PAIRWISE BALANCED, SEMI-REGULAR AND REGULAR GROUP DIVISIBLE DESIGNS

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By

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Abstract

From a regular group divisible design a method of constructing a pairwise balanced design and a semi-regular group divisible design is given.

1. Introduction

Sinha and Nigam [6] obtained balanced arrays and main-effect plans from regular group divisible designs. Here a method of constructing pairwise balanced designs and semi-regular group divisible designs from regular group divisible designs is given. The reference to design numbers are due to Clatworthy [2]. For definitions of some technical terms in this paper, refer to Raghavarao [5].

2. Statements

We present one construction method of semi-regular group divisible designs as follows.

THEOREM 1. *The existence of a regular group divisible design with parameters*

$$(1) \quad v=mn=b, r=m-1=k, \lambda_1=0, \lambda_2=1; m, n=m-2$$

implies the existence of a semi-regular group divisible design with parameters

$$(2) \quad v=m(n+1), b=mn+1, r=m-1, k=m, \lambda_1=0, \lambda_2=1; m^*=m, n^*=m-1$$

and a pairwise balanced design with parameters

$$(3) \quad v=m(n+1), b=m(n+1)+1, r=m, k_i=m \text{ or } m-1, \lambda=1.$$

PROOF. Let the mn treatments in the original regular group divisible design be arranged in an $n \times m$ array (i.e., m groups of n treatments each) as:

$$\begin{array}{cccc} 1, & 2, & \cdots, & m \\ m+1, & m+2, & \cdots, & 2m \\ \vdots & \vdots & & \vdots \\ (n-1)m+1, & (n-1)m+2, & \cdots, & nm \end{array}$$

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Any two treatments in the same group $\{i, m+i, \dots, (n-1)m+i\} (i=1, 2, \dots, m)$ are first associates, otherwise they are second associates. Now, to each block of a regular group divisible design with parameters (1), we add a treatment $(nm+i)$ if there is no treatment from the i -th group $(i=1, 2, \dots, m)$, and further add a new block $(nm+1, nm+2, \dots, nm+m)$ consisting of " m " new treatments to this arrangement. Thus, we get a semi-regular group divisible design with parameters (2) whose $m(n+1)$ treatments can now be arranged in an $(n+1) \times m$ array. Next, to the above semi-regular group divisible design with parameters (2), add m blocks each of size $(n+1)$ consisting of elements in m columns of the $(n+1) \times m$ array. Then we get a pairwise balanced design with parameters (3). Thus, the proof is completed.

REMARK: The existence of a regular group divisible design with parameters (1) is equivalent to the existence of a finite affine plane of order $n+1$. For, Theorem 8.6.2 in Raghavarao [5] gives the proof of Necessity of the result, while as Sufficiency part a reverse construction process is similarly provided. Thus, as a starting design in Theorem 1, one can choose a finite affine plane also.

3. Illustration

(i) Semi-regular group divisible design: Let us consider a regular group divisible design, R54, with parameters $v=8=b, r=3=k, \lambda_1=0, \lambda_2=1, m=4, n=2$, which yields a semi-regular group divisible design, SR41, with parameters $v=12, b=9, r=3, k=4, \lambda_1=0, \lambda_2=1, m=4, n=3$, having blocks, $(1, 2, 4, 11), (2, 3, 5, 12), (3, 4, 6, 9), (4, 5, 7, 10), (5, 6, 8, 11), (1, 6, 7, 12), (2, 7, 8, 9), (1, 3, 8, 10), (9, 10, 11, 12)$. The 3×4 array is

1	2	3	4
5	6	7	8.
9	10	11	12

(ii) Pairwise balanced design: By adding the following four blocks, $(1, 5, 9), (2, 6, 10), (3, 7, 11), (4, 8, 12)$, consisting of elements in the 4 columns of the above array to the above-constructed semi-regular group divisible design (i), one can obtain a pairwise balanced design with parameters $v=12, b=13, r=4, k_i=3$ or $4, \lambda=1$.

4. Some Semi-Regular Designs

Now corresponding to the regular group divisible designs, R—54, 114, 153, 183, 191, 202 in Clatworthy [2], we can obtain semi-regular group divisible designs with parameters as in SR—41, 58, 75, 96, 104, 110, respectively. The references to design numbers only indicate that a solution to each of these semi-regular group divisible designs is known in Clatworthy [2], but it can be checked that designs corresponding to SR—41, 58, 75 and 96 constructed here are also isomorphic to designs in Clatworthy [2]. The last two designs may yield non-isomorphic solutions to SR—104 and 110 in Clatworthy [2]. Corresponding to an unknown regular group divisible design with parameters

$$R-X: v=99=b, r=10=k, \lambda_1=0, \lambda_2=1, m=11, n=9,$$

there is possibility of getting an unknown semi-regular group divisible design with parameters

$$\text{SR-X: } v=110, b=100, r=10, k=11, \lambda_1=0, \lambda_2=1, m=11, n=10.$$

The existence of the design R-X is unknown in the sense that this is not found in Clatworthy [2], Freeman [3], Kageyama and Tanaka [4] and other available papers; and also their existence is not ruled out by the non-existence theorem for such symmetric designs in Bose and Connor [1]. Incidentally, the following two regular group divisible designs are also unknown within the scope of $r, k \leq 10$:

$$v=b=45, r=k=7, \lambda_1=0, \lambda_2=1; m=15, n=3,$$

$$v=b=75, r=k=9, \lambda_1=0, \lambda_2=1; m=25, n=3.$$

5. Some Pairwise Balanced Designs

Corresponding to the regular group divisible designs, R-54, 114, 153, 183, 191, 202, X, we will obtain the following pairwise balanced designs, respectively.

- (i) $v=12, b=13, r=4, k_1=4, k_2=3, \lambda=1,$
- (ii) $v=20, b=21, r=5, k_1=5, k_2=4, \lambda=1,$
- (iii) $v=30, b=31, r=6, k_1=6, k_2=5, \lambda=1,$
- (iv) $v=56, b=57, r=8, k_1=8, k_2=7, \lambda=1,$
- (v) $v=72, b=73, r=9, k_1=9, k_2=8, \lambda=1,$
- (vi) $v=90, b=91, r=10, k_1=10, k_2=9, \lambda=1,$
- (vii) $v=110, b=111, r=11, k_1=11, k_2=10, \lambda=1.$

The existence of the last design (vii) is unknown.

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