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Abstract

In this paper, we consider a matroid generalization of the popular matching problem introduced by Abraham, Irving, Kavitha and Mehlhorn. We present a polynomial-time algorithm for this problem.

1 Introduction

In this paper, we consider a problem of assigning applicants having preferences to posts. Such a matching problem naturally arises when a school assigns students to lectures or a firm assigns workers to tasks. For this matching problem, several solution concepts have been introduced. The concept of popularity introduced by Gärdenfors [7] is one of such solution concepts. Intuitively speaking, popularity of a matching M guarantees that there exists no other matching N such that more applicants prefer N to M than prefer M to N. Using the concept of popularity, Abraham, Irving, Kavitha and Mehlhorn [1] introduced the popular matching problem, and presented a linear-time algorithm for this problem. Several extensions of the popular matching problem have been investigated. For example, Manlove and Sng [10] considered a many-to-one variant of the popular matching problem, Mestre [11] considered a weighted variant, and Sng and Manlove [14] considered a weighted many-to-one variant. Furthermore, in the papers [3, 8, 9], the authors considered the popular matching problem in which posts also have preferences.

In this paper, we introduce a matroid generalization of the popular matching problem, and present a polynomial-time algorithm for this problem. Our model can represent the many-to-one variant of the popular matching problem introduced by Manlove and Sng [10] as a special case. A matroid generalization of the stable matching problem introduced by Fleiner [4] led to the discrete-convex generalization of the stable matching problem introduced by Fujishige and Tamura [6] and the matroid approach to the stable matching problem with lower quotas presented by Fleiner and Kamiyama [5]. We hope that our abstract model helps further progress in the field of the popular matching problem.

The rest of this paper is organized as follows. In Section 2, we formally define our problem. In Section 3, we give a characterization of a popular matching in our problem. In Section 4, we present our algorithm.

2 Preliminaries

Throughout this paper, let $\mathbb{Z}_+$ be the set of non-negative integers. For each subset X and each element x, we define $X + x := X \cup \{x\}$ and $X - x := X \setminus \{x\}$, respectively.

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An ordered pair $\mathcal{M} = (U, \mathcal{I})$ is called a *matroid*, if U is a finite set and $\mathcal{I}$ is a nonempty family of subsets of U satisfying the following conditions.

(I1) If $I \in \mathcal{I}$ and $J \subseteq I$, then $J \in \mathcal{I}$.

(I2) If $I, J \in \mathcal{I}$ and $|I| < |J|$, then there exists an element u in $J \setminus I$ with $I + u \in \mathcal{I}$

2.1 Problem formulation

Here we define the *popular matching problem under matroid constraints* (the PMuMC problem for short).

In the PMuMC problem, we are given a finite simple bipartite graph $G = (V, E)$ in which V is partitioned into two subsets A, P, and each edge in E connects a vertex in A and a vertex in P. We call a vertex in A an *applicant*, and a vertex in P a *post*. We denote by (a, p) the edge in E between an applicant a in A and a post p in P. For each vertex v in V and each subset M of E, we define $M(v)$ as the set of edges in M incident to v.

In addition, we are given an injective function $\pi \colon E \to \mathbb{Z}_+$. That is, $\pi(e) \neq \pi(e')$ for every distinct edges e, e' in E. Intuitively speaking, π represents preference lists of applicants. For each applicant a in A and each edges e, e' in $E(a)$, if $\pi(e) > \pi(e')$, then a prefers e to e'. Since π is injective, it represents "strict" preference lists of applicants. Without loss of generality, we assume that for each applicant a in A, there exists a post $p(a)$ in P such that $E(p(a))$ consists of only $(a, p(a))$ and $\pi(e) > \pi((a, p(a)))$ for every edge e in $E(a) - (a, p(a))$. Furthermore, for each post p in P, we are given a matroid $\mathcal{M}_p = (E(p), \mathcal{I}_p)$. For each applicant a in A, we assume that $\{(a, p(a))\} \in \mathcal{I}_{p(a)}$. Without loss of generality, we assume that for each applicant a in A, there exists a post p in $P - p(a)$ such that $(a, p) \in E$ and $\{(a, p)\} \in \mathcal{I}_p$.

A subset M of E is called a *matching* in G, if it satisfies the following two conditions.

- For every applicant a in A, we have $|M(a)| = 1$.
- For every post p in P, we have $M(p) \in \mathcal{I}_p$.

For each matching M in G and each applicant a in A, we denote by $\mu_M(a)$ the unique edge in $M(a)$. For each matchings M, N in G, we denote by $\mathsf{pre}_M(N)$ the number of applicants a in A with

$$\pi(\mu_N(a)) > \pi(\mu_M(a)),$$

i.e., $\mathsf{pre}_M(N)$ represents the number of applicants that prefer N to M. A matching M in G is said to be *popular*, if

$$\mathsf{pre}_N(M) \geq \mathsf{pre}_M(N)$$

for every matching N in G. That is, if a matching M in G is popular, then there exists no other matching N in G such that more applicants in A prefer N to M than prefer M to N. The goal of the PMuMC problem is to discern whether there exists a popular matching in G, and find it if one exists.

2.2 Examples

Here we give several examples that can be represented by the PMuMC problem.

We first consider the problem in which for each post p in P, we are given a matroid $\mathcal{M}_p = (E(p), \mathcal{I}_p)$ defined by

$$\mathcal{I}_p := \{\{e\} \mid e \in E(p)\} \cup \{\emptyset\}.$$

That is, at most one edge is assigned to each post. This problem is called the *popular matching problem*. Abraham, Irving, Kavitha and Mehlhorn [1] introduced this problem, and presented a linear-time algorithm for this problem. Our algorithm can be regarded as a matroid generalization of the algorithm presented in [1].

Next we consider the problem in which we are given a capacity function $c\colon P \to \mathbb{Z}_+$ and for each post p in P, we are given a matroid $\mathcal{M}_p = (E(p), \mathcal{I}_p)$ defined by

$$\mathcal{I}_p := \{E' \subseteq E(p) \mid |E'| \leq c(p)\}.$$

That is, for each p in P, at most $c(p)$ edges are assigned to p. Manlove and Sng [10] introduced this problem, and presented a polynomial-time algorithm by generalizing the algorithm presented in [1].

Finally, we consider a variant of the popular matching problem with laminar capacity constraints. For each post p in P, we are given a laminar family $\mathcal{C}_p$ of subsets of $E(p)$, i.e.,

$$\forall \text{ distinct } C_1, C_2 \in \mathcal{C}_p\colon\ C_1 \cap C_2 = \emptyset,\ \text{ or } C_1 \subseteq C_2,\ \text{ or } C_2 \subseteq C_1.$$

In addition, for each post p in P, we are given a capacity function $c_p\colon \mathcal{C}_p \to \mathbb{Z}_+$. For each post p in P, a matroid $\mathcal{M}_p = (E(p), \mathcal{I}_p)$ is defined by

$$\mathcal{I}_p := \{E' \subseteq E(p) \mid \forall C \in \mathcal{C}_p\colon\ |E' \cap C| \leq c_p(C)\}.$$

It is not difficult to see that $\mathcal{M}_p$ is a matroid for every post p in P. A laminar capacity constraint naturally arises when we assign students to projects (see, e.g., [2]). To the best of our knowledge, this problem has not been investigated.

2.3 Matroids

Here we give properties of matroids that are used in the sequel.

Let $\mathcal{M} = (U, \mathcal{I})$ be a matroid. A subset I in $\mathcal{I}$ is called an *independent set in* $\mathcal{M}$. A subset C of U is a *circuit in* $\mathcal{M}$, if $C \notin \mathcal{I}$, but every proper subset C' of C is an independent set in $\mathcal{M}$. It is known [12, Prop. 1.1.6] that if I is an independent set in $\mathcal{M}$ and u is an element in $U \setminus I$ with $I + u \notin \mathcal{I}$, then there exists the unique circuit $C_{\mathcal{M}}(u, I)$ in $\mathcal{M}$ which is a subset of $I + u$, and we have $u \in C_{\mathcal{M}}(u, I)$. We call $C_{\mathcal{M}}(u, I)$ the *fundamental circuit of* u *with respect to* I *in* $\mathcal{M}$. It is known [12, Ex. 6 in p. 21] that $C_{\mathcal{M}}(u, I)$ consists of all elements u' in $I + u$ with $I + u - u' \in \mathcal{I}$. For each subset X of U, a subset B of X is called a *base of* X *in* $\mathcal{M}$, if B is an inclusion-wise maximal subset of X that is an independent set in $\mathcal{M}$. We call a base of U in $\mathcal{M}$ a *base in* $\mathcal{M}$. The condition (I2) implies that for each subset X of U, every two bases of X in $\mathcal{M}$ have the same size, which is called the *rank of* X *in* $\mathcal{M}$ and denoted by $r_{\mathcal{M}}(X)$. It is known [12, Prop. 1.3.5] that a subset X of U is an independent set in $\mathcal{M}$ if and only if $|X| = r_{\mathcal{M}}(X)$.

Let $\mathcal{M} = (U, \mathcal{I})$ and S be a matroid and a subset of U, respectively. Define

$$\mathcal{I}|S := \{X \subseteq S \mid X \in \mathcal{I}\},$$

and $\mathcal{M}|S := (S, \mathcal{I}|S)$. It is not difficult to see that $\mathcal{M}|S$ is a matroid. Furthermore, we define a function $r'\colon 2^{U \setminus S} \to \mathbb{Z}_+$ by

$$r'(X) := r_{\mathcal{M}}(X \cup S) - r_{\mathcal{M}}(S).$$

In addition, we define

$$\mathcal{I}/S := \{X \subseteq U \setminus S \mid |X| = r'(X)\},$$

and $\mathcal{M}/S := (U \setminus S, \mathcal{I}/S)$. It is known [12, Prop. 3.1.6] that $\mathcal{M}/S$ is a matroid.

Let $\mathcal{M}_1 = (U_1, \mathcal{I}_1)$ and $\mathcal{M}_2 = (U_2, \mathcal{I}_2)$ be matroids with $U_1 \cap U_2 = \emptyset$. Define

$$\mathcal{I}_1 \oplus \mathcal{I}_2 := \{X \subseteq U_1 \cup U_2 \mid X \cap U_1 \in \mathcal{I}_1,\ X \cap U_2 \in \mathcal{I}_2\},$$

and $\mathcal{M}_1 \oplus \mathcal{M}_2 := (U_1 \cup U_2, \mathcal{I}_1 \oplus \mathcal{I}_2)$. It is known [12, Prop. 4.2.12] that $\mathcal{M}_1 \oplus \mathcal{M}_2$ is a matroid, and called the *direct sum of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$.

Although the following lemma easily follows from well-known facts about matroids, we give its proof for completeness.

Lemma 1. *Let $\mathcal{M} = (U, \mathcal{I})$ and S be a matroid and a subset of U, respectively.*

1. *Let B be an arbitrary base in $\mathcal{M}|S$. For every subset Y of $U \setminus S$, Y is an independent set in $\mathcal{M}/S$ if and only if $Y \cup B$ is an independent set in $\mathcal{M}$.*

2. *For every independent set I in $\mathcal{M}$ such that $I \cap S$ is a base in $\mathcal{M}|S$, I is an independent set in $\mathcal{M}|S \oplus \mathcal{M}/S$.*

3. *For every independent set I in $\mathcal{M}|S \oplus \mathcal{M}/S$ and every element u in S such that $(I+u) \cap S$ is an independent set in $\mathcal{M}|S$, $I+u$ is an independent set in $\mathcal{M}|S \oplus \mathcal{M}/S$.*

4. *For every independent set I in $\mathcal{M}|S \oplus \mathcal{M}/S$, I is an independent set in $\mathcal{M}$.*

Proof. **1.** Since B is a base in $\mathcal{M}|S$, we have $r_{\mathcal{M}}(S) = |B|$. We first prove that

$$r_{\mathcal{M}}(Y \cup S) = r_{\mathcal{M}}(Y \cup B). \tag{1}$$

The condition (I2) implies that there exists a base B' of $Y \cup B$ in $\mathcal{M}$ with $B \subseteq B'$. Assume that (1) does not hold, i.e., there exists an independent set I in $\mathcal{M}$ such that $I \subseteq Y \cup S$ and $|B'| < |I|$. It follows from the condition (I2) that there exists u in $I \setminus B'$ with $B' + u \in \mathcal{I}$. If u is in Y, then this contradicts the fact that B' is a base of $Y \cup B$ in $\mathcal{M}$. Thus, we have $u \in S$. However, this implies that

$$|(B' + u) \cap S| \geq |B| + 1 > |B|,$$

which contradicts the fact that B is a base of S in $\mathcal{M}$. This completes the proof of (1).

Assume that Y is an independent set in $\mathcal{M}/S$. It follows from (1) that

$$|Y| = r_{\mathcal{M}}(Y \cup S) - r_{\mathcal{M}}(S) = r_{\mathcal{M}}(Y \cup B) - |B|,$$

which implies that $|Y \cup B| = r_{\mathcal{M}}(Y \cup B)$. Thus, $Y \cup B \in \mathcal{I}$.

Conversely, we assume that $Y \cup B$ is an independent set in $\mathcal{M}$. It follows from (1) that

$$r_{\mathcal{M}}(Y \cup S) - r_{\mathcal{M}}(S) = r_{\mathcal{M}}(Y \cup B) - r_{\mathcal{M}}(S) = |Y \cup B| - |B| = |Y|,$$

which implies that Y is an independent set in $\mathcal{M}/S$.

2. It suffices to prove that $I \setminus S \in \mathcal{I}/S$. It follows from (1) that

$$\begin{aligned} r_{\mathcal{M}}((I \setminus S) \cup S) - r_{\mathcal{M}}(S) &= r_{\mathcal{M}}((I \setminus S) \cup (I \cap S)) - r_{\mathcal{M}}(S) \\ &= r_{\mathcal{M}}(I) - r_{\mathcal{M}}(S) = |I| - |I \cap S| = |I \setminus S|. \end{aligned}$$

This completes the proof.

3. Since $u \in S$, we have $(I + u) \setminus S = I \setminus S$. This completes the proof.

4. Let X and Y be subsets in $\mathcal{I}|S$ and $\mathcal{I}/S$, respectively. The condition (I2) implies that there exists a base B in $\mathcal{M}|S$ with $X \subseteq B$. It follows from (1) that

$$|Y \cup B| = |Y| + |B| = r_{\mathcal{M}}(Y \cup S) - r_{\mathcal{M}}(S) + r_{\mathcal{M}}(S) = r_{\mathcal{M}}(Y \cup B),$$

which implies that $Y \cup B \in \mathcal{I}$. Thus, the statement 4 follows from the condition (I1). $\square$

Let $\mathcal{M}_1 = (U, \mathcal{I}_1)$ and $\mathcal{M}_2 = (U, \mathcal{I}_2)$ be matroids on the same ground set U. A subset I of U is called a *common independent set of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$, if $I \in \mathcal{I}_1 \cap \mathcal{I}_2$. If I is a common independent set of $\mathcal{M}_1$ and $\mathcal{M}_2$ with maximum cardinality, we call I a *maximum-size common independent set of* $\mathcal{M}_1$ *and* $\mathcal{M}_2$. It is well known that if we can check whether a subset of U belongs to $\mathcal{I}_1$ (or $\mathcal{I}_2$) in time bounded by a polynomial in $|U|$, we can find a maximum-size common independent set of $\mathcal{M}_1$ and $\mathcal{M}_2$ in time bounded by a polynomial in $|U|$. See, e.g., [13, Sect. 41.2] for a survey of algorithms for finding a maximum-size common independent set.

3 Characterization

For each applicant a in A, we define the *f-edge $f(a)$ of a* as the unique element in

$$\arg\max\{\pi((a,p)) \mid (a,p) \in E(a),\ \{(a,p)\} \in \mathcal{I}_p\}.$$

For each post p in P, we denote by F_p the set of edges (a,p) in $E(p)$ with $(a,p) = f(a)$. For each applicant a is A, we define the *s-edge $s(a)$ of a* as the unique edge in

$$\arg\max\{\pi((a,p)) \mid (a,p) \in E(a) - f(a),\ \{(a,p)\} \in \mathcal{I}_p/F_p\}.$$

Notice that for each applicant a in A, the s-edge $s(a)$ of a is well-defined because there exists the post $p(a)$. Define the *reduced edge set* E_{re} by

$$E_{\mathsf{re}} := \{f(a), s(a) \mid a \in A\}.$$

The goal of this section is to prove the following Theorem 2. This theorem can be regarded as a matroid generalization of Theorem 2.5 in [1] and Theorem 1 in [10].

Theorem 2. *For every matching M in G, M is popular if and only if it satisfies the following two conditions.*

(P1) *For every post p in P, $M(p) \cap F_p$ is a base in $\mathcal{M}_p|F_p$.*

(P2) *M is a subset of E_{re}.*

For proving Theorem 2, we prove several lemmas. We first prove lemmas that are necessary for proving the *only if*-part.

Lemma 3. *Let M be a matching in G. If M is popular, then for every post p in P, $M(p) \cap F_p$ is a base in $\mathcal{M}_p|F_p$.*

Proof. Assume that there exists a post p in P such that $M(p) \cap F_p$ is not a base in $\mathcal{M}_p|F_p$. It follows from the condition (I2) that there exists an applicant a in A such that $f(a) \in F_p \setminus M(p)$ and

$$(M(p) \cap F_p) + f(a) \in \mathcal{I}_p|F_p. \tag{2}$$

If $M(p) + f(a) \in \mathcal{I}_p$, then we define

$$N := M + f(a) - \mu_M(a).$$

It follows from the condition (I1) that N is a matching. Furthermore, since $\pi(\mu_N(a)) > \pi(\mu_M(a))$ and $N(a') = M(a')$ for every applicant a' in $A - a$, we have

$$\mathsf{pre}_M(N) - \mathsf{pre}_N(M) = 1.$$

This contradicts the fact that M is a popular matching in G.

Next we consider the case where $M(p) + f(a) \notin \mathcal{I}_p$. Let C be the fundamental circuit of $f(a)$ with respect to $M(p)$ in $\mathcal{M}_p$. It follows from (2) that C contains an edge (b, p) in $M(p) \setminus F_p$ and we have $M(p) + f(a) - (b, p) \in \mathcal{I}_p$. Assume that $f(b) = (b, q)$. Notice that $q \neq p$ because (b, p) is not in F_p.

If $M(q) + f(b) \in \mathcal{I}_q$, then we define

$$N := (M \setminus \{\mu_M(a), (b, p)\}) \cup \{f(a), f(b)\}$$

(see Figure 1(a)). It follows from the condition (I1) that N is a matching. Since we have

$$\pi(\mu_N(a)) > \pi(\mu_M(a)), \quad \pi(\mu_N(b)) > \pi(\mu_M(b)), \quad \forall a' \in A \setminus \{a, b\}\colon\ N(a') = M(a'),$$

we have

$$\mathsf{pre}_M(N) - \mathsf{pre}_N(M) = 2,$$

which contradicts the fact that M is a popular matching in G.

Figure 1: Bold edges are removed and dashed edges are added.

Assume that $M(q) + f(b) \notin \mathcal{I}_q$. Since $\{f(b)\}$ is an independent set in $\mathcal{M}_q$, there exists an edge (c, q) in the fundamental circuit of $f(b)$ with respect to $M(q)$ in $\mathcal{M}_q$ such that $f(b) \neq (c, q)$. Define

$$N := (M \setminus \{\mu_M(a), (b, p), (c, q)\}) \cup \{f(a), f(b), (c, p(c))\}$$

(see Figure 1(b)). It follows from the condition (I1) that N is a matching. Since we have

$$\pi(\mu_N(a)) > \pi(\mu_M(a)), \quad \pi(\mu_N(b)) > \pi(\mu_M(b)), \quad \forall a' \in A \setminus \{a, b, c\}\colon\ N(a') = M(a'),$$

we have

$$\mathsf{pre}_M(N) - \mathsf{pre}_N(M) = 1,$$

which contradicts the fact that M is a popular matching in G. □

Lemma 4. *Let M be a matching in G. If M is popular, then for every post p in P, $M(p)$ does not contain an edge (a, p) in $E(p)$ with*

$$\pi(f(a)) > \pi((a, p)) > \pi(s(a)).$$

Proof. Assume that there exists an edge (a, p) in $M(p)$ satisfying the condition in this lemma. It follows from $M(p) \in \mathcal{I}_p$ and the condition (I1) that

$$(M(p) \cap F_p) + (a, p) \in \mathcal{I}_p. \tag{3}$$

Since Lemma 3 implies that $M(p) \cap F_p$ is a base in $\mathcal{M}_p | F_p$, it follows from the statement 1 of Lemma 1 that $\{(a, p)\} \in \mathcal{I}_p / F_p$. However, this observation contradicts the definition of $s(a)$, which completes the proof. □

Lemma 5. *Let M be a matching in G. If M is popular, then M is a subset of E_{re}.*

Proof. Assume that there exists an applicant a in A with $\mu_M(a) \notin E_{\mathsf{re}}$. Since it follows from the definition of $f(a)$ that $\{(a, p')\}$ is not an independent set in $\mathcal{M}_{p'}$ for every edge (a, p') in $E(a)$ with $\pi((a, p')) > \pi(f(a))$, Lemma 4 implies that $\pi(\mu_M(a)) < \pi(s(a))$. Assume that $s(a) = (a, p)$.

If $M(p) + s(a) \in \mathcal{I}_p$, then we define

$$N := M + s(a) - \mu_M(a).$$

It follows from the condition (I1) that N is a matching. Furthermore, since $\pi(\mu_N(a)) > \pi(\mu_M(a))$ and $N(a') = M(a')$ for every applicant a' in $A - a$, we have

$$\mathsf{pre}_M(N) - \mathsf{pre}_N(M) = 1.$$

This contradicts the fact that M is a popular matching in G.

Next we consider the case where $M(p) + s(a) \notin \mathcal{I}_p$. Let C be the fundamental circuit of $s(a)$ with respect to $M(p)$ in $\mathcal{M}_p$. Since $\{s(a)\}$ is an independent set in $\mathcal{M}_p/F_p$, Lemma 3 and the statement 1 of Lemma 1 imply that that

$$(M(p) \cap F_p) + s(a) \in \mathcal{I}_p. \tag{4}$$

It follows from (4) that $C \setminus F_p$ contains an edge (b, p) with $a \neq b$. Assume that $f(b) = q$. Notice that $p \neq q$ follows from $(b, p) \notin F_p$.

If $M(q) + f(b) \in \mathcal{I}_q$, then we define

$$N := (M \setminus \{\mu_M(a), (b, q)\}) \cup \{s(a), f(b)\}.$$

It follows from the condition (I1) that N is a matching. Since we have

$$\pi(\mu_N(a)) > \pi(\mu_M(a)), \quad \pi(\mu_N(b)) > \pi(\mu_M(b)), \quad \forall a' \in A \setminus \{a, b\}\colon\ N(a') = M(a'),$$

we have

$$\mathsf{pre}_M(N) - \mathsf{pre}_N(M) = 2,$$

which contradicts the fact that M is a popular matching in G.

Assume that $M(q) + f(b) \notin \mathcal{I}_q$. Since $\{f(b)\}$ is an independent set in $\mathcal{M}_q$, there exists an edge (c, q) in the fundamental circuit of $f(b)$ with respect to $M(q)$ in $\mathcal{M}_q$ such that $f(b) \neq (c, q)$. Define

$$N := (M \setminus \{\mu_M(a), (b, p), (c, q)\}) \cup \{s(a), f(b), (c, p(c))\}.$$

It follows from the condition (I1) that N is a matching. Since we have

$$\pi(\mu_N(a)) > \pi(\mu_M(a)), \quad \pi(\mu_N(b)) > \pi(\mu_M(b)), \quad \forall a' \in A \setminus \{a, b, c\}\colon\ N(a') = M(a'),$$

we have

$$\mathsf{pre}_M(N) - \mathsf{pre}_N(M) = 1,$$

which contradicts the fact that M is a popular matching in G. $\square$

Next we prove a lemma that is necessary for proving the *if*-part. For each matching M in G and each post p in P, we define $\mathsf{bet}_M(p)$ as the set of edges (a, p) in $E(p)$ such that $(a, p) \in M(p)$ and

$$\pi(f(a)) > \pi((a, p)) > \pi(s(a)).$$

Lemma 6. *Let M be a matching in G such that for every post p in P, $M(p) \cap F_p$ is a base in $\mathcal{M}_p|F_p$. For every matching N in G and every post p in P, we have*

$$|M(p) \cap F_p| \geq |N(p) \cap F_p| + |\mathsf{bet}_N(p)|.$$

Proof. Assume that

$$|M(p) \cap F_p| < |N(p) \cap F_p| + |\mathsf{bet}_N(p)|.$$

In this case, it follows from the condition (I2) that there exists an edge e in

$$[(N(p) \cap F_p) \cup \mathsf{bet}_N(p)] \setminus (M(p) \cap F_p)$$

with $(M(p) \cap F_p) + e \in \mathcal{I}_p$. If e is in F_p, then this contradicts the fact that $M(p) \cap F_p$ is a base in $\mathcal{M}_p|F_p$. Thus, we assume that e is in $\mathsf{bet}_N(p)$. Since $M(p) \cap F_p$ is a base in $\mathcal{M}_p|F_p$, it follows from the statement 1 of Lemma 1 that $\{e\}$ is in $\mathcal{I}_p/F_p$. However, this observation contradicts the definition of $s(a)$, which completes the proof. □

We are now ready to prove Theorem 2.

Proof of Theorem 2. Since the *only if*-part follows from Lemmas 3 and 5, we prove the *if*-part. Let M be a matching in G satisfying the conditions (P1) and (P2). Let us fix a matching N in G. Define Δ_M and Δ_N as the sets of applicants a in A such that

$$\pi(\mu_M(a)) > \pi(\mu_N(a)) \quad \text{and} \quad \pi(\mu_N(a)) > \pi(\mu_M(a)),$$

respectively. For proving the *if*-part, if suffices to prove that $|\Delta_M| \geq |\Delta_N|$. For proving this, we construct an injective function $\sigma\colon \Delta_N \to \Delta_M$ as follows.

Lemma 6 implies that for each post p in P, there exists an injective function φ_p from

$$[(N(p) \cap F_p) \cup \mathsf{bet}_N(p)] \setminus (M(p) \cap F_p) \tag{5}$$

to

$$(M(p) \cap F_p) \setminus [(N(p) \cap F_p) \cup \mathsf{bet}_N(p)].$$

We construct an injective function $\sigma\colon \Delta_N \to \Delta_M$ by using the injective functions φ_p for posts p in P. Let a be an applicant in Δ_N. Assume that $\mu_N(a) = (a, p)$. Recall that $\{(a, p')\} \notin \mathcal{I}_{p'}$ for every edge (a, p') in $E(a)$ with $\pi((a, p')) > \pi(f(a))$. It follows from $M \subseteq E_{\mathsf{re}}$ and $\pi(\mu_N(a)) > \pi(\mu_M(a))$ that $\mu_M(a) = s(a)$ and $\mu_N(a)$ is in (5). Assume that $\varphi_p(\mu_N(a)) = (b, p)$. Since (b, p) is in F_p, b is in Δ_M. Thus, we can define $\sigma(a) := b$. Since φ_p is an injective function for every post p in P and $|M(a')| = 1$ for every applicant a' in A, σ is also an injective function. This completes the proof. □

4 Algorithm

In this section, we present our algorithm for the PMuMC problem that is called the algorithm `PMuMC`. Define a matroid $\mathcal{M}_{\mathsf{ap}} = (E_{\mathsf{re}}, \mathcal{I}_{\mathsf{ap}})$ by

$$\mathcal{I}_{\mathsf{ap}} := \{M \subseteq E_{\mathsf{re}} \mid \forall p \in P\colon\ |M(p)| \leq 1\}.$$

Furthermore, define

$$\mathcal{M}_{\mathsf{po}} := \bigoplus_{p \in P} [(\mathcal{M}_p|F_p \oplus \mathcal{M}_p/F_p)|E_{\mathsf{re}}(p)],$$

i.e., $\mathcal{M}_{\mathsf{po}}$ is the direct sum of $(\mathcal{M}_p|F_p \oplus \mathcal{M}_p/F_p)|E_{\mathsf{re}}(p)$ for all posts p in P.

Lemma 7. *Let M be a matching in G. If M is popular, then M is a common independent set of $\mathcal{M}_{\mathsf{ap}}$ and $\mathcal{M}_{\mathsf{po}}$.*

Proof. Clearly, M is an independent set in $\mathcal{M}_{\mathsf{ap}}$. It follows from the condition (P2) of Theorem 2 that for every post p in P, $M(p)$ is a subset of $E_{\mathsf{re}}(p)$. Moreover, it follows from the condition (P1) of Theorem 2 and the statement 2 of Lemma 1 that $M(p)$ is an independent set in $\mathcal{M}_p|F_p \oplus \mathcal{M}_p/F_p$ for every post p in P, which completes the proof. □

The algorithm `PMuMC` is described as follows.

Algorithm `PMuMC`

Step 1. Find a maximum-size common independent set M of $\mathcal{M}_{\mathsf{ap}}$ and $\mathcal{M}_{\mathsf{po}}$.

Step 2. If there exists an applicant a in A with $M(a) = \emptyset$, then output **null**, i.e., there exists no popular matching in G. Otherwise, i.e., if $|M(a)| = 1$ for every applicant a in A, then go to **Step 3**.

Step 3 Set $i := 0$, $M_i := M$, and do the following.

(3-a) If $M_i(p) \cap F_p$ is a base in $\mathcal{M}_p|F_p$ for every post p in P, then go to **Step 4**.

(3-b) Arbitrarily choose a post p in P such that $M_i(p) \cap F_p$ is not a base in $\mathcal{M}_p|F_p$, and find an edge (a, p) in $F_p \setminus M_i(p)$ with $M_i(p) + (a, p) \in \mathcal{I}_p|F_p$. Furthermore, set

$$M_{i+1} := M_i + (a, p) - \mu_{M_i}(a).$$

(3-c) Update $i := i + 1$ and go to **Step (3-a)**.

Step 4. Output M_i, i.e., M_i is a popular matching in G.

End of Algorithm

Notice that in **Step (3-b)**, the condition (I2) guarantees that there exists an edge (a, p) in $F_p \setminus M_i(p)$ with $M_i(p) + (a, p) \in \mathcal{I}_p|F_p$. Moreover, it follows from the following two lemmas that the algorithm `PMuMC` is well-defined.

Lemma 8. *In* **Step (3-b)***, M_{i+1} is a common independent set of $\mathcal{M}_{\mathsf{ap}}$ and $\mathcal{M}_{\mathsf{po}}$.*

Proof. It suffices to prove that $M_{i+1}(p)$ is an independent set in $\mathcal{M}_p|F_p \oplus \mathcal{M}_p/F_p$. This follows from the statement 3 of Lemma 1. □

Lemma 9. *The number of iterations of* **Step (3-b)** *is at most $|E|$.*

Proof. Define δ_i by

$$\delta_i := \sum_{p \in P} |M_i(p) \cap F_p|.$$

In **Step (3-b)**, we have $\mu_{M_i}(a) \neq f(a)$, which implies that $\delta_i < \delta_{i+1}$. Thus, this lemma follows from $\delta_i \leq |E|$. □

We are now ready to prove a main result of this paper.

Theorem 10. *The algorithm `PMuMC` correctly solves the PMuMC problem.*

Proof. Lemma 7 implies that if the algorithm `PMuMC` outputs **null**, then there exists no popular matching in G. Assume that there exists a common independent set M of $\mathcal{M}_{\mathsf{ap}}$ and $\mathcal{M}_{\mathsf{po}}$ such that $|M(a)| = 1$ for every applicant a in A. It follows from the statement 4 of Lemmas 1 that M is a matching. Moreover, Theorem 2 implies that M is a popular matching. This completes the proof. $\square$

Here we analyze the time complexity of the algorithm `PMuMC`. We assume that for every post p in P, we can check whether a subset of $E(p)$ belongs to $\mathcal{I}_p$ in time bounded by a polynomial in $|E|$. Clearly, we can check whether a subset of F_p belongs to $\mathcal{I}_p|F_p$ in time bounded by a polynomial in $|E|$. Furthermore, the statement 1 of Lemma 1 implies that once we find a base in $\mathcal{M}_p|F_p$, we can check whether a subset of $E(p) \setminus F_p$ belongs to $\mathcal{I}_p/F_p$ in time bounded by a polynomial in $|E|$. Thus, we can check whether a subset of E is an independent set in $\mathcal{M}_{\mathsf{po}}$ in time bounded by a polynomial in $|E|$, which implies that we can do **Step 1** in time bounded by a polynomial in $|E|$. Since the other steps can be done in time bounded by a polynomial in $|E|$, the time complexity of the algorithm `PMuMC` is bounded by a polynomial in $|E|$.

References

[1] D. J. Abraham, R. W. Irving, T. Kavitha, and K. Mehlhorn. Popular matchings. *SIAM Journal on Computing*, 37(4):1030–1045, 2007.

[2] P. Biró, T. Fleiner, R. W. Irving, and D. F. Manlove. The college admissions problem with lower and common quotas. *Theoretical Computer Science*, 411(34-36):3136–3153, 2010.

[3] P. Biró, R. W. Irving, and D. F. Manlove. Popular matchings in the marriage and roommates problems. In *Proceedings of the 7th International Conference on Algorithms and Complexity*, volume 6078 of *Lecture Notes in Computer Science*, pages 97–108, 2010.

[4] T. Fleiner. A fixed-point approach to stable matchings and some applications. *Mathematics of Operations Research*, 28(1):103–126, 2003.

[5] T. Fleiner and N. Kamiyama. A matroid approach to stable matchings with lower quotas. In *Proceedings of the 23rd Annual ACM-SIAM Symposium on Discrete Algorithms*, pages 135–142, 2012.

[6] S. Fujishige and A. Tamura. A two-sided discrete-concave market with possibly bounded side payments: An approach by discrete convex analysis. *Mathematics of Operations Research*, 32(1):136–155, 2007.

[7] P. Gärdenfors. Match making: Assignments based on bilateral preferences. *Behavioral Science*, 20(3):166–173, 1975.

[8] C.-C. Huang and T. Kavitha. Popular matchings in the stable marriage problem. *Information and Computation*, 222:180–194, 2013.

[9] T. Kavitha. Popularity vs maximum cardinality in the stable marriage setting. In *Proceedings of the 23rd Annual ACM-SIAM Symposium on Discrete Algorithms*, pages 123–134, 2012.

[10] D. F. Manlove and C. T. S. Sng. Popular matchings in the capacitated house allocation problem. In *Proceedings of the 14th Annual European Symposium on Algorithms*, volume 4168 of *Lecture Notes in Computer Science*, pages 492–503, 2006.

[11] J. Mestre. Weighted popular matchings. In *Proceedings of the 33rd International Colloquium on Automata, Languages and Programming*, volume 4051 of *Lecture Notes in Computer Science*, pages 715–726, 2006.

[12] J. G. Oxley. *Matroid theory.* Oxford University Press, 2nd edition, 2011.

[13] A. Schrijver. *Combinatorial Optimization: Polyhedra and Efficiency.* Springer, 2003.

[14] C. T. S. Sng and D. F. Manlove. Popular matchings in the weighted capacitated house allocation problem. *Journal of Discrete Algorithms*, 8(2):102–116, 2010.

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