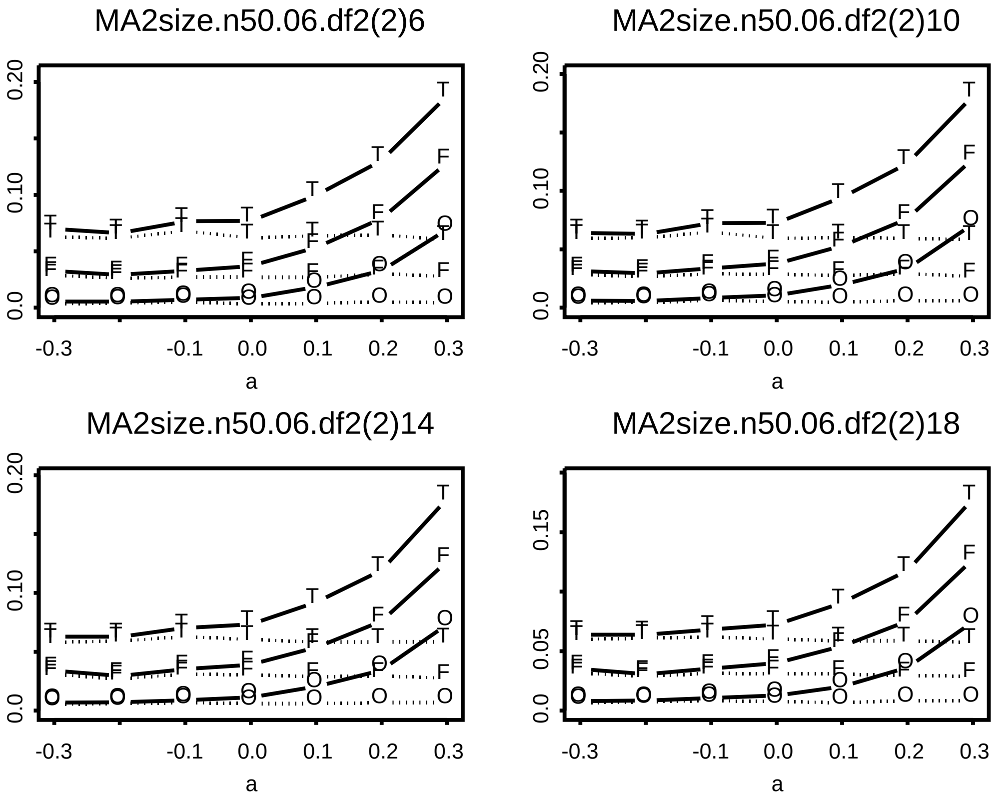

Figure 57: MA(2)b: $n = 50$, $DF = 2(2)6,\ 2(2)10,\ 2(2)14,\ 2(2)18$

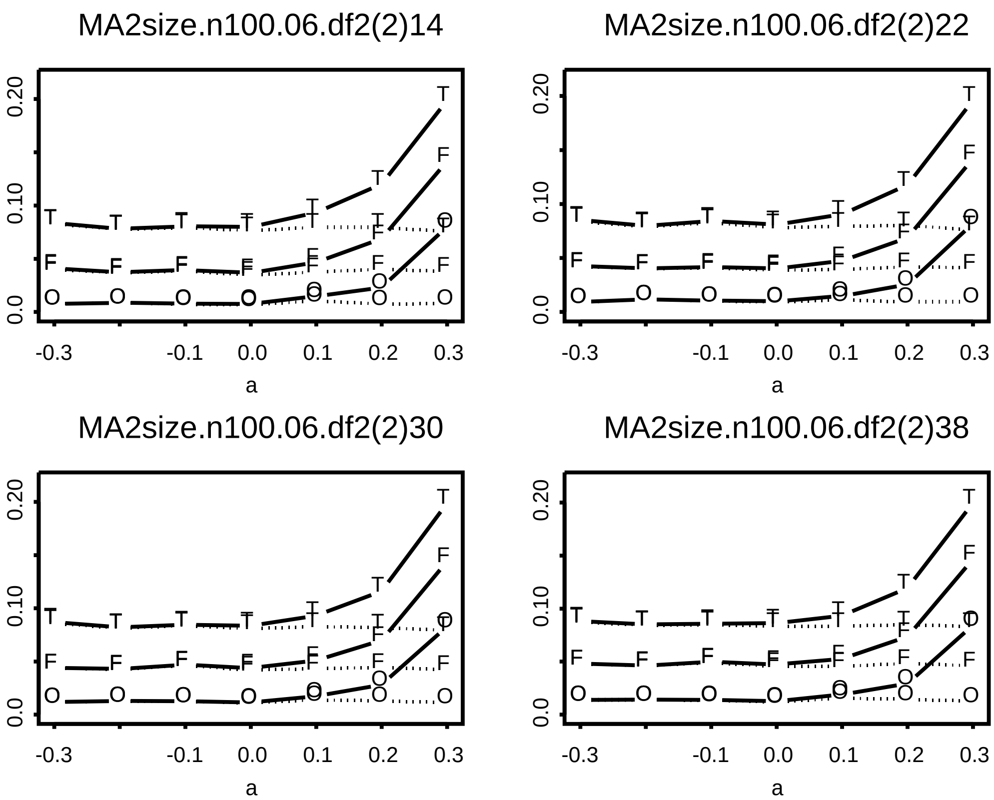

Figure 58: MA(2)b: $n = 100$, $DF = 2(2)14,\ 2(2)22,\ 2(2)30,\ 2(2)38$

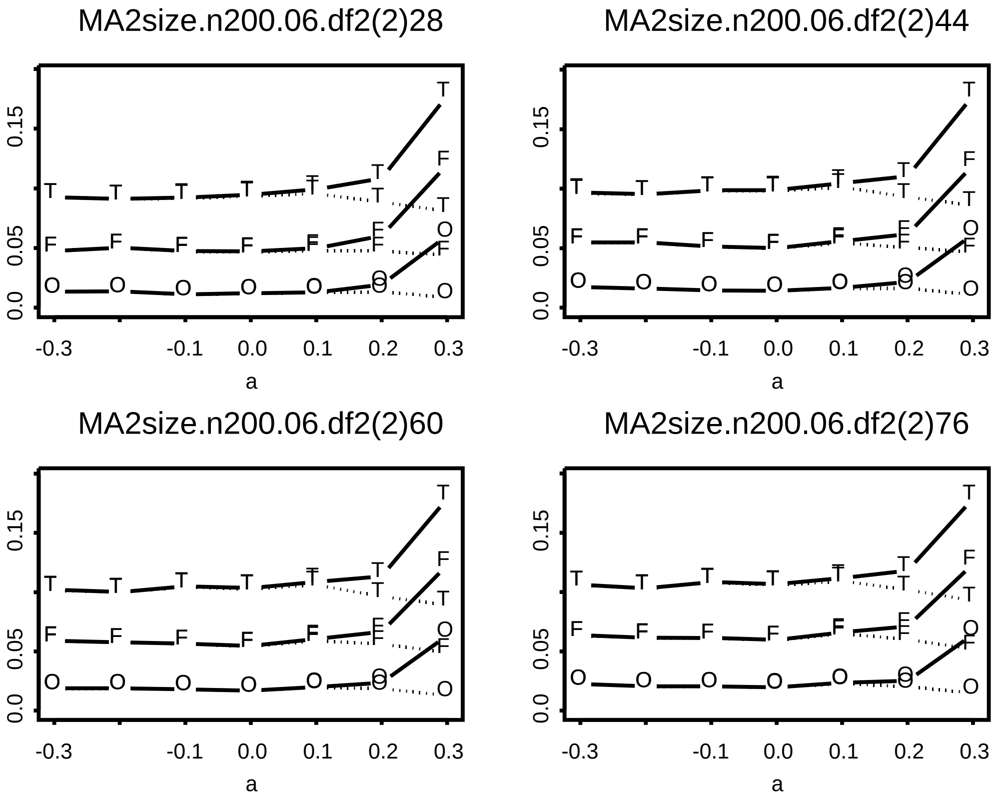

Figure 59: MA(2)b: $n = 200$, $DF = 2(2)28,\ 2(2)44,\ 2(2)60,\ 2(2)76$

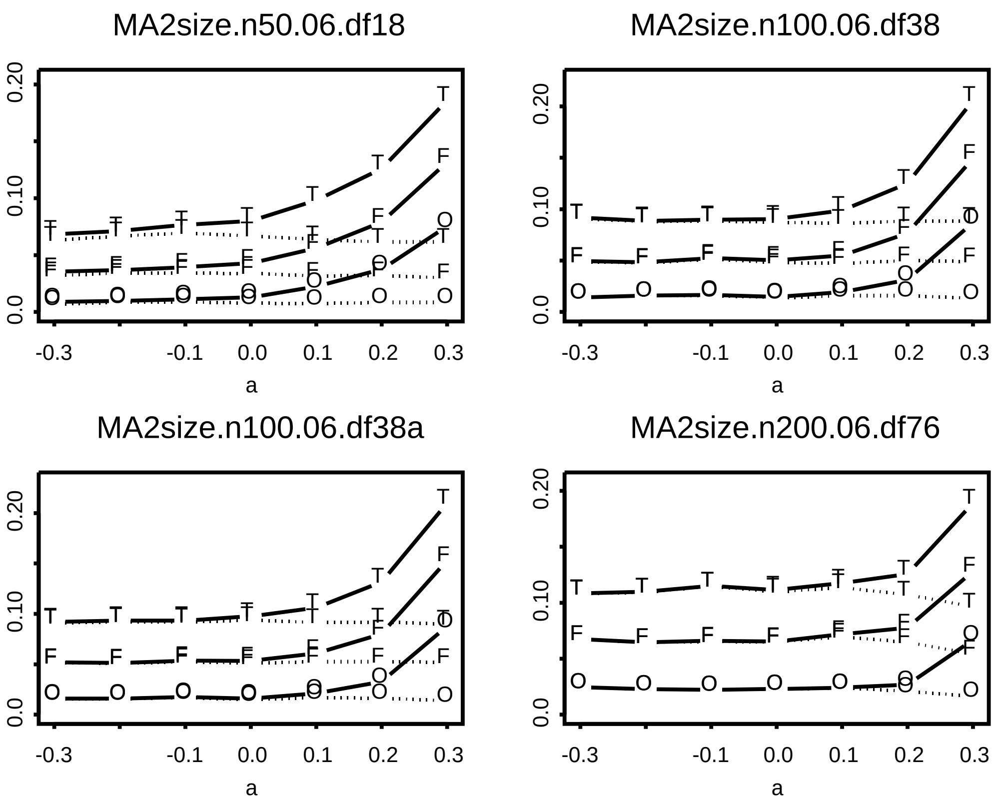

Figure 60: MA(2)b: $DF = (2, 10, 18),\ (2, 10, 20, 30, 38),\ (2, 20, 38),\ (2, 20, 40, 60, 76)$

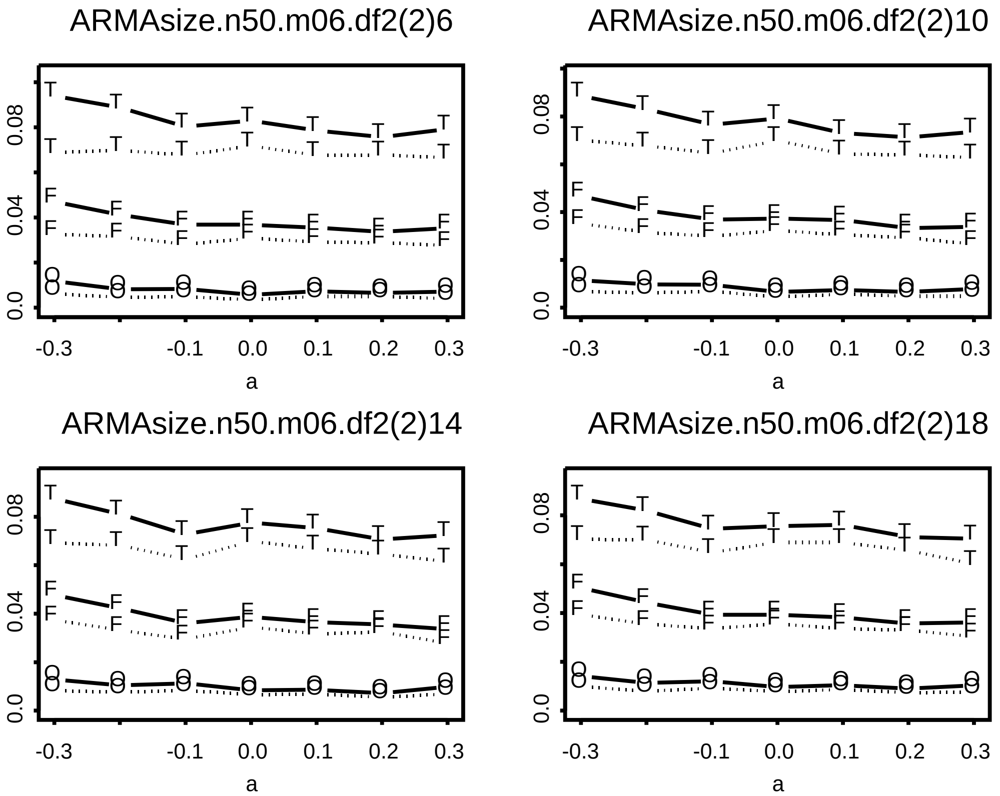

Figure 61: ARMA$(1,1)a$: $n = 50$, $DF = 2(2)6,\ 2(2)10,\ 2(2)14,\ 2(2)18$

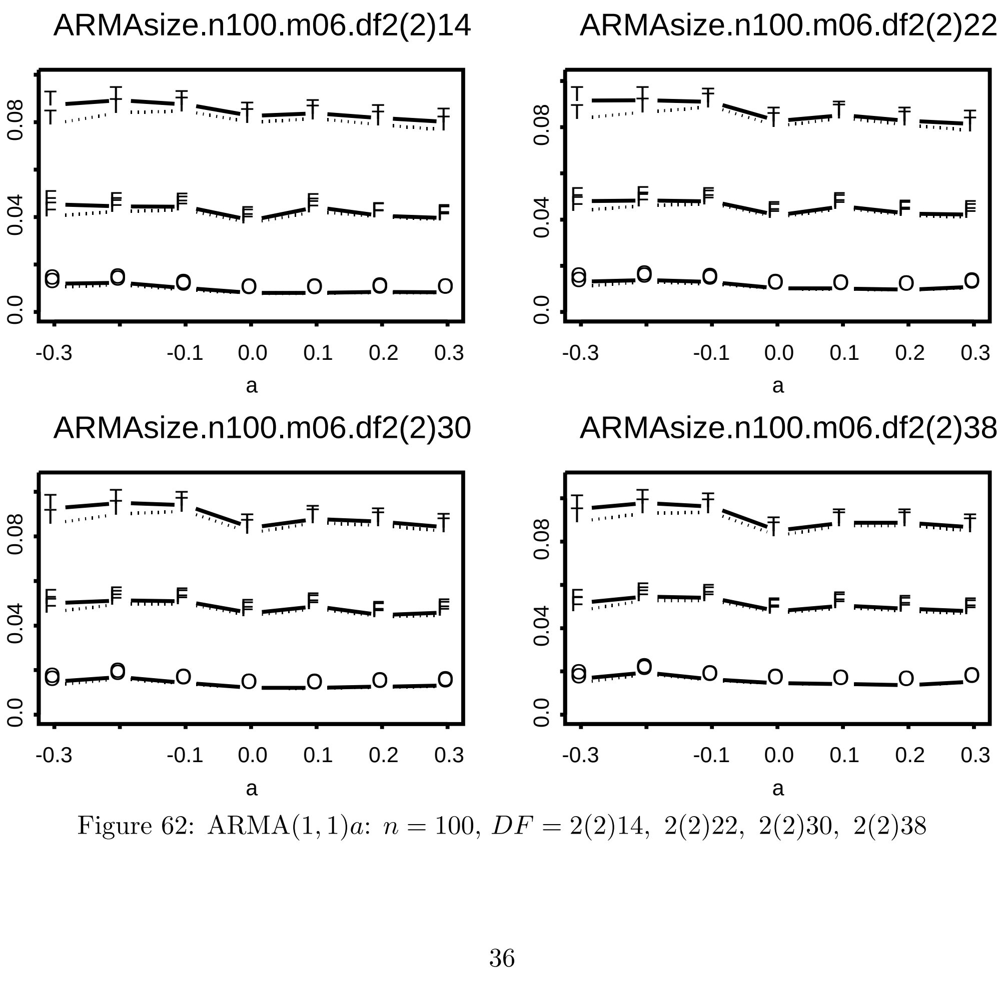

Figure 62: ARMA$(1,1)a$: $n = 100$, $DF = 2(2)14,\ 2(2)22,\ 2(2)30,\ 2(2)38$

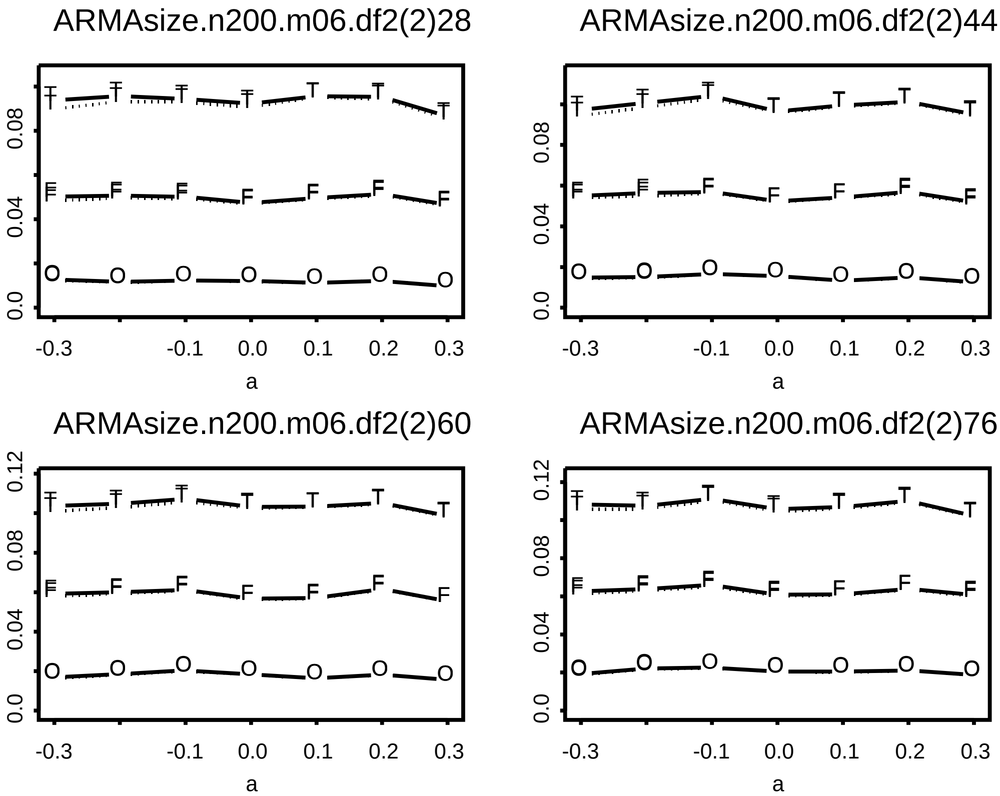

Figure 63: ARMA$(1,1)a$: $n = 200$, $DF = 2(2)28,\ 2(2)44,\ 2(2)60,\ 2(2)76$

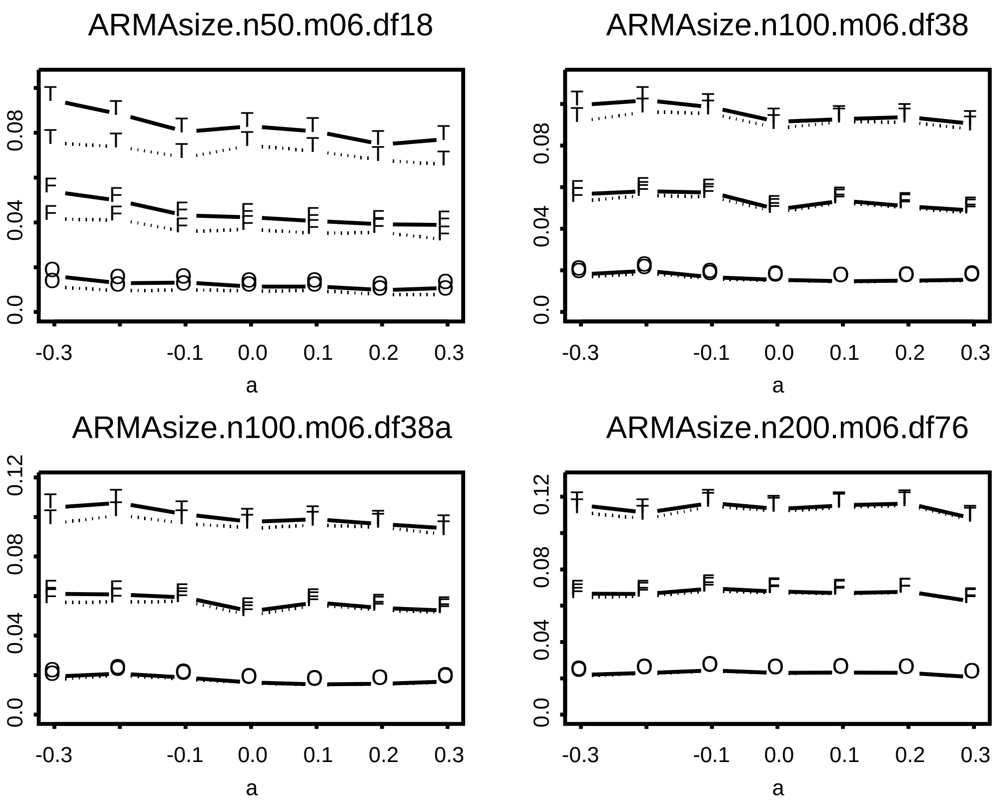

Figure 64: ARMA$(1,1)a$: $DF = (2,10,18),\ (2,10,20,30,38),\ (2,20,38),\ (2,20,40,60,76)$

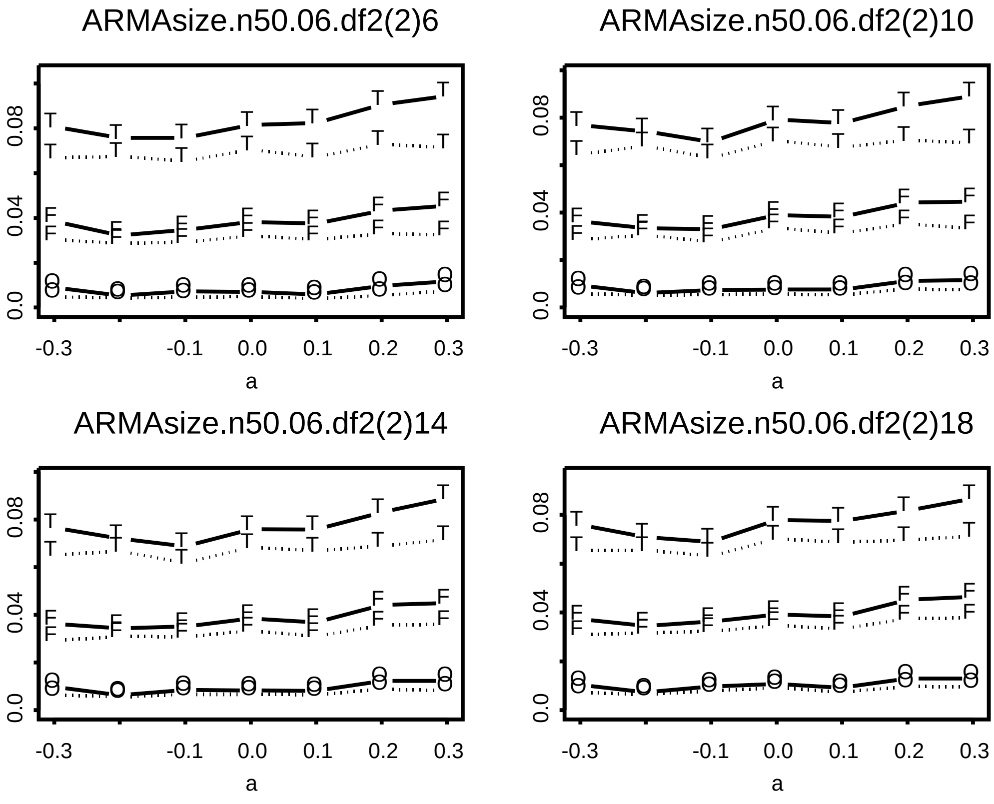

Figure 65: ARMA$(1,1)b$: $n = 50$, $DF = 2(2)6,\ 2(2)10,\ 2(2)14,\ 2(2)18$

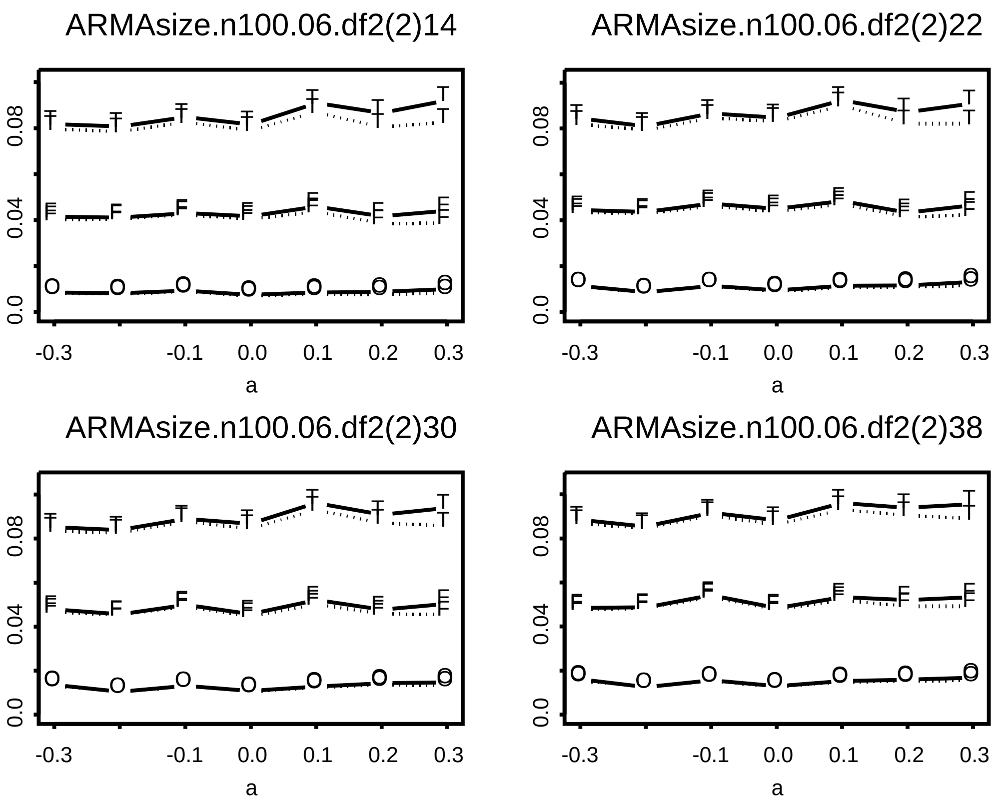

Figure 66: ARMA$(1,1)b$: $n = 100$, $DF = 2(2)14,\ 2(2)22,\ 2(2)30,\ 2(2)38$

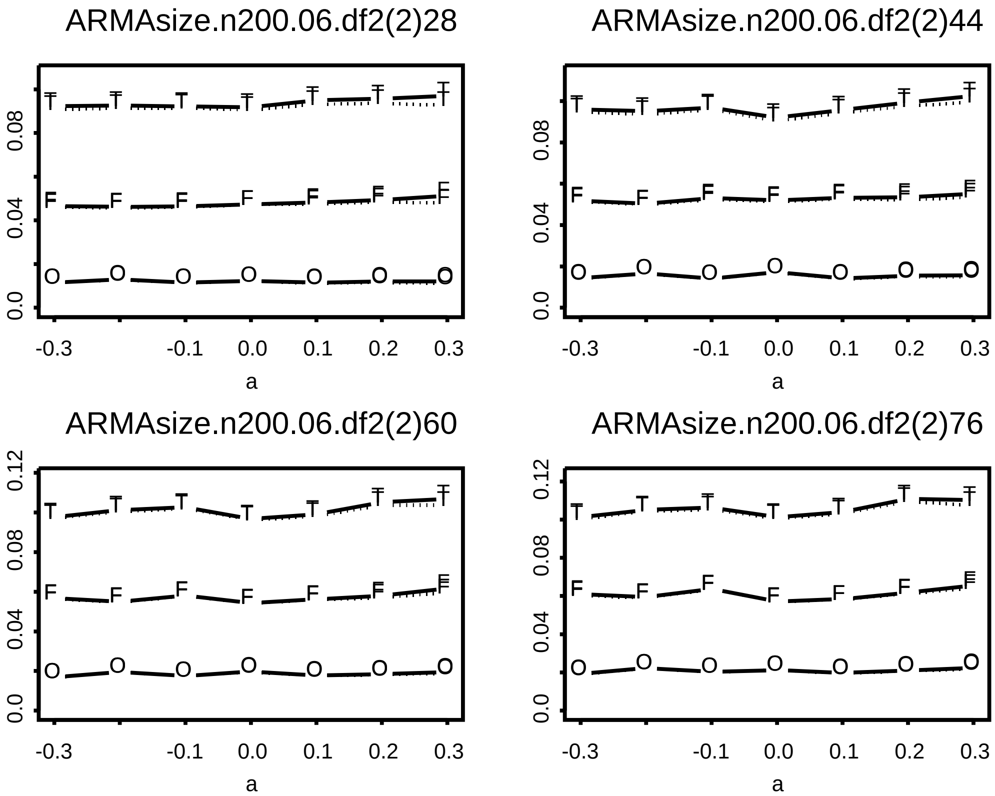

Figure 67: ARMA(1,1)b: $n = 200$, $DF = 2(2)28,\ 2(2)44,\ 2(2)60,\ 2(2)76$

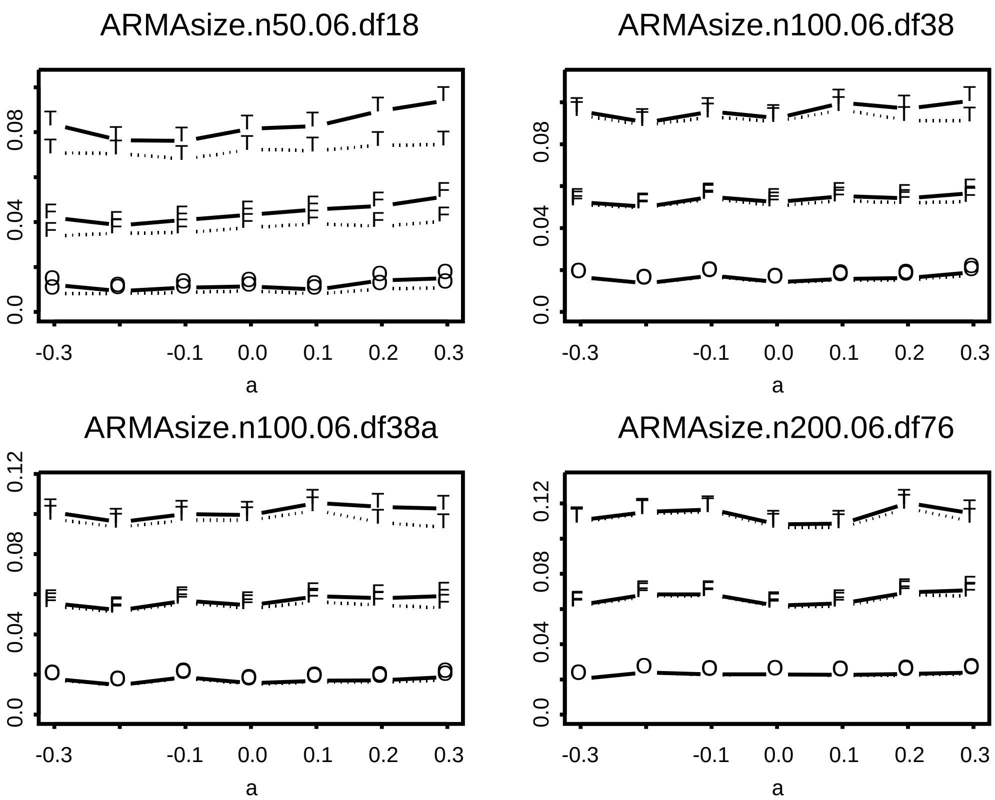

Figure 68: ARMA(1,1)b: $DF = (2,10,18),\ (2,10,20,30,38),\ (2,20,38),\ (2,20,40,60,76)$

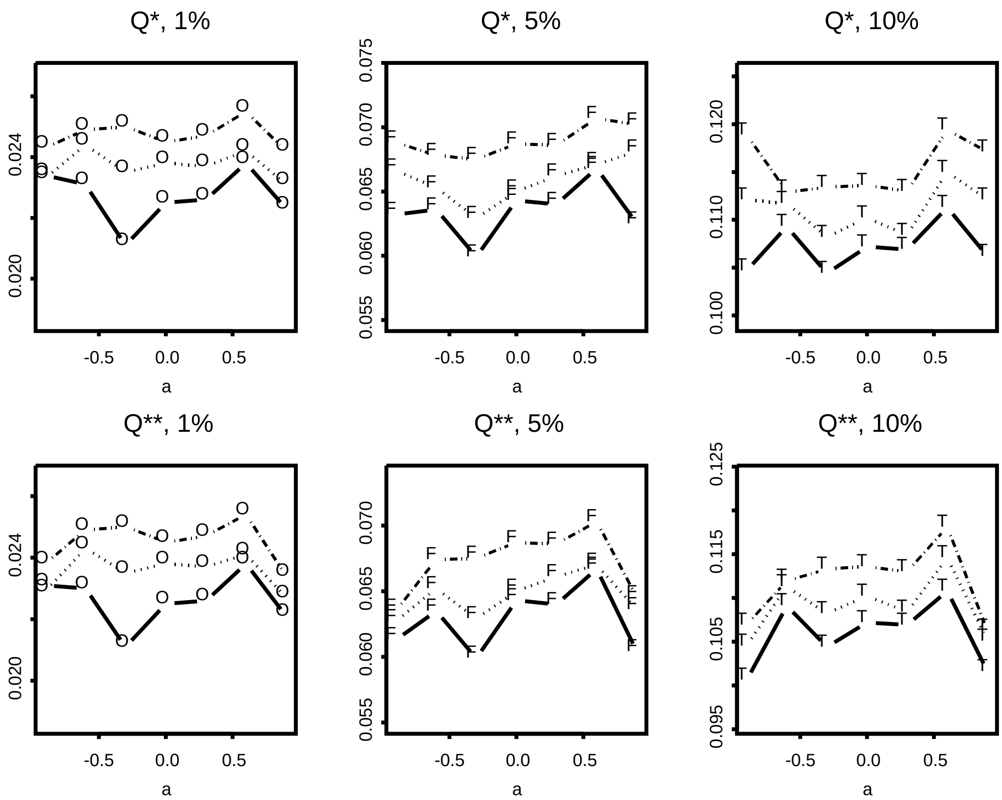

Figure 69: AR(1): $DF = 2(2)40$ (——), $(2, 10, 20, 30, 40)$ (······), $(2, 20, 40)$ (–·–·–)

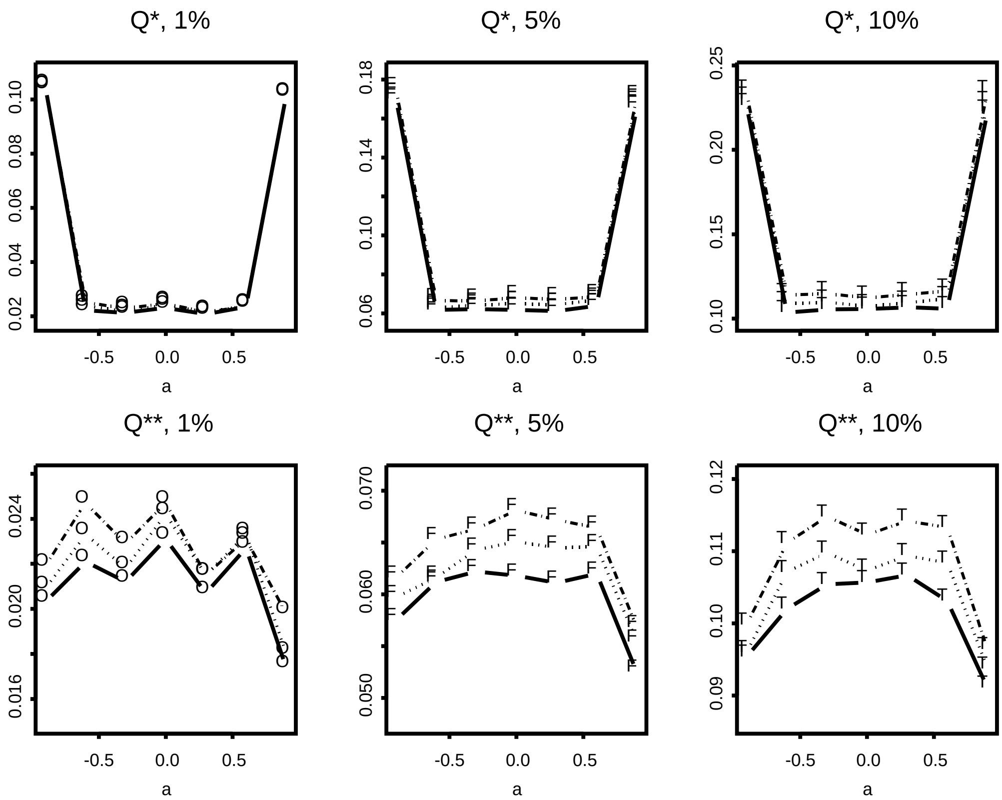

Figure 70: MA(1): $DF = 2(2)40$ (——), $(2, 10, 20, 30, 40)$ (······), $(2, 20, 40)$ (–·–·–)

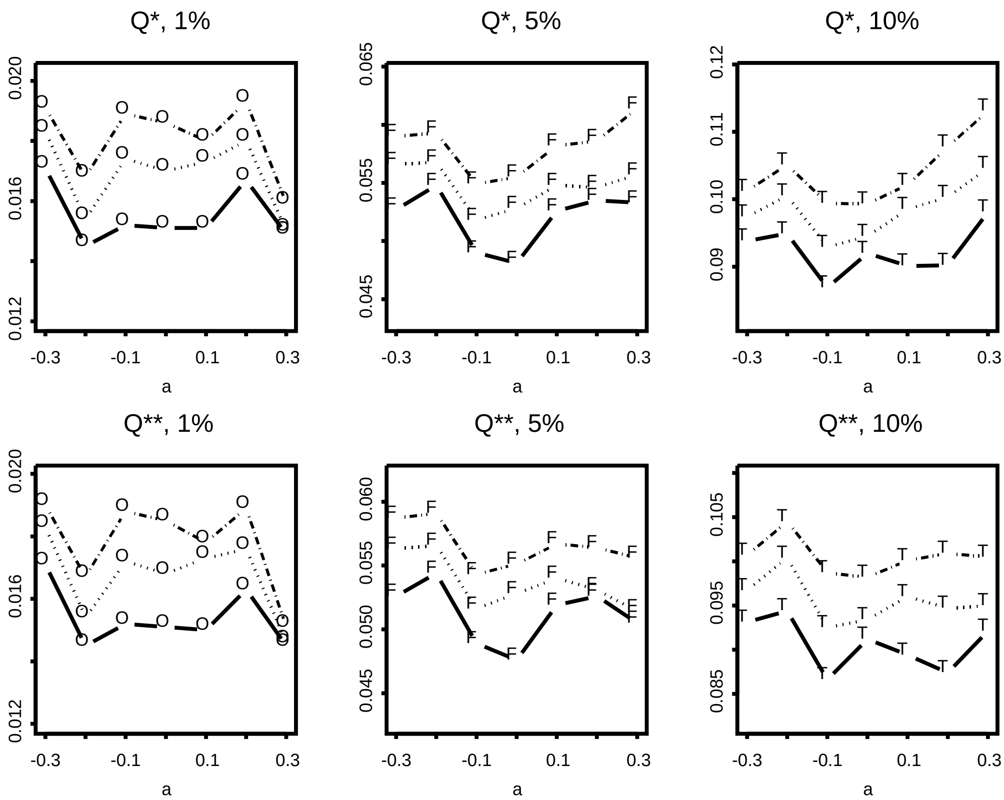

Figure 71: AR(2)a: $DF = 2(2)38$ (——), $(2, 10, 20, 30, 38)$ $(\cdots\cdots)$, $(2, 20, 38)$ (–·–·–)

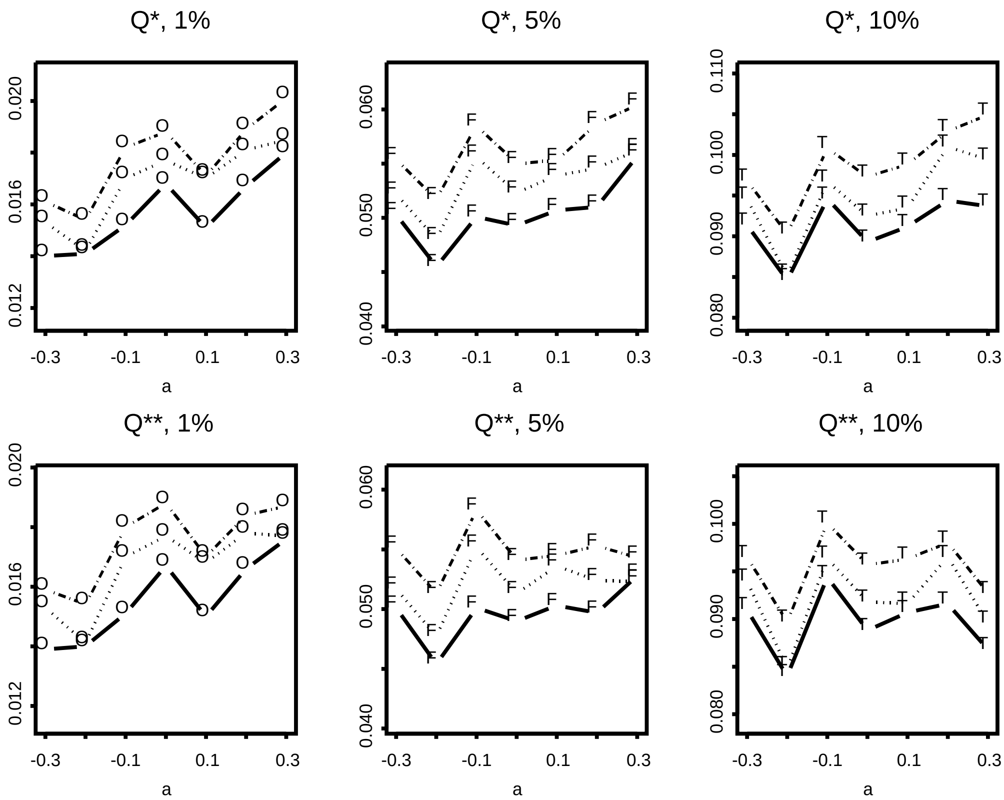

Figure 72: AR(2)b: $DF = 2(2)38$ (——), $(2, 10, 20, 30, 38)$ $(\cdots\cdots)$, $(2, 20, 38)$ (–·–·–)

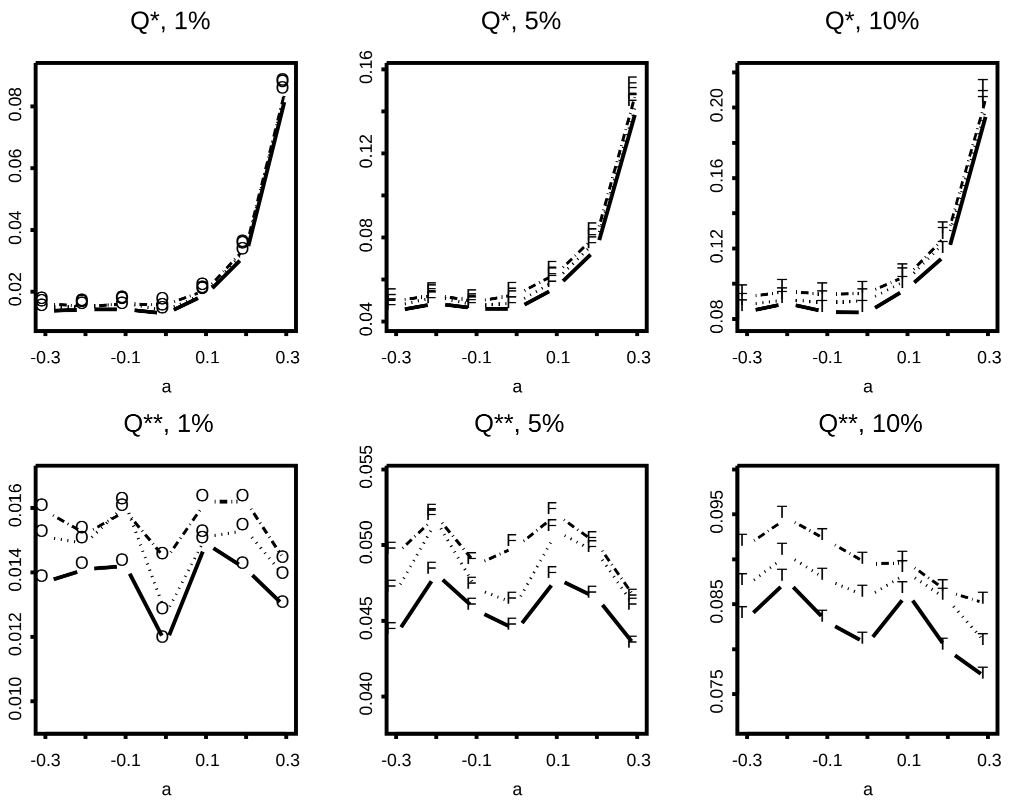

Figure 73: MA(2)a: $DF = 2(2)38$ (——), $(2, 10, 20, 30, 38)$ $(\cdots\cdots)$, $(2, 20, 38)$ (–·–·–)

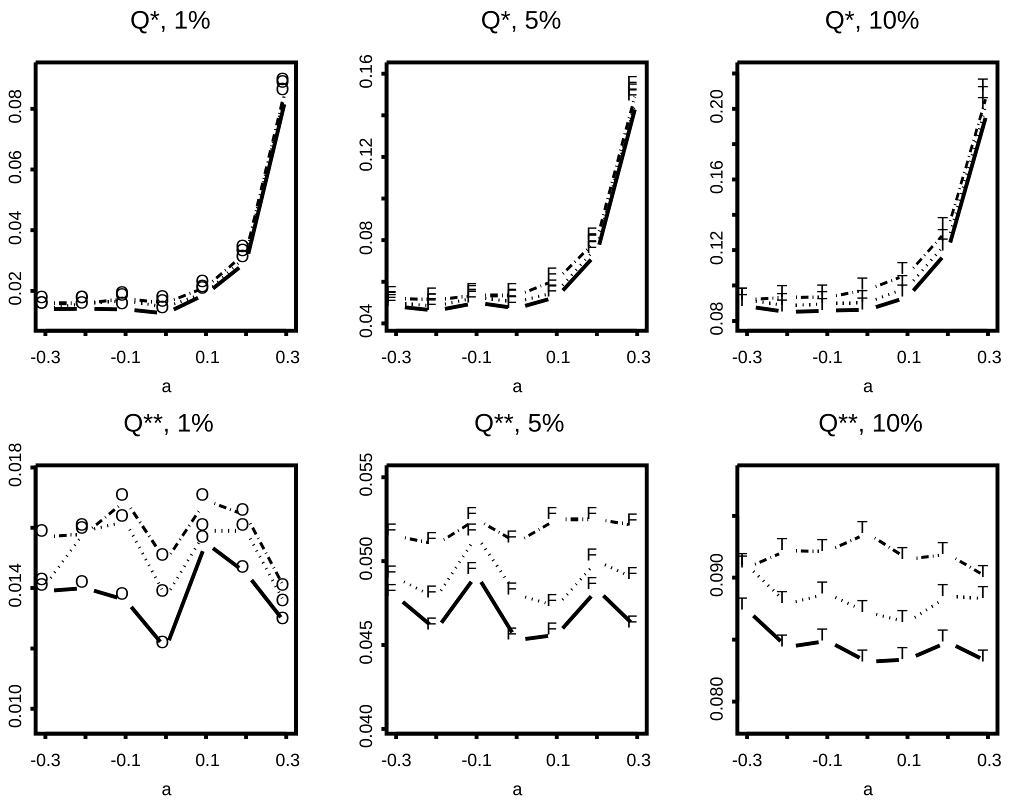

Figure 74: MA(2)b: $DF = 2(2)38$ (——), $(2, 10, 20, 30, 38)$ $(\cdots\cdots)$, $(2, 20, 38)$ (–·–·–)

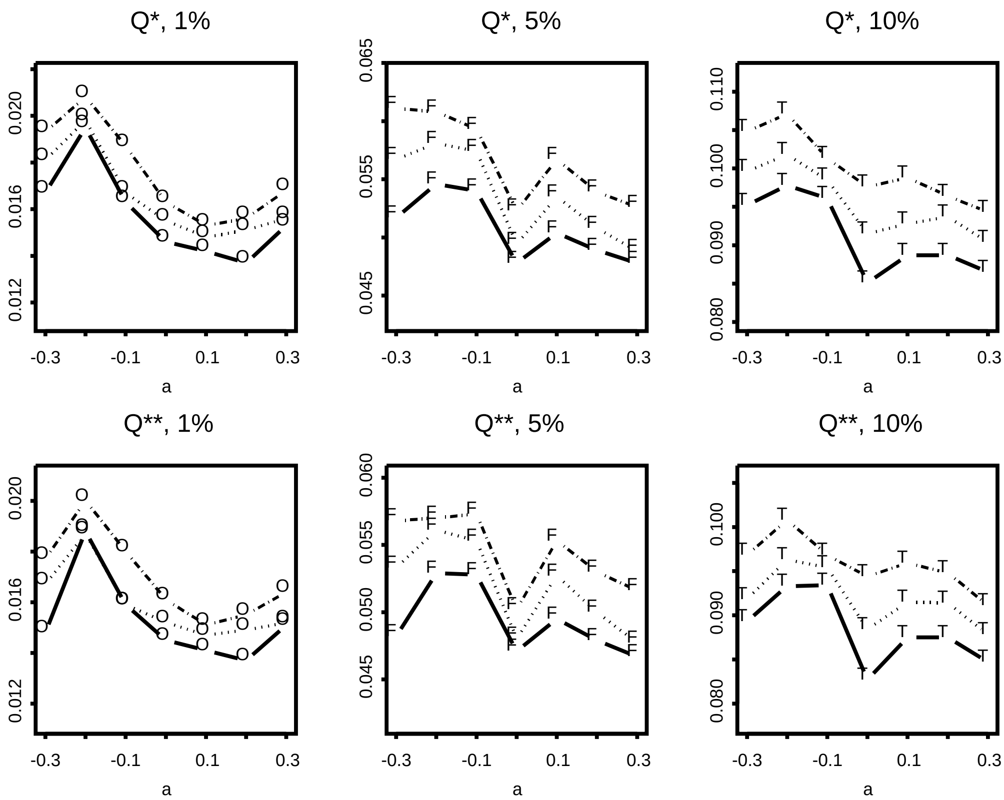

Figure 75: ARMA$(1,1)a$: $DF = 2(2)38$ (——), $(2, 10, 20, 30, 38)$ (······), $(2, 20, 38)$ (–·–·–)

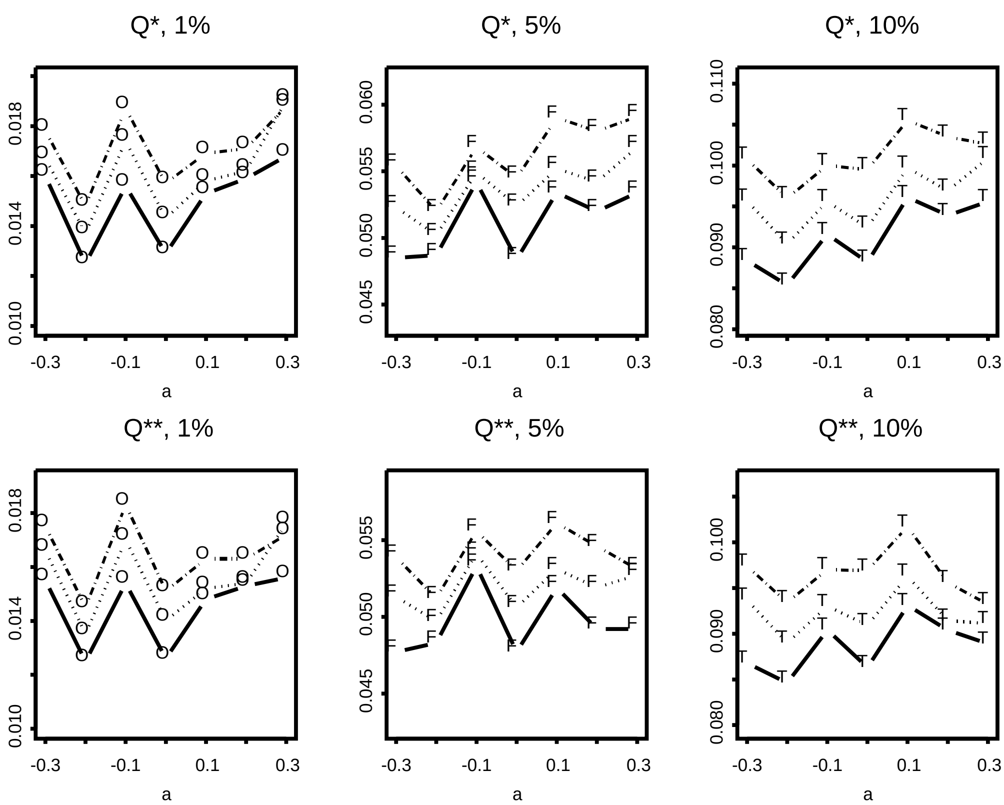

Figure 76: ARMA$(1,1)b$: $DF = 2(2)38$ (——), $(2, 10, 20, 30, 38)$ (······), $(2, 20, 38)$ (–·–·–)

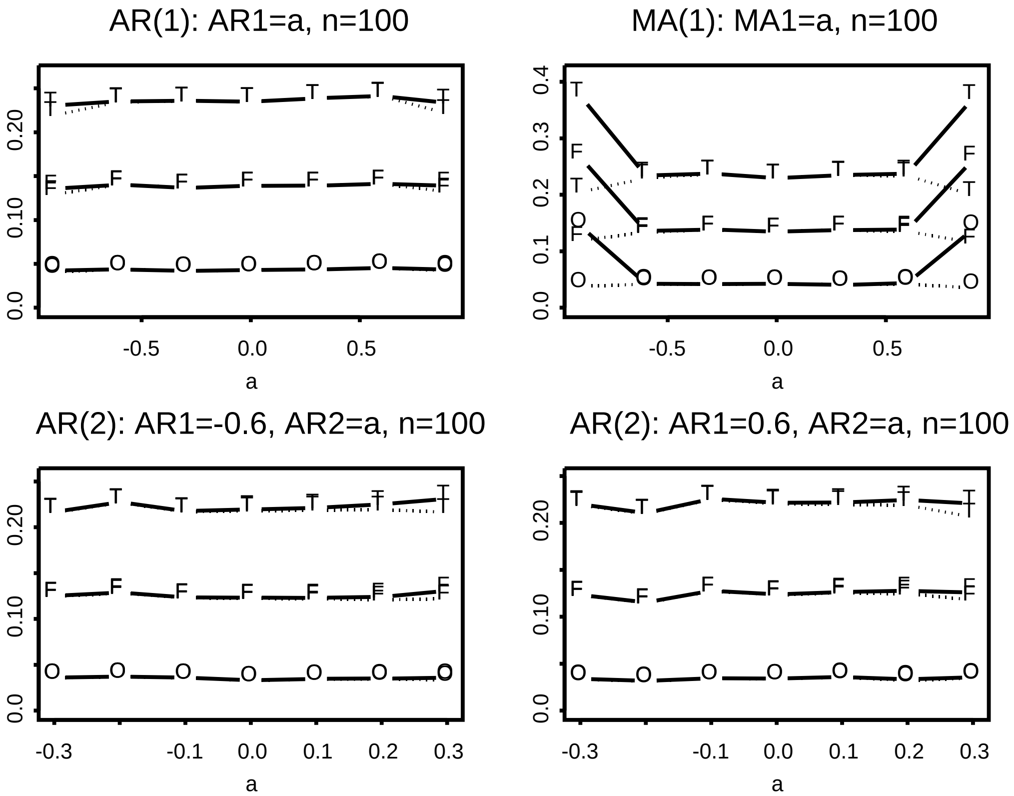

Figure 77: $n = 100$, $DF = (3, 5, 9, 15, 21, 27)$, $(\alpha_L, \alpha_U) = (0.02, 0.05)$, $(0.09, 0.21)$, $(0.18, 0.34)$.

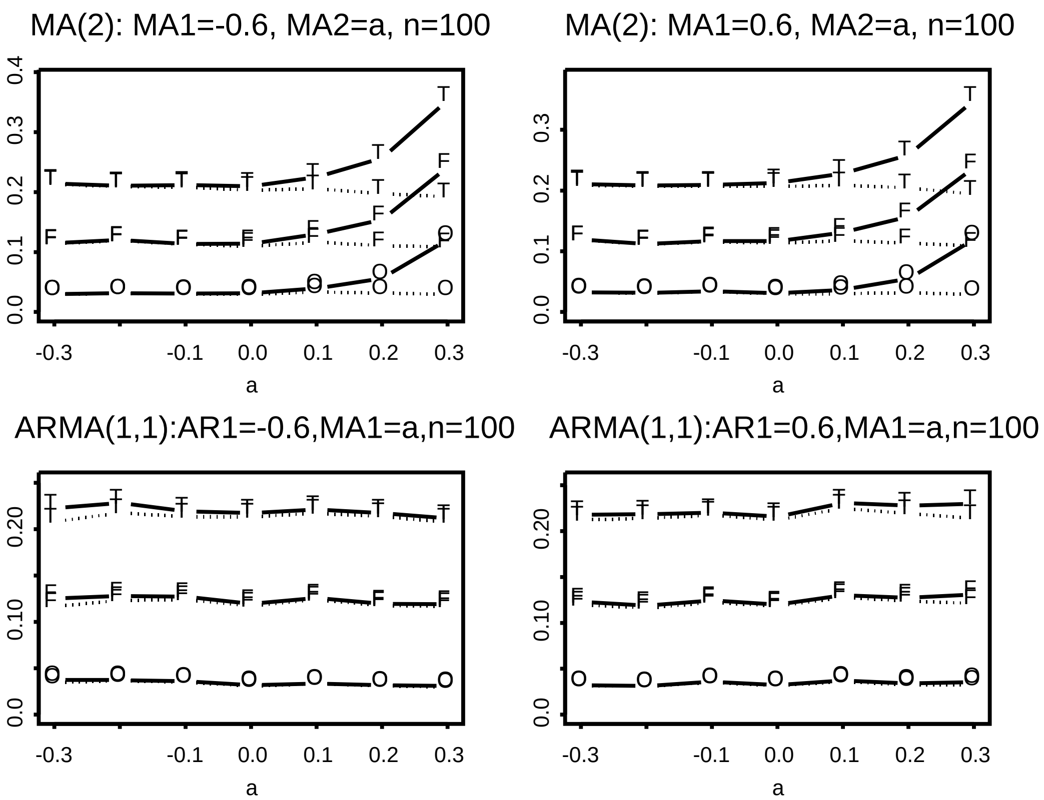

Figure 78: $n = 100$, $DF = (3, 5, 9, 15, 21, 27)$, $(\alpha_L, \alpha_U) = (0.02, 0.05)$, $(0.09, 0.21)$, $(0.18, 0.34)$.

3.2 Empirical powers

The rest of this section considers the power of portmanteau statistics. The true models are given by (3). For the following figures, Figures 79–102, O denotes $100\alpha = 1\%$, F denotes $100\alpha = 5\%$ and T denotes $100\alpha = 10\%$,and Q_m^* is shown by a solid line and Q_m^{**} by a dashed line for different sets of DF. We fitted an AR(1) model for the AR(2) models and the ARMA(1, 1) models, and a MA(1) model for the MA(2) models. Thereafter, we computed the marginal significance level β and d_js by (II) in Katayama (2008). We found the following:

1. The tests for DF(2, 4, 6) possess the highest power, as we expected.
2. The differences in the powers, between DF(2, 4, 6) and DF(2, 4, . . . , M), increase as M increases or $|a|$ increases.
3. Nevertheless, the multiple tests for DF(2, 4, . . . , M) for large values of M possess comparable power to the power of the case for DF(2, 4, 6).
4. We also tested the case for DF of discontinuous even integers. For example, see Figures 82, 86, 90 and so on. These also possess comparable power to the power of the case for DF(2, 4, 6) as we expected.

4 Conclusion

In summary, we conclude that the multiple portmanteau tests by $Q_{m(i)}^{**}$, $m(i) - p - q = 2i$, $i = 1, 2, \ldots, s$ possess efficient empirical size and power even if $M = m(s)$ become moderately large. One reasonable M should be given by (5) to retain the joint significance levels.

ACKNOWLEDGMENTS

The author would like to convey his sincere thanks to Professor Wai Keung Li for valuable suggestions made when the author visited the University of Hong Kong in March 2008 and March 2009. The financial support of the Japan Society for the Promotion of Science is gratefully acknowledged.

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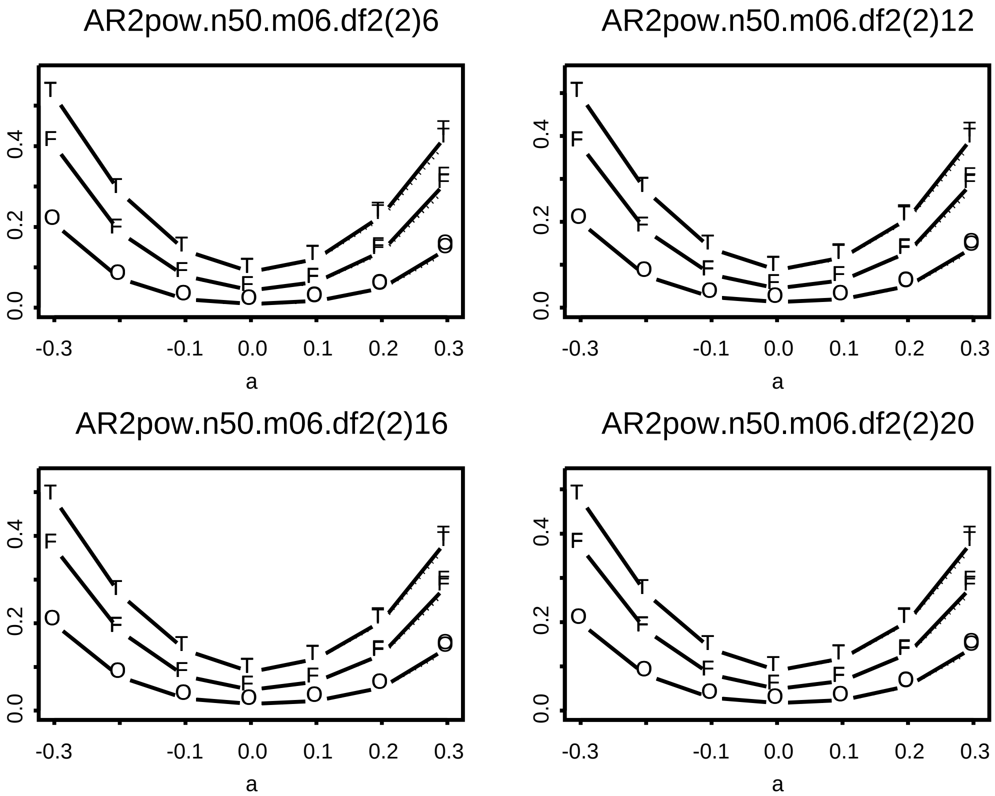

Figure 79: AR(2)a: $n = 50$, $DF = 2(2)6,\ 2(2)12,\ 2(2)16,\ 2(2)20$

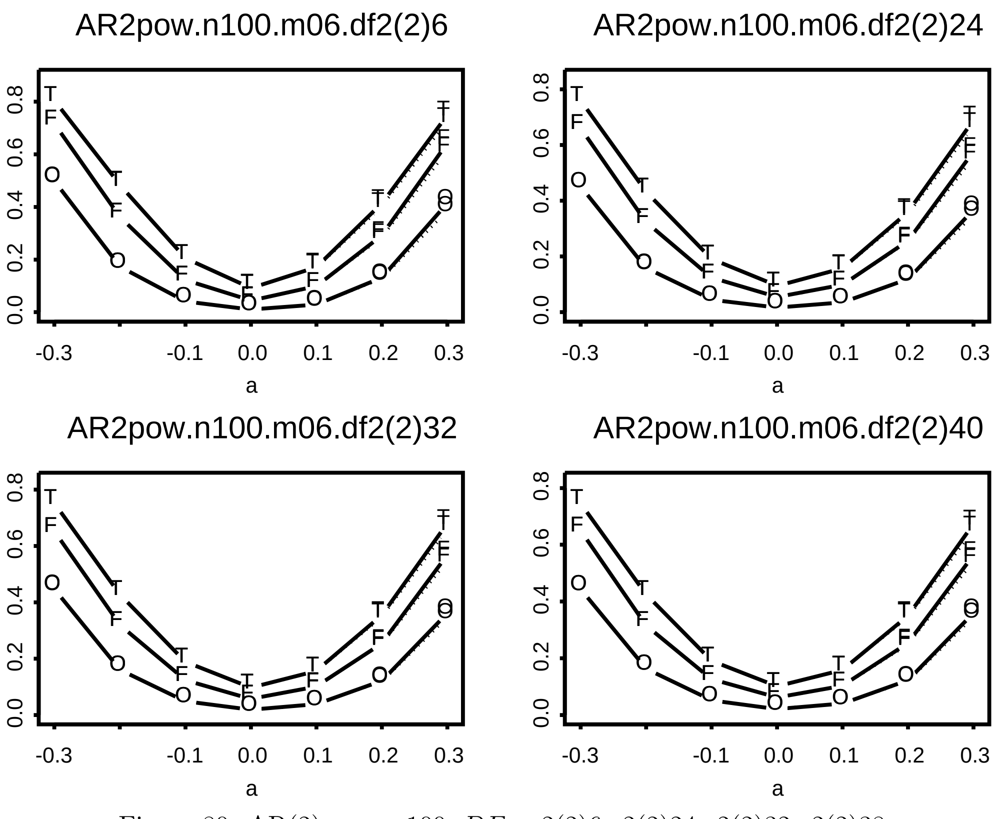

Figure 80: AR(2)a: $n = 100$, $DF = 2(2)6,\ 2(2)24,\ 2(2)32,\ 2(2)38$

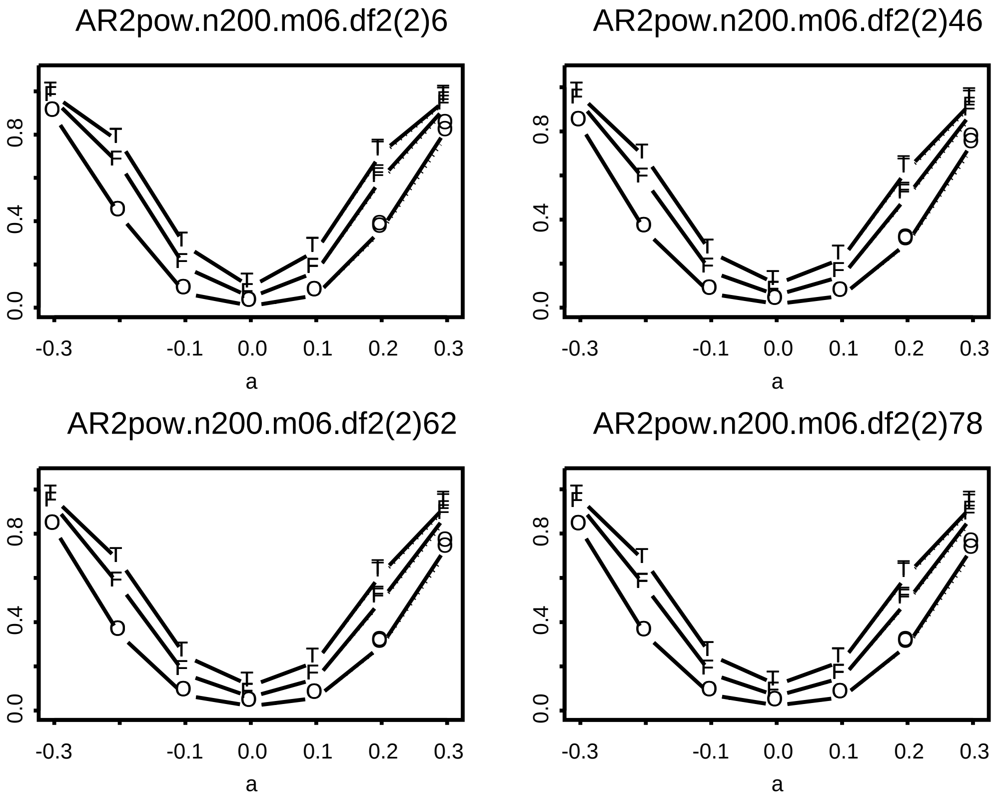

Figure 81: AR(2)a: $n = 200$, $DF = 2(2)6,\ 2(2)46,\ 2(2)62,\ 2(2)78$

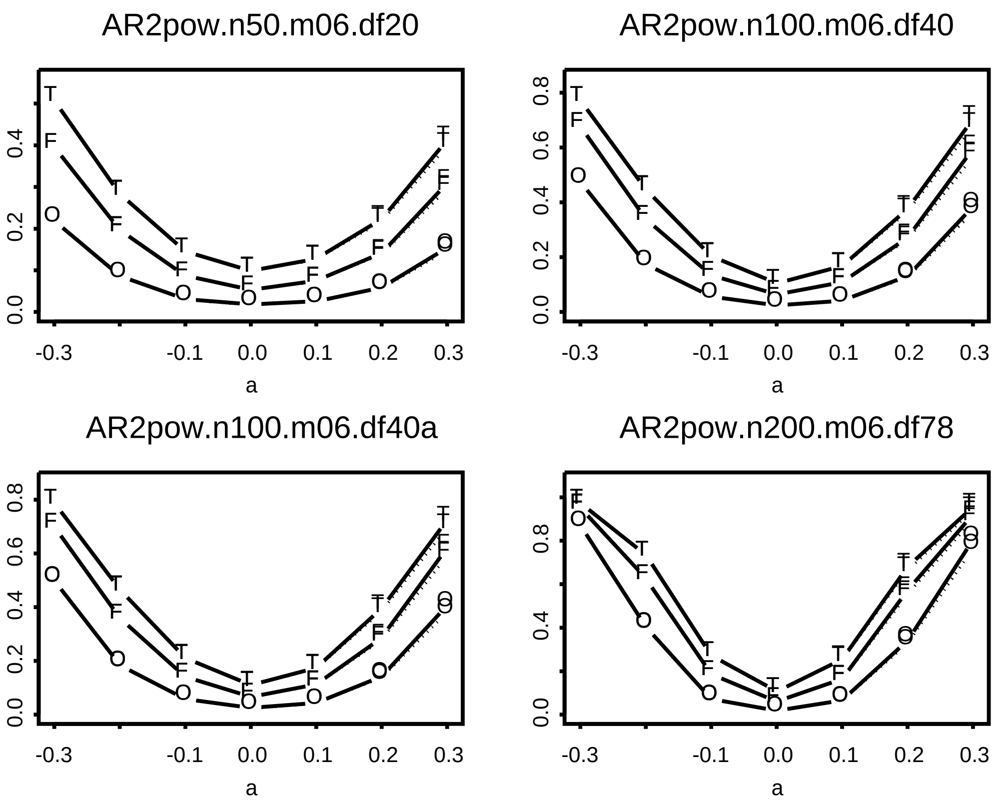

Figure 82: AR(2)a: $DF = (2, 10, 20),\ (2, 10, 20, 30, 40),\ (2, 20, 40),\ (2, 20, 40, 60, 78)$

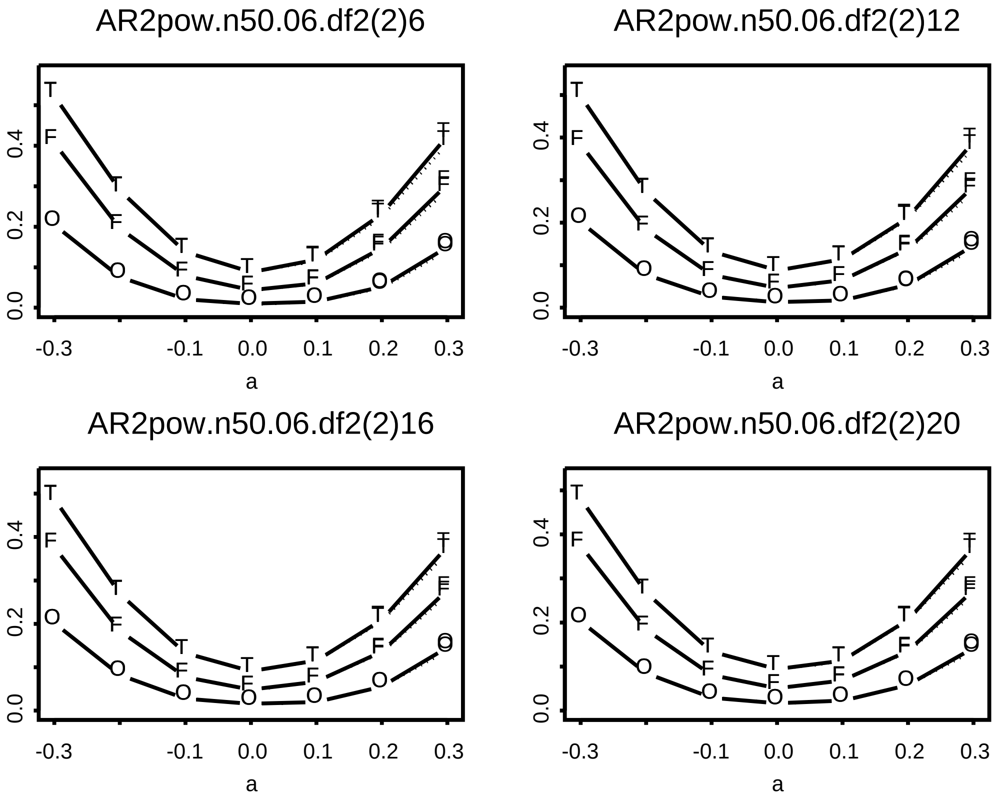

Figure 83: AR(2)b: $n = 50$, $DF = 2(2)6,\ 2(2)12,\ 2(2)16,\ 2(2)20$

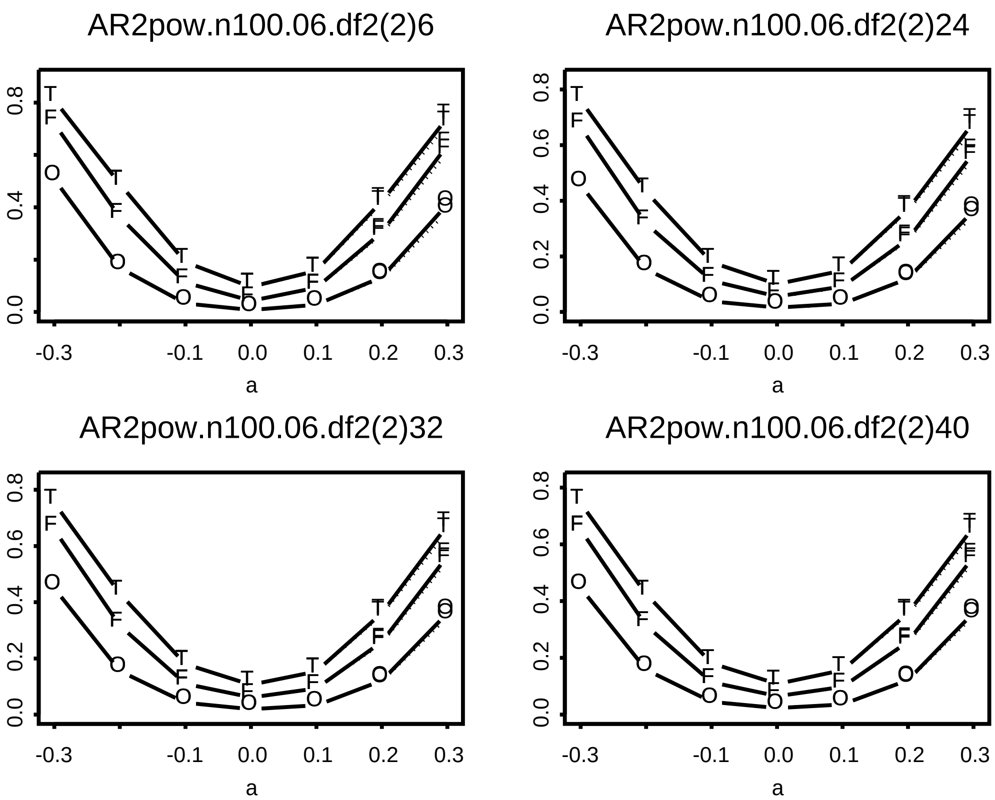

Figure 84: AR(2)b: $n = 100$, $DF = 2(2)6,\ 2(2)24,\ 2(2)32,\ 2(2)40$

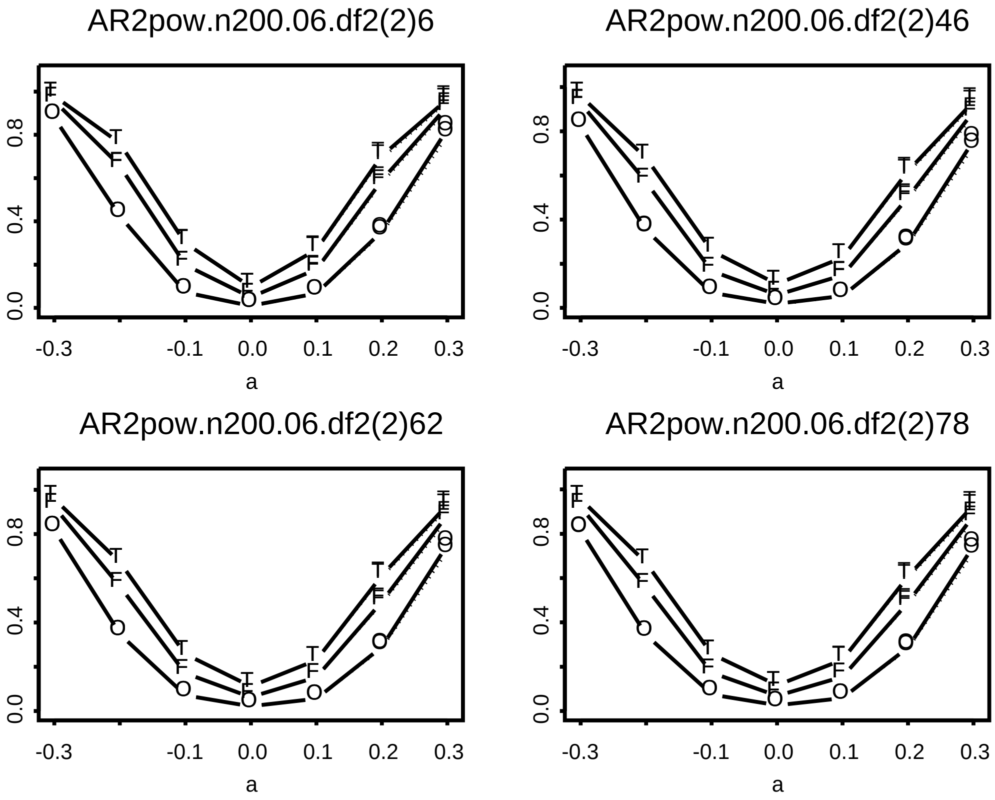

Figure 85: AR(2)b: $n = 200$, $DF = 2(2)6,\ 2(2)46,\ 2(2)62,\ 2(2)78$

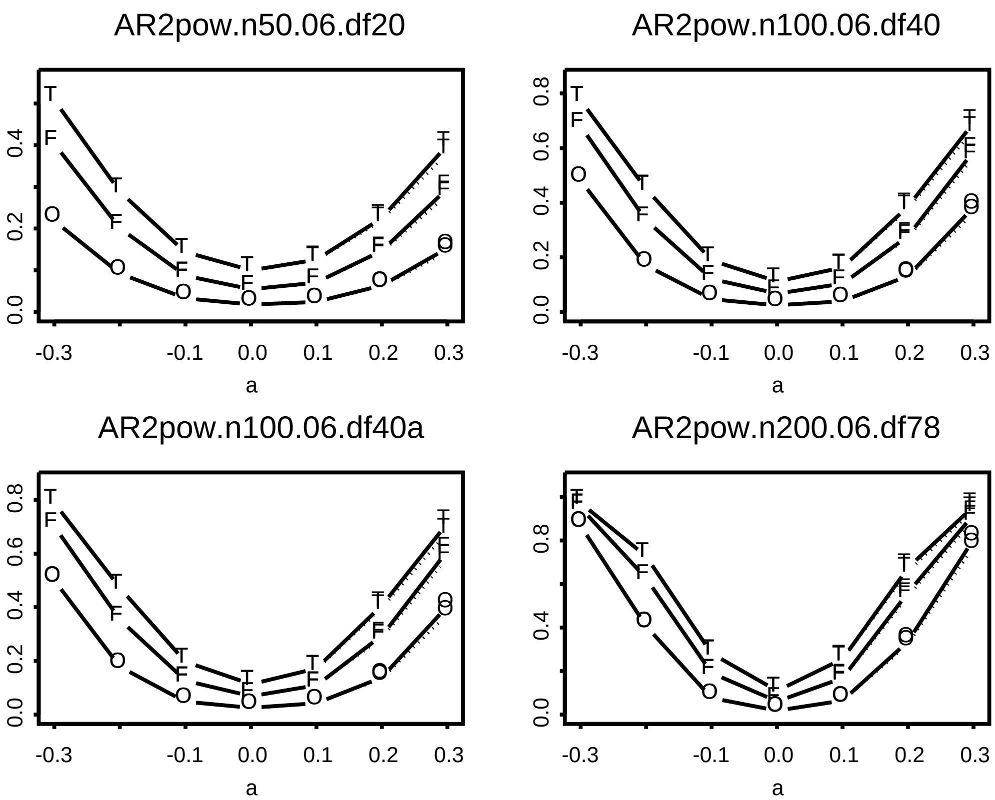

Figure 86: AR(2)b: $DF = (2, 10, 20),\ (2, 10, 20, 30, 40),\ (2, 20, 40),\ (2, 20, 40, 60, 78)$

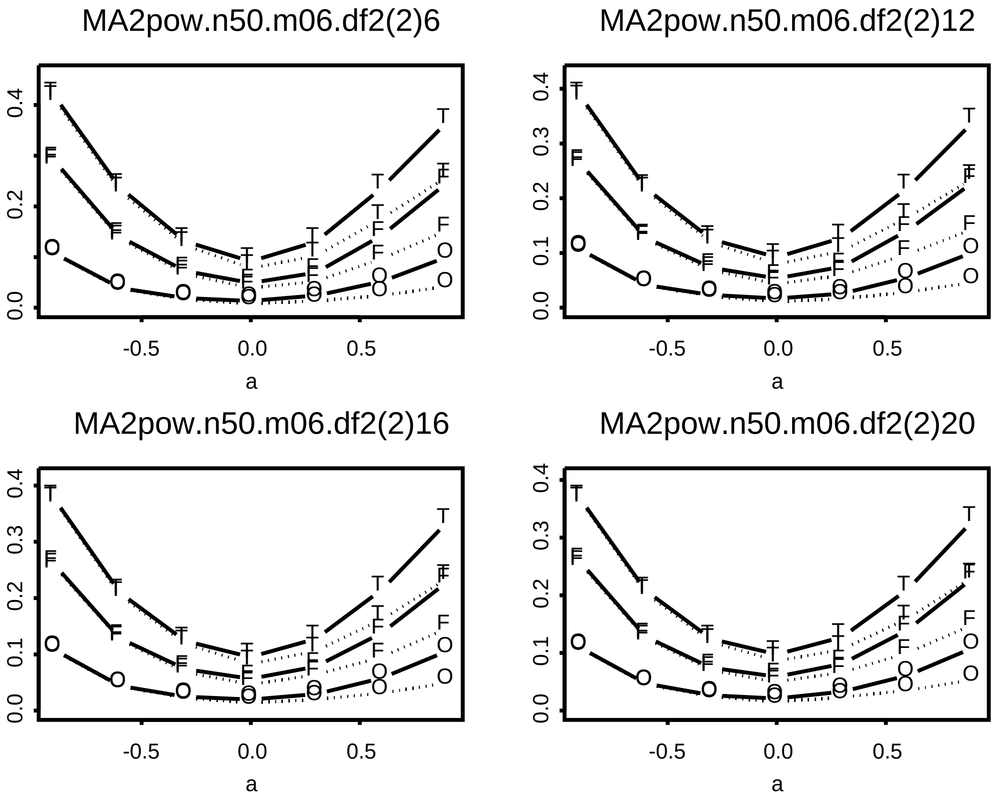

Figure 87: MA(2)a: $n = 50$, $DF = 2(2)6,\ 2(2)12,\ 2(2)16,\ 2(2)20$

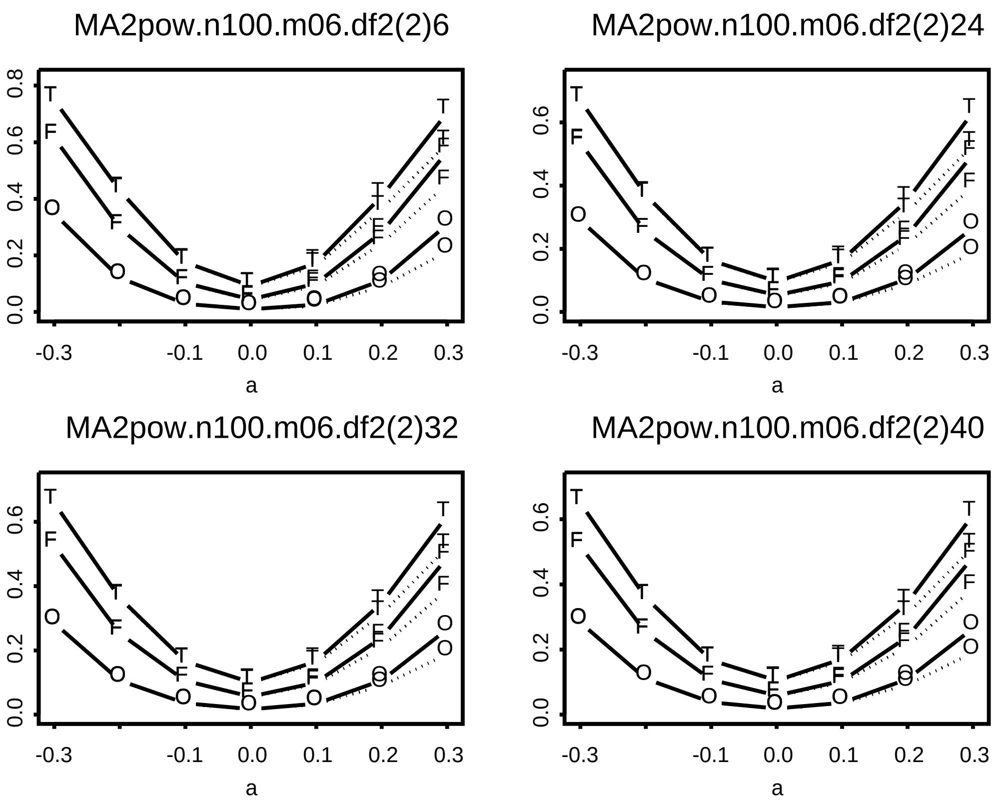

Figure 88: MA(2)a: $n = 100$, $DF = 2(2)6,\ 2(2)24,\ 2(2)32,\ 2(2)40$

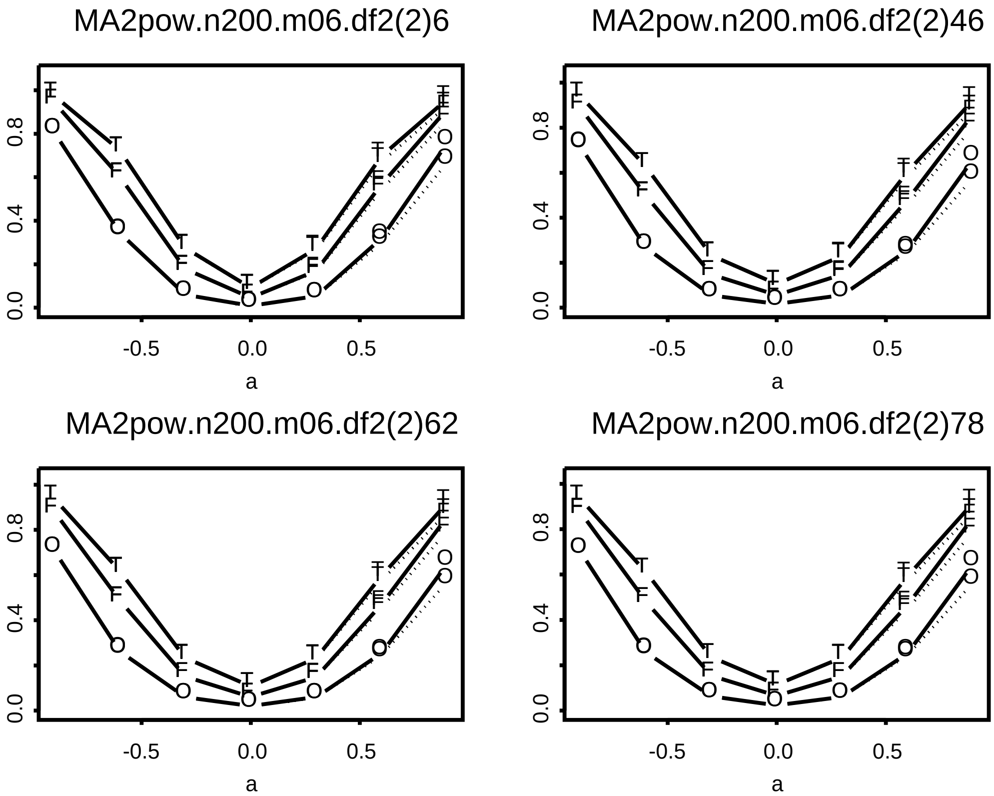

Figure 89: MA(2)a: $n = 200$, $DF = 2(2)6,\ 2(2)46,\ 2(2)62,\ 2(2)78$

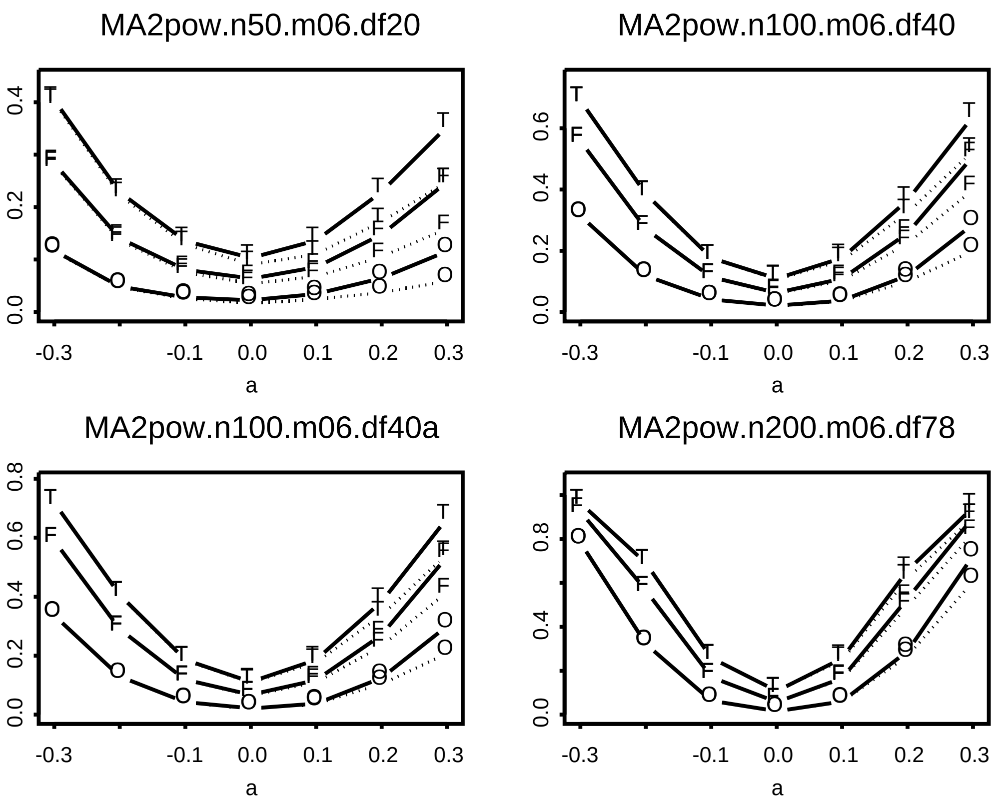

Figure 90: MA(2)a: $DF = (2, 10, 20),\ (2, 10, 20, 30, 40),\ (2, 20, 40),\ (2, 20, 40, 60, 78)$

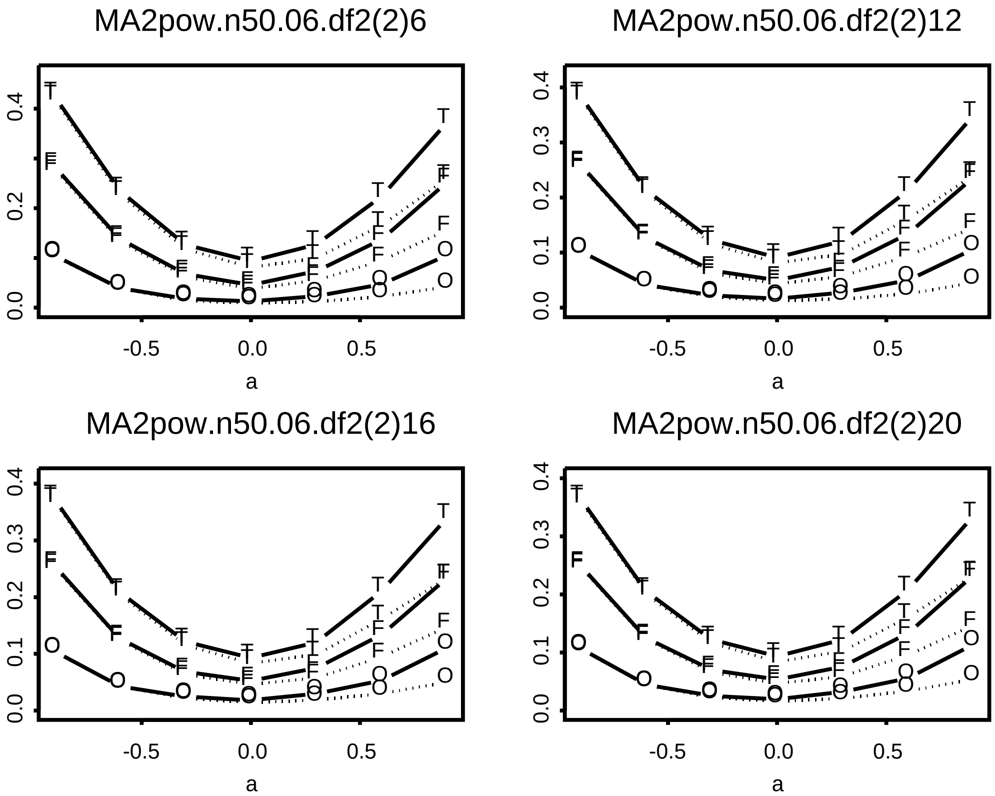

Figure 91: MA(2)b: $n = 50$, $DF = 2(2)6,\ 2(2)12,\ 2(2)16,\ 2(2)20$

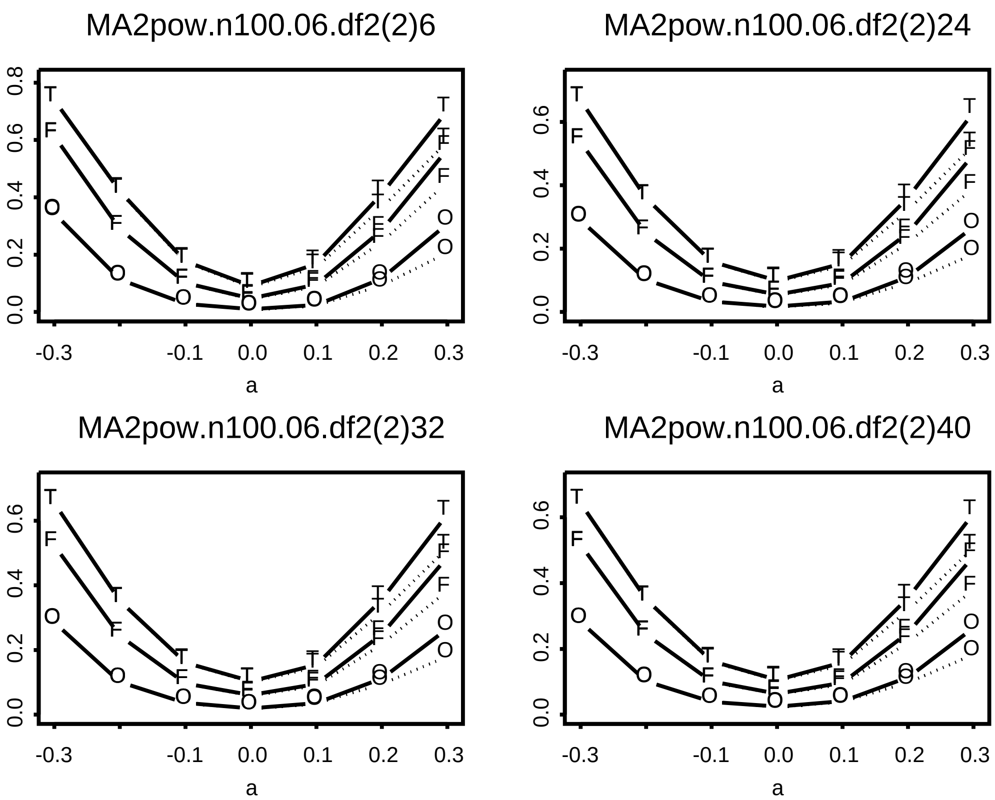

Figure 92: MA(2)b: $n = 100$, $DF = 2(2)6,\ 2(2)24,\ 2(2)32,\ 2(2)40$

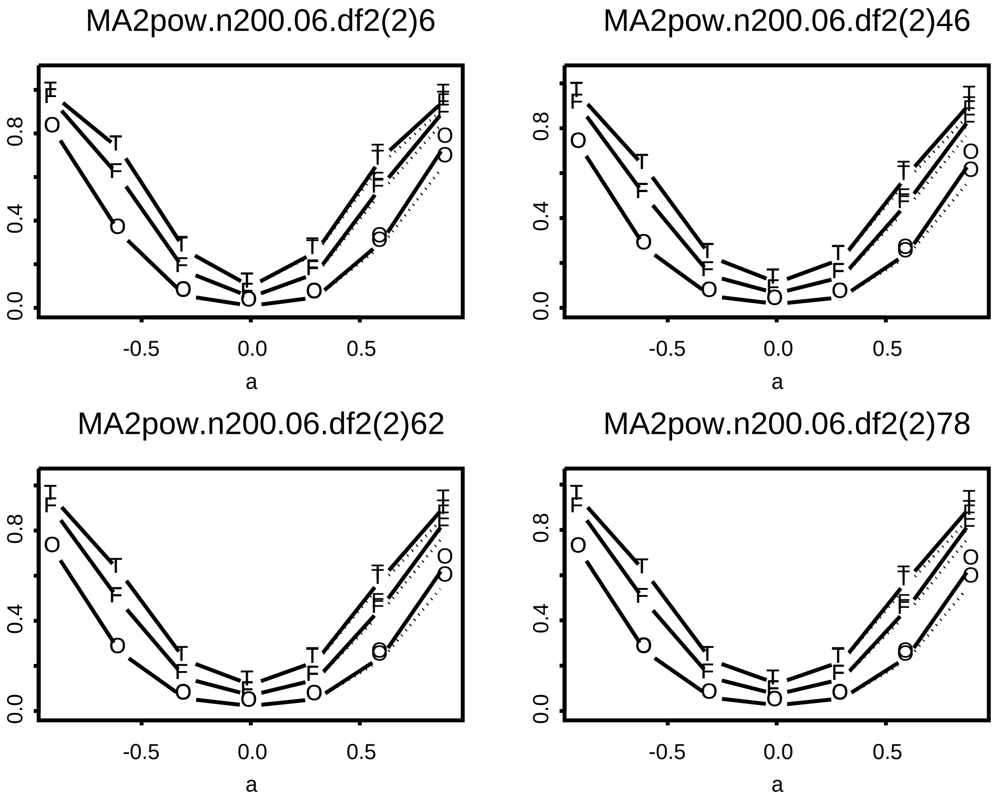

Figure 93: MA(2)b: $n = 200$, $DF = 2(2)6,\ 2(2)46,\ 2(2)62,\ 2(2)78$

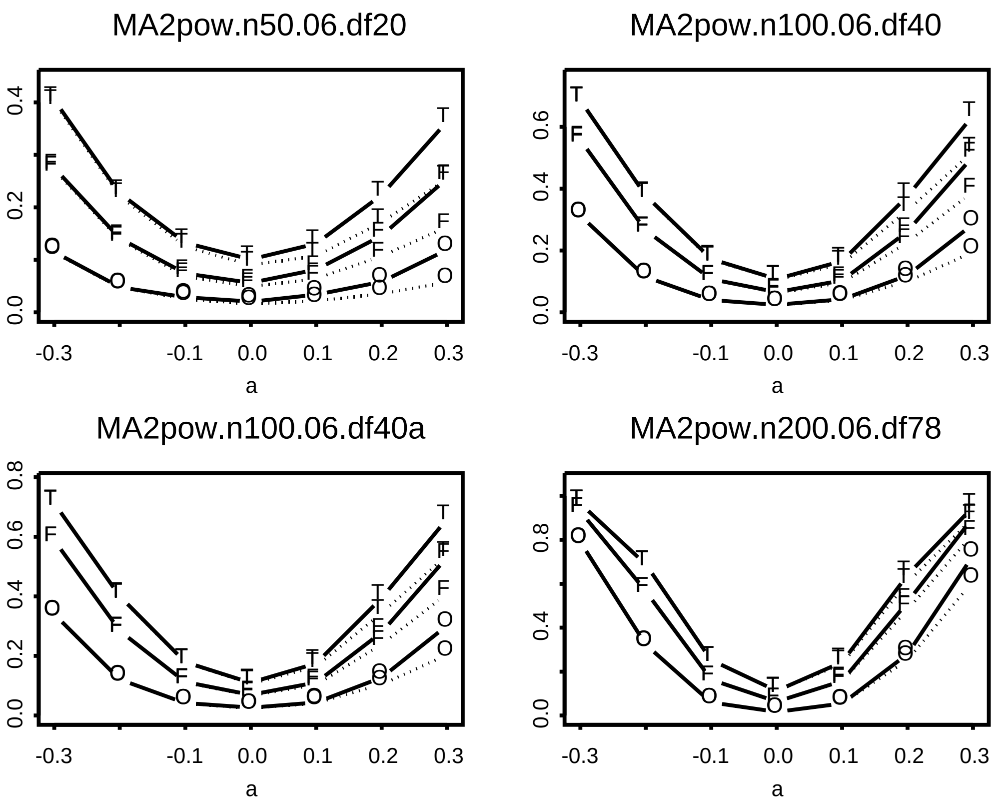

Figure 94: MA(2)b: $DF = (2, 10, 20),\ (2, 10, 20, 30, 40),\ (2, 20, 40),\ (2, 20, 40, 60, 78)$

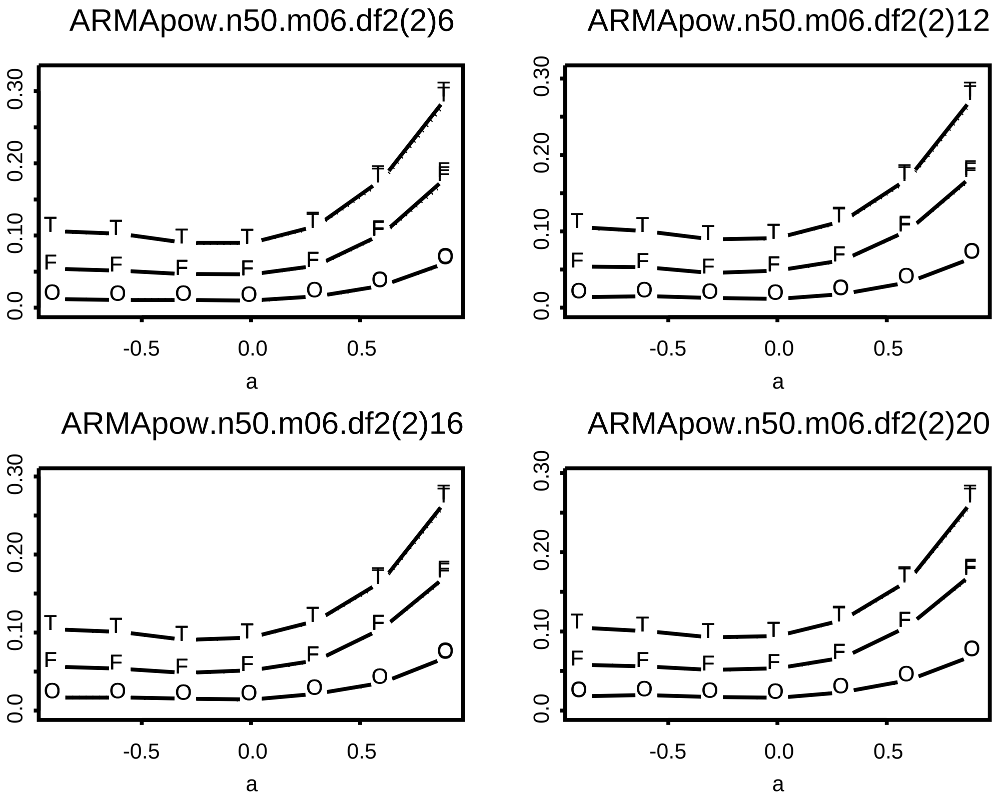

Figure 95: ARMA$(1, 1)a$: $n = 50$, $DF = 2(2)6,\ 2(2)12,\ 2(2)16,\ 2(2)20$

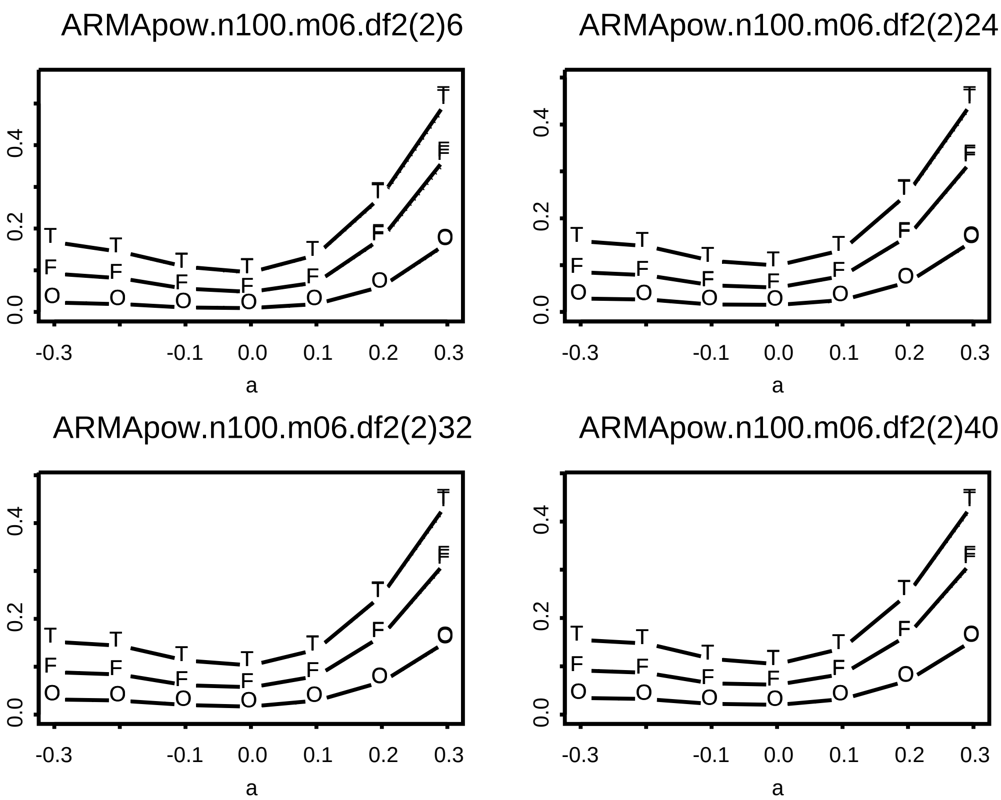

Figure 96: ARMA$(1, 1)a$: $n = 100$, $DF = 2(2)6,\ 2(2)24,\ 2(2)32,\ 2(2)40$

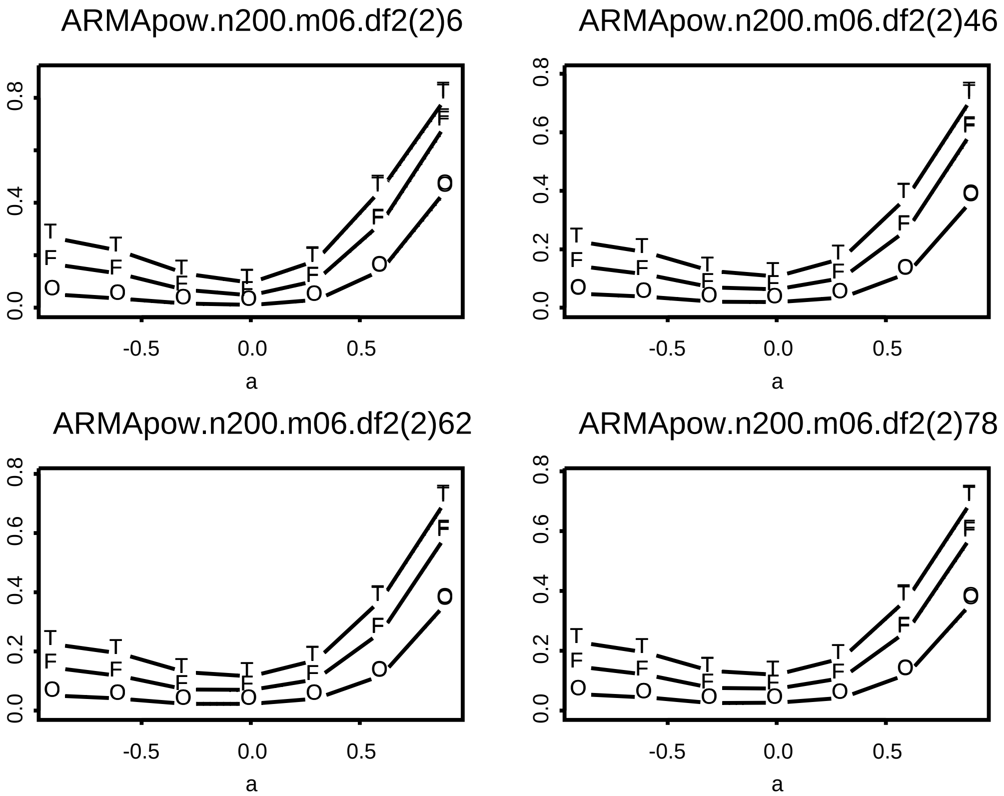

Figure 97: ARMA(1, 1)a: $n = 200$, $DF = 2(2)6,\ 2(2)46,\ 2(2)62,\ 2(2)78$

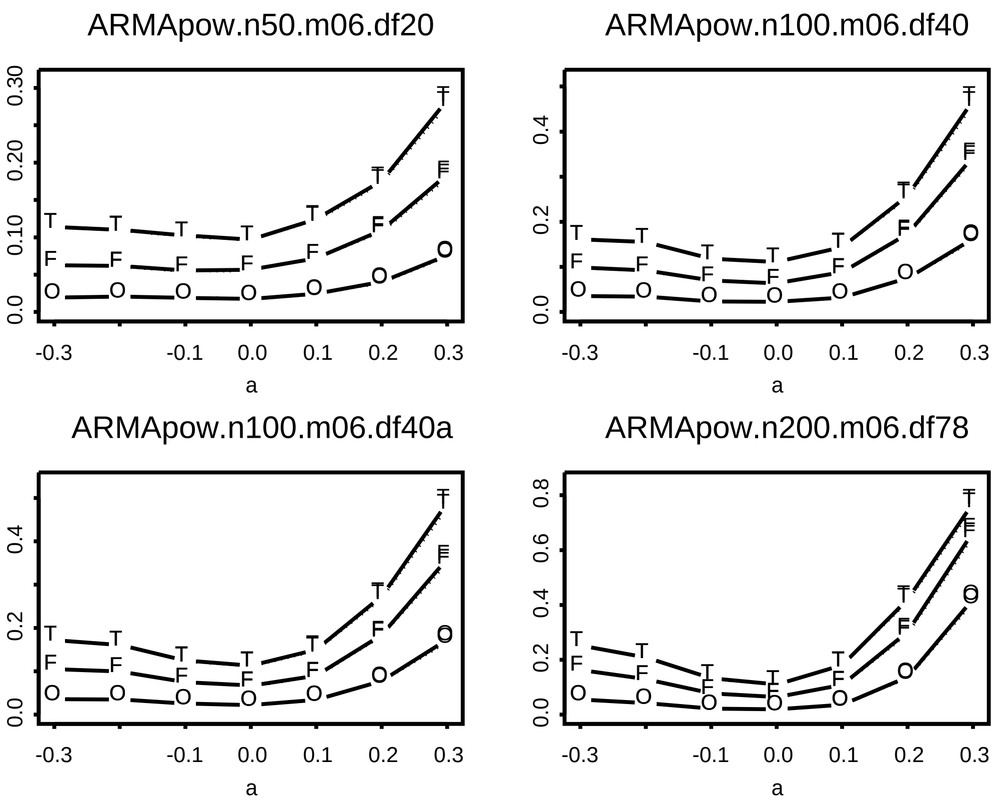

Figure 98: ARMA(1, 1)a: $DF = (2, 10, 20),\ (2, 10, 20, 30, 40),\ (2, 20, 40),\ (2, 20, 40, 60, 78)$

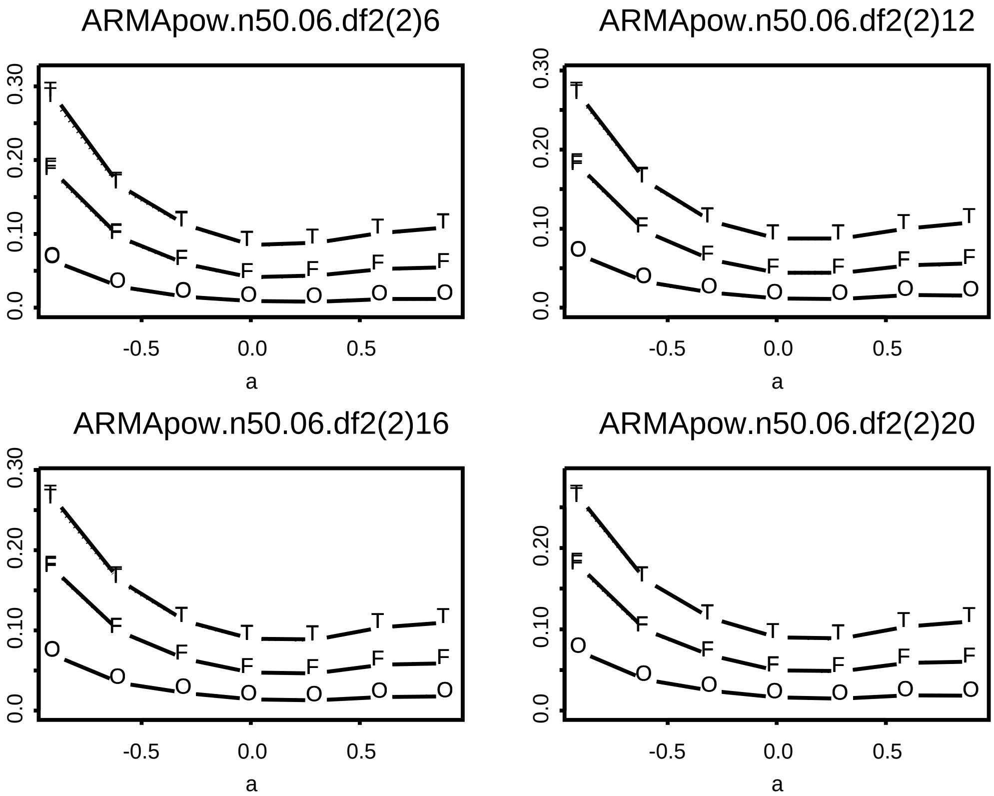

Figure 99: ARMA$(1,1)b$: $n = 50$, $DF = 2(2)6,\ 2(2)12,\ 2(2)16,\ 2(2)10$

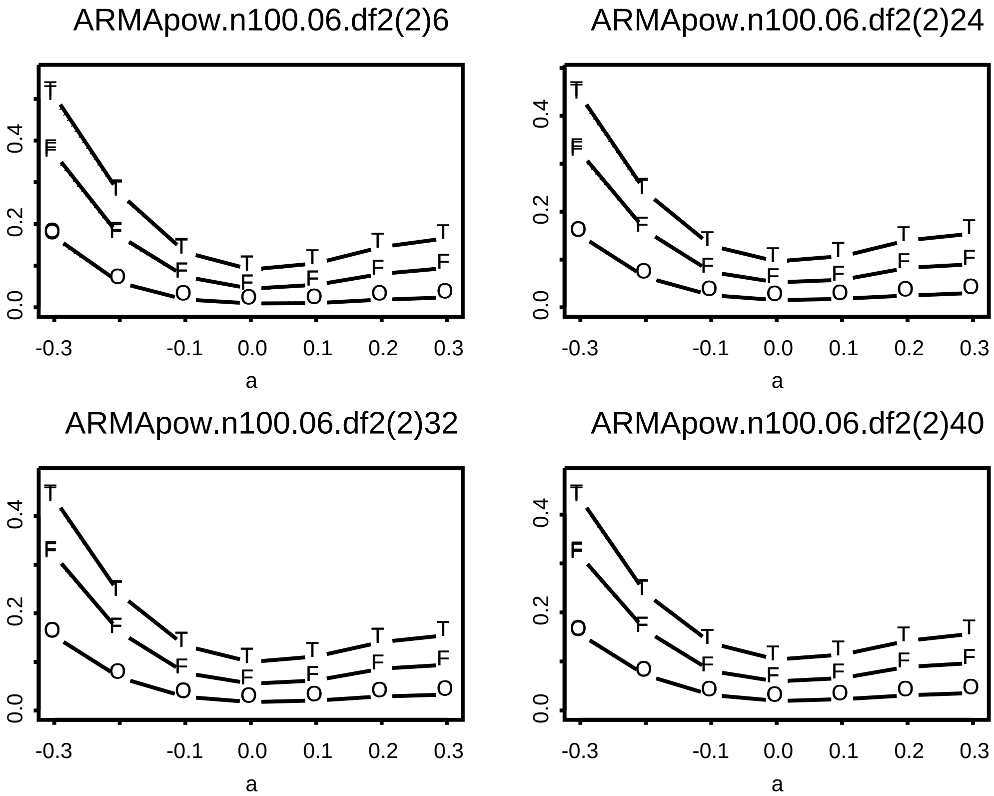

Figure 100: ARMA$(1,1)b$: $n = 100$, $DF = 2(2)6,\ 2(2)24,\ 2(2)32,\ 2(2)40$

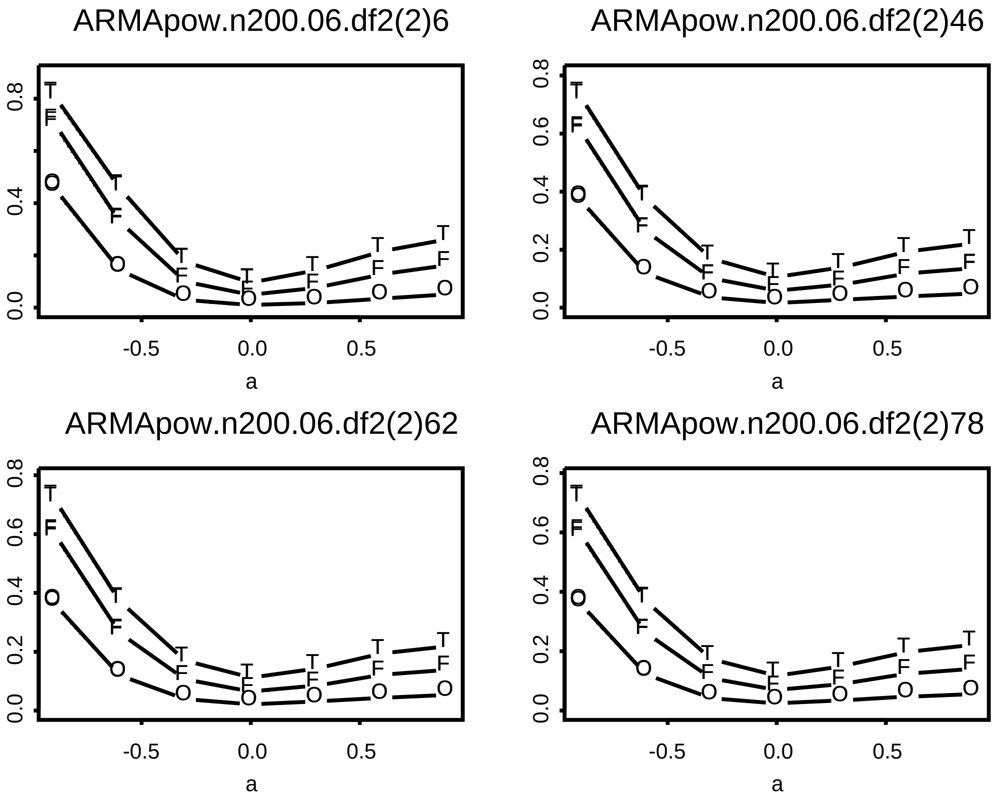

Figure 101: ARMA$(1,1)b$: $n=200,\ DF=2(2)6,\ 2(2)46,\ 2(2)62,\ 2(2)78$

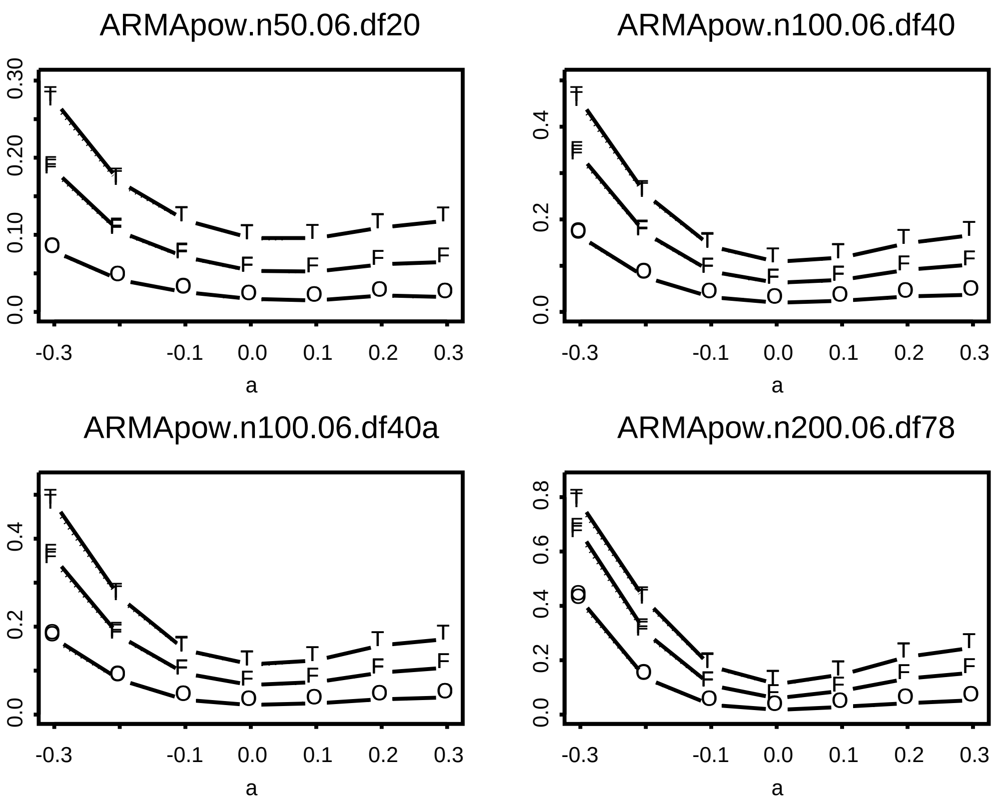

Figure 102: ARMA$(1,1)b$: $DF=(2,10,20),\ (2,10,20,30,40),\ (2,20,40),\ (2,20,40,60,78)$