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GIBBS DIFFERENTIATIONS OF p-ADIC HARMONIZABLE PROCESSES AND THEIR APPLICATIONS TO LINEAR p-ADIC SYSTEMS

By

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Abstract

One of the aims of this paper is to give a sufficient conidtion for sample Gibbs differentiability of p-adic harmonizable processes, which are represented by stochastic integrals of the Chrestenson functions with respect to random measures. Other aims are to apply the result to linear Gibbs differential equations, which model p-adic linear systems, and to express their solutions explicitly for driving functions of p-adic harmonizable processes.

1. Introduction

In the previous paper (Endow 1995) we discuss the use of Gibbs derivatives in linear system theory. The quadratic mean and sample Gibbs differentiability conditions of dyadic and p-adic stationary processes are shown by (Endow 1989) and (Endow 1995), respectively. He also presented at EMCSR'96 the conditions of sample Gibbs differentiation of dyadic harmonizable processes which are not necessarily stationary (Endow 1996). In this paper we will extend these results to p-adic harmonizable processes, which include p-adic stationary processes in the special case, and give explicit solutions to linear Gibbs differential equations which are moldels of p-adic linear systems (Endow 1996).

Let $\{X(t); t \in \mathbf{R}_+ := [0,\infty)\}$ be a p-adic harmonizable process with the representation

$$X(t) = \int_0^\infty \psi(t,x)\xi(dx), \tag{1}$$

where the integral above is understood in the q.m. sense, ψ denotes the Chrestenson function (Chrestenson 1955, Selfridge 1955) that is a generalization of Walsh function, and ξ is a random measure not necessarily orthogonal. Its covariance function is also expressed by

$$r_X(t,u) := \mathrm{Cov}(X(t), X(u)) = \int_0^\infty \int_0^\infty \psi(t,x)\overline{\psi(u,y)}F(dx,dy), \tag{2}$$

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where $F(dx,dy) = \mathbf{E}\xi(dx)\overline{\xi(dy)}$ is a complex-valued measure of bounded variation over $\mathbf{R}_+^2$ and is called the two dimensional spectral distribution function of the process. We note that a class of p-adic (harmonizable) stationary processes is included in the class of p-adic harmonizable processes. In fact, if the whole mass of the spectral distribution function is real and positive, and concentrates on the diagonal line $x = y$, then the corresponding harmonizable process is p-adic stationary. In the stationary case ξ becomes othogonal and $F(dx) = \mathbf{E}|\xi(dx)|^2$. The covariance function thus reduces to

$$r_X(t,u) = \int_0^\infty \psi(t,x)\overline{\psi(u,x)}F(dx). \tag{3}$$

The harmonizability of p-adic stationary processes was discussed by (Endow 1994).

Now we briefly review p-adic continuity and Gibbs differentiation of a function f defined on $\mathbf{R}_+$. A function f is called p-adic continuous at $t \in \mathbf{R}_+$, if for any $\varepsilon > 0$ there exists some $\delta > 0$ such that

$$|t \oplus u| < \delta \Rightarrow \ |f(t) - f(u)| < \varepsilon,$$

where $\oplus$ means p-adic addition. If the sum

$$\Delta_N f(t) = \sum_{k=-N}^{N} p^k \sum_{j=1}^{p-1} A_j f(t \oplus jp^{-k-1})$$

with $A_0 = (p-1)/2,\ A_j = \theta^j/(1-\theta^j),\ j = 1,2,\cdots,p-1$, has a finite limit as $N \to \infty$, then f is called pointwise Gibbs differentiable at $t \in \mathbf{R}_+$, where $\theta = \exp\{2\pi\sqrt{-1}/p\}$ (c.f. Butzer-Wagner 1973, Weiyi 1991). Note that "dyadic" is a special case of $p = 2$. In the following we shall sometimes omit the adjective "p-adic" if it is clear from the context.

For $f \in L^r(\mathbf{R}_+)$, $1 \le r < \infty$, the L^r-limit of $\Delta_N f$, if it exists, is called the L^r-strong Gibbs derivative. It follows from definition that the Chrestenson functions are Gibbs differentiable in both senses, and for $k \in \mathbf{N}_0 := \{0,1,2,\cdots\}$,

$$D^k\psi(t,x) = x^k\psi(t,x) \quad (t \in \mathbf{R}_+), \tag{4}$$

which shows that the Chrestenson functions are the eigenfunctions of the Gibbs differential operator. Hence it will be shown (c.f. Weiyi 1991) that if $f,\ D^k f \in L^r(\mathbf{R}_+)$; $k \in \mathbf{N}_0$, $1 \le r < \infty$, then

$$(\widehat{D^k f})(x) = x^k \widehat{f}(x) \ a.e., \tag{5}$$

where $\widehat{f}$ denotes the Chrestenson-Fourier transform.

The convolution theorem is also hold for the Chrestenson-Fourier transform; For $f, g \in L^1(\mathbf{R}_+)$

$$(\widehat{f * g})(t) = \widehat{f}(x)\widehat{g}(x) \quad (x \in \mathbf{R}_+), \tag{6}$$

where $f * g$ denotes the p-adic convolution of f and g, and is defined by

$$f * g := \int_0^\infty f(s)g(t \oplus s)ds. \tag{7}$$

In the p-adic group world the Chrestenson functions and Gibbs differentiation play similar roles to the Euler functions and the Newton-Leibniz differentiation in the real world. It is well known that differentiable functions are continuous in the real world. However, in the p-adic group world Gibbs differentiable functions are not p-adic continuous, nor are p-adic continuous functions Gibbs differentiable.

2. Gibbs differentiations of stohastic processes

Let $\{X(t); t \in \mathbf{R}_+\}$ be a process. If the sum

$$\Delta_N X(t) = \sum_{k=-N}^{N} p^k \sum_{j=0}^{p-1} A_j X(t \oplus jp^{-k-1})$$

converges almost surely as $N \to \infty$, then the process $X(t)$ is sample Gibbs differentiable at t and the limit denoted by $DX(t)$ is called the sample Gibbs derivative of the process. The higher order derivatives are defined similarly by induction. For a second order process $X(t)$, if the mean square limit of $\Delta_N X(t)$ exists as $N \to \infty$, then the process is quadratic mean (q.m.) Gibbs differentiable at t, and the limit denoted by $D^{[1]}X(t)$ is called the q.m. Gibbs derivative of the process.

Now we will consider q.m. Gibbs differentiation of p-adic harmonizable processes.

THEOREM 2.1. *Let $X(t)$ be a p-adic harmonizable process with representations* (1) *and* (2). *If it is satisfied that for $r \in \mathbf{N}_0$*

$$\int_0^\infty \int_0^\infty (xy)^r |F(dx, dy)| < \infty, \tag{8}$$

then the process is r-times q.m. Gibbs differentiable and

$$D^{[r]}X(t) = \int_0^\infty x^r \psi(t, x) \xi(dx). \tag{9}$$

The covariance function of $D^{[h]}X(t)$ and $D^{[k]}X(u)$ $(0 \le h, k \le r)$ is expressed by

$$\begin{aligned} \operatorname{Cov}(D^{[h]}X(t), D^{[k]}X(u)) &= \int_0^\infty \int_0^\infty x^h \psi(t, x) \overline{y^k \psi(u, y)} F(dx, dy) \\ &= D_t^{[h]} D_u^{[k]} r_X(t, u). \end{aligned} \tag{10}$$

We omit the proof since it is similar to the demonstration of Theorem 3.1 in (Endow 1995). Remark that in the p-adic stationary case the condition (8) reduces to

$$\int_0^\infty x^{2r} F(dx) < \infty,$$

i.e., $x^r \in L^2(F)$, and it is a necessary and sufficient condition for r-times q.m. Gibbs differentiable.

Next we will consider sample properties of p-adic harmonizable processes. Let us put that for $I_{n,k} := [np^{-k}, (n+1)p^{-k});\ k, n \in \mathbf{N}_0$

$$\xi(I_{n,k}) := \int_{I_{n,k}} \xi(dx)$$

and

$$X_k := \sum_{n=0}^{\infty} \psi(t, np^{-k})\xi(I_{n,k}). \tag{11}$$

The series above converges in q.m., since

$$\begin{aligned}
&\mathbf{E}\left|\sum_{n=0}^{\infty} \psi(t, np^{-k})\xi(I_{n,k})\right|^2 \\
&\quad = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \psi(t, mp^{-k})\overline{\psi(t, np^{-k})} \int_{I_{m,k}}\int_{I_{n,k}} F(dx, dy) \\
&\quad \le \int_0^{\infty}\int_0^{\infty} |F(dx, dy)| < \infty.
\end{aligned}$$

Here we will consider sample p-adic continuity of the processes.

THEOREM 2.2. *If it is satisfied that for some $\alpha > 1$*

$$\int_0^{\infty}\int_0^{\infty} (xy)^{\frac{\alpha}{2}} |F(dx, dy)| < \infty, \tag{12}$$

then the series (11) *absolutely converges almost surely, and $X_k(t)$ converges uniformly for every finite interval as $k \to \infty$ almost surely to a p-adic harmonizable process $\widetilde{X}(t)$, which is sample p-adic continuous and is equivalent to $X(t)$.*

Proof. Let us break the demonstration into three parts. Firstly, we will show that the series (11) absolutely converges almost surely, and the sum $X_k(t)$ consequently will be sample p-adic continuous. It is easy to see that for $\alpha > 1$

$$\begin{aligned}
\mathbf{E}\sum_{n=0}^{\infty} |\xi(I_{n,k})| &\le \left(\int_{I_{0,k}}\int_{I_{0,k}} F(dx, dy)\right)^{\frac{1}{2}} \\
&\quad + p^{\frac{\alpha k}{2}} \left(\sum_{n=1}^{\infty} n^{-\alpha}\right)^{\frac{1}{2}} \left(\sum_{n=1}^{\infty} n^{\alpha} p^{-\alpha k} \int_{I_{n,k}}\int_{I_{n,k}} F(dx, dy)\right)^{\frac{1}{2}}
\end{aligned} \tag{13}$$

in which the right side is finite since

$$\sum_{n=1}^{\infty} n^{\alpha} p^{-\alpha k} \int_{I_{n,k}}\int_{I_{n,k}} F(dx, dy) \le \int_0^{\infty}\int_0^{\infty} (xy)^{\frac{\alpha}{2}} |F(dx, dy)| < \infty. \tag{14}$$

Hence the series (11) absolutely converges almost surely.

Next let us show that $X_k(t)$ converges uniformly for every finite interval $0 \le t \le A$ $(0 < A < \infty)$ as$k \to \infty$ almost surely. Since

$$\xi(I_{n,k}) = \sum_{l=0}^{p-1} \xi(I_{mp+l,k+1}),$$

we have that

$$\begin{aligned}
&X_{k+1}(t) - X_k(t) \\
&= \sum_{m=0}^{\infty} \sum_{l=0}^{p-1} \left(\psi(t,(mp+l)p^{-k-1}) - \psi(t,(mp)p^{-k-1})\right) \xi(I_{mp+l,k+1}) \\
&= \sum_{m=0}^{\infty} \psi(t, mp^{-k}) \sum_{l=0}^{p-1} \left(\psi(t, lp^{-k-1}) - 1\right) \xi(I_{mp+l,k+1}).
\end{aligned}$$

Hence noting that $|\psi| = 1$ and $\psi(t,x) = 1$ $(tx < 1/p)$, we will see that for any fixed $A > 1$

$$\sup_{0 \le t \le A} |X_{k+1}(t) - X_k(t)| \le C(A,k) \sum_{n=0}^{\infty} |\xi(I_{n,k+1})|,$$

where $C(A,k) = 0$ when $k > 1 + \log_p A$; $= 2$ otherwise. It follows from (13) and (14) that for some constant C_1

$$\begin{aligned}
Q_k &:= \Pr\left\{ \sup_{0 \le t \le A} |X_{k+1}(t) - X_k(t)| > \varepsilon_k \right\} \\
&\le \frac{C(A,k)}{\varepsilon_k} \mathbf{E} \sum_{n=0}^{\infty} |\xi(I_{n,k+1})| = C_1 \frac{C(A,k)}{\varepsilon_k}.
\end{aligned}$$

Hence for a sequence $\{\varepsilon_k\}$ of positive numbers with $\sum_{k=1}^{\infty} \varepsilon_k < \infty$, the series $\sum_{k=1}^{\infty} Q_k$ converges, since it has substantially finite number of nonzero terms. Thus Borel-Cantelli's theorem shows that $X_k(t)$ converges almost surely as $k \to \infty$ uniformly for $0 \le t \le A$.

Lastly let us show that $X_k(t)$ q.m. converges to $X(t)$. Remind that for $0 \le t < p^k$, $\psi(t,x) = \psi(t, mp^{-k})$ $(x \in I_{m,k})$. Then for $0 \le t \le A$ and $k > \log_p A$

$$\begin{aligned}
&\mathbf{E}|X(t) - X_k(t)|^2 \\
&= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \int_{I_{m,k}} \int_{I_{n,k}} (\psi(t,x) - \psi(t, mp^{-k}))\overline{(\psi(t,y) - \psi(t, np^{-k}))} F(dx, dy) \\
&= 0.
\end{aligned}$$

With above discussion we can conclude in the following way. The sequence of sample p-adic continuous process $X_k(t)$ converges almost surely as $k \to \infty$ uniformly for every finite interval, and it also q.m. converges to the original process $X(t)$. Hence the convergent limit is sample p-adic continuous and is equivalent to the p-adic harmonizable process $X(t)$. □

Now let us prove the following theorem on sample Gibbs differentiability of p-adic harmonizable processes.

THEOREM 2.3. *If a p-adic harmonizable process with the representations* (1) *and* (2) *satisfies that for some* $1 < \alpha < 2$ *and* $r \in \mathbf{N}_0$

$$\int_0^\infty \int_0^\infty (xy)^{r+\frac{\alpha}{2}} |F(dx, dy)| < \infty, \tag{15}$$

then $X(t)$ *has a version which has the sample p-adic continuous r-th Gibbs derivative.*

Before proving this theorem we need a lemma.

LEMMA 2.4. *If the condition* (15) *is satisfied then* $X_k(t)$ *defined by* (11) *is equivalent to a p-adic harmonizable process which has the sample p-adic continuous r-th Gibbs derivative.*

Proof. We shall prove it only for $r = 1$ since similar argument will be applied to the higher cases. After a familier manipulation we see that

$$\begin{aligned}
&\mathbf{E}\sum_{n=0}^{\infty} n|\xi(I_{n,k})| \\
&\quad \le p^{k(1+\frac{\alpha}{2})}\left(\sum_{n=1}^{\infty} n^{-\alpha}\right)^{\frac{1}{2}} \left(\sum_{n=1}^{\infty}(np^{-k})^{2+\alpha})\mathbf{E}|\xi(I_{n,k})|^2\right)^{\frac{1}{2}} \\
&\quad \le p^{k(1+\frac{\alpha}{2})}\left(\sum_{n=1}^{\infty} n^{-\alpha}\right)^{\frac{1}{2}} \left(\sum_{n=1}^{\infty}\int_{I_{n,k}}\int_{I_{n,k}} (xy)^{1+\frac{\alpha}{2}}|F(dx,dy)|\right)^{\frac{1}{2}} \\
&\quad \le \infty,
\end{aligned}$$

which shows that $\sum_{n=0}^{\infty} n|\xi(I_{n,k})| < \infty$ *a.s.* Accordingly since the series on the right hand side of

$$\Delta_N X_k(t) = \sum_{n=0}^{\infty}\left\{\sum_{l=-N}^{N} p^{-l}\sum_{j=0}^{p-1} A_j\psi(jp^{-l-1}, np^{-k})\right\}\psi(t, np^{-k})\xi(I_{n,k})$$

is dominated in absolute value by $p^{-k}\sum n|\xi(I_{n,k})|$ almost surely, and since each term is converges boundedly to $np^{-k}\psi(t, np^{-k})\xi(I_{n,k})$ as $N \to \infty$, the limit of $\Delta_N X_k(t)$ as $N \to \infty$ should exist almost surely and is expressed by

$$DX_k(t) = p^{-k}\sum_{n=0}^{\infty} n\psi(t, np^{-k})\xi(I_{n,k}),$$

which has a p-adic continuous version as shown in Theorem 2. □

Proof of Theorem 3. Note that Theorem 3 reduces to Theorem 2 in the case of $r = 0$. So we shall prove the theorem only for $r = 1$. By definition we rewrite that

$$\begin{aligned}
&\Delta_N X_{k+1}(t) - \Delta_N X_k(t) \\
&\quad = \sum_{n=0}^{\infty}\sum_{i=1}^{p-1}\sum_{l=-N}^{N} p^l \sum_{j=0}^{p-1} A_j\psi(t, np^{-k})\left(\psi(t, ip^{-k-1}) - 1\right)\xi(I_{np+i,k+1}),
\end{aligned}$$

because of $\xi(I_{n,k}) = \sum_{i=0}^{p-1} \xi(I_{np+i,k+1})$ *a.s.* Noting that $\psi(t, ip^{-k}) = 1$ $(0 \le t < p^k,\ i \in \mathbf{N})$ and the Chrestenson functions are Gibbs differentiable and (4), we have that for $0 \le t < p^k$ and uniformly in N

$$\begin{aligned}|\Delta_N X_{k+1}(t) - \Delta_N X_k(t)| &\le \sum_{n=0}^{\infty}\sum_{i=0}^{p-1} |\xi(I_{np+i,k+1})||(np+i)p^{-k-1} - np^{-k}| \\ &\le p^{-k}\sum_{n=0}^{\infty} |\xi(I_{n,k+1})|.\end{aligned}$$

It follows from (13) and (14) that for any $\varepsilon_k > 0$ and some $\alpha > 1$,

$$\begin{aligned}Q_k &:= \Pr\left\{\sup_{0\le t\le A} |\Delta_N X_{k+1}(t) - \Delta_N X_k(t)| \ge \varepsilon_k\right\} \\ &\le \varepsilon_k^{-1} p^{-k} \sum_{n=0}^{\infty} |\xi(I_{n,k+1})| = O(\varepsilon_k^{-1} p^{-k(1-\frac{\alpha}{2})}).\end{aligned}$$

If ε_k is choson to be $p^{-\beta k}$ with $0 < \beta < 1 - \alpha/2$, then $\sum_{k=0}^{\infty} \varepsilon_k < \infty$ and $\sum_{k=0}^{\infty} Q_k < \infty$. Borel-Cantelli's theorem therefore shows that

$$\sup_{0\le t\le A} |\Delta_N X_{k+1}(t) - \Delta_N X_k(t)| < \varepsilon_k \ a.s.,$$

except for a finite number of k. For any $\varepsilon > 0$ we can take $n_0 \in \mathbf{N}$ such that $\sum_{k=n_0}^{\infty} \varepsilon_k < \varepsilon$. Hence we see that for $m, n \ge n_0$

$$\sup_{0\le t\le A} |\Delta_N X_m(t) - \Delta_N X_n(t)| < \varepsilon \ a.s., \tag{16}$$

uniformly in N. Consequently, $\Delta_N X_m(t)$ almost surely converges as $m \to \infty$ uniformly for $0 \le t \le A$ and N. On account of Theorem 2, letting $m \to \infty$ in the first term of (16), we will see that

$$\sup_{0\le t\le A} \left|\Delta_N \widetilde{X}(t) - \Delta_N X_n(t)\right| \le \varepsilon \ a.s.$$

Then from Lemma 1 with $r = 1$, $\Delta_N X_m(t)$ converges almost surely as $N \to \infty$ and hence $\widetilde{X}(t)$, which is equivalent to $X(t)$, is Gibbs differentiable almost surely.

Finaly (16) implies that the Gibbs derivative $DX_k(t)$ of $X_k(t)$ converges uniformly to the derivative $D\widetilde{X}(t)$ of $\widetilde{X}(t)$. Since Lemma 1 gives us that $DX_k(t)$ is p-adic continuous almost surely, $D\widetilde{X}(t)$ is also sample p-adic continuous for every $0 \le t \le A$. This completes the proof. □

Remark that this theorem is a generalization of Theorem 4.4 in (Endow 1995), and of Theorem 3 in (Endow 1996). We note that from Theorems 1 and 3 if a p-adic harmonizable process is sample Gibbs differentiable then it is also q.m. Gibbs differentiable and, accordingly its derivatives are expressed by (9). Hereafter we always choose from a class of equivalent processes such a version that has well properties, such as a sample p-adic continuity, whenever possible.

3. Linear Gibbs differential equations

In this section we shall consider stochastic linear Gibbs differential equation, i.e., linear Gibbs differential systems with random signals as their inputs.

If a p-adic harmonizable process $X(t)$ has m-times sample Gibbs differentiable then

$$P(D)X(t) = \sum_{k=0}^{m} a_k D^k X(t) \tag{17}$$

is well defined, where $P(s) = \sum_{k=0}^{m} a_k s^k$. If the representation (1) is also assumed, then by (4)

$$P(D)X(t) = \int_0^\infty \psi(t,x)P(x)\xi(dx) \tag{18}$$

and its covariance function is given by

$$\begin{aligned}&\mathrm{Cov}(P(D)X(t), P(D)X(u))\\ &= \int_0^\infty \int_0^\infty \psi(t,x)\overline{\psi(u,y)}P(x)\overline{P(y)}F(dx,dy).\end{aligned} \tag{19}$$

Therefore $P(D)X(t)$ is also p-adic harmonizable process.

Now let us introduce linear Gibbs differential equations. A linear Gibbs differential equation is defined by

$$Q(D)Y(t) = P(D)X(t), \tag{20}$$

where $X(t)$ is a given random process, $Y(t)$ is an unknown process, and $Q(s) = \sum_{k=0}^{n} b_k s^k$. The equation is considered as a model of a p-adic linear system and the given process is called a driving process, which is an input for the system. If a process $Y(t)$ satisfies the equation (20) then it is called a solution of the equation.

THEOREM 3.1. *Let an input process $X(t)$ be a p-adic harmonizable process with the representation* (1). *Suppose that it is m-times sample Gibbs differentiable. If the polynomial $Q(s)$ has no zeros on $\mathbf{R}_+$, and*

$$\int_0^\infty \int_0^\infty (xy)^{n+\frac{\alpha}{2}} \left|\frac{P(x)P(y)}{Q(x)Q(y)}\right| |F(dx,dy)| < \infty \tag{21}$$

for some $1 < \alpha < 2$, then

$$Y(t) = \int_0^\infty \psi(t,x)\frac{P(x)}{Q(x)}\xi(dx) \tag{22}$$

is a sample p-adic continuous solution of the equation (20). *Then its covariance function is given by*

$$r_Y(t,u) = \int_0^\infty \int_0^\infty \psi(t,x)\overline{\psi(u,y)}\frac{P(x)}{Q(x)}\left(\overline{\frac{P(y)}{Q(y)}}\right) F(dx,dy). \tag{23}$$

Proof. Remind that the integral in (22) is well defined by (21). It follows from (21) and Theorem 2.3 that $Y(t)$ has the sample continuous n-th p-adic Gibbs derivative, and

$$\begin{aligned} Q(D)Y(t) &= Q(D)\int_0^\infty \psi(t,x)\frac{P(x)}{Q(x)}\xi(dx) \\ &= \int_0^\infty \psi(t,x)P(x)\xi(dx) \\ &= P(D)X(t). \end{aligned}$$

Hence $Y(t)$ satisfies the equation (20). □

Notice that Theorem 4 is valid with $\alpha = 0$ for the q.m. cases, and when $X(t)$ is p-adic stationary process it reduces to Theroem 5.1 in (Endow 1995). We will give a simple example.

Example. Let ξ, ζ be random variables with zero means and the variance

$$\mathrm{Var}\begin{pmatrix} \xi \\ \zeta \end{pmatrix} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{pmatrix}.$$

Define a process $\{X(t); t \in \mathbf{R}_+\}$ by

$$X(t) := \psi(t,x)\xi + \psi(t,y)\zeta.$$

Then it is a p-adic harmonizable process and its covariance function is expressed by

$$\begin{aligned} r_X(t,u) &= \psi(t,x)\overline{\psi(u,x)}\sigma_1^2 + \psi(t,x)\overline{\psi(u,y)}\sigma_{12} \\ &\quad +\psi(t,y)\overline{\psi(u,x)}\sigma_{21} + \psi(t,y)\overline{\psi(u,y)}\sigma_2^2. \end{aligned}$$

It is easy to see that the solution of the linear Gibbs differential equation (20) is given by

$$Y(t) = \psi(t,x)\frac{P(x)}{Q(x)}\xi + \psi(t,y)\frac{P(y)}{Q(y)}\zeta,$$

and its covariance is also expressed by

$$\begin{aligned} &r_Y(t,u) \\ &= \psi(t,x)\overline{\psi(u,x)}\left|\frac{P(x)}{Q(x)}\right|^2 \sigma_1^2 + \psi(t,x)\overline{\psi(u,y)}\frac{P(x)}{Q(x)}\overline{\left(\frac{P(y)}{Q(y)}\right)}\sigma_{12} \\ &\quad +\psi(t,y)\overline{\psi(u,x)}\frac{P(y)}{Q(y)}\overline{\left(\frac{P(x)}{Q(x)}\right)}\sigma_{21} + \psi(t,y)\overline{\psi(u,y)}\left|\frac{P(y)}{Q(y)}\right|^2 \sigma_2^2. \end{aligned}$$

At the end, we will point out some advantages of p-adic linear systems modeled by Gibbs differential equarions, by recalling that the use of Fourier analysis in linear system theory is based on the convolution theorem and the relationship with the Newton-Leibniz derivative. As in many other areas, the application of Fourier analysis in linear system theorey supported by the existence of the fast Fourier transform, and related algorithms for efficient calculation of Fourier coefficients and some other parameters useful in practical applications.

Similarly, the Gibbs deirvatives possess most of the useful properties of the Newton-Leibniz derivative, and therefore the role of the Gibbs derivative incooparation with the Chrestenson-Fourier analysis in p-adic linear system theory can be compared to that of Newton-Leibniz with the Fourier analysis in linear system theory. Hence p-adic linear systems modeled by Gibbs differential equations will be useful in the theory and practical applications.

The concept of p-adic harmonizability can be extended to that of the Vilenkin harmonizability. Therfore, it is possible to say that our results obtained above will be valid for the Vilenkin harmonizable processes, and use of Gibbs differentiations will play considerable roles in linear Vilenkin stochastic systems.

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