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A TWO-PERSON INFINITE GAME SUGGESTED FROM A QUIZ OF GUESSING A NUMBER

By

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Abstract

This paper is purposed to formulate and analyze a two-person infinite game suggested from a quiz of guessing a number which is applicable to a plan of production under an uncertain demand, waiting for a person who arrives at a random time, etc. Shown are the two types of model, *shoppihg* and *shooting*.

1. Introduction

The problem discussed in this paper concerns the two-person infinite game suggested from the following example.

Firstly an umpire chooses a random number T in $[0, 1]$ which has *cdf* $H(t)=Pr\{T\leqq t\}$ and does not inform the two participants (Players I and II) in this game of the realized value of the *r. v.* T. Then each of I and II chooses a number in $[0, 1]$ independently from each other. Here two criteria which determine the winning player in this game are defined as follows:

(i) To choose a number which is greater than what his opponent chooses but does not exceed the realized value of the *r. v.* T.

(ii) To choose a number which is nearest to the realized value of the *r. v.* T.

Related to this example, there are many applications. i. e., a plan of production level under an uncertain demand, a formulation of on budget, waiting for a person who arrives at a random time and so on. The first type of the above criteria is termed *shopping* and the second, *shooting*.

Next, the settlement of two kinds of payoffs in this game is arranged as follows:

(a) If Player I wins, he receives units a from II and if II wins, on the other hand, he receives units b from I.
(b) The winning player only receives one unit from their umpire, that is, each player wisher to maximize his winning probability.

The former produces the zero-sum game and the latter leads to the non-zero-sum game.

For the sake of arguments in the remaining sections, we assume that *cdf* $H(t)$ is continuous, increasing $H(0)=0$ and $H(1)=1$, and has *pdf* $h(t)>0$ on $[0, 1]$. We shall

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employ usual notations on expectation of payoff $M(x, y)$ defined on the unit squase as follows:

$$M(F. G)=\int_0^1\int_0^1 M(x, y)dF(x)dG(y), \quad \text{and}$$

$$M(x, G)=\int_0^1 M(x, y)dG(y); \quad M(F, y)=\int_0^1 M(x, y)dF(x)$$

where $F(x)$ and $G(y)$ are the mixed strategies (*cdf* on [0, 1]) for Players I and II respectively.

The model mentioned above is very simple, but the author believes that it is highly applicable to various decision problems. In relation to this work, Teraoka [3] formulated and analyzed a two-person game of timing in which the appearance of the object is random and whether the players are able to obtain the object or not is uncertain.

2. The Shopping Model

In this model, each player wishes to choose the smallest possible number, since $H(t)$ increases with t. However, if the player chooses too small a number, his opponent may choose such an appropriate number that is greater than his number but yet does not exceed the realized value of the $r.v.\ T$.

2.1 Two-person zero-sum shopping game

Let x and y be the numbers which are chosen by Players I and II respectively, the assumption (a) in Section 1 gives the expected payoff kernel $M(x, y)$ as follows:

$$M(x, y)=\begin{cases} (a+b)H(y)-aH(x)-b, & x<y \\ (a-b)\{1-H(x)\} & x=y \\ bH(y)-(a+b)H(x)+a, & x>y. \end{cases} \tag{2.1}$$

Since $M(x, y)$ is strictly increasing in x and strictly increasing in y over the domains $0\leqq x<y\leqq 1$ and $0\leqq y<x\leqq 1$ respectively, we suppose that the mixed strategy $F(x)$ for I consists of a density part $f(x)>0$ over an interval $(0, u)$ and a mass part α at $x=0$, and that $G(y)$ for II consists of a denslty part $g(y)>0$ over the same interval and a pass part β at $y=0$. Then we have

$$M(F, y)=\begin{cases} \alpha(a-b)-\int_0^u \{-(a+b)H(x)+a\}f(x)dx, & y=0 \\ \alpha\{(a+b)H(y)-b\}+\int_0^y \{(a+b)H(y)-aH(x)-b\}f(x)dx & \\ \qquad +\int_y^u \{bH(y)-(a+b)H(x)+a\}f(x)dx, & 0<y\leqq u \\ \alpha\{(a+b)H(y)-b\}+\int_a^u \{(a+b)H(y)-aH(x)-b\}f(x)dx, & u<y\leqq 1. \end{cases} \tag{2.2}$$

which is continuous in $y\in(0, u]$. Supposing that $M(F, y)\equiv v$ for $y\in(0, u]$, we get

$$h(y)/\{1-H(y)\}=\left\{(a+b)f(y)\right\}\Big/\left\{a\left(\alpha+\int_a^y f(x)dx\right)+b\right\} \quad \text{for} \quad 0<y\leqq u \tag{2.3}$$

which leads to

$$aF(y)+b=K[1-H(y)]^{-a/(a+b)}, \qquad 0<y\leqq u$$

where $F(y)=\alpha+\int_0^y f(x)dx$ so that

(2.4) $$F(x)=(\alpha+b/a)[1-H(x)]^{-a/(a+b)}-b/a \qquad \text{for} \quad 0\leqq x\leqq u.$$

Considering $F(u)=1$ and $0\leqq\alpha<1$, u satisfies the equation

$$(\alpha+b/a)[1-H(u)]^{-a/(a+b)}=1+b/a\,.$$

Thus we obtain

(2.5) $$H(u)=1-\{(a+b)/(a\alpha+b)\}^{-(1+b/a)}$$

which has a unique root in $[0, 1]$. When $\alpha=0$ we denote the unique root of equation (2.5) by u_1, that is,

$$u_1=H^{-1}(1-\{1+(a/b)\}^{-(1+b/a)})$$

Substituting (2.4) by (2.2), we find that

(2.6) $$M(F, y)\begin{cases} =v+\alpha a\,, & y=0 \\ =v\,, & 0<y\leqq u \\ >v\,, & u<y\leqq 1\,. \end{cases}$$

Similar arguments on $M(x, G)$ give us

(2.7) $$G(y)=(a/b+\beta)[1-H(y)]^{-b/(a+b)}-a/b \qquad \text{for} \quad 0\leqq y\leqq u\,,$$

(2.8) $$H(u)=1-\{(a+b)/(a+b\beta)\}^{-(1+a/b)}$$

and when $\beta=0$

$$u_2=H^{-1}(1-\{1+(b/a)\}^{-(1+a/b)})$$

which is the unique root of (2.8). Moreover, we find that

(2.9) $$M(x, G)\begin{cases} =v-\beta b\,, & x=0 \\ =v\,, & 0<x\leqq u \\ <v\,, & u<x\leqq 1\,. \end{cases}$$

Thus we arrive at $\alpha\beta=0$, which leads to $u=\min(u_1, u_2)$. After all we have the following:

(i) When $a>b$, since $u=u_2$ we get $\beta=0$ and

$$\begin{aligned}\alpha &= \{1+(b/a)\}[1-H(u_2)]^{a/(a+b)}-b/a \\ &= \{1+(b/a)\}^{1-(a/b)}-b/a>0\,.\end{aligned}$$

(ii) When $a=b$, since $u=u_1=u_2$ we get $\alpha=\beta=0$.

(iii) When $a<b$, since $u=u_1$ we get $\alpha=0$ and

$$\beta=\{1+(a/b)\}^{1-(b/a)}-a/b>0\,.$$

Since the Karlin's theorem [2; Chapter 6] holds, we have obtained the unique optimal strategies $F^*(x)$ for I and $G^*(y)$ for II of the zero-sum game (2.1).

Here we shall derive the value of the game v. When $\alpha=0$, since

$$M(F,\ 0)=a-(a+b)\int_0^u H(x)f(x)dx \qquad \text{and}$$

$$M(F,\ u)=(a+b)H(u)-b-a\int_0^u H(x)f(x)dx$$

we have

$$\int_0^u H(x)f(x)dx=\{(a+b)/b\}\,\{1-H(u)\}$$

by using $M(F,\ y)\equiv v$ for $y\in[0,\ u]$. Hence we get

$$v=M(F,\ 0)=a-\{(a+b)^2/b\}\,\{1-H(u)\}.$$

Considering $u=u_2$, we obtain

$$v=a-\{(a+b)^2/b\}(1+a/b)^{-(1+b/a)}, \quad \text{when} \quad \alpha=0.$$

In a similar fashion, we have

$$v=-b+\{(a+b)^2/a\}(1+b/a)^{-(1+a/b)}, \quad \text{when} \quad \beta=0.$$

The above considerations lead to the following theorem.

THEOREM 1. *Let*

$$u=H^{-1}(1-\{(1+b/a)^{-(1+a/b)}\vee(1+a/b)^{-(1+b/a)}\}),$$

then the optimal strategies $F^*(x)$ *for I and* $G^*(y)$ *for II of the zero-sum game* (2.1) *are the following mixed strategies:*

$$F^*(x)=\begin{cases}(\alpha+b/a)[1-H(x)]^{-a/(a+b)}-b/a\,, & 0\leqq x\leqq y\\ 1\,, & u<x\leqq 1\end{cases}\ ;$$

$$G^*(y)=\begin{cases}(\beta+a/b)[1-H(y)]^{-b/(a+b)}-a/b\,, & 0\leqq y\leqq u\\ 1\,, & u<y\leqq 1\,,\end{cases}$$

where two mass parts α *and* β *at zero are determined as follows:*

If $a>b$, *then* $\alpha=\{1+(b/a)\}^{1-a/b}-b/a>0$ *and* $\beta=0$.

If $a=b$, *then* $\alpha=0$ *and* $\beta=0$

If $a<b$, *then* $\alpha=0$ *and* $\beta=\{1+(a/b)\}^{1-b/a}-a/b>0$.

Then value of the game v^* *is given by*

$$v^*=\begin{cases}-b+\{(a+b)^2/a\}(1+b/a)^{-(1+a/b)}>0 & \text{if} \quad a>b\\ 0 & \text{if} \quad a=b\\ a-\{(a+b)^2/b\}(1+a/b)^{-(1+b/a)}<0 & \text{if} \quad a<b\,.\end{cases}$$

REMARK: (i) According to the above theorem, the profitable player is forced to behave more timidly than his opponent, since his remaining probability at point 0 is positive.

(ii) When $a=b$, we have

$$F^*(x)=G^*(y)=\begin{cases}\{1-H(x)\}^{-1/2}-1\,, & 0\leqq x\leqq H^{-1}(3/4)\\ 1. & H^{-1}(3/4)<x\leqq 1\end{cases}$$

and

$$v^*=0\,.$$

2.2 Two person non-zero-sum shopping game

Let $M_i(x, y)$ be the expected payoff kernel to Player i when x and y are the pure strategies for I and II respectively, the assumption (b) in Section 1 leads to the following:

$$M_1(x, y)=\begin{cases} H(y)-H(x), & x<y \\ 0, & x=y\ ; \\ 1-H(x), & x>y \end{cases} \tag{2.10}$$

$$M_2(x, y)=\begin{cases} H(x)-H(y), & y<x \\ 0, & y=x \\ 1-H(y), & y>x\,. \end{cases} \tag{2.11}$$

We observe that $M_1(x, y)$ is strictly decreasing in x over the domains $0\leqq x<y\leqq 1$ and $0\leqq y<x<1$ and strictly increasing in y over the domain $0\leqq x<y\leqq 1$. Since both the payoff kernels (2.10) and (2.11) are symmetric, the equilibrium strategy for one player remains also the equilibrium strategy for his opponent.

Here we shall employ the following Lemma without proof.

LEMMA 1. *For the non-zero-sum infinite game $M_1(x, y)$ and $M_2(x, y)$ defined on the unit square, if there exist the distribution functions $F^0(x)$ and $G^0(y)$, values v_1^0 and v_2^0, and intervals $[l_1, u_1]\subset[0, 1]$ and $[l_2, u_2]\subset[0, 1]$ such that*

$$\int_{l_2}^{u_2} M_1(x, y)dG^0(y)\begin{Bmatrix} = \\ < \end{Bmatrix}v_1^0 \quad if \quad x\in\begin{Bmatrix} [l_1, u_1] \\ [l_1, u_1]^c \end{Bmatrix},$$

where $G^0(l_2)=0$ and $G^0(u_2)=1$;

$$\int_{l_1}^{u_1} M_2(x, y)dF^0(x)\begin{Bmatrix} = \\ < \end{Bmatrix}v_2^0 \quad if \quad y\in\begin{Bmatrix} [l_2, u_2] \\ [l_2, u_2]^c \end{Bmatrix},$$

where $F^0(l_1)=0$ and $F^0(u_1)=0$, then (F^0, G^0) is the equilibribrium point for $M_1(x, y)$ and $M_2(x, y)$, and v_1^0 and v_2^0 satisfy

$$v_1^0=\int_0^1\int_0^1 M_1(x, y)dF^0(x)dG^0(y);$$

$$v_2^0=\int_0^1\int_0^1 M_2(x, y)dF^0(x)dG^0(y) \quad respectively.$$

Now we shall define the following mixed strategy:

$$F^0(x)=\begin{cases} -\log\{1-H(x)\}, & 0\leqq x<H^{-1}(1-1/e)\doteqdot H^{-1}(0.632) \\ 1, & H^{-1}(1-1/e)<x\leqq 1. \end{cases} \tag{2.12}$$

Then we shall state Theorem 2.2.

THEOREM 2. *A pair of equilibrium strategies of non-zero-sum game* (2.10) *and* (2.11) *is given by $(F^0(x), F^0(y))$. The equilibrium payoff of the game to each player is $1/e$ $\doteqdot 0.632$.*

PROOF. Supposing that both of the mixed strategies $F(x)$ for I and $F(y)$ for II consist of a density function $f(\cdot)$ over the support $[0, u]$, we get

$$M_1(x, F)=\begin{cases} F(x)-H(x)+\int_x^u H(y)f(y)dy\,, & 0\leqq x\leqq u \\ 1-H(x)\,, & u<x\leqq 1. \end{cases} \tag{2.13}$$

We also suppose that

$$M_1(x, y)=v_1^0 \qquad \text{for all} \quad x\in[0, u]\,, \tag{2.14}$$

which yields

$$f(x)=h(x)/\{1-H(x)\} \qquad \text{for} \quad x\in[0, u]\,.$$

since $F(0)=0$, we have

$$F(x)=-\log\{1-H(x)\} \qquad \text{for} \quad x\in[0, u]\,.$$

The boundary condition $F(u)=1$ gives $u=H^{-1}(1-1/e)\doteqdot H^{-1}(0.632)$. Thus we have derived the mixed strategy $F^0(x)$ defined in (2.12) which satisfies the intergral equation (2.14) which has a unique solution under an appropriate boundary condition. Then we get

$$M_1(x, F^0)=\begin{cases} M_1(u, F^0)=1-H(u)=1/e\,, & 0\leqq x\leqq u \\ 1-H(x)<1-H(u)=1/e\,, & 0<x\leqq 1 \end{cases}$$

from (2.13).

Since both the payoff kernels (2.10) and (2.11) are symmetric with respect to x and y and Lemma 1 holds, we have proved Theorem 2.

REMARK: Though the proof of Theorem 1 is very simple, the implication is very interesting, that is, $1-1/e$ is the lower bound of the neglecting region for $H(t)$ and $1/e$ is the expectation of the winning probability under the equilibrium condition. We are interested in the extention to n-person model. Perhaps the equilibrium payoff to each player will be $(1/e)^n$ and its equilibrium mixed strategy will be $F^0(x)$ for all players.

3. The Shooting Model

In this model, each player wishes to choose his number which is nearest to the realized value of $r.v.\ T.$, however, both the two have no information about cdf $H(t)$. Intuitively we can conjecture that the optimal number will relate to one of the three values of cdf $H(t)$, $E(T)$, $H^{-1}(1/2)$, and $\max h(t)$. Moreover, from the game-theoretical viewpoint, if the optimal strategy exists then it will be given by $H^{-1}(1/2)$. Thus we shall define t_0 as follows:

$$H(t_0)=1/2\,, \quad \text{i.e.,} \quad t_0=H^{-1}(1/2)\,, \tag{3.1}$$

which exists uniquely in $[0, 1]$.

3.1 Two-person zero-sum shooting game

Let x and y be the pure strategies for I and II respectively, the expected payoff kernel $M(x, y)$ to Player I is given by

$$M(x, y)=\begin{cases} (a+b)H((x+y)/2)-b\,, & x<y \\ \Phi(a, b)\,, & x=y \\ a-(a+b)H((x+y)/2)\,, & x>y\,, \end{cases} \tag{3.2}$$

where $\Phi(a, b)$ depends only on a and b but not x and y. Since $M(x, y)$ is strictly increasing in each of x and over the domain $0 \leqq x < y \leqq 1$ and strictly decreasing in each of x and y over the domain $0 \leqq y < x \leqq 1$, we have

$$M(t_0, y) > \left\{ \begin{matrix} (a+b)H(t_0)-b \\ a-(a+b)H(t_0) \end{matrix} \right\} = \frac{a-b}{2} \quad \text{if} \quad y \left\{ \begin{matrix} > \\ < \end{matrix} \right\} t_0 :$$

$$M(x, t_0) < \left\{ \begin{matrix} (a+b)H(t_0)-b \\ a-(a+b)H(t_0) \end{matrix} \right\} = \frac{a-b}{2} \quad \text{if} \quad x \left\{ \begin{matrix} < \\ > \end{matrix} \right\} t_0 ,$$

from the definition on t_0, i.e., Equation (3.1).

Here we shall define the mixed strategy $\xi(x)$ for any $\varepsilon > 0$ as follows:

$$\xi(x) = \begin{cases} 0, & 0 \leqq x < t_0 - \delta \\ \int_{t_0-\delta}^{x} (1/2\delta)dt, & t_0 - \delta \leqq x < t_0 + \delta \\ 1, & t_0 + \delta \leqq x \leqq 1 \end{cases} \tag{3.3}$$

where

$$\delta = \min[t_0 - H^{-1}(t_0 - \varepsilon/(a+b)),\ H^{-1}(t_0 + \varepsilon/(a+b)) - t_0].$$

Then the definitions on $\xi(x)$ and t_0 lead to

$$M(\xi, y) = \begin{cases} \int_{t_0-\delta}^{t_0+\delta} [a-(a+\dot{b})H((x+y)/2)](1/2\delta)dx, & y \leqq t_0 - \delta \\ \int_{t_0-\delta}^{y} [(a+b)H((x+y)/2)-b](1/2\delta)dx & \\ \quad + \int_{y}^{t_0+\delta} [a-(a+b)H((x+y)/2)](1/2\delta)dx, & t_0 - \delta \leqq y \leqq t_0 + \delta \\ \int_{t_0-\delta}^{t_0+\delta} [(a+b)H((x+y)/2)-b](1/2\delta)dx, & y \geqq t_0 + \delta . \end{cases}$$

An analysis of the first case ($y \leqq t_0 - \delta$) yields

$$M(\xi, y) \geqq \int_{t_0-\delta}^{t_0+\delta} [a-(a+b)H(t_0)](1/2\delta)dx = (a-b)/2 .$$

The second case ($t_0 - \delta \leqq y \leqq t_0 + \delta$) gives

$$\begin{aligned} M(\xi, y) &\geqq (1/2\delta)[\{(a+b)H(t_0-\delta)-b\}(y-t_0+\delta) + \{a-(a+b)H(t_0+\delta)\}(t_0+\delta-y)] \\ &\geqq \frac{1}{2\delta}\left[\left\{(a+b)\left(\frac{1}{2}-\frac{\varepsilon}{a+b}\right)-b\right\}(y-t_0+\delta) + \left\{a-(a+b)\left(\frac{1}{2}+\frac{\varepsilon}{a+b}\right)\right\}(t_0+\delta-y)\right] \\ &= (a-b)/2 - \varepsilon . \end{aligned}$$

The third case ($y \geqq t_0 + \delta$) leads to

$$M(\xi, y) \geqq \int_{t_0-\delta}^{t_0+\delta} [(a+b)H(t_0)-b](1/2\delta)dx = (a-b)/2 .$$

Thus we obtain

$$M(\xi, y) \geqq (a-b)/2 - \varepsilon \qquad \text{for all} \quad y \in [0, 1] .$$

Similar arguments on $M(x, \xi)$ give

$$M(x, \xi) \leqq (a-b)/2 + \varepsilon \qquad \text{for all} \quad x \in [0, 1] .$$

The above considerations lead us to the following theorem.

THEOREM 3. *Let* t_0 *and* $\xi(\cdot)$ *be defined by* (3.1) *and* (3.3) *respectively, then*:

(i) *The game has value* $(a-b)/2$.

(ii) *If* $\Phi(a, b)=(a-b)/2$, *then* (t_0, t_0) *is the saddle point of the game.*

(iii) *If* $\Phi(a, b)<(a-b)/2$, *then* $\xi(x)$ *is an* ε*-optimal mixed strategy for Player I and* t_0 *is the optimal pure strategy for Player II.*

(iv) *If* $\Phi(a, b)>(a-b)/2$, *then* t_0 *is the optimal pure strategy for Player I and* $\xi(y)$ *is an* ε*-optimal mixed strategy for Player II.*

NOTE: (a) Usually, $\Phi(a, b)$ equals to 0, $a-b$ or $(a-b)/2$. (b) The difference between a and b has no inference on the strategies stated in Theorem 3, but relates to the value of the game.

3.2 Two-person non-zero-sum shooting game

According to the assumption (b) in Section 1, we have the expected payoff kernel $M_i(x, y)$ for Player i as follows:

$$M_1(x, y)=\begin{cases} H((x+y)/2), & x<y \\ 0, & x=y; \\ 1-H((x+y)/2), & x>y \end{cases} \tag{3.4}$$

$$M_2(x, y)=\begin{cases} H((x+y)/2), & y<x \\ 0, & y=x \\ 1-H((x+y)/2), & y>x. \end{cases} \tag{3.5}$$

Then we get

$$M_1(x, t_0)\begin{Bmatrix} <H(t_0) \\ =0 \\ <1-H(t_0) \end{Bmatrix}; \quad M_2(x, t_0)\begin{Bmatrix} >1-H(t_0) \\ =0 \\ >H(t_0) \end{Bmatrix} \quad \text{if} \quad x\begin{Bmatrix} < \\ = \\ > \end{Bmatrix}t_0$$

and

$$M_1(t_0, y)\begin{Bmatrix} <H(t_0) \\ =0 \\ >1-H(t_0) \end{Bmatrix}; \quad M_2(t_0, y)\begin{Bmatrix} <1-H(t_0) \\ =0 \\ <H(t_0) \end{Bmatrix} \quad \text{if} \quad y\begin{Bmatrix} > \\ = \\ < \end{Bmatrix}t_0,$$

where $H(t_0)=1-H(t_0)=1/2$ from (3.1).

As in section 3.1, we also define the mixed strategy $\eta(x)$ for any $\varepsilon>0$ as follows:

$$\eta(x)=\begin{cases} 0, & 0\leqq x<t_0-\delta \\ \int_{t_0-\delta}^{x}(1/2\delta)dt, & t_0-\delta<x\leqq t_0+\delta \\ 1, & t_0+\delta\leqq x\leqq 1, \end{cases} \tag{3.6}$$

where δ is given by

$$\delta=\min[t_0-H^{-1}(t_0-\varepsilon), H^{-1}(t_0+\varepsilon)-t_0].$$

Then, for any $\varepsilon>0$ the following inequalities are held:

$$\begin{cases} M_1(x, \eta)\leqq 1/2+\varepsilon & \text{for all} \quad x\in[0, 1] \\ M_1(\eta, y)\geqq 1/2-\varepsilon & \text{for all} \quad y\in[0, 1] \end{cases}; \tag{3.7}$$

$$\begin{cases} M_2(x, \eta)\geqq 1/2-\varepsilon & \text{for all} \quad x\in[0, 1] \\ M_2(\eta, y)\leqq 1/2+\varepsilon & \text{for all} \quad y\in[0, 1], \end{cases} \tag{3.8}$$

By putting $a=1$ and $b=0$ in the proof of Theorem 3. Thus we have Theorem 4.

THEOREM 4. (i) *The two-pevson non-zero-sum game* (3.4) *and* (3.5) *are the almost strictly competitive game.*

(ii) *Let* t_0 *and* $\eta(\cdot)$ *be defined by* (3.1) *and* (3.6) *respectirely, then* $(\eta(x), \eta(y))$ *is a pair of* ε*-equilibrium mixed strategies and a pair of* ε*-twisted equilibrium mixed stratigies, that is,* $\eta(x)$ *and* $\eta(y)$ *are the* ε*-optimal mixed strategies fov I and II respectively.*

(iii) *The value of the game is* $1/2$.

NOTE: The *almost strictly competive game* was discovered by Aumann [1]. And then he defined the *twisted equilibrium point* and suggested a reasonable optimal strategy and value of the game for some classes of two-person non-zero-sum games.

4. Concluding Remarks

In this paper four problems for a two-person infinite game suggested from a quiz of guessing a number have been formulated and analyzed. Theorems 1-4 mention that all the four values of the game are dependent only on the returns to both players and not on cdf $H(t)$. This fact is convincible, since $H(t)$ has the same weight to both players. All the optimal strategy pairs, however, are dependent on the returns to both players and $H(t)$ intrinsically. Now the validity of convergence is assumed as follows:

Letting $a/b\to\infty$ *then* $G^*(y)\to F^0(y)$ *and* $v^*\to 1/e$, without proof, since $M_1(x, y)$ of Section 2.2 is formulated by puttiing $a=1$ and $b=0$ in $M(x, y)$ of Section 2.1.

Here, it is remarked that the problem discussed in this paper is closely related with the competitive bidding problems and games of timing, especially our shopping model further relates to the problem where the highest bid wins, and to the model of silent duel. Though the structure of payoff in our model is very simple, the form of competition is essential to the problems mentioned above.

Finally, it is noted that we were left with more interesting cases such as follows:

(a) The winning player receives the return which is a function of the difference (or its absolute value) between his selected number and the realized value of *r.v.* T, especially in the case where the return is increasing (or decreasing) function with respect to the difference.

(b) Generalization to multi-person game, which is more realistic but very difficult even for three-person model.

(c) Extension to a multi-stage game suggested from incnrporating the learning structure and information about the realized value of *r.v.* T. by selecting a number, such as search theory.

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