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Kamiyama, Naoyuki
Institute of Mathematics for Industry, Kyushu University

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Naoyuki Kamiyama

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Institute of Mathematics for Industry
Graduate School of Mathematics
Kyushu University
Fukuoka, JAPAN

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Naoyuki Kamiyama*

Institute of Mathematics for Industry, Kyushu University
kamiyama@imi.kyushu-u.ac.jp

Abstract

In this paper, we consider the fault-tolerant facility location problem with submodular penalties that is a common generalization of the fault-tolerant facility location problem and the facility location problem with submodular penalties. For this problem, we present a combinatorial $3 \cdot H_R$-approximation algorithm, where R is the maximum connectivity requirement and H_R is the R-th harmonic number. Our algorithm is a common generalization of the algorithm for the the fault-tolerant facility location problem presented by Jain and Vazirani (2003) and that for the facility location problem with submodular penalties presented by Du, Lu and Xu (2012).

1 Introduction

The facility location problem is one of the most important problems in combinatorial optimization. Unfortunately, this problem is **NP**-hard, and thus most of the research has been focusing on designing approximation algorithms with good performance. So far, the best approximation ratio for the facility location problem is 1.488 due to Li [8].

Many variants of the facility location problem have appeared. The fault-tolerant facility location problem is one of variants of the facility location problem. In this problem, each client has a connectivity requirement and we have to connect each client to as many open facilities as its connectivity requirement. This problem was introduced by Jain and Vazirani [7], and several approximation algorithms were presented [7, 5, 11, 1]. So far, the best approximation ratio for the fault-tolerant facility location problem is 1.725 due to Byrka, Srinivasan and Swamy [1].

The facility location problem with submodular penalties is another variant of the facility location problem. In this problem, not all clients are connected to open facilities and unconnected clients incur a penalty cost determined by a monotone submodular function on the client set. This problem was introduced by Hayrapetyan, Swamy and Tardos [6], and several approximation algorithms were presented [6, 2, 3, 9]. So far, the best approximation ratio for the facility location problem with submodular penalties is 2 due to Li, Du, Xiu and Xu [9].

In this paper, we consider the fault-tolerant facility location problem with submodular penalties that is a common generalization of the above two variants of the facility location problem. For this problem, we present a combinatorial $3 \cdot H_R$-approximation algorithm, where R is the maximum connectivity requirement and H_R is the R-th harmonic number. Our algorithm is a common generalization of the algorithm for the the fault-tolerant facility location problem presented by Jain and Vazirani [7] and that for the facility location problem with submodular penalties presented by Du, Lu and Xu [3].

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Organization. In Section 2, we give a formal definition of the fault-tolerant facility location problem with submodular penalties. In Section 3, we present an algorithm for the fault-tolerant facility location problem with submodular penalties. In Section 4, we analyze an approximation ratio of our algorithm. Section 5 concludes this paper.

Notation. We denote by $\mathbb{R}$, $\mathbb{R}_+$ and $\mathbb{Z}_+$ the sets of real numbers, non-negative real numbers and non-negative integers, respectively.

Assume that we are given a set U. For each subset X of U, we define a characteristic vector χ_X in $\{0,1\}^U$ by

$$\chi_X(u) := \begin{cases} 1 & \text{if } u \in X \\ 0 & \text{if } u \in U \setminus X. \end{cases}$$

Let d_1, d_2 be vectors in $\mathbb{R}^U$. Define a vector $d_1 \pm d_2$ in $\mathbb{R}^U$ by

$$(d_1 \pm d_2)(u) := d_1(u) \pm d_2(u).$$

In addition, we write $d_1 \geq d_2$, if $d_1(u) \geq d_2(u)$ for every element u in U.

2 Problem Formulation

The *fault-tolerant facility location problem with submodular penalties* is defined as follows. We are given a finite set F of facilities and a finite set D of clients. For each facility i in F, an opening cost f_i in $\mathbb{R}_+$ is given. For each client j in D, a connectivity requirement r_j in $\mathbb{Z}_+$ is given. We assume that $r_j \leq |F|$ for every client j in D. For each facility i in F and each client j in D, a connecting cost $c_{i,j}$ in $\mathbb{R}_+$ is given. We assume that connecting costs satisfy the triangle inequality, i.e.,

$$c_{i,j} + c_{i,j'} + c_{i',j'} \geq c_{i',j}$$

for every facilities i, i' in F and every clients j, j' in D. In addition, we are given a penalty function $h\colon \mathbb{R}_+^D \to \mathbb{R}_+$, which is the Lovász extension of a non-negative monotone submodular function $\rho\colon 2^D \to \mathbb{R}_+$ with $\rho(\emptyset) = 0$. We will give formal definitions of a submodular function and its Lovász extension later.

An *assignment* is a triple (X, d, φ) of a subset X of F, functions $d\colon D \to \mathbb{Z}_+$ and $\varphi\colon D \to 2^F$. An assignment is said to be *feasible*, if

$$\forall j \in D\colon\ \varphi(j) \subseteq X \text{ and } |\varphi(j)| + d(j) = r_j.$$

The *cost* $\xi(X, d, \varphi)$ of an assignment (X, d, φ) is defined by

$$\xi(X, d, \varphi) := \sum_{i \in X} f_i + \sum_{j \in D} \sum_{i \in \varphi(j)} c_{i,j} + h(d).$$

The goal of the fault-tolerant facility location problem with submodular penalties is to find a feasible assignment with minimum cost.

Here we give formal definitions of a submodular function and its Lovász extension. A function $\rho\colon 2^D \to \mathbb{R}_+$ is said to be *submodular*, if

$$\forall X, Y \subseteq D\colon\ \rho(X) + \rho(Y) \geq \rho(X \cup Y) + \rho(X \cap Y). \tag{1}$$

It is well known that (1) is equivalent to

$$\forall X, Y \subseteq D \text{ s.t. } X \subseteq Y,\ \forall j \in D \setminus Y\colon\ \rho(X \cup \{j\}) - \rho(X) \geq \rho(Y \cup \{j\}) - \rho(Y). \tag{2}$$

A submodular function $\rho\colon 2^D \to \mathbb{R}_+$ is said to be *monotone*, if

$$\forall X, Y \subseteq D \text{ s.t.} X \subseteq Y\colon\ \rho(X) \le \rho(Y).$$

The *Lovász extension* $h\colon \mathbb{R}_+^D \to \mathbb{R}_+$ of a submodular function $\rho\colon 2^D \to \mathbb{R}_+$ is defined as follows. Assume that we are given a vector d in $\mathbb{R}_+^D$. We denote by $\hat{d}_1 > \hat{d}_2 > \cdots > \hat{d}_k$ the distinct values of its components and define

$$U_l := \{j \in D \mid d(j) \ge \hat{d}_l\} \tag{3}$$

for each $l = 1, 2, \ldots, k$. We define $h(d)$ by

$$h(d) := \sum_{l=1}^{k-1} (\hat{d}_l - \hat{d}_{l+1})\rho(U_l) + \hat{d}_k \cdot \rho(U_k).$$

It is not difficult to see that $\rho(X) = h(\chi_X)$ for every subset X of D. This implies that the fault-tolerant facility location problem with submodular penalties is a generalization of the facility location problem with submodular penalties.

Here we give properties of the function h that will be used in the sequel. It is known [4] that $h(d)$ is equal to the optimal value of the following linear programming (4).

$$\begin{array}{lll} \max & \sum_{j \in D} d(j) \cdot p_j & \\ \text{s.t.} & \sum_{j \in X} p_j \le \rho(X) & (X \subseteq D) \\ & p_j \in \mathbb{R} & (j \in D). \end{array} \tag{4}$$

In addition, it follows from the monotonicity of ρ that h is also "monotone".

Lemma 1. *For every vectors d, d' in $\mathbb{Z}_+^D$ with $d \ge d'$, we have $h(d) \ge h(d')$.*

Proof. We prove this lemma by induction on

$$\|d\| := \sum_{j \in D} d(j).$$

If $\|d\| = 0$, then $h(d) \ge h(d')$ clearly holds for every vector d' in $\mathbb{Z}_+^D$ with $d \ge d'$. Assuming that this lemma holds for every vector d in $\mathbb{Z}_+^D$ with $\|d\| = \Delta$, we consider the case of $\|d\| = \Delta + 1$. If we can prove that

$$\forall j \in D \text{ s.t.} d(j) > 0\colon\ h(d) \ge h(d - \chi_{\{j\}}),$$

then this lemma follows from the induction hypothesis. Let us fix a client j in D with $d(j) > 0$. We denote by $\hat{d}_1 > \hat{d}_2 > \cdots > \hat{d}_k$ the distinct values of the components of d and define U_l by (3) for each $l = 1, 2, \ldots, k$. Assume that $d(j) = \hat{d}_s$. We have

$$\begin{aligned} h(d - \chi_{\{j\}}) := &\sum_{l=1}^{s-1} (\hat{d}_l - \hat{d}_{l+1})\rho(U_l) + \rho(U_s \setminus \{j\}) + (\hat{d}_s - \hat{d}_{s+1} - 1)\rho(U_s) \\ &+ \sum_{l=s+1}^{k-1} (\hat{d}_l - \hat{d}_{l+1})\rho(U_l) + \hat{d}_k \cdot \rho(U_k). \end{aligned}$$

Hence, we have

$$h(d) - h(d - \chi_{\{j\}}) = \rho(U_s) - \rho(U_s \setminus \{j\}) \ge 0,$$

where the inequality follows from the monotonicity of ρ. This completes the proof. □

3 Algorithm

In this section, we explain our algorithm for the fault-tolerant facility location problem with submodular penalties. For this, we first define a subproblem $\mathsf{S}(H, K, \sigma)$ for each subset H of F, each subset K of D and each function $\sigma\colon K \to 2^H$ as follows. Intuitively, in $\mathsf{S}(H, K, \sigma)$, facilities in H are already open, and each client j of K is not allowed to be connected to facilities in $\sigma(j)$. Under this constraint, this subproblem asks for opening facilities in $F \setminus H$ and connecting each client j in K_t to a newly opened facility or a facility in $H \setminus \sigma(j)$. Notice that in this problem, a connectivity requirement of each client in K is equal to one.

A feasible solution of $\mathsf{S}(H, K, \sigma)$ is a triple (Y, P, ψ) of a subset Y of $F \setminus H$, a subset P of K and a function $\psi\colon K \setminus P \to F$ such that

$$\forall j \in K \setminus P\colon\ \psi(j) \in (H \cup Y) \setminus \sigma(j).$$

The *cost* $\xi_{\mathsf{S}}(Y, P, \psi)$ of a feasible solution (Y, P, ψ) of $\mathsf{S}(H, K, \sigma)$ is defined by

$$\xi_{\mathsf{S}}(Y, P, \psi) := \sum_{i \in Y} f_i + \sum_{j \in K \setminus P} c_{\psi(j), j} + \rho(P).$$

The goal of $\mathsf{S}(H, K, \sigma)$ is to find a feasible solution with minimum cost.

Now we are ready to present our algorithm for the fault-tolerant facility location problem with submodular penalties, called $\mathsf{FTFLwSP}$. Define

$$R := \max\{r_j \mid j \in D\}.$$

The algorithm $\mathsf{FTFLwSP}$ is described as follows.

Step 1: Set $t := R$, $X_{t+1} := \emptyset$, $d_{t+1}(j) := 0$ and $\varphi_{t+1}(j) := \emptyset$ for each client j in D.

Step 2: If $t \geq 1$, do the following (2-a) to (2-d).

(2-a) Set $H_t := X_{t+1}$, $K_t := \{j \in D \mid r_j \geq t\}$ and $\sigma_t(j) := \varphi_{t+1}(j)$ for each client j in K_t.

(2-b) Find a feasible solution (Y_t, P_t, ψ_t) of $\mathsf{S}(H_t, K_t, \sigma_t)$.

(2-c) Set $X_t := X_{t+1} \cup Y_t$, $d_t := d_{t+1} + \chi_{P_t}$, and

$$\varphi_t(j) := \begin{cases} \varphi_{t+1}(j) \cup \{\psi_t(j)\} & \text{if } j \in K_t \setminus P_t \\ \varphi_{t+1}(j) & \text{if } j \in (D \setminus K_t) \cup P_t. \end{cases}$$

(2-d) Update $t := t - 1$ and go to **Step 2**.

Step 3: Output (X_1, d_1, φ_1).

In **Step 2** of the t-th iteration, we connect a client j in D with $r_j \geq t$ to some open facility or increase its penalty by one. Hence, the ouput (X_1, d_1, φ_1) is clearly a feasible assignment. Notice that an approximation ratio of the algorithm $\mathsf{FTFLwSP}$ depends on the quality of (Y_t, P_t, ψ_t) in **Step (2-b)**. In addition, if we can find (Y_t, P_t, ψ_t) in polynomial time, then the algorithm $\mathsf{FTFLwSP}$ is also a polynomial-time algorithm. We will discuss these points in the next section.

4 Analysis

In this section, we analyze an approximation ratio of the algorithm FTFLwSP. An IP formulation of the fault-tolerant facility location problem with submodular penalties is described as follows.

$$\begin{aligned} \min \quad & \sum_{i \in F} f_i y_i + \sum_{i \in F} \sum_{j \in D} c_{i,j} x_{i,j} + h(d) \\ \text{s.t.} \quad & \sum_{i \in F} x_{i,j} + d(j) \geq r_j \qquad (j \in D) \\ & x_{i,j} \leq y_i \qquad (i \in F,\, j \in D) \\ & x_{i,j} \in \{0,1\} \qquad (i \in F,\, j \in D) \\ & y_i \in \{0,1\} \qquad (i \in F) \\ & d \in \mathbb{Z}_+^D. \end{aligned} \tag{5}$$

Notice that it follows from Lemma 1 that there exists an optimal solution of (5) such that the first constraint holds with equality for every client j in D. Denote by OPT the optimal value of (5), i.e., the fault-tolerant facility location problem with submodular penalties.

Here we consider an LP relaxation of (5). The dual problem of (4) is described as follows.

$$\begin{aligned} \min \quad & \sum_{X \subseteq D} \rho(X) \cdot q_X \\ \text{s.t.} \quad & \sum_{X \subseteq D \colon j \in X} q_X = d(j) \qquad (j \in D) \\ & q_X \geq 0 \qquad (X \subseteq D). \end{aligned} \tag{6}$$

It follows from (6) that an LP relaxation of (5) can be described as follows.

$$\begin{aligned} \min \quad & \sum_{i \in F} f_i y_i + \sum_{i \in F} \sum_{j \in D} c_{i,j} x_{i,j} + \sum_{X \subseteq D} \rho(X) \cdot q_X \\ \text{s.t.} \quad & \sum_{i \in F} x_{i,j} + \sum_{X \subseteq D \colon j \in X} q_X \geq r_j \qquad (j \in D) \\ & x_{i,j} \leq y_i \qquad (i \in F,\, j \in D) \\ & x_{i,j} \geq 0 \qquad (i \in F,\, j \in D) \\ & 0 \leq y_i \leq 1 \qquad (i \in F) \\ & q_X \geq 0 \qquad (X \subseteq D). \end{aligned} \tag{7}$$

Denote by $\mathsf{OPT}_{\mathsf{LP}}$ the optimal value of (7). Notice that $\mathsf{OPT}_{\mathsf{LP}} \leq \mathsf{OPT}$ holds. The dual problem of (7) is described as follows.

$$\begin{aligned} \max \quad & \sum_{j \in D} r_j \alpha_j - \sum_{i \in F} z_i \\ \text{s.t.} \quad & \alpha_j - \beta_{i,j} \leq c_{i,j} \qquad (i \in F,\, j \in D) \\ & \sum_{j \in D} \beta_{i,j} \leq f_i + z_i \qquad (i \in F) \\ & \sum_{j \in X} \alpha_j \leq \rho(X) \qquad (X \subseteq D) \\ & \alpha_j \geq 0 \qquad (j \in D) \\ & \beta_{i,j} \geq 0 \qquad (i \in F,\, j \in D) \\ & z_i \geq 0 \qquad (i \in F). \end{aligned} \tag{8}$$

Next we consider an IP formulation of $\mathsf{S}(H_t, K_t, \sigma_t)$. Define a new function $\rho_{K_t} \colon 2^{K_t} \to \mathbb{R}_+$ by $\rho_{K_t}(X) := \rho(X)$ for each subset X of K_t. It is clear that ρ_{K_t} is submodular. Let $h_{K_t} \colon \mathbb{R}_+^{K_t} \to \mathbb{R}_+$ be the Lovász extension of ρ_{K_t}. An IP formulation of $\mathsf{S}(H_t, K_t, \sigma_t)$ is described as follows.

$$\begin{aligned}
\min \quad & \sum_{i \in F \setminus H_t} f_i y_i + \sum_{i \in F} \sum_{j \in K_t} c_{i,j} x_{i,j} + h_{K_t}(d) \\
\text{s.t.} \quad & \sum_{i \in F \setminus \sigma_t(j)} x_{i,j} + d(j) \geq 1 \qquad (j \in K_t) \\
& x_{i,j} \leq y_i \qquad (i \in F \setminus H_t,\ j \in K_t) \\
& x_{i,j} \in \{0,1\} \qquad (i \in F,\ j \in K_t) \\
& y_i \in \{0,1\} \qquad (i \in F \setminus H_t) \\
& d \in \{0,1\}^{K_t}.
\end{aligned} \tag{9}$$

Similarly to (7), an LP relaxation of (9) is described as follows.

$$\begin{aligned}
\min \quad & \sum_{i \in F \setminus H_t} f_i y_i + \sum_{i \in F} \sum_{j \in K_t} c_{i,j} x_{i,j} + \sum_{X \subseteq K_t} \rho_{K_t}(X) \cdot q_X \\
\text{s.t.} \quad & \sum_{i \in F \setminus \sigma_t(j)} x_{i,j} + \sum_{X \subseteq K_t \colon j \in X} q_X \geq 1 \qquad (j \in K_t) \\
& x_{i,j} \leq y_i \qquad (i \in F \setminus H_t,\ j \in K_t) \\
& x_{i,j} \geq 0 \qquad (i \in F,\ j \in K_t) \\
& y_i \geq 0 \qquad (i \in F \setminus H_t) \\
& q_X \geq 0 \qquad (X \subseteq K_t).
\end{aligned} \tag{10}$$

Denote by $\mathsf{OPT}_{\mathsf{SLP}}(t)$ the optimal value of (10). The dual problem of (10) is described as follows.

$$\begin{aligned}
\max \quad & \sum_{i \in K_t} \alpha_i \\
\text{s.t.} \quad & \alpha_j - \beta_{i,j} \leq c_{i,j} \qquad (j \in K_t,\ i \in F \setminus H_t) \\
& \alpha_j \leq c_{i,j} \qquad (j \in K_t,\ i \in H_t \setminus \sigma_t(j)) \\
& \sum_{j \in K_t} \beta_{i,j} \leq f_i \qquad (i \in F \setminus H_t) \\
& \sum_{j \in X} \alpha_j \leq \rho_{K_t}(X) \qquad (X \subseteq K_t) \\
& \alpha_j \geq 0 \qquad (j \in K_t) \\
& \beta_{i,j} \geq 0 \qquad (i \in F \setminus H_t,\ j \in K_t).
\end{aligned} \tag{11}$$

From now on, we analyze an approximation ratio of the algorithm $\mathsf{FTFLwSP}$.

Lemma 2. *For every $t = 1, 2, \ldots, R$, we can find a feasible solution (Y_t, P_t, ψ_t) of $\mathsf{S}(H_t, K_t, \sigma_t)$ such that*

$$\xi_{\mathsf{S}}(Y_t, P_t, \psi_t) \leq 3 \cdot \mathsf{OPT}_{\mathsf{SLP}}(t)$$

in polynomial time.

We will give the proof of Lemma 2 in the next subsection.

Lemma 3. *For every $t = 1, 2, \ldots, R$, we have*

$$\mathsf{OPT}_{\mathsf{SLP}}(t) \leq \frac{1}{t} \cdot \mathsf{OPT}_{\mathsf{LP}}.$$

Proof. It follow from the strong duality theorem that there exists a feasible solution

$$\alpha_j \ (j \in K_t), \qquad \beta_{i,j} \ (i \in F \setminus H_t,\, j \in K_t)$$

of (11) with

$$\sum_{j \in K_t} \alpha_j = \mathsf{OPT}_{\mathsf{SLP}}(t).$$

To prove the theorem, we construct a feasible solution

$$\hat{\alpha}_j \ (j \in D), \qquad \hat{\beta}_{i,j} \ (i \in F,\, j \in D), \qquad \hat{z}_i \ (i \in F)$$

of (8) with

$$\sum_{j \in D} r_j \hat{\alpha}_j - \sum_{i \in F} \hat{z}_i \geq t \cdot \mathsf{OPT}_{\mathsf{SLP}}(t).$$

It follows from the weak duality theorem that

$$\sum_{j \in D} r_j \hat{\alpha}_j - \sum_{i \in F} \hat{z}_i \leq \mathsf{OPT}_{\mathsf{LP}},$$

which completes the proof.

We first define $\hat{\alpha}_j$ for each client j in D by

$$\hat{\alpha}_j := \begin{cases} \alpha_j & \text{if } j \in K_t \\ 0 & \text{if } j \in D \setminus K_t. \end{cases}$$

Next we define $\hat{\beta}_{i,j}$ for each facility i in F and each client j in D by

$$\hat{\beta}_{i,j} := \begin{cases} \beta_{i,j} & \text{if } j \in K_t \text{ and } i \in F \setminus H_t \\ 0 & \text{if } j \in K_t \text{ and } i \in H_t \setminus \sigma_t(j) \\ \alpha_j & \text{if } j \in K_t \text{ and } i \in \sigma_t(j) \\ 0 & \text{if } j \in D \setminus K_t \text{ and } i \in F. \end{cases}$$

Finally, we define $\hat{z}_i$ for each facility i in F by

$$\hat{z}_i := \begin{cases} \displaystyle\sum_{j \in K_t \colon i \in \sigma_t(j)} \alpha_j & \text{if } i \in H_t \\ 0 & \text{if } i \in F \setminus H_t. \end{cases}$$

Here we prove that $\hat{\alpha}_i$, $\hat{\beta}_{i,j}$ and $\hat{z}_i$ are a feasible solution of (8). We first consider the first constraint. For each client j in K_t and each facility i in $F \setminus H_t$,

$$\hat{\alpha}_j - \hat{\beta}_{i,j} = \alpha_j - \beta_{i,j} \leq c_{i,j}.$$

For each client j in K_t and each facility i in $H_t \setminus \sigma_t(j)$,

$$\hat{\alpha}_j - \hat{\beta}_{i,j} = \alpha_j - 0 = \alpha_j \leq c_{i,j}.$$

For each client j in K_t and each facility i in $\sigma_t(j)$,

$$\hat{\alpha}_j - \hat{\beta}_{i,j} = \alpha_j - \alpha_j = 0 \leq c_{i,j}.$$

For each client j in $D \setminus K_t$ and each facility i in F,

$$\hat{\alpha}_j - \hat{\beta}_{i,j} = 0 - 0 = 0 \leq c_{i,j}.$$

Next we consider the second constraint. For each facility i in H_t,

$$\sum_{j \in D} \hat{\beta}_{i,j} = \sum_{j \in K_t} \hat{\beta}_{i,j} = \sum_{j \in K_t \colon i \in \sigma_t(j)} \alpha_j = \hat{z}_i \leq f_i + \hat{z}_i.$$

For each facility i in $F \setminus H_t$,

$$\sum_{j \in D} \hat{\beta}_{i,j} = \sum_{j \in K_t} \beta_{i,j} \leq f_i \leq f_i + \hat{z}_i.$$

Finally, we consider the third constraint. For each subset X of K_t.

$$\sum_{j \in X} \hat{\alpha}_j = \sum_{j \in X} \alpha_j \leq \rho_{K_t}(X) = \rho(X).$$

For each subset X of D with $X \setminus K_t \neq \emptyset$.

$$\sum_{j \in X} \hat{\alpha}_j = \sum_{j \in X \cap K_t} \alpha_j \leq \rho_{K_t}(X \cap K_t) = \rho(X \cap K_t) \leq \rho(X),$$

where the last inequality follows from the monotonicity of ρ.

Next we consider the objective value.

$$\begin{aligned}
\sum_{j \in D} r_j \hat{\alpha}_j - \sum_{i \in F} \hat{z}_i &= \sum_{j \in K_t} r_j \alpha_j - \sum_{i \in H_t} \sum_{j \in K_t \colon i \in \sigma_t(j)} \alpha_j \\
&= \sum_{j \in K_t} r_j \alpha_j - \sum_{j \in K_t} \sum_{i \in \sigma_t(j)} \alpha_j \\
&= \sum_{j \in K_t} r_j \alpha_j - \sum_{j \in K_t} |\sigma_t(j)| \alpha_j \\
&= \sum_{j \in K_t} (r_j - |\varphi_{t+1}(j)|) \alpha_j \qquad (\text{by } \sigma_t(j) = \varphi_{t+1}(j)) \\
&\geq \sum_{j \in K_t} (r_j - (r_j - t)) \alpha_j \qquad (\text{by } |\varphi_{t+1}(j)| \leq r_j - t) \\
&= \sum_{j \in K_t} t \cdot \alpha_j \\
&= t \cdot \mathsf{OPT}_{\mathsf{SLP}}(t).
\end{aligned}$$

This completes the proof. □

Lemma 4. *For every vector d in $\mathbb{Z}_+^D$ and every subset X of D, we have*

$$h(d + \chi_X) - h(d) \leq h(\chi_X) = \rho(X). \tag{12}$$

Proof. If $d(j) = 0$ for every client j in D, then (12) clearly holds. Assume that there exists a client j in D with $d(j) > 0$. To prove (12), it suffices to prove that there exists a client j^* in D such that $d(j^*) > 0$ and

$$h(d + \chi_X) - h(d) \leq h(d^* + \chi_X) - h(d^*), \tag{13}$$

where the vector d^* in $\mathbb{R}^D_+$ is define by $d^* := d - \chi_{\{j^*\}}$.

We denote by $\hat{d}_1 > \hat{d}_2 > \cdots > \hat{d}_k$ the distinct values of the components of d and define

$$\begin{aligned} U_l &:= \{j \in D \mid d(j) \geq \hat{d}_l\} \\ X_l &:= \{j \in X \mid d(j) = \hat{d}_l\} \\ \overline{X}_l &:= \{j \in D \setminus X \mid d(j) = \hat{d}_l\} \\ U_l^+ &:= U_{l-1} \cup X_l \end{aligned}$$

for each $l = 1, 2, \ldots, k$, where define $U_0 := \emptyset$. Define j^* in D by

$$j^* := \begin{cases} \text{a client in } \overline{X}_k & \text{if } \hat{d}_k \neq 0 \text{ and } \overline{X}_k \neq \emptyset \\ \text{a client in } X_k & \text{if } \hat{d}_k \neq 0 \text{ and } \overline{X}_k = \emptyset \\ \text{a client in } \overline{X}_{k-1} & \text{if } \hat{d}_k = 0 \text{ and } \overline{X}_{k-1} \neq \emptyset \\ \text{a client in } X_{k-1} & \text{if } \hat{d}_k = 0 \text{ and } \overline{X}_{k-1} = \emptyset. \end{cases}$$

First we calculate the left-hand side of (13). Since

$$\begin{aligned} h(d) &= \sum_{l=1}^{k-1} (\hat{d}_l - \hat{d}_{l+1}) \rho(U_l) + \hat{d}_k \cdot \rho(U_k) \\ h(d + \chi_X) &= \sum_{l=1}^{k-1} \Big(\rho(U_l^+) + (\hat{d}_l - \hat{d}_{l+1} - 1) \rho(U_l) \Big) + \rho(U_k^+) + \hat{d}_k \cdot \rho(U_k), \end{aligned}$$

we have

$$h(d + \chi_X) - h(d) = \sum_{l=1}^{k-1} \Big(\rho(U_l^+) - \rho(U_l) \Big) + \rho(U_k^+).$$

Next we consider the right-hand side of (13). Assume that $\hat{d}_k \neq 0$ and $\overline{X}_k \neq \emptyset$. In this case,

$$\begin{aligned} h(d^*) =& \sum_{l=1}^{k-1} (\hat{d}_l - \hat{d}_{l+1}) \rho(U_l) + \rho(U_k \setminus \{j^*\}) + (\hat{d}_k - 1) \cdot \rho(U_k) \\ h(d^* + \chi_X) =& \sum_{l=1}^{k-1} \Big(\rho(U_i^+) + (\hat{d}_l - \hat{d}_{l+1} - 1) \rho(U_l) \Big) \\ & + \rho(U_k^+) + \rho(U_k \setminus \{j^*\}) + (\hat{d}_k - 1) \cdot \rho(U_k). \end{aligned}$$

Hence, we have

$$h(d^* + \chi_X) - h(d^*) = \sum_{l=1}^{k-1} \Big(\rho(U_l^+) - \rho(U_l) \Big) + \rho(U_k^+),$$

which implies that (13) holds.

Assume that $\hat{d}_k \neq 0$ and $\overline{X}_k = \emptyset$. In this case,

$$\begin{aligned} h(d^*) =& \sum_{l=1}^{k-1} (\hat{d}_l - \hat{d}_{l+1}) \rho(U_l) + \rho(U_k \setminus \{j^*\}) + (\hat{d}_k - 1) \cdot \rho(U_k) \\ h(d^* + \chi_X) =& \sum_{l=1}^{k-1} \Big(\rho(U_l^+) + (\hat{d}_l - \hat{d}_{l+1} - 1) \rho(U_i) \Big) + \rho(U_k^+ \setminus \{j^*\}) + \hat{d}_k \cdot \rho(U_k^+). \end{aligned}$$

Hence, since $U_k = U_k^+$, we have

$$h(d^* + \chi_X) - h(d^*) = \sum_{l=1}^{k-1} \Big(\rho(U_l^+) - \rho(U_l)\Big) + \rho(U_k^+),$$

which implies that (13) holds.

Assume that $\hat{d}_k = 0$ and $\overline{X}_{k-1} \neq \emptyset$. We first consider the case of $\hat{d}_{k-1} > 1$. In this case,

$$\begin{aligned} h(d^*) &= \sum_{l=1}^{k-2} (\hat{d}_l - \hat{d}_{l+1})\rho(U_l) + \rho(U_{k-1} \setminus \{j^*\}) + (\hat{d}_{k-1} - 1) \cdot \rho(U_{k-1}) \\ h(d^* + \chi_X) &= \sum_{l=1}^{k-2} \Big(\rho(U_l^+) + (\hat{d}_l - \hat{d}_{l+1} - 1)\rho(U_l)\Big) \\ &\quad + \rho(U_{k-1}^+) + \rho(U_{k-1} \setminus \{j^*\}) + (\hat{d}_{k-1} - 2) \cdot \rho(U_{k-1}) + \rho(U_k^+). \end{aligned}$$

Hence, we have

$$h(d^* + \chi_X) - h(d^*) = \sum_{l=1}^{k-1} \Big(\rho(U_l^+) - \rho(U_l)\Big) + \rho(U_k^+),$$

which implies that (13) holds. If $\hat{d}_{k-1} = 1$, then we have

$$\begin{aligned} h(d^*) &= \sum_{l=1}^{k-2} (\hat{d}_l - \hat{d}_{l+1})\rho(U_l) + \rho(U_{k-1} \setminus \{j^*\}) \\ h(d^* + \chi_X) &= \sum_{l=1}^{k-2} \Big(\rho(U_l^+) + (\hat{d}_l - \hat{d}_{l+1} - 1)\rho(U_l)\Big) + \rho(U_{k-1}^+) + \rho(U_k^+ \setminus \{j^*\}). \end{aligned}$$

Hence, we have

$$\begin{aligned} h(d^* + \chi_X) - h(d^*) &= \sum_{l=1}^{k-1} \Big(\rho(U_l^+) - \rho(U_l)\Big) + \rho(U_{k-1}) + \rho(U_k^+ \setminus \{j^*\}) - \rho(U_{k-1} \setminus \{j^*\}) \\ &\geq \sum_{l=1}^{k-1} \Big(\rho(U_l^+) - \rho(U_l)\Big) + \rho(U_k^+), \end{aligned}$$

where the inequality follows from (2) and $U_{k-1} \subseteq U_k^+$. This implies that (13) holds.

Assume that $\hat{d}_k = 0$ and $\overline{X}_{k-1} = \emptyset$. In this case,

$$\begin{aligned} h(d^*) &= \sum_{l=1}^{k-2} (\hat{d}_l - \hat{d}_{l+1})\rho(U_l) + \rho(U_{k-1} \setminus \{j^*\}) + (\hat{d}_{k-1} - 1) \cdot \rho(U_{k-1}) \\ h(d^* + \chi_X) &= \sum_{l=1}^{k-2} \Big(\rho(U_l^+) + (\hat{d}_l - \hat{d}_{l+1} - 1)\rho(U_l)\Big) \\ &\quad + \rho(U_{k-1}^+ \setminus \{j^*\}) + (\hat{d}_{k-1} - 1)\rho(U_{k-1}^+) + \rho(U_k^+). \end{aligned}$$

Hence, since $U_{k-1} = U_{k-1}^+$, we have

$$\begin{aligned} h(d^* + \chi_X) - h(d^*) &= \sum_{l=1}^{k-2} \Big(\rho(U_l^+) - \rho(U_l)\Big) + \rho(U_k^+) \\ &= \sum_{l=1}^{k-1} \Big(\rho(U_l^+) - \rho(U_l)\Big) + \rho(U_k^+). \end{aligned}$$

This completes the proof. □

Now we are ready to prove the main result of this paper.

Theorem 5. *We have*

$$\xi(X_1, d_1, \varphi_1) \leq 3 \cdot H_R \cdot \mathsf{OPT},$$

where $H_R := 1 + \frac{1}{2} + \cdots + \frac{1}{R}$.

Proof. It follows from Lemma 4 that

$$\forall t = 1, \ldots, R\colon\ \xi(X_t, d_t, \varphi_t) - \xi(X_{t+1}, d_{t+1}, \varphi_{t+1}) \leq \xi_{\mathsf{S}}(Y_t, P_t, \psi_t). \tag{14}$$

Thus, it follows from (14) that

$$\xi(X_1, d_1, \varphi_1) \leq \sum_{t=1}^{R} \xi_{\mathsf{S}}(Y_t, P_t, \psi_t). \tag{15}$$

Hence, it follows from (15) and Lemmas 3, 4 that

$$\xi(X_1, d_1, \varphi_1) \leq \sum_{t=1}^{R} 3 \cdot \mathsf{OPT}_{\mathsf{SLP}}(t) \leq 3 \cdot H_R \cdot \mathsf{OPT}_{\mathsf{LP}} \leq 3 \cdot H_R \cdot \mathsf{OPT}.$$

This completes the proof. □

4.1 Proof of Lemma 2

In this subsection, we prove Lemma 2. Our algorithm for $\mathsf{S}(H_t, K_t, \sigma_t)$ is essentially the same as the algorithm proposed by Du, Lu and Xu [3] except the following differences.

- The opening cost of a facility in H_t is zero.
- The connection cost between a client j in K_t and a facility i in $\sigma_t(j)$ is infinite.

For completeness, we reproduce the algorithm of Du, Lu and Xu [3] in our setting. Define a modified opening cost $\hat{f}_i$ for each facility i in F by

$$\hat{f}_i := \begin{cases} f_i & \text{if } i \in F \setminus H_t \\ 0 & \text{if } i \in H_t. \end{cases}$$

Define a modified connecting cost $\hat{c}_{i,j}$ for each facility i in F and each client j in K_t by

$$\hat{c}_{i,j} := \begin{cases} c_{i,j} & \text{if } i \in F \setminus \sigma(j) \\ \infty & \text{if } i \in \sigma(j). \end{cases}$$

Notice that connecting costs do not necessarily satisfy the triangle inequality.

Our algorithm consists of two phases. In the first phase, we use a concept of time δ. The algorithm starts at $\delta = 0$. Initially, we set $\alpha_j := 0$ for each client j in K_t and $\beta_{i,j} := 0$ for each facility j in $F \setminus H_t$ and each client j in K_t. Facilities i in F with $\hat{f}_i > 0$ are *closed*, and facilities i in F with $\hat{f}_i = 0$ are *open*. Every client in K_t is *unfrozen*. Let P be the set of penalized client, and set $P := \emptyset$.

The algorithm increases α_j for all unfrozen clients j in K_t uniformly at the unit rate δ, and declares the pair (i, j) of a facility i in F and a client j in K_t *tight*, if $\alpha_j = \hat{c}_{i,j}$. Once the pair (i, j) is tight, it increases $\beta_{i,j}$ at the same rate as α_j so that $\alpha_j - \beta_{i,j} = \hat{c}_{i,j}$ is satisfied. The algorithm keeps increasing δ until there exists no unfrozen client. As δ increases, the following events may occur.

Event 1. If

$$\sum_{j \in K_t} \beta_{i,j} = \hat{f}_i$$

for a closed facility i in F, then i is *temporarily open*. In addition, the algorithm *freezes* unfrozen clients j in K_t with $\beta_{i,j} > 0$ and we call i the *witness* for j.

Event 2. If $\alpha_j = \hat{c}_{i,j}$ for an open/temporarily open facility i and an unfrozen client j, then the algorithm freezes j and we call i the witness for j.

Event 3. If

$$\sum_{j \in X} \alpha_j = \rho_{K_t}(X)$$

for a subset X of K_t, then the algorithm freezes unfrozen clients in X and adds all elements in X to P.

If several events occur simultaneously, the algorithm executes them in an arbitrary order.

Next we explain the second phase. Denote by T the set of temporarily open facilities in F. Facilities i, i' in F are said to be *dependent*, if there exists a client j in K_t such that $\beta_{i,j} > 0$ and $\beta_{i',j} > 0$. In this phase, we first choose a maximal independent subset T' of T, and facilities in T' are open. Then, the algorithm outputs (Y_t, P_t, ψ_t) defined as follows.

- Define Y_t as the set of open facilities in $F \setminus H_t$.
- Define $P_t := P$.
- For each client j in $K_t \setminus P_t$, define $\psi_t(j)$ as an open facility i in F minimizing $\hat{c}_{i,j}$.

In the same as the proof of Lemma 3.1 of [3], we can prove that this algorithm can be implemented in polynomial time. Furthermore, since the pair (i, j) of a client j in K_t and a facility i in $\sigma_t(j)$ never be tight, (Y_t, P_t, ψ_t) is a feasible solution of $\mathsf{S}(H_t, K_t, \sigma_t)$.

From now on, we analyze an approximation ratio of the algorithm. In the same as Lemma 3.2 of [3], we can prove that during the algorithm's execution, we have

$$\sum_{j \in P} \alpha_j = \rho_{K_t}(P).$$

It follows from this observation that

$$\alpha_j \ (j \in K_t), \qquad \beta_{i,j} \ (i \in H_t \setminus K_t, \ j \in K_t) \tag{16}$$

obtained in the first phase is a feasible solution of (11), which implies

$$\sum_{j \in K_t} \alpha_j \leq \mathsf{OPT}_{\mathsf{SLP}}(t).$$

We denote by F_{op} the set of open facilities in F. For each client j in $K_t \setminus P_t$, we denote by $\mathsf{w}(j)$ the witness for j. For each open facility j in F, we denote by N_i the set of clients j in K_t with $\beta_{i,j} > 0$. Notice that $N_i \cap N_{i'}$ is empty for every distinct facilities i, i' in F_{op}. Define

$$\begin{aligned} D_{\mathsf{po}} &:= \{j \in K_t \setminus P_t \mid \exists i \in F_{\mathsf{op}} \colon \ j \in N_i\} \\ D_1 &:= \{j \in K_t \setminus (P_t \cup D_{\mathsf{po}}) \mid \mathsf{w}(j) \in F_{\mathsf{op}}\} \\ D_2 &:= K_t \setminus (P_t \cup D_{\mathsf{po}} \cup D_1). \end{aligned}$$

Now we prove

$$\sum_{j \in K_t \setminus P_t} c_{\psi_t(j),j} = \sum_{j \in K_t \setminus P_t} \hat{c}_{\psi_t(j),j} \leq \sum_{i \in F_{\mathsf{op}}} \sum_{j \in N_i \setminus P_t} \hat{c}_{i,j} + \sum_{j \in D_1} \alpha_j + \sum_{j \in D_2} 3 \cdot \alpha_j. \tag{17}$$

The first inequality follows from the fact that no client j in K_t is not connected to facilities in $\sigma(j)$. For proving the second inequality, we consider the following three cases.

Case 1. We first consider the connecting costs for clients in D_{po}. For each client j in D_{po}, we denote by $\mathsf{p}(j)$ the unique facility i in F_{op} with $j \in N_i$. We have

$$\sum_{j \in D_{\mathsf{po}}} \hat{c}_{\psi_t(j),j} \leq \sum_{j \in D_{\mathsf{po}}} \hat{c}_{\mathsf{p}(j),j} \leq \sum_{i \in F_{\mathsf{op}}} \sum_{j \in N_i} \hat{c}_{i,j}.$$

Case 2. Next we consider the connecting cost for a client j in D_1. Since $\mathsf{w}(j)$ is open and j is not in D_{po}, we have $\beta_{\mathsf{w}(j),j} = 0$. This implies that the event 2 occurred when the algorithm froze j, i.e., $\alpha_j = \hat{c}_{\mathsf{w}(j),j}$. Thus, since $\mathsf{w}(j)$ is open, we have

$$\sum_{j \in D_1} \hat{c}_{\psi_t(j),j} \leq \sum_{j \in D_1} \hat{c}_{\mathsf{w}(j),j} = \sum_{j \in D_1} \alpha_j.$$

Case 3. Here we consider the connecting cost for a client j in D_2. Define $i := \mathsf{w}(j)$. In this case, there exist an open facility i' in F and a client j' in K_t such that $\beta_{i,j'} > 0$ and $\beta_{i',j'} > 0$. Since $\beta_{i,j'}$ and $\beta_{i',j'}$ are positive, i and i' are not in H_t. This implies that the triangle inequality holds for $\hat{c}_{i,j}$, $\hat{c}_{i',j}$, $\hat{c}_{i,j'}$ and $\hat{c}_{i',j'}$. Let t_i and $t_{i'}$ be the times at which i and i' are temporarily open, respectively. In addition, the following facts immediately follow.

- Since i is the witness for j, we have $\alpha_j \geq t_i$ and $\alpha_j \geq \hat{c}_{i,j}$.
- Since the pairs (i, j') and (i', j') are tight, we have $\alpha_{j'} \geq \hat{c}_{i,j'}$ and $\alpha_{j'} \geq \hat{c}_{i',j'}$.
- Since j' is frozen earlier than the time $\min\{t_i, t_{i'}\}$, we have $\alpha_{j'} \leq \min\{t_i, t_{i'}\}$.

It follows from these facts and the triangle inequality that

$$\hat{c}_{\psi_t(j),j} \leq \hat{c}_{i',j} \leq \hat{c}_{i,j} + \hat{c}_{i,j'} + \hat{c}_{i',j'} \leq 2\alpha_{j'} + \alpha_j \leq 3\alpha_j.$$

Hence, we have

$$\sum_{j \in D_2} \hat{c}_{\psi_t(j),j} \leq \sum_{j \in D_2} 3 \cdot \alpha_j,$$

which completes the proof of (17).

Since $N_i \cap N_{i'}$ is empty for every distinct facilities i, i' in F_{op}, we have

$$\sum_{i \in Y_t} f_i = \sum_{i \in F_{\mathsf{op}}} \sum_{j \in N_i} \beta_{i,j}$$

In addition, we have

$$\begin{aligned}\sum_{i \in F_{\mathsf{op}}} \sum_{j \in N_i} \beta_{i,j} + \sum_{i \in F_{\mathsf{op}}} \sum_{j \in N_i \setminus P_t} \hat{c}_{i,j} &\leq \sum_{i \in F_{\mathsf{op}}} \sum_{j \in N_i} (\beta_{i,j} + \hat{c}_{i,j}) \\ &\leq \sum_{j \in D_{\mathsf{po}}} \alpha_j + \sum_{j \in P_t} \alpha_j.\end{aligned}$$

It follows from these observations and (17) that

$$\begin{aligned}\sum_{i\in Y_t} f_i + \sum_{j\in K_t\setminus P_t} c_{\psi_t(j),j} + \rho_{K_t}(P_t) &\le \sum_{j\in D_{\mathsf{po}}} \alpha_j + \sum_{j\in D_1} \alpha_j + \sum_{j\in D_2} 3\cdot\alpha_j + \sum_{j\in P_t} 2\cdot\alpha_j \\ &\le 3\sum_{j\in K_t} \alpha_j \\ &\le 3\cdot \mathsf{OPT}_{\mathsf{SLP}}(t).\end{aligned}$$

This completes the proof.

5 Conclusion

In this paper, we introduced the fault-tolerant facility location problem with submodular penalties, and presented a combinatorial $3 \cdot H_R$-approximation algorithm, where R is the maximum connectivity requirement. One direction of future work is to improve an approximation ratio. To discern whether we can extend a constant approximation algorithm for the fault-tolerant facility location problem to our problem is interesting. Another direction is to generalize a penalty function. In discrete convex analysis, it is known that the Lovász extensions of submodular functions coincide with polyhedral L-convex functions that are positively homogenous (see [10] for discrete convex analysis). Thus, it is interesting to consider the problem in which the Lovász extension is replaced by a more general discrete convex function.

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