PAIRWISE BALANCED, SEMI-REGULAR AND REGULAR GROUP DIVISIBLE DESIGNS

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By

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Abstract

From a regular group divisible design a method of constructing a pairwise balanced design and a semi-regular group divisible design is given.

1. Introduction

Sinha and Nigam [6] obtained balanced arrays and main-effect plans from regular group divisible designs. Here a method of constructing pairwise balanced designs and semi-regular group divisible designs from regular group divisible designs is given. The reference to design numbers are due to Clatworthy [2]. For definitions of some technical terms in this paper, refer to Raghavarao [5].

2. Statements

We present one construction method of semi-regular group divisible designs as follows.

THEOREM 1. *The existence of a regular group divisible design with parameters*

(1) $v=mn=b,\ r=m-1=k,\ \lambda_1=0,\ \lambda_2=1;\ m,\ n=m-2$

implies the existence of a semi-regular group divisible design with parameters

(2) $v=m(n+1),\ b=mn+1,\ r=m-1,\ k=m,\ \lambda_1=0,\ \lambda_2=1;\ m^*=m,\ n^*=m-1$

and a pairwise balanced design with parameters

(3) $v=m(n+1),\ b=m(n+1)+1,\ r=m,\ k_i=m$ *or* $m-1,\ \lambda=1.$

PROOF. Let the mn treatments in the original regular group divisible design be arranged in an $n\times m$ array (i.e., m groups of n treatments each) as:

$$\begin{array}{cccc} 1, & 2, & \cdots, & m \\ m+1, & m+2, & \cdots, & 2m \\ \vdots & \vdots & & \vdots \\ (n-1)m+1, & (n-1)m+2, & \cdots, & nm \end{array}.$$

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Any two treatments in the same group $\{i, m+i, \cdots, (n-1)m+i\}(i=1, 2, \cdots, m)$ are first associates, otherwise they are second associates. Now, to each block of a regular group divisible design with parameters (1), we add a treatment $(nm+i)$ if there is no treatment from the i-th group $(i=1, 2, \cdots, m)$, and further add a new block $(nm+1, nm+2, \cdots, nm+m)$ consisting of "m" new treatments to this arrangment. Thus, we get a semi-regular group divisible design with parameters (2) whose $m(n+1)$ treatments can now be arranged in an $(n+1)\times m$ array. Next, to the above semi-regular group divisible design with parameters (2), add m blocks each of size $(n+1)$ consisting of elements in m columns of the $(n+1)\times m$ array. Then we get a pairwise balanced design with parameters (3). Thus, the proof is completed.

REMARK: The existence of a regular group divisible design with parameters (1) is equivalent to the existence of a finite affine plane of order $n+1$. For, Theorem 8.6.2 in Raghavarao [5] gives the proof of Necessity of the result, while as Sufficiency part a reverse construction process is similarly provided. Thus, as a starting design in Theorem 1, one can choose a finite affine plane also.

3. Illustration

(i) Semi-regular group divisible design: Let us consider a regular group divisible design, R54, with parameters $v=8=b$, $r=3=k$, $\lambda_1=0$, $\lambda_2=1$, $m=4$, $n=2$, which yields a semi-regular group divisible design, SR41, with parameters $v=12$, $b=9$, $r=3$, $k=4$, $\lambda_1=0$, $\lambda_2=1$, $m=4$, $n=3$, having blocks, (1, 2, 4, 11), (2, 3, 5, 12), (3, 4, 6, 9), (4, 5, 7, 10), (5, 6, 8, 11), (1, 6, 7, 12), (2, 7, 8, 9), (1, 3, 8, 10), (9, 10, 11, 12). The 3×4 array is

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8. \\ 9 & 10 & 11 & 12 \end{array}$$

(ii) Pairwise balanced design: By adding the following four blocks, (1, 5, 9), (2, 6, 10), (3, 7, 11), (4, 8, 12), consisting of elements in the 4 columns of the above array to the above-constructed semi-regular group divisible design (i), one can obtain a pairwise balanced design with parameters $v=12$, $b=13$, $r=4$, $k_i=3$ or 4, $\lambda=1$.

4. Some Semi-Regular Designs

Now corresponding to the regular group divisible designs, R—54, 114, 153, 183, 191, 202 in Clatworthy [2], we can obtain semi-regular group divisible designs with parameters as in SR—41, 58, 75, 96, 104, 110, respectively. The references to design numbers only indicate that a solution to each of these semi-regular group divisible designs is known in Clatworthy [2], but it can be checked that designs corresponding to SR —41, 58, 75 and 96 constructed here are also isomorphic to designs in Clatworthy [2]. The last two designs may yield non-isomorphic solutions to SR—104 and 110 in Clatworthy [2]. Corresponding to an unknown regular group divisible design with parameters

$$\text{R—X}: v=99=b,\ r=10=k,\ \lambda_1=0,\ \lambda_2=1,\ m=11,\ n=9,$$

there is possibility of getting an unknown semi-regular group divisible design with parameters

$$\text{SR—X}: v=110,\ b=100,\ r=10,\ k=11,\ \lambda_1=0,\ \lambda_2=1,\ m=11,\ n=10.$$

The existence of the design R—X is unknown in the sense that this is not found in Clatworthy [2], Freeman [3], Kageyama and Tanaka [4] and other available papers; and also their existence is not ruled out by the non-existence theorem for such symmetric designs in Bose and Connor [1]. Incidentally, the following two regular group divisible designs are also unknown within the scope of r, $k \leqq 10$:

$$v=b=45,\ r=k=7,\ \lambda_1=0,\ \lambda_2=1;\ m=15,\ n=3,$$

$$v=b=75,\ r=k=9,\ \lambda_1=0,\ \lambda_2=1;\ m=25,\ n=3.$$

5. Some Pairwise Balanced Designs

Corresponding to the regular group divisible designs, R—54, 114, 153, 183, 191, 202, X, we will obtain the following pairwise balanced designs, respectively.

(i) $v=12,\ b=13,\ r=4,\ k_1=4,\ k_2=3,\ \lambda=1,$

(ii) $v=20,\ b=21,\ r=5,\ k_1=5,\ k_2=4,\ \lambda=1,$

(iii) $v=30,\ b=31,\ r=6,\ k_1=6,\ k_2=5,\ \lambda=1,$

(iv) $v=56,\ b=57,\ r=8,\ k_1=8,\ k_2=7,\ \lambda=1,$

(v) $v=72,\ b=73,\ r=9,\ k_1=9,\ k_2=8,\ \lambda=1,$

(vi) $v=90,\ b=91,\ r=10,\ k_1=10,\ k_2=9,\ \lambda=1,$

(vii) $v=110,\ b=111,\ r=11,\ k_1=11,\ k_2=10,\ \lambda=1.$

The existence of the last design (vii) is unknown.

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