ON SEMANTIC SPACE OF DICTIONARIES

Shiraishi, Shuji
Department of Information Systems, Interdisciplinary Graduate School of Engineering Sciences, Kyushu University

https://doi.org/10.5109/13356

出版情報：Bulletin of informatics and cybernetics. 21 (1/2), pp.55-63, 1984-03. Research Association of Statistical Sciences
バージョン：

Bulletin of Informatics and Cybernetics Vol. 21, No. 1∼2, 1984

ON SEMANTIC SPACE OF DICTIONARIES

By

Shuji SHIRAISHI*

Abstract

In this paper we first present a formal definition of dictionaries and introduce a semantic space of dictionaries which is used for giving formal meanings of entry words. Then we show that the semantic space is uniquely determined up to isomorphism. We also construct the semantic space which consists of infinite trees with no leaf and some other trees where only leaves are labeled. Hence we can simply take this semantic space when we discuss semantics of dictionaries. Finally we point out that our framework of dictionaries gives a mathematical foundation to Quillian's word concept problem.

1. Introduction

Dictionaries are indispensable not only for our daily lives but also for computerized systems such as database systems and knowledge information systems [1, 2].

According to the Random House Dictionary [3], a dictionary is "*a book containing a selection of the words of a language, usually arranged alphabetically giving information about their meanings, pronounciations, etymologies, inflected forms, etc., expressed in either the same or another language.*" However in the present paper we simply take a dictionary as *a book containing words of a language and their meanings expressed in the same language or another language.*

Although there may be several standpoints to treat meanings of words in a dictionary, we take them in the following manner. Consider our consulting a dictionary for the meaning of a word. The explanation of the word is expressed with a finite sequence of words in the dictionary. If we find unknown words in it, we may again consult the dictionary for them. In this way we will get the meaning of a word by consulting a dictionary finitely many times.

As far as the author's knowledge is concerned, no formal definition of a dictionary has been given and semantics of dictionaries has not yet been studied in a mathematical way. So in this paper we first give a formal definition of a dictionary and a semantic space of dictionaries. A dictionary is made up of a set of entry words, their explanations which are expressed with finite sequences of entry words in the dictionary and possibly undefined words. A semantic space is made up of a set Y and a bijection $\sharp$ from Y^* (the union of one or more products of Y's) to Y satisfying some commutative diagram.

* Department of Information Systems, Interdisciplinary Graduate School of Engineering Sciences, Kyushu University 39, Kasuga, Fukuoka 816, Japan.

In Section 2 we present a formal definition of a dictionary, which gives a framework of dictionaries and enables a mathematical discussion. In Section 3 we introduce a semantic space to treat formal meanings of words in a dictionary. Then we show that the semantic space is uniquely determined up to isomorphism. In Section 4 we construct the semantic space which is made up of infinite trees with no leaf and some other trees where only leaves are labeled. In Section 5 we discuss that our framework of dictionaries gives a mathematical solution to Quillian's word concept problem.

2. Dictionary

We give in this section a formal definition of a dictionary. In general explanations of entry words in a dictionary are expressed with finite sequences of entry words and possibly undefined words.

NOTATION. Let X be a set. Then X^* is defined by:

$$X^*=X^0+X^1+\cdots+X^n+\cdots,$$

where X^n is n-fold product of X and $+$ is disjoint union, and X^0 consists of just one special element λ, which corresponds to the empty string in language theory.

DEFINITION 1. A *dictionary* is a triple $DIC=(X, A, D)$, where

(1) X is a non-empty set of *entry words*,

(2) A is a set of *undefined words*,

(3) D is a mapping from X to $(X+A)^*$.

We call words other than entry words *undefined words* and also call $D(x)$ the *explanation* of x. The mapping D corresponds to the action of consulting a dictionary. If A is empty, we call a dictionary self-contained.

EXAMPLE. We give an example of a dictionary.

D(*concept*)=(*a, general, notion, or, idea*),

D(*notion*)=(*a, general, or, vague, idea*),

D(*idea*)=(*any, conception, existing, in, the, mind*),

D(*general*)=(*of, or, pertaining, to, all, persons, or, things, belonging, to, a, group, or, category*),

D(*mind*)=(*the, part, in, a, human, being, that, reasons, understands, perceives*).

For clearness, the expression $D(x)=(x_1, x_2, \cdots, x_n)$ may be illustrated by using the following tree (Fig. 2.1):

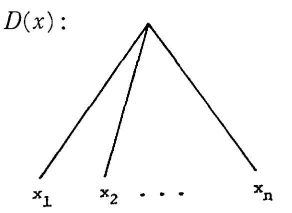

Fig. 2.1 $D(x)=(x_1, x_2, \cdots, x_n)$.

Next we consider semantics of dictionaries. Several approaches to the semantics may be considered from the various standpoints, such as linguistic, philosophical and mathematical standpoints. But we here take the semantics formally (mathematically) in the following way, that is, a formal meaning of any word in a dictionary is obtained by combining those of the words which appear in the explanation.

For example let us consider the semantics of very tiny dictionary $DIC=(X, A, D)$, where

$$X=\{x_1, x_2, x_3\},$$

$$A=\{a\},$$

and D is defined by:

$$D(x_1)=(x_1, x_2),$$

$$D(x_2)=(a),$$

$$D(x_3)=(x_3, a).$$

As $D(x_1)=(x_1, x_2)$, the formal meaning of x_1 is the combination of those of x_1 and x_2. Now let us see the process of getting the meaning of x_1 (Fig. 2.2). If we consult the dictionary for the x_1 one time, we get the first step, that is, $D(x_1)=(x_1, x_2)$. If we continue to consult it for the explanation (x_1, x_2), we get the second step. In the same

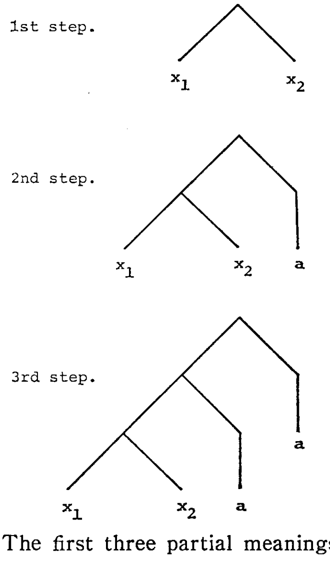

Fig. 2.2 The first three partial meanings of x_1.

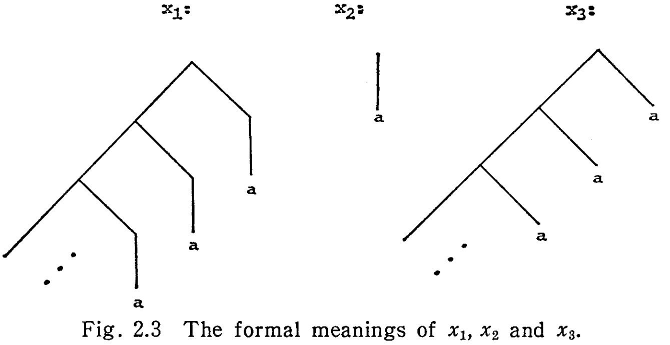

Fig. 2.3 The formal meanings of x_1, x_2 and x_3.

way we get the third step. That is, the (total) meaning of x_1 is obtained by consulting the dictionary repeatedly. This may be the process by which we acquire meanings of words. If we encounter undefined words "a", then this process partially stop there. Since we cannot consult the dictionary for them any more. This is the same for x_2, x_3. So we may take the meanings of x_1, x_2, x_3 as the following tree diagrams respectively (Fig. 2.3.):

As we can see in the above figure, in order to consider semantics of dictionaries, we are necessary some space which is used for giving formal meanings of words such as trees illustrated above. So in the next section we introduce a semantic space of dictionaries.

3. Semantic Space

We first define a semantic space of dictionaries and state the reason why this definition is sound. Then we show that the semantic space is isomorphic.

Now we define a semantic space formally.

DEFINITION 2. A *semantic space* for dictionaries is a pair $(Y, \sharp)$ with the following conditions:

(1) Y is a set.

(2) $\sharp : Y^* \to Y$ is a bijection.

(3) For any dictionary $DIC=(X, A, D)$, let $s: X \to Y$ and $\tilde{s} : X+A \to Y$ be mappings such that $\tilde{s}|X=s$ and $\tilde{s}(a)$ is some element of Y for any a in A, then the following diagram commutes and s, called a *semantic mapping*, is uniquely determined:

$$\begin{tikzcd} X \arrow[r, "s"] \arrow[d, "D"'] & Y \\ (X+A)^* \arrow[r, "\tilde{s}^*"'] & Y^* \arrow[u, "\sharp"'] \end{tikzcd}$$

where $\tilde{s}^* = s^0 + s^1 + \cdots + s^n \cdots$.

We explain briefly the reason why this definition is sound. It should be natural that we acquire the semantics $s(x)$ of an entry word x in X is uniquely determined. This semantics $s(x)$ is successively obtained along the commutative diagram above. Firstly, the word x in X is expressed with the finite sequence $x_1, \cdots, x_n$ $(n \geqq 1)$ of entry words and possibly undefined words by using the mapping D, that is,

$$D(x)=(x_1, \cdots, x_n).$$

Secondly the semantics of the explanation of x is defined by:

$$\tilde{s}^*(D(x))=\tilde{s}^n(x_1, \cdots, x_n)=(\tilde{s}(x_1), \cdots, \tilde{s}(x_n)).$$

Here if x_i is in X, then $\tilde{s}(x_i)=s(x_i)$. Otherwise, that is, if x_i is undefined words (x_i in A), the semantics may be any element of Y as long as our definition is satisfied.

Lastly the semantics $s(x)$ of x in X is obtained by combining these semantics

$s(x_1), \cdots, s(x_n)$ using the mapping $\sharp$, that is,

$$s(x)=\sharp(\tilde{s}(x_1), \cdots, \tilde{s}(x_n)).$$

The mapping $\sharp$ should be a bijection, since if the semantics of the explanations of words are mutually distinct, then so should be the semantics of the words, and since extra semantics is not in Y, that is, any semantics of Y is always the combination of some finite number of semantics of Y.

By this definition, we get the following proposition, which asserts that any semantic spaces are isomorphic.

PROPOSITION 3. *Let* $(Y_i, \sharp_i)$ $(i=1, 2)$ *be semantic spaces. Then*

$$(Y_1, \sharp_1)\cong(Y_2, \sharp_2)$$

holds.

PROOF. $\sharp_i^{-1}: Y_i \to Y_i^*$ $(i=1, 2)$ are considered to be self-contained dictionaries. Let $f: Y_1 \to Y_2$ and $g: Y_2 \to Y_1$ be the semantics mappings. Then the commutative diagram

$$\begin{tikzcd}
Y_1 \arrow[r, "f"] \arrow[d, "\sharp_1^{-1}"'] & Y_2 \arrow[r, "g"] \arrow[d, shift left, "\sharp_2^{-1}"] & Y_1 \\
Y_1^* \arrow[r, "\tilde{f}^*"'] & Y_2^* \arrow[r, "\tilde{g}^*"'] \arrow[u, shift left, "\sharp_2"] & Y_1^* \arrow[u, "\sharp_1"']
\end{tikzcd}$$

holds and the commutative diagram

$$\begin{tikzcd}
Y_1 \arrow[r, "i_{Y_1}"] \arrow[d, "\sharp_1^{-1}"'] & Y_1 \\
Y_1^* \arrow[r, "\tilde{i}^*_{Y_1}"'] & Y_1^* \arrow[u, "\sharp_1"']
\end{tikzcd}$$

also holds, where i_{Y_1} the identity mapping. By the uniqueness of semantic mapping we have $gf=i_{Y_1}$. Similarly we have $fg=i_{Y_2}$. Therefore the proposition is obtained. □

4. A Semantic Space Construction

We now construct the semantic space which consists of infinite trees with no leaf and some other trees where only leaves are labeled. For this end, we first give some notions on trees.

DEFINITION. Let $J_n=\{1, \cdots, n\}$ and let $J^*=J_1^*+\cdots+J_n^*+\cdots$. Then $J^*\supset\alpha$ is said to be a *tree* if the following conditions are satisfied:

(1) $st\in\alpha$ implies $s\in\alpha$.

(2) $sk\in\alpha$ and $k\in N$ imply $s\{1, \cdots, k\}\subset\alpha$, where N denotes the set of all natural numbers.

Any tree is finitely branching. Δ and $L(\alpha)$ denote the set of all trees and the set of all leaves of a tree α, respectively.

DEFINTIION.

α is a *partial tree* iff $L(\alpha)\neq\emptyset$.

α is a *total tree* iff $L(\alpha)=\emptyset$.

Partial trees are trees with leaves. Total trees are (infinite) trees with no leaf. We denote by Δ_p and Δ_t the set of all partial trees and the set of all total trees, respectively. Clearly $\Delta=\Delta_p+\Delta_t$. We also define labeled trees.

DEFINITION. Let Z be a labeled set. Then the set of all Z-labeled trees is

$${}_Z\Delta=\bigcup_{\alpha\in\Delta}\{m \mid m:\alpha\rightarrow Z\}\ ,$$

and the set of all trees where only leaves are Z-labeled is

$${}_Z\tilde{\Delta}=\bigcup_{\alpha\in\Delta}\{\alpha-L(\alpha)+m \mid m:L(\alpha)\rightarrow Z\}\ .$$

EXAMPLE. We give an example of a tree α in Fig. 4.1,

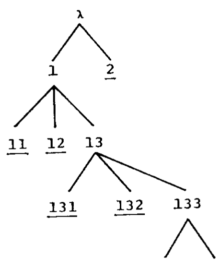

Fig. 4.1 Tree

where $L(\alpha)=\{2, 11, 12, 131, 132, \cdots\}$ are underlined.

We also give an example of the tree where only leaves are Z-labeled in Fig. 4.2,

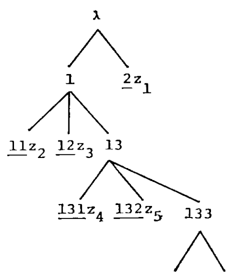

Fig. 4.2 Tree where only leaves are labeled

where xz_i means that leaf x is labeled by z_i.

Now we define a pair $(T, \sharp)$ as follows:

(1) $T=\Delta_t+{}_{\{\perp\}}\tilde{\Delta}$,

(2) $\sharp$ is a mapping from T^* to T such that

$$\sharp(\alpha_1, \cdots, \alpha_n)=\{\lambda\}+1\alpha_1+\cdots+n\alpha_n\ ,$$

$$\sharp(\lambda)=\{\perp\}\ .$$

Then we get:

Theorem 4. *$(T, \sharp)$ is a semantic space.*

PROOF. Clearly $\sharp$ is a bijection. So it suffices to show the uniqueness and existence of the mapping s satisfying the following commutative diagram:

$$\begin{tikzcd} X \arrow[r, "s"] \arrow[d, "D"'] & T \\ (X+A)^* \arrow[r, "\tilde{s}^*"'] & T^* \arrow[u, "\sharp"'] \end{tikzcd}$$

where we define for any a in A, $\tilde{s}^*(a)=\{\perp\}$.

First we show the uniqueness of the mapping s. Let t and s be the semantic mappings satisfying the above commutative diagram. Then for any x in X with $D(x)=(x_1, \cdots, x_n)$, we get

$$t(x)=\{\lambda\}+1\tilde{t}(x_1)+2\tilde{t}(x_2)+\cdots+n\tilde{t}(x_n),$$

$$s(x)=\{\lambda\}+1\tilde{s}(x_1)+2\tilde{s}(x_2)+\cdots+n\tilde{s}(x_n).$$

As $t(x)$ and $s(x)$ are the trees with the same root $\{\lambda\}$, we can say that $t(x)$ and $s(x)$ are the same trees if all the children of the root $\{\lambda\}$ are the same. Indeed if x_i is in A, then by the definition of $\tilde{s}$, $\tilde{s}(x_i)=\tilde{t}(x_i)=\{\perp\}$. Otherwise, that is, if x_i is in X, then $\tilde{t}(x_i)=t(x_i)$ and $\tilde{s}(x_i)=s(x_i)$. But $t(x_i)$ and $s(x_i)$ are the trees with the same root $\{\lambda\}$, so all the children are the same.

Next we show the existence of the mapping s. We consider the following commutative diagram:

$$\begin{tikzcd} X \arrow[r, "s_m"] \arrow[d, "D"'] & T_X \\ (X+A)^* \arrow[r, "\tilde{s}^*_{m-1}"'] & T_X^* \arrow[u, "\sharp_X"'] \end{tikzcd}$$

where (1) $T_X=\Delta_\iota+_{X+\{\perp\}}\Delta$,

(2) $\sharp_X$ is a bijection from T_X^* to T_X (like $\sharp$),

(3) $\tilde{s}_m$ ($m\geqq 0$) is a mapping from $X+A$ to T_X such that

(i) $\tilde{s}_m | X=s_m$,

(ii) for any a in A, $s_m(a)=\{\perp\}$,

(iii) for any x in X, $s_0(x)=\{x\}$.

Thus for any x in X with $D(x)=(x_1, \cdots, x_n)$, the following expression holds for any $m\geqq 1$:

$$(*) \qquad s_m(x)=\{\lambda\}+1\tilde{s}_{m-1}(x_1)+\cdots+n\tilde{s}_{m-1}(x_n).$$

Note here that the order in T_X is defined as follows. Let $\alpha(x_1, \cdots, x_n)$ denote the tree α with the leaves $x_1, \cdots, x_n$, and let $\omega=\alpha(x_1, \cdots, \alpha_i, \cdots, x_n)$ denote the tree replacing

x_i with the tree α_i. Then for any ω, α in T_X, we define the order $>$ by: $\omega>\alpha$ iff $\omega=\alpha(x_1, \cdots, \alpha_i, \cdots, x_n)$ for some x_i and α_i in T_X. Then it is easy to show that $(T_X, >)$ is a poset.

By this definition and the equation (*), we have $s_m(x)>s_{m-1}(x)$ for any $m\geqq 1$, and we get

$$s(x)=\sup_m s_m(x) \quad \text{in} \quad T.$$

Therefore by taking the sup of (*), we get

$$s(x)=\{\lambda\}+1\tilde{s}(x_1)+\cdots+n\tilde{s}(x_n). \qquad \square$$

5. Discussion

In this section we point out that our approach also gives a mathematical foundation to Quillian's word concept problem [4]. First we recall Quillian's definition of a word meaning. He dealt with the "meaning" of commonplace words, such as "machine", "family", "chair", and so on. The basis of his word definition is that the meaning of a word is expressed with the relation of other words. This definition is called a "plane" or "immediate definition". That is, a plane is made up of a word and its explanation which have an associative structure.

He also considered a word's full concept as all the words that can be reached by an exhaustive tracing process. This process starts at the "patriarch" (entry) words in a dictionary, and then moves to every word in each of its planes, and again starts at every word found in each of them, and so on. Reentries or loops within a full concept are permitted. Thus there is no hierarchy of superclasses and subclasses. That is, there are no word concepts as such that are "primitive". Every word is defined in terms of ordered configuration of other words.

Now we state the relationship between Quillian's definition and ours. The plane corresponds, for example, to $D(x)=(x_1, \cdots, x_n)$. The plane has an associative structure, but ours has no structure other than the order $(x_1, \cdots x_n)$. Sets of planes may be taken as our dictionaries. The word's full concept corresponds to some element of our semantic space. That is, if x is a word in a dictionary, then its full concept corresponds to $s(x)$, where s is a semantic mapping, which is the element of our semantic space. As we showed in Section 3 and 4, the semantic space of dictionaries is uniquely determined up to isomorphism and made up of trees, we can take a word's full concept to be the tree.

In summary we have given a framework of a dictionary in which semantics of words are specified by elements of our semantic space. Our idea is based on the observation that any word x in a dictionary is expressed with a finite sequence $x_1, \cdots, x_n$ of words in it and that the semantics $s(x)$ of a word x is obtained by combining the semantics $s(x_1), \cdots, s(x_n)$ of words $x_1, \cdots, x_n$.

This approach may be the first trial on semantics of dictionary and will also support the semantic description of data models. We will apply this framework to the semantic description of our data model called bottom-free data model. Another interesting mathematical problems are to study a rational property of dictionary semantics [5] and to solve the dictionary domain equation: $X^n+\cdots+X\cong X$ [6].

Acknowledgements

The author would like to express his sincere thanks to Dr. Setsuo Arikawa for his valuable suggestion, constructive comments and encouragement during the course of this study. The author wishes to thank Mr. Satoru Miyano for his critical reading the manuscript and helpful comments. The author also wishes to thank Prof. Setsuo Ohsuga at University of Tokyo for his helpful comments on Quillian's work.

References

[1] BORKIN, S. A.: *Data Models: A Semantic Approach for Database Systems*, MIT press, Cambridge, (1980).
[2] WINOGRAD, T.: *Understanding Natural Language*, Academic Press, New York, (1972).
[3] STEIN, J. and URDANG, L. (Eds.): *The Random House Dictionary of the English Language*, Random House, New York, (1967).
[4] QUILLIAN, R.: *Word Concepts: A Theory and Simulation of Some Basic Semantic Capabilities*, Behavial Science, 12, (1967).
[5] SATO, M.: *Algebraic Structure of Symbolic Expressions*, TR-82-05, Fac. of Sci., Univ. of Tokyo (1982).
[6] SCOTT, D.: *Domain Equations*, Proceedings of The Sixth IBM Symposium on Mathematical Foundations of Computer Science, Logic Aspect of Programs, (1981).

Communicated by S. Arikawa
Received September 10, 1983