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Symbolic-numeric hybrid optimization for plant/controller integrated design in $\mathcal{H}_\infty$ loop-shaping design

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Abstract. This paper proposes a plant/controller design integration method for $\mathcal{H}_\infty$ loop-shaping design based on symbolic-numeric hybrid optimization. This approach firstly employs parametric polynomial spectral factorization to accomplish parametric optimization and derive an expression for the optimal cost. Owing to the obtained expression, sensitivity analysis of the achievable performance level with respect to plant parameters is amenable, which allows numerical optimization methods to seek the optimal set of parameter values.

Keywords. $\mathcal{H}_\infty$ loop-shaping design, plant/controller design integration, symbolic-numeric hybrid optimization, parametric polynomial spectral factorization

1. Introduction

Complicated systems of today heavily rely on computers so as to achieve shorter lead time in the development process and also higher performance and efficiency during operation. When systems involve dynamics, control of such dynamics is one of crucial factors that determine the performance of the systems. Micro computers are the key ingredient to achieve good control, but the increased computational power and the development of design algorithms are indispensable for control system design as well.

Modern control design formulates control problems as optimization problems. Cost functions typically employed involve system norms such as the $\mathcal{H}_2$-norm and the $\mathcal{H}_\infty$-norm. The use of these norms is motivated by the facts that they are easy to deal with and that they admit practical engineering interpretations. Solution approaches have been developed for $\mathcal{H}_2$ control and $\mathcal{H}_\infty$ control, and they usually involve solution of Riccati equations [15] and optimization based on linear matrix inequalities (LMIs) [2], which are solved numerically on computers.

Ordinary solution approaches for control problems however assume that dynamical systems to be controlled (*plants*) are given in advance (e.g., already designed and fixed). This assumption is not necessarily satisfactory in that the plant may not be designed appropriately for control purposes and, consequently, that even the best control strategy may only achieve mediocre performance. This motivates plant/controller design integration, an approach that designs both the plant and the controller simultaneously [4, 9].

Conventional solution approaches are not suited for integrated design. Numerical solution of algebraic Riccati equations requires that all the plant data should be given as numbers and the LMI-based optimization formulation yields bilinear matrix inequalities (BMIs) which are not convex problems and tend to be hard to solve [8].

In order to resolve the computational issue, the authors have proposed a symbolic-numeric hybrid optimization approach for $\mathcal{H}_2$ control problems [10]. The key technique is parametric polynomial spectral factorization [1] that exploits the power of algebraic methods, which allows solution of parametric Riccati equations. By means of parametric polynomial spectral factorization, the optimal value of the cost function in $\mathcal{H}_2$ control can be expressed in terms of plant parameters in an algebraic manner. As the consequence of this parametric optimization, one can employ numerical optimization techniques and optimize the optimal cost with respect to plant parameters, obtaining the optimal controller for the plant that is designed to be easy to control. Although the integrated design problem is nonconvex, the proposed approach can achieve local optima.

This paper attempts to extend this symbolic-numeric hybrid optimization approach to $\mathcal{H}_\infty$ control problems. The main difference, which is the cause of the difficulty, is the way the optimal cost is characterized. In the $\mathcal{H}_2$ control case, the optimal cost can be expressed as a rational function in plant parameters and the *Sum of Roots*, a quantity introduced in polynomial spectral factorization by means of algebraic approaches and characterized in an algebraic manner. It is thus straightforward to carry out analysis of the sensitivity of the optimal cost with respect to parameters, allowing orthodox optimization methods such as Newton's method to be utilized. The solution of $\mathcal{H}_\infty$ control is typically more involved, and one has to have the optimal cost as a parameter and change its value iteratively to see whether associated equations have feasible solutions. Here, instead of striving for general $\mathcal{H}_\infty$ control problems, solu-

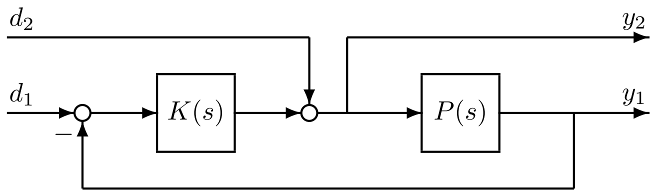

Figure 1: $\mathcal{H}_\infty$ loop-shaping design formulation.

tion of a special class of $\mathcal{H}_\infty$ control, known as $\mathcal{H}_\infty$ loop-shaping design, is aimed at. The reason is twofold. Firstly, the optimal cost in this formulation is expressed as the largest real eigenvalue of a matrix and thus can be characterized algebraically. Secondly, this formulation more often than not helps to design satisfactory controllers in practice.

The rest of the paper is organized as follows. In Section 2, the problem formulation of $\mathcal{H}_\infty$ loop-shaping design and its solution approach are reviewed. Section 3 reviews parametric polynomial spectral factorization based on algebraic methods and its extension to solution of Riccati equations. In Section 4, the integrated $\mathcal{H}_\infty$ loop-shaping design problem is formulated, and a symbolic-numeric hybrid optimization solution approach is proposed. The approach is demonstrated on a design example in Section 5. Some concluding remarks are made in Section 6.

2. $\mathcal{H}_\infty$ Loop-shaping Design

The $\mathcal{H}_\infty$ loop-shaping design problem is formulated in the following way [12]. In the feedback configuration in Figure 1, given a plant $P(s)$, the task is to find, from the set $\mathcal{K}_s$ of all stabilizing real rational controllers, a controller $K(s)$ that minimizes the $\mathcal{H}_\infty$-norm of the transfer function matrix from $(d_1\ d_2)^T$ to $(y_1\ y_2)^T$. An explicit expression for the optimal cost

$$\gamma_{opt} := \inf_{K\in\mathcal{K}_s} \left\| \begin{bmatrix} I \\ K \end{bmatrix} (I+PK)^{-1} \begin{bmatrix} I & P \end{bmatrix} \right\|_\infty$$

can be computed by solving Riccati and Lyapunov equations.

Let $P(s)$ be given in minimal realization state-space representation

$$P(s) = \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right] .$$

Then,

$$\gamma_{opt} = \sqrt{1+\lambda_{max}(XY)} = \frac{1}{\sqrt{1-\lambda_{max}(YQ)}} , \qquad (1)$$

where X and Y are the unique stabilizing solutions of the Riccati equations

$$\begin{aligned} A^T X + XA - XBB^T X + C^T C = 0 , \\ AY + YA^T - YC^T CY + BB^T = 0 , \end{aligned} \qquad (2)$$

respectively, and Q is the unique solution of the Lyapunov equation

$$Q(A - YC^T C) + (A - YC^T C)^T Q + C^T C = 0 . \qquad (3)$$

These matrices are related as

$$Q = (I + XY)^{-1} X .$$

It is noted that Riccati equations are nonlinear equations while Lyapunov equations are linear equations and easier to solve.

It is known that finding the optimal controllers is numerically and theoretically complicated, and obtaining suboptimal controllers which are easier to find is usually sufficient. For any $\gamma > \gamma_{opt}$, a controller achieving

$$\left\| \begin{bmatrix} I \\ K \end{bmatrix} (I+PK)^{-1} \begin{bmatrix} I & P \end{bmatrix} \right\|_\infty < \gamma$$

is given by

$$K(s) = \left[\begin{array}{c|c} A - BB^T X_\infty - YC^T C & YC^T \\ \hline B^T X_\infty & 0 \end{array}\right] ,$$

where

$$X_\infty = \frac{\gamma^2}{\gamma^2-1} Q \Big(I - \frac{\gamma^2}{\gamma^2-1} YQ \Big)^{-1} .$$

The problem setting and the solution are in the modern control framework, but the achievable performance level γ_{opt} admits classical control interpretations and gives lower bounds for gain and phase margins. See [14] for details.

3. Parametric Polynomial Spectral Factorization and Solution of Riccati Equations

The task of polynomial spectral factorization is stated as follows. Given a $2n$-th order even polynomial $f(s)$ in s with no roots on the imaginary axis,

$$f(s) = s^{2n} + a_{2n-2}s^{2n-2} + \cdots + a_2 s^2 + a_0 , \qquad (4)$$

the task is to find a unique polynomial

$$g(s) = s^n + \sigma s^{n-1} + b_{n-2}s^{n-2} + \cdots + b_0 \qquad (5)$$

that satisfies the relationship

$$f(s) = (-1)^n g(s) g(-s) \qquad (6)$$

and moreover has roots in the open left half plane only (i.e., is stable). The polynomial $g(s)$ that is sought is called the spectral factor.

A number of numerical approaches have been developed [6, 13], but such numerical approaches are not applicable for parametric polynomial spectral factorization. Recently, an algebraic approach was developed by the authors [1], which can be utilized for the parametric case where a_i in (4) are expressed as polynomials/rational functions of parameters. More specifically, the problem of polynomial spectral factorization reduces to finding the algebraic relationship between parameters and a quantity σ called the

Sum of Roots (SoR). Then the coefficients of the spectral factor $g(s)$ are expressed in polynomial form in the SoR and rational form in parameters. The following theorem is the basis of the algebraic approach.

Theorem 1 ([1]). *Given $f(s)$ and $g(s)$ as in* (4) *and* (5), *respectively, consider σ, b_i, $i = 0, \ldots, n-2$, as variables. A system of algebraic equations in terms of σ and b_i's is obtained by comparing the coefficients of* (6). *Then the set $\mathcal{G}$ of the polynomials obtained from the polynomial parts of the equations forms the reduced Gröbner basis of the ideal generated by itself with respect to the graded reverse lexicographic order $\sigma \succ b_{n-2} \succ \cdots \succ b_0$. Moreover, in the generic case, σ is a separating element, and one can get a special Gröbner basis called* shape basis *with respect to any elimination ordering $\{b_0, \ldots, b_{n-2}\} \succ\succ \sigma$:*

$$\left\{ S_f(\sigma),\, b_{n-2} - h_{n-2}(\sigma),\, \ldots,\, b_0 - h_0(\sigma) \right\} ,$$

where S_f is a polynomial of degree exactly 2^n and h_i's are polynomials of degree strictly less than 2^n.

For Gröbner bases and associated ideas such as the graded reverse lexicographic order, readers are referred to standard textbooks, e.g., [5].

This fact allows the shape basis to be effectively obtained by means of the basis conversion (change-of-order) technique [5, 7], since all is needed is a conversion from a particular Gröbner basis to another Gröbner basis. Once the shape basis is computed, it is straightforward to compute the spectral factor, and one has to find the largest real root of $S_f(\sigma)$ and then substitute it into $h_i(\sigma)$ [1]. In the parametric case, S_f is a polynomial in σ and parameters, while h_i's are polynomials in σ but in general rational functions in parameters. The result indicates that all the coefficients of the spectral factor can be related with the SoR and parameters in an algebraic manner.

On solving Riccati equation (2), it is customary to consider the so-called Hamiltonian matrix

$$H = \begin{bmatrix} A^{\mathrm{T}} & -C^{\mathrm{T}}C \\ -BB^{\mathrm{T}} & -A \end{bmatrix}$$

that is associated to it [15]. The eigenvalues of H are located symmetrically about the real and imaginary axes, with no eigenvalues being on the imaginary axis. Namely, the characteristic polynomial of H, $\det(sI - H)$, satisfies the condition for $f(s)$ in (4). Then, polynomial spectral factorization is executed for the characteristic polynomial. It is shown [11] that the solution Y of (2) is expressed as rational functions in the coefficients of the spectral factor, and, consequently, in the SoR. In this way, Riccati equations with parameters can be solved.

4. Proposed Approach

4.1. Plant/Controller Integrated Design Problem

When the plant does not contain any parameters, there is an established method for optimal design where two Riccati equations, or a Riccati equation and a Lyapunov equation, are solved numerically. If the plant has some parameters that can be adjusted, one may wish to tune these parameters so that the plant may be easier to control and, consequently, a better closed-loop system with the optimally designed controller may be achieved. Namely, not only is optimal controller design aimed at, but also freedom in the plant is exploited, leading to preferable overall design. This paper considers the latter control design problem.

Assume that the plant have some parameters that can be tuned and that, given some values for these parameters, optimal control design is always possible. The task considered in this paper is to find the smallest value of the optimal cost γ_{opt} (i.e., find the *best of the best*) and parameter values that achieve this. More specifically, let $\boldsymbol{q} = (q_i)$ be the parameter vector and denote its feasible set by $\mathcal{Q}$. The task is to find

$$\inf_{\boldsymbol{q}\in\mathcal{Q}} \gamma_{\mathrm{opt}} = \inf_{\boldsymbol{q}\in\mathcal{Q}} \inf_{K\in\mathcal{K}_{\mathrm{s}}} \left\| \begin{bmatrix} I \\ K \end{bmatrix} (I+PK)^{-1} \begin{bmatrix} I & P \end{bmatrix} \right\|_{\infty}$$

and $\boldsymbol{q}_{\mathrm{opt}}$ that achieves this. Since both the plant and the controller are optimized, this problem is a *plant/controller design integration problem.* It is noted that this is in general a non-convex optimization problem and that there may be many local optima. It is difficult to find the global optimum with guarantee, and the proposed approach only attempts to find a local minimum. Also, parameters q_i are design parameters and thus can be chosen, i.e., they are not uncertain parameters as considered in the robust control setting. It is a standing assumption that parameters enter the plant coefficients in polynomial/rational function form.

As reviewed in Section 2, γ_{opt} is expressed in terms of the largest eigenvalue $\lambda_{\max}(XY)$ of XY (or the largest eigenvalue $\lambda_{\max}(YQ)$ of YQ) as in (1). Minimization of γ_{opt} is equivalent to minimizing $\lambda_{\max}(XY)$ (or $\lambda_{\max}(YQ)$). In order to simplify the computation, the approach proposed here minimizes $\lambda_{\max}(YQ)$ instead of γ_{opt}.

4.2. Outline of the Approach

The suggested approach is first outlined. The basic strategy is reminiscent of that of the approach [10] proposed by the authors for $\mathcal{H}_2$ control.

1. Solve Riccati equation (2), keeping parameters as they are, by means of parametric polynomial spectral factorization [1] and its extension to solution to Riccati equations [11]. Further solve Lyapunov equation (3) symbolically.

2. Based on the algebraic relationship obtained in the previous step, compute the sensitivity of $\lambda_{\max}(YQ)$ with respect to plant parameters.

3. Using the sensitivity computed in the previous step, compute the minimal value of $\lambda_{\max}(YQ)$ and parameter values that achieve this by means of typical numerical optimization approaches such as the steepest descent method and Newton's method [3].

It is noted that Steps 1 and 2 are executed symbolically and that Step 3 involves numerical optimization. In this way, a symbolic-numeric hybrid approach is realized. Step 1 can be executed by the approaches referenced above. Step 2 is the crucial step in the proposed approach and is described in detail in the next subsection. Step 3 may seem nothing but a simple application of optimization techniques of textbook level, which is partially the case. It is, however, emphasized that this only becomes possible because of the result in Step 2 which allows direct sensitivity analysis of the optimal cost with respect to parameters by exploiting the power of algebraic methods.

4.3. Sensitivity Analysis Step

The crucial difference between the $\mathcal{H}_2$ control problem [10] and the $\mathcal{H}_\infty$ loop-shaping design problem considered here is the way the optimal costs are expressed in terms of parameters and the SoR. The optimal cost in the $\mathcal{H}_2$ control case is expressed as a rational function in terms of the elements of the solution matrices of Riccati equations, whereas the optimal cost in $\mathcal{H}_\infty$ loop-shaping design is characterized as the largest real eigenvalue of the solution matrices of Riccati/Lyapunov equations, which complicates analysis of the sensitivity of the optimal cost to parameters. One can nevertheless employ the implicit function theorem for this purpose.

In fact, when executing parametric polynomial spectral factorization, the result is not expressed in explicit form in terms of parameters. Instead, what is obtained is an algebraic relationship where the SoR plays a role of connecting parameters and the result. Therefore, for the $\mathcal{H}_2$ control case, the implicit function theorem is indeed used to compute the partial derivatives of the SoR with respect to parameters and further the partial derivatives of the optimal cost with respect to parameters. In $\mathcal{H}_\infty$ loop-shaping design, what one has to do is to use the implicit function theorem repeatedly.

More specifically, parametric polynomial spectral factorization in Step 1 relates the parameters $\boldsymbol{q}$ and the SoR σ algebraically:

$$S_f(\sigma;\boldsymbol{q}) = 0 \ .$$

Further, the solution Y of Riccati equation (2) is expressed in $\boldsymbol{q}$ and σ. By symbolically solving Lyapunov equation (3), its solution Q is obtained in terms of $\boldsymbol{q}$, σ and the elements of Y (which are expressed in $\boldsymbol{q}$ and σ). By using these solution and the characteristic equation of the matrix YQ, $\det(\lambda I - YQ) = 0$, the largest real eigenvalue $\lambda_{\max}(YQ)$ is related to $\boldsymbol{q}$ and σ in an algebraic manner.

With the above preparation, the sensitivity of $\lambda_{\max}(YQ)$ to the parameters $\boldsymbol{q}$ can be analysed in the following way. Firstly, compute the partial derivative of $S_f(\sigma;\boldsymbol{q}) = 0$ with respect to q_i, regarding σ as a function of $\boldsymbol{q}$. The resulting expression involves σ, $\boldsymbol{q}$ and $\frac{\partial\sigma}{\partial q_i}$, In fact, it is linear in $\frac{\partial\sigma}{\partial q_i}$, and one can solve it for $\frac{\partial\sigma}{\partial q_i}$, i.e., $\frac{\partial\sigma}{\partial q_i}$ is expressed explicitly in terms of $\boldsymbol{q}$ and σ.

In a similar fashion, one can carry out sensitivity analysis of $\lambda_{\max}(YQ)$ to $\boldsymbol{q}$, which allows one to perform plant/controller integrated design for $\mathcal{H}_\infty$ loop-shaping design. Again partial differentiating $\det(\lambda I - YQ) = 0$ in terms of q_i, regarding λ and σ as a function of $\boldsymbol{q}$ this time, one will get a linear equation for $\frac{\partial\lambda}{\partial q_i}$ and thus obtain an expression for $\frac{\partial\lambda}{\partial q_i}$ in terms of $\boldsymbol{q}$, σ, $\frac{\partial\sigma}{\partial q_i}$ and λ, which are all computable once $\boldsymbol{q}$ is fixed. If one further computes the partial derivative of $\frac{\partial\lambda}{\partial q_i}$, expressions for $\frac{\partial^2\lambda}{\partial q_i^2}$ and other 2nd order partial derivatives of λ can be computed. Those expressions enable the steepest descent method and Newton's method to be performed.

5. Design Example

In this section, the proposed approach is applied to the following plant with two parameters, q_1 and q_2, to demonstrate the approach:

$$P(s;q_1,q_2) = \frac{q_2(s-q_1)}{s^2(s-3)} = \left[\begin{array}{ccc|c} 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_2 \\ 0 & 0 & 0 & -q_1q_2 \\ \hline 1 & 0 & 0 & 0 \end{array}\right] ,$$

$$\mathcal{Q} = \left\{(q_1,q_2) \;\middle|\; 0.1 \le q_1 \le 1,\ 2 \le q_2 \le 4\right\} .$$

The task is to find parameter values that minimize $\lambda_{\max}(YQ)$, which is essentially equivalent to γ_{opt}.

Firstly, parameters q_1, q_2 and $\lambda_{\max}(YQ)$ are algebraically related. The solution of (2) can be written as

$$Y = \begin{bmatrix} \sigma+3 & b_1 & b_0 \\ b_1 & b_1\sigma - b_0 & b_0\sigma \\ b_0 & b_0\sigma & Y_{3,3} \end{bmatrix} ,$$
$$Y_{3,3} = b_0\sigma^2 - b_0b_1 - 9b_0 + q_1q_2^2 ,$$

where σ, b_1 and b_0 are obtained from parametric polynomial spectral factorization. The SoR σ is the largest real root of

$$\begin{aligned} S_f(\sigma;q_1,q_2) = \sigma^8 - 36\sigma^6 &+ \left(-8q_2^2 + 486\right)\sigma^4 \\ &+ \left(-64q_1^2q_2^2 + 144q_2^2 - 2916\right)\sigma^2 \\ &+ 16q_2^4 - 648q_2^2 + 6561 , \end{aligned}$$

and b_1 and b_0 are expressed in closed-form in σ and q_i:

$$b_1 = \frac{1}{2}\left(\sigma^2 - 9\right) , \tag{7}$$
$$\begin{aligned} b_0 = &\frac{\sigma}{8\left(4q_2^2 - 81\right)} \\ &\times \left\{\sigma^6 - 36\sigma^4 + \left(-4q_2^2 + 405\right)\sigma^2 \right. \\ &\left. + \left(-64q_1^2q_2^2 + 72q_2^2 - 1458\right)\right\} . \end{aligned} \tag{8}$$

Lyapunov equation (3) yields a set of linear equations in the elements of Q, and it is thus straightforward to solve it symbolically:

$$Q = \frac{1}{2b_0(b_1\sigma - b_0)} \begin{bmatrix} b_0b_1 & 0 & -b_0 \\ 0 & b_0 & 0 \\ -b_0 & 0 & \sigma \end{bmatrix} .$$

From (7) and (8), expressions for the elements of Q are obtained as rational functions in σ and q_i.

Using the obtained expressions for Y and Q, one can compute the characteristic polynomial of YQ:

$$\begin{aligned}\det(\lambda I - YQ) = \lambda^3 \\ + \big(3b_0^2 - b_0b_1\sigma - 3b_0b_1 - b_0\sigma^3 + 9b_0\sigma - q_1q_2^2\sigma\big)\lambda^2 \\ + b_0\big(3b_0^3 - 3b_0^2\sigma^3 - 3b_0^2\sigma^2 + 18b_0^2\sigma + 27b_0^2 - b_0b_1^3 \\ - b_0b_1^2\sigma^2 + 2b_0b_1\sigma^4 + 3b_0b_1\sigma^3 - 18b_0b_1\sigma^2 - 27b_0b_1\sigma \\ - 2q_1q_2^2b_0\sigma - 3q_1q_2^2b_0 + 2q_1q_2^2b_1\sigma^2 + 3q_1q_2^2b_1\sigma\big)\lambda \\ - b_0^2(b_1\sigma - b_0)\big(b_0^3 + 2b_0^2b_1\sigma + 3b_0^2b_1 - 2b_0^2\sigma^3 - 6b_0^2\sigma^2 \\ + 9b_0^2\sigma + 27b_0^2 + b_0b_1^3 - 2b_0b_1^2\sigma^2 - 3b_0b_1^2\sigma + 9b_0b_1^2 \\ + b_0b_1\sigma^4 + 3b_0b_1\sigma^3 - 9b_0b_1\sigma^2 - 27b_0b_1\sigma - q_1q_2^2b_0\sigma \\ - 3q_1q_2^2b_0 - q_1q_2^2b_1^2 + q_1q_2^2b_1\sigma^2 + 3q_1q_2^2b_1\sigma\big) ,\end{aligned}$$

which is a 3rd order polynomial in λ. By using (7) and (8) and clearing the denominators, its coefficients are expressed as polynomials in σ and q_i. The quantity to be sought, namely, $\lambda_{\max}(YQ)$, is the largest real root of this polynomial.

Using the relationship $S_f(\sigma; q_1, q_2) = 0$, one can compute the partial derivatives of σ, $\frac{\partial\sigma}{\partial q_i}$, by way of the implicit function theorem. Moreover, from $\det(\lambda I - YQ) = 0$, the sensitivity of $\lambda_{\max}(YQ)$ to changes in parameters q_i can be analysed. For instance, at $(q_1, q_2) = (0.4, 3)$, they are computed as

$$\begin{aligned}\frac{\partial}{\partial q_1}\lambda_{\max}(YQ) &= 0.002033515157 ,\\ \frac{\partial}{\partial q_2}\lambda_{\max}(YQ) &= -0.0001402638544 ,\\ \frac{\partial^2}{\partial q_1^2}\lambda_{\max}(YQ) &= 0.01037328705 ,\\ \frac{\partial^2}{\partial q_1\partial q_2}\lambda_{\max}(YQ) &= -0.001604806724 ,\\ \frac{\partial^2}{\partial q_2^2}\lambda_{\max}(YQ) &= 0.0005480235616 .\end{aligned}$$

Furthermore, starting from $(q_1, q_2) = (0.4, 3)$, Newton's method computes the optimal value

$$\begin{aligned}\lambda_{\max}(XY) &= 0.9972422498 ,\\ \boldsymbol{q}_{\mathrm{opt}} &= (0.27004, 2.7002) ,\end{aligned}$$

after 10 iterations (Figure 2).

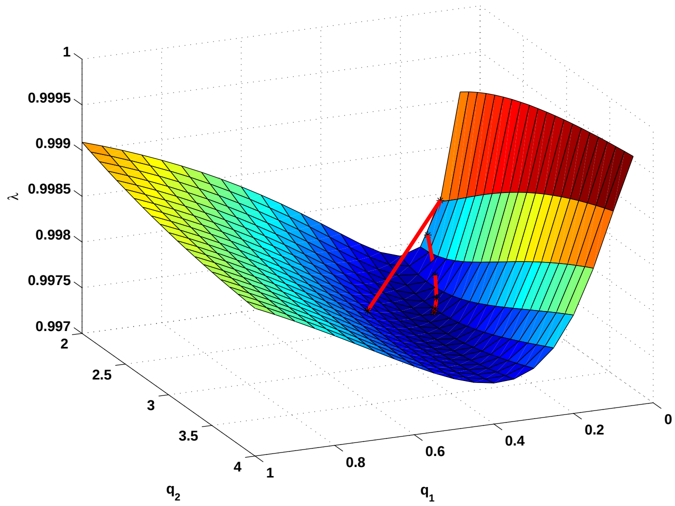

Figure 2: Optimization result by Newton's method.

6. Concluding Remarks

This paper has established a symbolic-numeric hybrid optimization approach for plant/controller design integration in $\mathcal{H}_\infty$ loop-shaping design. The approach first carries out parametric optimization and obtains an expression for the optimal cost with parameters by way of parametric polynomial spectral factorization. The result is then exploited to analyse the sensitivity of the optimal cost to parameters. Numerical optimization is thus amenable. As a consequence, the approach finds a pair of a plant and a controller that minimizes the cost function, achieving the best of the best design.

Further work includes its extension to more general $\mathcal{H}_\infty$ control problems, where the characterization of the optimal cost is more involved. A method of effectively characterizing the optimal cost may be a crucial step.

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