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CATEGORICAL RELATIONAL DATABASE MODELS

By

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Abstract

A category of relational models and its quotient category are defined and their basic properties are discussed. Moreover a categorical join dependency and a category of adjoint models, which dualize the notion of relational models, are studied.

1. Introduction

Theory of categories [1, 8] founded by S. Mac Lane and S. Eilenberg can be applied to various fields of mathematics and has the advantage that it serves a global point of view for logical structures of many mathematical objects. For instance category theory has been a useful tool in studying theories of automata, mathematical languages, systems, graphs and programmings [1, 2, 3, 4, 5] as mathematical foundations of computer science.

The purpose of this paper is to investigate a categorical aspect of relational database models. This categorical viewpoint for theory of database models was initiated by the work of A. Kato [6, 7]. Given an image factorization system $(\mathcal{E}, \mathcal{M})$ of a category \mathcal{C} and a functor $X: \mathcal{X} \rightarrow \mathcal{C}$, we define relational models and their morphisms in section 2. Then we can naturally obtain a category $R(X)$ of relational models and morphisms between them. Also we show some basic properties of the category $R(X)$ of relational models and consider an equivalence relation on the class of all morphisms in $R(X)$ to obtain a quotient category $R[X]$ of $R(X)$. In section 3 a join dependency associated with a family of natural transformations $k_\alpha: X \rightarrow X_\alpha$ ($\alpha \in A$) is introduced. The categorical join dependency is a reasonable abstraction of the usual join dependency and it induces a left adjoint functor from $R(X)$ into a full subcategory $D(X)$ of $R(X)$ consisting of all join dependent models. As in Kato [6] it turns out that the join dependency of a relational model is regarded as a notion in the quotient category $R[X]$. In the final section we introduce a notion of adjoint models as a dual of relational models and their morphisms in case that a functor $X: \mathcal{X} \rightarrow \mathcal{C}$ has a right adjoint functor $Y: \mathcal{C} \rightarrow \mathcal{X}$ and \mathcal{X} has an image factorization system $(\mathcal{E}^*, \mathcal{M}^*)$. Then the category $R\{Y\}$ of adjoint models is naturally constructed and we extend Kato's result [6] that $R\{Y\}$ is equivalent to $R[X]$ as categories.

2. Categories of Relational Models

Relational database models due to E.F. Codd are defined as subsets of a cartesian product of sets indexed by a set of attributes. In this section we will generalize the

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notion of relational database models and define a category consisting of them. Throughout the rest of the paper we assume that an image factorization system $(\mathcal{E}, \mathcal{M})$ in a category \mathcal{C} and a functor $X: \mathcal{X} \rightarrow \mathcal{C}$ are given.

DEFINITION 2.1. A (relational) model (with respect to $X: \mathcal{X} \rightarrow \mathcal{C}$) is defined to be a pair (Q, q) of an object Q in \mathcal{X} and a monomorphism $q: \bar{Q} \rightarrow XQ$ in \mathcal{C} such that $q \in \mathcal{M}$. A morphism $f: (Q, q) \rightarrow (R, r)$ of a model (Q, q) into another model (R, r) is defined to be a morphism $f: Q \rightarrow R$ in \mathcal{X} such that there exists a morphism $\bar{f}: \bar{Q} \rightarrow \bar{R}$ in \mathcal{C} rendering the following square commutative:

$$\begin{array}{ccc} \bar{Q} & \xrightarrow{q} & XQ \\ \bar{f} \downarrow & & \downarrow Xf \\ \bar{R} & \xrightarrow{r} & XR. \end{array}$$

(The uniqueness of $\bar{f}: \bar{Q} \rightarrow \bar{R}$ follows from the injectivity of $r: \bar{R} \rightarrow XR$. An arrow “ \rightarrow ” represents a monomorphism in \mathcal{M} .)

Let $R(X)$ be the category of all models and all morphisms between them. Identity morphisms and the composition of morphisms in $R(X)$ are given in a trivial fashion.

LEMMA 2.2. A morphism $f: (Q, q) \rightarrow (R, r)$ is an isomorphism in $R(X)$ if and only if $f: Q \rightarrow R$ is an isomorphism in \mathcal{X} and $\bar{f}: \bar{Q} \rightarrow \bar{R}$ is an epimorphism in \mathcal{E} .

By analogy with A. Kato [6] we define an equivalence relation on the class of all morphisms in category $R(X)$ as follows:

DEFINITION 2.3. Let $f_1, f_2: (Q, q) \rightarrow (R, r)$ be two morphisms in $R(X)$. We say that f_1 is equivalent to f_2 , denoted by $f_1 \sim f_2: (Q, q) \rightarrow (R, r)$, if $\bar{f}_1: \bar{Q} \rightarrow \bar{R}$ and $\bar{f}_2: \bar{Q} \rightarrow \bar{R}$ (in Definition 2.1) are identical, that is, $\bar{f}_1 = \bar{f}_2$.

It is obvious that the relation “ \sim ” is an equivalence relation. The next proposition states that this equivalence relation “ \sim ” is preserved by the composition of morphisms in $R(X)$.

PROPOSITION 2.4. If $f_1 \sim f_2: (Q, q) \rightarrow (R, r)$ and $g_1 \sim g_2: (R, r) \rightarrow (S, s)$ in $R(X)$ then $g_1 f_1 \sim g_2 f_2: (Q, q) \rightarrow (S, s)$.

We are now ready to obtain a quotient category [8; p. 51] of $R(X)$ classifying morphisms by the equivalence relation “ \sim ” defined above. We denote the quotient category by $R[X]$. That is, objects in $R[X]$ are the same ones as in $R(X)$ and a morphism $[f]: (Q, q) \rightarrow (R, r)$ in $R[X]$ is an equivalence class of a morphism $f: (Q, q) \rightarrow (R, r)$ in $R(X)$.

PROPOSITION 2.5. Let $f: (Q, q) \rightarrow (R, r)$ be a morphism in $R(X)$. If $f: Q \rightarrow R$ is an isomorphism in \mathcal{X} and if there exists a morphism $g: (R, r) \rightarrow (Q, q)$ with $fg \sim 1_R$, then $f: (Q, q) \rightarrow (R, r)$ is an isomorphism in $R(X)$.

In the above proposition the existence of a morphism $g: (R, r) \rightarrow (Q, q)$ with $fg \sim 1_R: (R, r) \rightarrow (R, r)$ means that $[f]: (Q, q) \rightarrow (R, r)$ is a retraction [5; p. 19] in $R[X]$.

COROLLARY 2.6. Let $f: (Q, q) \rightarrow (R, r)$ be a morphism in $R(X)$. If $f: Q \rightarrow R$ is an isomorphism in \mathcal{X} and if $[f]: (Q, q) \rightarrow (R, r)$ is an isomorphism in $R[X]$, then $f: (Q, q) \rightarrow (R, r)$ is an isomorphism in $R(X)$.

By using the diagonal fill-in lemma [1, p. 39] we have the following lemma.

LEMMA 2.7. Let $f:(Q, q) \rightarrow (R, r)$ be a morphism in $R(X)$. If there exists a morphism $g: R \rightarrow Q$ with $gf = 1_Q$ and if $\bar{f}: \bar{Q} \rightarrow \bar{R}$ is in \mathcal{E} , then $[f]: (Q, q) \rightarrow (R, r)$ is an isomorphism in $R[X]$.

EXAMPLE 2.8. Let \mathcal{C} be a category with products and A a set. A is regarded as a discrete category. Denote by \mathcal{C}^A the functor category of all functors from A into \mathcal{C} , i.e., \mathcal{C}^A is a product category of A copies of \mathcal{C} . Let $\pi_A: \mathcal{C}^A \rightarrow \mathcal{C}$ be the product functor $(\langle S_a \rangle_{a \in A} \mapsto \pi_a S_a)$. Then the category $R(\pi_A)$ of relational models is obtained and the case $\mathcal{C} = \mathbf{Set}$ was considered by Kato [6].

3. Generalized Join Dependencies

In [6] Kato described that observations of join dependencies about relational models as stated in Example 2.8 are sufficient if they are considered in quotient category $R[X]$. This section generalizes the result due to Kato [6] for the category of relational models defined in the previous section.

As in the section 2, we assume that an image factorization system $(\mathcal{E}, \mathcal{M})$ in a category \mathcal{C} and a functor $X: \mathcal{X} \rightarrow \mathcal{C}$ are given. Moreover supposed that category \mathcal{C} has products and pullbacks, and supposed that a family of functors $X_\alpha: \mathcal{X} \rightarrow \mathcal{C}$ ($\alpha \in A$) and a family of natural transformations $k_\alpha: X \rightarrow X_\alpha$ ($\alpha \in A$) are given.

Let (Q, q) be an object of category $R(X)$. Then consider an $(\mathcal{E}, \mathcal{M})$ -factorization of the composite $k_\alpha q: \bar{Q} \rightarrow XQ \rightarrow X_\alpha Q$ ($\alpha \in A$), which is rendered by the commutative square

$$\begin{array}{ccc} \bar{Q} & \xrightarrow{q} & XQ \\ e_\alpha \downarrow & & \downarrow k_\alpha \\ \bar{Q}_\alpha & \xrightarrow{m_\alpha} & X_\alpha Q \end{array}$$

(Notice that $k_\alpha = k_{\alpha, Q}: XQ \rightarrow X_\alpha Q$. An arrow “ \rightarrow ” represents an epimorphism in \mathcal{E} .) Let $pr_\alpha: \pi_\alpha X_\alpha Q \rightarrow X_\alpha Q$ ($\alpha \in A$) and $\bar{pr}_\alpha: \pi_\alpha \bar{Q}_\alpha \rightarrow \bar{Q}_\alpha$ ($\alpha \in A$) be systems of projections of products $\pi_\alpha X_\alpha Q$ and $\pi_\alpha \bar{Q}_\alpha$, respectively. We denote by $\top_\alpha k_\alpha: XQ \rightarrow \pi_\alpha X_\alpha Q$ a unique morphism with $pr_\alpha(\top_\alpha k_\alpha) = k_\alpha$ for each $\alpha \in A$ and by $\pi_\alpha m_\alpha: \pi_\alpha \bar{Q}_\alpha \rightarrow \pi_\alpha X_\alpha Q$ a unique morphism with $pr_\alpha(\pi_\alpha m_\alpha) = m_\alpha \bar{pr}_\alpha$ for each $\alpha \in A$. Construct a pullback of two morphisms $\top_\alpha k_\alpha$ and $\pi_\alpha m_\alpha$ with common codomain.

$$\begin{array}{ccc} \hat{Q} & \xrightarrow{\quad q^* \quad} & XQ \\ \top_\alpha e'_\alpha \downarrow & (PB) & \downarrow \top_\alpha k_\alpha \\ \pi_\alpha \bar{Q}_\alpha & \xrightarrow{\quad \pi_\alpha m_\alpha \quad} & \pi_\alpha X_\alpha Q \end{array}$$

Trivially q^* is in \mathcal{M} [1, Exercise 2.4.11; 2, Proposition 2.11] since m_α is in \mathcal{M} for each $\alpha \in A$. Hence we have a new object (Q, q^*) in $R(X)$. By the universal property of pullbacks there exists a unique morphism $\bar{\eta}: \bar{Q} \rightarrow \hat{Q}$ in \mathcal{X} making the following daigram commute.

$$\begin{array}{ccccc}
 \bar{Q} & & & & \\
 \searrow q & & & & \\
 & \bar{Q} & \xrightarrow{q^*} & XQ & \\
 \swarrow \tau_{\alpha} e_{\alpha} & \downarrow \tau_{\alpha} e'_{\alpha} & (PB) & \downarrow \tau_{\alpha} k_{\alpha} & \\
 & \pi_{\alpha} \bar{Q}_{\alpha} & \xrightarrow{\pi_{\alpha} m_{\alpha}} & \pi_{\alpha} X_{\alpha} Q &
 \end{array}$$

This implies that $1_Q : (Q, q) \rightarrow (Q, q^*)$ is a morphism in $R(X)$. Note that $1_Q : Q \rightarrow Q$ is an identity morphism in \mathcal{X} but $1_Q : (Q, q) \rightarrow (Q, q^*)$ is not always an identity morphism in $R(X)$. It follows at once from the construction of q^* that $q^{**} = q^*$ since e'_{α} is in \mathcal{E} for each $\alpha \in A$.

PROPOSITION 3.1. *Let (Q, q) and (R, r) be two objects in $R(X)$. If $f : (Q, q) \rightarrow (R, r^*)$ is a morphism in $R(X)$, then so is $f : (Q, q^*) \rightarrow (R, r^*)$.*

COROLLARY 3.2. *If $f : (Q, q) \rightarrow (R, r)$ is a morphism in $R(X)$, then so is $f : (Q, q^*) \rightarrow (R, r^*)$.*

The last corollary shows that $*$ gives a functor called the join functor:

$$\begin{array}{ccc}
 * : R(X) & \longrightarrow & R(X) \\
 (Q, q) & & (Q, q^*) \\
 f \downarrow & \longmapsto & \downarrow f \\
 (R, r) & & (R, r^*) .
 \end{array}$$

During the rest of this section we assume that $\tau_{\alpha} k_{\alpha} : XQ \rightarrow \pi_{\alpha} X_{\alpha} Q$ is a monomorphism for each object Q in \mathcal{X} . The following results are basic properties of the join functor $* : R(X) \rightarrow R(X)$.

LEMMA 3.3. *Let $f, g : (Q, q) \rightarrow (R, r)$ be two morphisms in $R(X)$. If $f \sim g : (Q, q) \rightarrow (R, r)$, then $f \sim g : (Q, q^*) \rightarrow (R, r^*)$.*

COROLLARY 3.4. *An object (Q, q) in $R(X)$ is join dependent (JD), that is, $q = q^*$ as a subobject of XQ if and only if $[1_Q] : (Q, q) \rightarrow (Q, q^*)$ is an isomorphism in $R[X]$.*

THEOREM 3.5. *Let (Q, q) and (R, r) be two object in $R(X)$. If (R, r) is isomorphic to (Q, q) in $R[X]$ and if $q = q^*$ (JD), then $r = r^*$ (JD).*

Let $D(X)$ be the full subcategory of $R(X)$ consisting of all objects (Q, q) with $q = q^*$ (JD). Then Proposition 3.1 indicates that the inclusion functor $I : D(X) \subset R(X)$ is a right adjoint of the join functor $* : R(X) \rightarrow D(X)$:

$$\frac{(Q, q) \xrightarrow{f} I(R, r^*)}{(Q, q^*) \xrightarrow{f} (R, r^*)} .$$

EXAMPLE 3.6. Let $\{A_{\alpha} \mid \alpha \in A\}$ be a family of sets with $A = \bigcup_{\alpha} A_{\alpha}$. Moreover, let $\pi_{\alpha} : \mathbf{Set}^A \rightarrow \mathbf{Set}$ be a functor $(\langle S_{\alpha} \rangle_{\alpha \in A} \mapsto \pi_{\alpha \in A_{\alpha}} S_{\alpha})$ and $k_{\alpha} : \pi_A \rightarrow \pi_{\alpha}$ a natural transformation of projections ($\alpha \in A$). Under this situation the join dependency discussed in the present section is the usual join dependency [6].

4. Categories of Adjoint Models

In this section we will define a category of adjoint models in case that a functor $X: \mathcal{X} \rightarrow \mathcal{C}$ has a left adjoint functor $Y: \mathcal{C} \rightarrow \mathcal{X}$. Also a sufficient condition for the category of adjoint models to be isomorphic to the quotient category $R[X]$ of relational models will be given. In addition we assume that $Y \dashv X: \mathcal{C} \rightarrow \mathcal{X}$ is an adjunction [8, p. 78] and that $(\mathcal{C}^*, \mathcal{M}^*)$ is an image factorization system of \mathcal{C} .

DEFINITION 4.1. An adjoint model (with respect to an adjunction $Y \dashv X: \mathcal{C} \rightarrow \mathcal{X}$) is defined to be a pair $\langle \bar{Q}, \hat{q} \rangle$ of an object \bar{Q} in \mathcal{C} and an epimorphism $\hat{q}: Y\bar{Q} \rightarrow Q$ in \mathcal{X} such that $\hat{q} \in \mathcal{E}^*$ and an adjoint $q: \bar{Q} \rightarrow XQ$ of $\hat{q}: Y\bar{Q} \rightarrow Q$ is in \mathcal{M} . A morphism $\bar{f}: \langle \bar{Q}, \hat{q} \rangle \rightarrow \langle \bar{R}, \hat{r} \rangle$ of an adjoint model $\langle \bar{Q}, \hat{q} \rangle$ into another adjoint model $\langle \bar{R}, \hat{r} \rangle$ is defined to be a morphism $\bar{f}: \bar{Q} \rightarrow \bar{R}$ in \mathcal{C} such that there exists a morphism $f: Q \rightarrow R$ in \mathcal{X} rendering the following square commutative:

$$\begin{array}{ccc} Y\bar{Q} & \xrightarrow{\hat{q}} & Q \\ Y\bar{f} \downarrow & & \downarrow f \\ Y\bar{R} & \xrightarrow{\hat{r}} & R \end{array}$$

(The uniqueness of $f: Q \rightarrow R$ follows from the surjectivity of $\hat{q}: Y\bar{Q} \rightarrow Q$. Arrows " \dashrightarrow " and " \twoheadrightarrow " in \mathcal{X} represent monomorphisms in \mathcal{M}^* and epimorphisms in \mathcal{E}^* , respectively.)

Let $R\{Y\}$ be the category of all adjoint models and all morphisms between them. Identity morphisms and the composition of morphisms in $R\{Y\}$ are given as usual.

Taking an adjoint of the last commutative square we have the following commutative diagrams:

$$\begin{array}{ccc} \langle \bar{Q}, \hat{q} \rangle & \xrightarrow{\bar{f}} & \langle \bar{R}, \hat{r} \rangle \\ Y\bar{Q} & \xrightarrow{Y\bar{f}} & Y\bar{R} \\ \hat{q} \downarrow & & \downarrow \hat{r} \\ Q & \xrightarrow{f} & R \\ & \text{---} f \text{---} & \\ \hline \bar{Q} & \xrightarrow{\bar{f}} & \bar{R} \\ q \downarrow & & \downarrow r \\ XQ & \xrightarrow{Xf} & XR \end{array}$$

Thus we naturally obtain a functor

$$\begin{array}{ccc} F: R\{Y\} & \longrightarrow & R(X) \\ \langle \bar{Q}, \hat{q} \rangle & (Q, q) & \\ \bar{f} \downarrow \longmapsto \downarrow f & & \\ \langle \bar{R}, \hat{r} \rangle & (R, r) & \end{array}$$

The functor $F: R\{Y\} \rightarrow R(X)$ is called the *Armstrong functor* because it corresponds to a construction [9] of characteristic relations (data models) from data models defined by first order predicates satisfying equivalence laws.

Let (R, r) be an object in $R(X)$ and let

$$\begin{array}{ccc} Y\bar{R} & \xrightarrow{\hat{r}} & R \\ & \searrow \hat{e} & \nearrow m \\ & R' & \end{array}$$

be an $(\mathcal{E}^*, \mathcal{M}^*)$ -factorization of an adjoint \hat{r} of r . Consider an adjoint of the last commutative triangle:

$$\begin{array}{ccc} Y\bar{R} & \xrightarrow{\hat{r}} & R \\ & \searrow \hat{e} & \nearrow m \\ & R' & \end{array}$$

$$\begin{array}{ccc} \bar{R} & \xrightarrow{r} & XR \\ & \searrow e & \nearrow Xm \\ & XR' & \end{array}$$

Since $Xm \cdot e = r \in \mathcal{M}$ by definition of (R, r) it follows that $e \in \mathcal{M}$ and hence we obtain an object $\langle \bar{R}, \hat{e} \rangle$ in $R\{Y\}$. It is trivial that $F\langle \bar{R}, \hat{e} \rangle = (R', e)$ and $m: (R', e) \rightarrow (R, r)$ is a morphism in $R(X)$, where $\bar{m} = 1_{\bar{R}}$. The next lemma shows that morphism $m: F\langle \bar{R}, \hat{e} \rangle \rightarrow (R, r)$ in $R(X)$ is a universal arrow [8] from F to (R, r) and the object $\langle \bar{R}, \hat{e} \rangle$ in $R\{Y\}$ is cofree [1] over (R, r) .

LEMMA 4.2. *Let $\langle \bar{Q}, \hat{q} \rangle$ be an object in $R\{Y\}$ and (R, r) an object in $R(X)$. For any morphism $f: F\langle \bar{Q}, \hat{q} \rangle \rightarrow (R, r)$ in $R(X)$ there exists a unique morphism $\bar{g}: \langle \bar{Q}, \hat{q} \rangle \rightarrow \langle \bar{R}, \hat{e} \rangle$ in $R\{Y\}$ such that the triangle*

$$\begin{array}{ccc} \langle \bar{Q}, \hat{q} \rangle & & F\langle \bar{Q}, \hat{q} \rangle \\ \downarrow \bar{g} & & \downarrow F\bar{g} \\ \langle \bar{R}, \hat{e} \rangle & \xrightarrow{m} & (R, r) \end{array} \quad \begin{array}{ccc} & & \nearrow f \\ & & \end{array}$$

is commutative.

COROLLARY 4.3. *Armstrong functor $F: R\{Y\} \rightarrow R(X)$ has a right adjoint functor $G: R(X) \rightarrow R\{Y\}$ ($(R, r) \mapsto \langle \bar{R}, \hat{e} \rangle$) and $m: (R', e) \rightarrow (R, r)$ is a component of counit $FG \rightarrow 1_{R(X)}$ of adjunction $F \dashv G: R\{Y\} \rightarrow R(X)$.*

Denote by $\tilde{F}: R\{Y\} \rightarrow R[X]$ the composite of Armstrong functor $F: R\{Y\} \rightarrow R(X)$

followed by quotient functor $P: R(X) \rightarrow R[X]$ (cf. §2). From definition of functor G there exist a unique functor $\tilde{G}: R[X] \rightarrow R\{Y\}$ rendering the triangle of functors

$$\begin{array}{ccc}
 R(X) & & \\
 \downarrow P & \searrow G & \\
 R[X] & \xrightarrow{\tilde{G}} & R\{Y\}
 \end{array}$$

commutative. Since $GF = 1_{R\{Y\}}$ (the identity functor of $R\{Y\}$), it is immediate that $\tilde{G}\tilde{F} = 1_{R[X]}$. The totality of all morphisms $[m]: \tilde{F}\tilde{G}(R, r) = (R', e) \rightarrow (R, r)$ for all object (R, r) in $R(X)$ constitutes a natural transformation $[m]: \tilde{F}\tilde{G} \rightarrow 1_{R[X]}$.

THEOREM 4.4. *If every monomorphism in \mathcal{M}^* is a split monomorphisms [8; p. 19], then $R\{Y\}$ is equivalent to $R[X]$, i. e., $[m]: \tilde{F}\tilde{G} \rightarrow 1_{R[X]}$ is a natural isomorphism.*

The functor $\pi_A: \mathcal{C}^A \rightarrow \mathcal{C}$ in Example 2.8 has a left adjoint $\Delta_A: \mathcal{C} \rightarrow \mathcal{C}^A$ since \mathcal{C} has products. In the case $\mathcal{C} = \mathbf{Set}_0$, the category of nonempty sets, we have $\mathbf{Set}_0[\pi_A] \cong \mathbf{Set}_0[\Delta_A]$ (an equivalence of categories) because every monomorphism in $\mathcal{M}^* = \langle \mathbf{Mon} \rangle_{a \in A}$ is a split monomorphism. This case was initially observed by Kato [6, Theorem 5.1].

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