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NP-HARD ASPECTS IN ANALOGICAL REASONING

By

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Abstract

Analogy is described in terms of predicate logic. This paper considers the complexity of analogical reasoning in which no function symbols except constants are allowed. We show that the problem of deciding whether a given atomic formula can be inferred by analogy is NP-hard.

1. Introduction

Analogical reasoning is an inference method that acquires unknown facts or knowledge by finding similarities among given objects and then converting the facts or knowledge holding in one object to the other. The inference of this kind has been recognized to give a key to a problem or to yield a new discovery or a prediciton.

Some theoretical formulations have been proposed to realize analogical reasoning on a computer [1, 3, 4, 5, 6, 7, 8]. But the computational complexity in analogical reasoning has not yet been studied very much. This paper takes the theory by Haraguchi and Arikawa [1, 3, 4, 5] for the formal discussion of analogy. We consider a problem of verifying whether a specified fact can be obtained by analogy among two objects. Their theory is developed in terms of predicate logic. In order to focus on the issues from analogical reasoning itself, we deal with the case where no function symbols are allowed. Even in such a simple case, we show that deciding whether a given atomic formula can be inferred by analogy is NP-hard.

Analogical reasoning is not usually aimed to solve the problem of our discussion but for finding new facts or knowledge. However, our NP-hard result suggests that the search space is exponentially large and if facts with some specified restriction are to be searched then the computational procedure may be at least as hard as finding a truth assignment satisfying a Boolean formula.

2. Principle of Analogy

A *definite clause* is a formula of the form

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$$q_0(t_1^0, \ldots, t_{n_0}^0) \leftarrow q_1(t_1^1, \ldots, t_{n_1}^1), \ldots, q_r(t_1^r, \ldots, t_{n_r}^r) \quad (r \geq 0),$$

where t_j^i are terms and q_i are predicate symbols. In Haraguchi and Arikawa's analogy theory [1, 3, 4, 5], an object of analogy is the minimal model M represented by a finite set S of definite clauses. In this paper we concentrate on the case where no function symbols are allowed except constant symbols. Therefore terms are constants or variables. We call each element in M a *fact*. An atomic formula containing no variables is simply called an *atom*.

Let S_i be a finite set of definite clauses and let $C(S_i)$ be the set of constants in S_i for $i = 1, 2$. A *partial identity* between S_1 and S_2 is a subset φ of $C(S_1) \times C(S_2)$ such that for each $a \in C(S_1)$ (resp., $a' \in C(S_2)$) there is at most one $a' \in C(S_2)$ (resp., $a \in C(S_1)$) with $\langle a, a' \rangle \in \varphi$. Hence φ gives a one-to-one correspondence between some subsets of $C(S_1)$ and $C(S_2)$.

Let $t_j \in C(S_1)$, $t_j' \in C(S_2)$ for $j = 1, 2, \ldots, n$ and let α, α' be atoms in S_1, S_2, respectively. For a partial identity φ, we say that α and α' are *identified* by φ, denoted by $\alpha\varphi\alpha'$, if they are written as

$$\begin{aligned}\alpha &= p(t_1, t_2, \ldots, t_n),\\ \alpha' &= p(t_1', t_2', \ldots, t_n'),\end{aligned}$$

and $\langle t_j, t_j' \rangle \in \varphi$ for $i = 1, 2, \ldots, n$.

Haraguchi and Arikawa's analogy is explained with these terminologies as follows. We assume that there exist facts $\beta_1, \beta_2, \ldots, \beta_n$ in S_1 such that $\alpha \leftarrow \beta_1, \beta_2, \ldots, \beta_n$ holds in S_1. Then if there exist facts $\beta_1', \beta_2', \ldots, \beta_n'$ in S_2 with $\beta_i\varphi\beta_i'$ for $i = 1, 2, \ldots, n$ then we infer α' in S_2 by identifying it with α.

An atom α' inferred in this way is not always a fact in S_2. But the partial identity φ gives a reason of possibility that α' holds in S_2. Then we can continue to infer by analogy, assuming such α' to be a fact in S_2. Conversely, we also infer atoms in S_1 from S_2 by analogy in the same way. Let $M_i(*)$ be the set of atoms in S_i which can be inferred in this way. Formally, $M_i(*)$ is defined inductively as follows.

DEFINITION. Let S_i be a finite set of definite clauses and let M_i be the minimal model of S_i for $i = 1, 2$. For a partial identity φ, we define $M_i(*)$ as follows, where we set i (resp., i') to 1 (resp., 2) or 2 (resp., 1).

$$\begin{aligned}
M_i(*) &= \cup_k M_i(k),\\
M_i(0) &= M_i,\\
R_i(k) &= \{\alpha \leftarrow \beta_1, \beta_2, \ldots, \beta_n \mid \beta_j \in M_i(k),\ \beta_j' \in M_{i'}(k)\ (j = 1, 2, \ldots, n) \text{ and}\\
&\qquad\qquad \alpha' \leftarrow \beta_1', \beta_2', \ldots, \beta_n' \text{ holds in } S_{i'}\\
&\qquad\qquad \text{and } \alpha\varphi\alpha',\ \beta_j\varphi\beta_j'\},\\
M_i(k+1) &= \{\alpha \mid R_i(k) \cup M_i(k) \cup S_i \vdash \alpha\}.
\end{aligned}$$

EXAMPLE. Consider the following sets S_1 and S_2 of definite clauses, where upper-case letters are variables and lower-case letters are constants or predicate symbols.

$$S_1 = \{p(a, b), q(b, c),$$
$$r(Y, X) \leftarrow q(X, Y),$$
$$s(X, Z) \leftarrow p\ (X, Y), r(Z, Y)\}$$
$$S_2 = \{p(a', b'), q(b', c')\ \}$$

Then, take the following partial identity φ:

$$\varphi = \{\langle a, a'\rangle, \langle b, b'\rangle, \langle c, c'\rangle\}.$$

For S_1, S_2 and φ, the inference by analogy goes as follows. First, we obtain $M_2(0) = \{p(a', b'), q(b', c')\}$. Next we get $r(c, b) \leftarrow q\ (b, c)$ from $r(Y, X) \leftarrow q(X, Y) \in S_1$. Then we get $r(c', b') \leftarrow q(b', c') \in R_2(0)$ by $q(b, c)\varphi q(b', c')$ and $r(c', b') \in M_2(1)$. Moreover, we get $s(a, c) \leftarrow p(a, b), r(c, b)$ from $s(X, Z) \leftarrow p(X, Y), r(Z, Y), \in S_1$. Then we get $s(a', c') \leftarrow p(a', b'), r'(c', b') \in R_2(1)$ by $p(a, b)\varphi p(a', b')$, $r(c, b)\varphi r(c', b')$ and $s(a', b') \in M_2(2)$. No more atoms can be inferred by analogy. Hence $M_2(2) = M_2(*)$.

3. Complexity of Analogy

Given sets S_1, S_2 of definite clauses, the inference by analogy consists of two phases. One is to find an appropriate partial identity φ which gives a similarity among S_1 and S_2. The other is to compute $M_1(*)$ and $M_2(*)$ based on φ. The complexity of the last depends on the size of $M_1(*)$ and $M_2(*)$.

We consider the following decision problem where a specified formula is searched.

ANALOGY

Instance: Two finite sets S_1, S_2 of definite clauses and an atom $p(t_1, t_2, \ldots, t_n)$.

Problem: Decide whether there exists a partial identity φ between S_1 and S_2 such that $p(t_1, t_2, \ldots, t_n)$ is in $M_2(*)$.

We obtain the following theorem about the complexity of searching a partial identity.

THEOREM. ANALOGY *is* NP-*hard*.

PROOF. We give a reduction from 3-SAT (3-satisfiability problem) [2] to ANALOGY. For a Boolean formula $F = C_1C_2 \ldots C_m$ in three conjunctive normal form (3-CNF), S_1 and S_2 are constructed as follows, where $x_1, \ldots, x_n$ are the variables in F. For $i = 1, 2, \ldots, n$,

$$h_i(a_i),\ h_i(\bar{a}_i) \in S_1 \text{ and } h_i(a_i') \in S_2,$$

where a_i and $\bar{a}_i$ are constant symbols in S_1, a_i' is a constant symbol in S_2 and h_i is a predicate symbol.

Next, for each clause C_j, we use predicate symbols p_j of zero argument and q_j of one argument. Let α be a literal in C_j.

If $\alpha_j = x_u$, then

$$q_j(a_u) \in S_1,\quad q_j(a_u') \in S_2 \text{ and } p_j \leftarrow h_u(X), q_j(X) \in S_1.$$

If $\alpha_j = \bar{x}_u$, then

$$q_j(\bar{a}_u) \in S_1, \quad q_j(a'_u) \in S_2 \text{ and } p_j \leftarrow h_u(X), q_j \ \ldots, q_j(X) \in S_1.$$

Moreover,

$$p \leftarrow p_1, p_2, \ldots, p_m \in S_1,$$

where p is a predicate symbol of zero argument. Then we show that F is satisfiable if and only if there exists a partial identity φ such that p is in $M_2(*)$.

First, if F is satisfiable, then let $\hat{x}_1, \ldots, \hat{x}_n$ be a truth assignment to the variables $x_1, \ldots, x_n$ that satisfies each clause C_j of F. We define the partial identity φ by

$$\langle a_i, a'_i \rangle \in \varphi \quad \text{if } \hat{x}_i = 1$$
$$\langle \bar{a}_i, a'_i \rangle \in \varphi \quad \text{if } \hat{x}_i = 0$$

for each $i = 1, \ldots, n$. Then we can infer each p_j as follows. If C_j contains a literal $\hat{x}_i$ with $\hat{x}_i = 1$, then φ contains $\langle a_i, a'_i \rangle$. We get $p_j \leftarrow h_i(a_i), q_j(a_i) \in S_1$ from $p_j \leftarrow h_i(X), q_j(X) \in S_1$. Then we get $p_j \leftarrow h_i(a'_i), q_j(a'_i) \in R_2(0)$ by $h_i(a_i)\varphi h_i(a'_i)$ and $q_j(a_i)\varphi q_j(a'_i)$. Therefore $p_j \in M_2(1)$. If C_j is satisfiable by a literal $\hat{x}_i$ with $\hat{x}_i = 0$, we can show in a similar way that p_j is inferred by analogy. Hence p is in $M_2(2)$.

Conversely, assume that there exists a partial identity φ such that p is in $M_2(*)$. For each $i = 1, \ldots, n$, we define a truth assignment $\hat{x}_1, \ldots, \hat{x}_n$ as follows: If φ contains $\langle a_i, a'_i \rangle$, then $\hat{x}_i = 1$. If φ contains $\langle \bar{a}_i, a'_i \rangle$, then $\hat{x}_i = 0$. Otherwise $\hat{x}_i$ is arbitrary. If p is $M_2(*)$, each p_j must be inferred by analogy using φ since it is not in S_2. Then, there exists i such that $p_j \leftarrow h_i(a'_i), q_j(a'_i)$ is in $R_2(0)$ and $p_j \leftarrow h_i(a_i), q_j(a_i)$ or $p_j \leftarrow h_i(\bar{a}_i), q_j(\bar{a}_i)$ holds in S_1 since both h_i and q_j are not in the left side of definite clauses. Therefore φ must contain either $\langle a_i, a'_i \rangle$ or $\langle \bar{a}_i, a'_i \rangle$. If $\langle a_i, a'_i \rangle \in \varphi$, then we can satisfy C_j by $\hat{x}_i = 1$. If $\langle \bar{a}_i, a'_i \rangle \in \varphi$, then we can satisfy C_j by $\hat{x}_i = 0$. It is not hard to see that this reduction is computable in polynomial time or log space. Hence ANALOGY is NP-hard. □

REMARK. If the argument of each predicate symbol is bounded by a fixed constant and if the number of atomic formulas containing variables is also bounded by a fixed constant in each definite clause, then $M_1(*)$ and $M_2(*)$ are polynomial-time computable for a given φ. Therefore we can see that ANALOGY is solvable in NP by guessing a partial identity. The definite clauses constructed in our reduction satisfies these conditions. Moreover, S_2 consists of only facts in the proof.

4. Conclusion

Our analysis shows that searching a partial identity φ such that a given atom can be inferred with φ is at least as hard as finding a truth assignment that satisfies a given 3-CNF formula. This means that the search space is huge and suggests that analogical reasoning for searching new facts or knowledge requires some constraint on the search space such as by weight or something similar to it. Otherwise, it is just like searching a exponentially large space.

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