

MODELING OF NERVOUS SYSTEMS BY DOUBLY STOCHASTIC POISSON PROCESSES

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MODELING OF NERVOUS SYSTEMS BY DOUBLY STOCHASTIC POISSON PROCESSES

By

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Abstract

Neuron impulse sequences are represented by doubly stochastic Poisson Processes and Kalman filtering theory is applied to estimate the random intensity functions of the processes. Models of nervous systems in which excitatory and inhibitory synapses have elementary operation, are studied by simultaneous states equations in the theory. The method of estimation is applied to neuron impulse sequences simultaneously recorded from preoptic area of a monkey by a micro-electrode.

1. Introduction

In recent years, physiological experiments with micro-electrode method have made it clear that information transmission and processing in the brain are carried out through neuron impulse sequences. However we can not identify the operation of a neuron impulse sequence only by the shapes or patterns of the sequence apart from original neuron. Each neuron impulse sequence fired from an original neuron has two roles. One is function of the neuron in brain and the other is concerned with the information about activity of the neuron. For example, positions of auditory and visual areas are different in brain and neurons in their areas have proper roles about auditory or visual sense, but shapes of their neuron impulse sequences may be equal. This fact makes it possible to integrate neural information through corresponding neuron impulse sequences. Observed impulse sequences in experiments show somewhat complicated patterns which are based on their impulse frequencies. To study nervous systems we have to consider the frequencies of impulse sequences as basic variables of the system.

In this paper, some mathematical models of nervous systems are given and estimation for time dependent fluctuation of impulse frequencies is performed on the models.

For analysis of nervous systems, N. Wiener's stochastic nonlinear theory [1] has been used by the paper [2] and the others. The theory gives powerful tool for continuous data of nerve action potential or brain waves. For discrete data of neuron impulse sequences we had no method to analyze. We apply Kalman filtering theory

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to random intensity functions of doubly stochastic Poisson processes which are models of neuron impulse sequences. The method of this paper is considered to be valuable for analyses of nervous systems by the impulse sequence and to increase the effectiveness of physiological experiments with micro-electrodes still more.

2. Stochastic models of neuron impulse sequences

We assume that neuron impulse sequence conducted by each axon in a nervous system forms a stochastic point process termed neuron point process in this paper. We have neuron input and output point processes for input and output impulse sequences of a nervous system denoted by $\{M_t; t \geq t_0\}$ and $\{N_t; t \geq t_0\}$, respectively; or $\{M_t\}$ and $\{N_t\}$, for abbreviation. For multi-dimensional input and output point processes, we use $\mathbf{M}_t = (M_t^{(1)}, \dots, M_t^{(m)})'$ and $\mathbf{N}_t = (N_t^{(1)}, \dots, N_t^{(n)})'$, respectively. Let intensity of neuron point process correspond to frequency of neuron impulse sequence and we denote those of input and output one-dimensional point processes by π_t and λ_t , and $\Pi_t = (\pi_t^{(1)}, \dots, \pi_t^{(m)})'$ and $\Lambda_t = (\lambda_t^{(1)}, \dots, \lambda_t^{(n)})'$, for multi-dimensional point processes, respectively. In this paper, neuron point processes are assumed to be doubly stochastic Poisson processes, which are defined as follows according to [3].

DOUBLY STOCHASTIC POISSON PROCESS. *A stochastic point process $\{N_t; t \geq t_0\}$ is a doubly stochastic Poisson process with intensity process $\{\lambda_t(\mathbf{x}_t); t \geq t_0\}$ if conditionally stochastic point process $\{N_t\}$ for given stochastic process $\{\mathbf{x}_t; t \geq t_0\}$ is a Poisson process with intensity $\lambda_t(\mathbf{x}_t)$. N -dimensional doubly stochastic Poisson process $\{\mathbf{N}_t; t \geq t_0\}$ with intensity process $\{\Lambda_t(\mathbf{x}_t); t \geq t_0\}$ is a stochastic point process such that its component processes $\{N_t^{(i)}; t \geq t_0\}$, $i=1, \dots, n$, are doubly stochastic Poisson processes with intensity processes $\{\lambda_t^{(i)}(\mathbf{x}_t); t \geq t_0\}$, $i=1, \dots, n$, respectively, and are conditionally independent for given stochastic process $\{\mathbf{x}_t; t \geq t_0\}$.*

From this definition we have

$$(2.1) \quad E(N_t) = E\{\lambda_t(\mathbf{x}_t)\}, \quad E(\mathbf{N}_t) = E\{\Lambda_t(\mathbf{x}_t)\}.$$

3. Modeling of nervous systems

We assume that stochastic process $\{\mathbf{x}_t\}$ which gives the intensity function $\Lambda_t(\mathbf{x}_t)$ of doubly stochastic Poisson process is intensity function of input neuron point process. Then, output of a nervous system is doubly stochastic Poisson process with intensity $\Lambda_t(\mathbf{x}_t)$ and structure of the nervous system is represented by a relation of $\{\mathbf{x}_t\}$ or an equation of $\{\mathbf{x}_t\}$. It is reasonable that as the relation stochastic differential equations are applied. Using the definition of Kalman filtering theory, the differential equation is called states equation and $\{\mathbf{x}_t\}$ called states. Fitness of the equation to the nervous system is studied by estimation in which inputs $\{\mathbf{x}_t\}$ are estimated by output data $\{\mathbf{N}_t\}$.

3.1. Estimation of Π_t .

It is assumed that intensity Π_t of input process $\{\mathbf{M}_t\}$ is stochastic process which satisfies the stochastic differential equation,

1) M' represents transpose of vector or matrix M .

$$(3.1) \quad d\Pi_t = A_t \Pi_t dt + \mathbf{b}_t dt + \mathbf{u}_t dt, \quad \Pi_{t_0} = \Pi_0,$$

and intensity A_t of output process $\{N_t\}$ is given by the stochastic equation,

$$(3.2) \quad A_t = S_t \Pi_t + \mathbf{h}_0,$$

where

A_t ; known $m \times m$ matrix,

\mathbf{b}_t ; known m -dimensional vector,

Π_0 ; m -dimensional random variable with known mean $\bar{\Pi}_0$ and known covariance matrix Σ_0 ,

\mathbf{u}_t ; m -dimensional normal white noise with zero-mean and covariance matrix $E(\mathbf{u}_t \mathbf{u}_\tau') = U_t \delta(t - \tau)$,

S_t ; known $n \times m$ matrix,

\mathbf{h}_0 ; known n -dimensional constant vector.

Then, following assertion is shown.

ASSERTION 1 (*estimation of Π_t*). Let Π_t^i be a linear estimate of Π_t in terms of output data $\{N_t\}$. Denote the linear estimate Π_t^i that minimizes error covariance matrix $E[(\Pi_t - \Pi_t^i)(\Pi_t - \Pi_t^i)']$ by Π_t^* : here the minimization means that $E[(\Pi_t - \Pi_t^i)(\Pi_t - \Pi_t^i)'] - E[(\Pi_t - \Pi_t^*)(\Pi_t - \Pi_t^*)']$ is nonnegative definite for all choices Π_t^i . Then, using Kalman filtering theory in [4], we can obtain the estimate Π_t^* by following equations:

$$(3.3a) \quad d\tilde{\Pi}_t^* = \tilde{A}_t \tilde{\Pi}_t^* dt + \tilde{\Sigma}_t \tilde{C}_t' \text{diag}(E(\lambda_t^{(i)}))^{-1} [dN_t - \tilde{C}_t \tilde{\Pi}_t^* dt], \quad \tilde{\Pi}_{t_0}^* = (\bar{\Pi}_0', 1)',$$

$$(3.3b) \quad d\tilde{\Sigma}_t = (\tilde{U}_t + \tilde{A}_t \tilde{\Sigma}_t + \tilde{\Sigma}_t \tilde{A}_t' - \tilde{\Sigma}_t \tilde{C}_t' \text{diag}(E(\lambda_t^{(i)}))^{-1} \tilde{C}_t \tilde{\Sigma}_t) dt, \quad \tilde{\Sigma}_{t_0} = \tilde{\Sigma}_0,$$

where

$$\tilde{A}_t = \begin{pmatrix} A_t & \mathbf{b}_t \\ \mathbf{0} & 0 \end{pmatrix}; \quad (m+1) \times (m+1) \text{ matrix},$$

$$\tilde{\Pi}_t^* = (\Pi_t^{*'}, 1)'; \quad (m+1)\text{-dimensional vector},$$

$$\tilde{C}_t = (S_t, \mathbf{h}_0); \quad n \times (m+1) \text{ matrix},$$

$$\tilde{U}_t = \begin{pmatrix} U_t & \mathbf{0} \\ \mathbf{0} & 0 \end{pmatrix}; \quad (m+1) \times (m+1) \text{ matrix},$$

$$\tilde{\Sigma}_0 = \begin{pmatrix} \Sigma_0 & \mathbf{0} \\ \mathbf{0} & 0 \end{pmatrix}; \quad (m+1) \times (m+1) \text{ matrix},$$

$$\tilde{\Sigma}_t = \begin{pmatrix} \Sigma_t & \mathbf{0} \\ \mathbf{0} & 0 \end{pmatrix}; \quad (m+1) \times (m+1) \text{ matrix},$$

$$\Sigma_t; \text{ error covariance matrix of } \Pi_t^*.$$

The assertion is shown as follows.

Equation (3.1) is rewritten as

$$(3.4) \quad d\tilde{\Pi}_t = \tilde{A}_t \tilde{\Pi}_t dt + \tilde{\mathbf{u}}_t dt, \quad \tilde{\Pi}_{t_0} = (\Pi_0', 1)',$$

where

$$\tilde{\Pi}_t = (\Pi'_t, 1)', \quad \tilde{\mathbf{u}}_t = (\mathbf{u}'_t, 0)'.$$

Here, (3.4) is states equation of Kalman filter. Next, we give an observed process of the states by

$$(3.5) \quad \mathbf{r}_t = \tilde{C}_t \tilde{\Pi}_t + \mathbf{w}_t,$$

where $\{\mathbf{w}_t\}$ is n -dimensional normal white noise process and independent of $\{\mathbf{u}_t\}$ such that $E(\mathbf{w}_t) = \mathbf{0}$ and $E(\mathbf{w}_t \mathbf{w}'_s) = \text{diag}(E(\lambda_t^{(i)})) \delta(t-s)$. From (3.2) and (3.5),

$$E(\mathbf{r}_t dt) = E(A_t + \mathbf{w}_t) dt.$$

Since $E(\mathbf{w}_t) = \mathbf{0}$, we have from (2.1) that

$$(3.6) \quad E(\mathbf{r}_t dt) = E(d\mathbf{N}_t).$$

And from (3.2), (3.5) and independency of \mathbf{w}_t and A_t ,

$$E[(\mathbf{r}_t dt)(\mathbf{r}_t dt)'] = E[(A_t dt)(A_t dt)'] + E[(\mathbf{w}_t dt)(\mathbf{w}_t dt)'].$$

Here, from the fact that normal white noise \mathbf{w}_t can be expressed as

$$\mathbf{w}_t = E(\mathbf{w}_t) + \text{diag}(E(\lambda_t^{(i)}))^{1/2} \frac{d}{dt} \boldsymbol{\beta}_t,$$

where $\{\boldsymbol{\beta}_t\}$ is n -dimensional Brownian motion process, we have

$$(3.7) \quad E[(\mathbf{w}_t dt)(\mathbf{w}_t dt)'] = \text{diag}(E(\lambda_t^{(i)})) dt.$$

Furthermore, since from definition in Section 2 $\{\mathbf{N}_t\}$ is conditionally Poisson process and its component processes $\{N_t^{(i)}\}$, $i=1, \dots, n$, are conditionally independent, we can get

$$(3.8) \quad E[(A_t dt)(A_t dt)'] + \text{diag}(E(\lambda_t^{(i)})) dt = E[(d\mathbf{N}_t)(d\mathbf{N}_t)'].$$

Therefore, we have from (3.6), (3.7) and (3.8) that

$$(3.9) \quad \text{Cov}(\mathbf{r}_t dt) = \text{Cov}(d\mathbf{N}_t).$$

Consequently, from (3.6) and (3.9), mean and covariance matrix of $\mathbf{r}_t dt$ coincide with those of $d\mathbf{N}_t$. Thus we can use observed value $d\mathbf{N}_t$ as $\mathbf{r}_t dt$ in applying Kalman filtering theory to (3.4) and (3.5). That is, the estimate Π_t^* of Π_t in terms of output data $\{\mathbf{N}_t\}$ is given by equations (3.3).

3.2. Estimation of parameters in model systems

It is assumed that intensity Π_t of input point process is given by a stochastic differential equation,

$$(3.10) \quad d\Pi_t = A_t \Pi_t dt + \mathbf{b}_t dt + \mathbf{u}_t dt, \quad \Pi_{t_0} = \Pi_0,$$

where

$A_t = (a_t^{(ij)}), i, j = 1, \dots, m$; unknown $m \times m$ matrix,
 $b_t = (b_t^{(1)}, \dots, b_t^{(m)})'$; unknown m -dimensional vector,
 $u_t = (u_t^{(1)}, \dots, u_t^{(m)})'$; $u_t^{(i)}, i = 1, \dots, m$, are mutually independent normal white noise
 with zero-mean and covariance $E\{u_t^{(i)} u_{t-\tau}^{(i)'}\} = V_u^{(i)}(t) \delta(t-\tau)$, respectively,
 Π_0 ; m -dimensional random variable with known mean $\bar{\Pi}_0$ and known covariance
 matrix Σ_0 .

Then, following assertion is shown.

ASSERTION 2 (estimation of parameters.) Assume that parameters $a_t^{(ij)}$ and $b_t^{(i)}$ are constant value, and that observed value $\tilde{\pi}_t^{(i)}$ of i -component $\pi_t^{(i)}$ of Π_t is given by

$$(3.11) \quad \tilde{\pi}_t^{(i)} = \pi_t^{(i)} + w_t^{(i)}, \quad i = 1, \dots, m,$$

where $w_t^{(i)}$ is normal white noise with zero-mean and covariance $E\{w_t^{(i)} w_{t-\tau}^{(i)'}\} = V_w^{(i)}(t) \delta(t-\tau)$, respectively. Then mean-square estimates $a_t^{*(ij)}$ and $b_t^{*(i)}$ of parameters $a_t^{(ij)}$ and $b_t^{(i)}$ in terms of observed value of $\{\Pi_t\}$ are given by following equations:

$$(3.12a) \quad d\tilde{K}_t^{*(i)} = \Sigma_t^{(i)} C_t' V_v^{(i)}(t)^{-1} [d\tilde{\pi}_t^{(i)} - C_t \tilde{K}_t^{*(i)} dt], \quad i = 1, \dots, m,$$

$$(3.12b) \quad \Sigma_t^{(i)} = -(\Sigma_t^{(i)} C_t' V_v^{(i)}(t)^{-1} C_t \Sigma_t^{(i)}) dt, \quad i = 1, \dots, m,$$

where

$$\tilde{K}_t^{*(i)} = (a_t^{*(i1)}, \dots, a_t^{*(im)}, b_t^{*(i)})',$$

$$C_t = (\tilde{\pi}_t^{(1)}, \dots, \tilde{\pi}_t^{(m)}, 1),$$

$$V_v^{(i)}(t) = V_u^{(i)}(t) + \text{Var}(dw_t^{(i)}/dt) + \sum_{j=1}^m [(a_t^{(ij)})^2 V_w^{(j)}(t)].$$

The assertion is shown as follows.

Rearranging (3.10) with regard to i -component of Π_t , we have following m equations:

$$(3.13) \quad d\pi_t^{(i)} = (\pi_t^{(1)}, \dots, \pi_t^{(m)}, 1)(a_t^{(i1)}, \dots, a_t^{(im)}, b_t^{(i)})' dt + u_t^{(i)} dt, \quad i = 1, \dots, m.$$

Using equations (3.11), we have

$$(3.14) \quad d\tilde{\pi}_t^{(i)} = (\tilde{\pi}_t^{(1)}, \dots, \tilde{\pi}_t^{(m)}, 1)(a_t^{(i1)}, \dots, a_t^{(im)}, b_t^{(i)})' dt + v_t^{(i)} dt, \quad i = 1, \dots, m,$$

where

$$v_t^{(i)} = u_t^{(i)} + dw_t^{(i)}/dt - \sum_{j=1}^m a_t^{(ij)} w_t^{(j)}, \quad i = 1, \dots, m.$$

Since parameters $a_t^{(ij)}$ and $b_t^{(i)}$ are constant,

$$(3.15) \quad \frac{d}{dt} \begin{pmatrix} a_t^{(i)} \\ b_t^{(i)} \end{pmatrix} = 0, \quad i = 1, \dots, m,$$

where $a_t^{(i)} = (a_t^{(i1)}, \dots, a_t^{(im)})'$. Applying Kalman filtering theory to (3.14) and (3.15), we have equations (3.12).

3.3. Estimation of A_t

It is assumed that output process $\{N_t\}$ is n -dimensional doubly stochastic Poisson process with intensity process $\{A_t\}$ and that mean $E(A_t)$ and covariance matrix $K_A(t, u)$ are known. Let A_t^i be a linear estimate of A_t in terms of output data $\{N_t\}$ given by

$$(3.16) \quad A_t^i = \mathbf{a}_t + \int_{t_0}^t H(t, u) d\mathbf{N}_u,$$

where \mathbf{a}_t is n -dimensional vector and $H(t, u)$ is $n \times n$ impulse response matrix. Then, following assertion holds by [3].

ASSERTION 3 (*estimation of A_t*) Let the states equation

$$(3.17) \quad d\mathbf{x}_t = A_t \mathbf{x}_t dt + B_t d\boldsymbol{\beta}_t, \quad \mathbf{x}_{t_0} = \mathbf{x}_0,$$

where A_t is known $m \times m$ matrix, B_t is known $m \times r$ matrix, $\{\boldsymbol{\beta}_t\}$ is r -dimensional Brownian motion process and \mathbf{x}_0 is random variable with zero-mean and known covariance matrix Σ_0 , satisfy

$$E(\mathbf{x}_t) = \mathbf{0}, \quad \text{for } t \geq t_0$$

and

$$K_A(t, u) = C_t E(\mathbf{x}_t \mathbf{x}_u') C_u', \quad \text{for } t, u \geq t_0,$$

where C_t is known $n \times m$ matrix, that is, $A_t = E(A_t) + C_t \mathbf{x}_t$. Denote the estimate A_t^i that minimizes error covariance matrix $E[(A_t - A_t^i)(A_t - A_t^i)']$ by A_t^* . Then, the estimate A_t^* is given by following equations:

$$(3.18a) \quad d\mathbf{x}_t^* = A_t \mathbf{x}_t^* dt + \Sigma_t C_t' \text{diag}(E(\lambda_t^{(i)}))^{-1} [dN_t - A_t^* dt], \quad \mathbf{x}_{t_0}^* = \mathbf{0},$$

$$(3.18b) \quad d\Sigma_t = (A_t \Sigma_t + \Sigma_t A_t' + B_t B_t' - \Sigma_t C_t' \text{diag}(E(\lambda_t^{(i)}))^{-1} C_t \Sigma_t) dt, \quad \Sigma_{t_0} = \Sigma_0,$$

$$(3.18c) \quad A_t^* = E(A_t) + C_t \mathbf{x}_t^*.$$

Furthermore, error covariance matrix of A_t^* is given by

$$E[(A_t - A_t^*)(A_t - A_t^*)'] = C_t \Sigma_t C_t'.$$

The assertion is shown in [3] as follows.

The estimate A_t^i in the form of (3.16) that minimizes error covariance matrix $E[(A_t - A_t^i)(A_t - A_t^i)']$ is given by

$$(3.19) \quad A_t^* = E(A_t) + \int_{t_0}^t H_0(t, u) [d\mathbf{N}_u - E(A_u) du],$$

where the optimum impulse response matrix $H_0(t, u)$ satisfies

$$(3.20) \quad H_0(t, u) \text{diag}(E(\lambda_u^{(i)})) + \int_{t_0}^t H_0(t, \sigma) K_A(\sigma, u) d\sigma = K_A(t, u), \quad \text{for } t_0 \leq u < t.$$

Seeing (3.19) and (3.20), two intensity processes lead to the same linear intensity estimator if their means and covariance matrices are identical. So we can give outside stochastic process of A_t by $\{\mathbf{x}_t\}$ that satisfies states equation (3.17). Now, we give an observed process of the states by

$$(3.21) \quad \mathbf{r}_t = \mathbf{A}_t + \text{diag} (E(\lambda_t^{(i)}))^{1/2} \mathbf{w}_t,$$

where \mathbf{w}_t is n -dimensional normal white noise and independent of \mathbf{A}_t such that $E(\mathbf{w}_t) = \mathbf{0}$ and $E(\mathbf{w}_t \mathbf{w}_\tau) = \mathbf{I}_n \delta(t - \tau)$. Then, it is shown that mean and covariance matrix of $\mathbf{r}_t dt$ and those of dN_t are identical by the same way as Section 3.1. Therefore, we can use observed value dN_t as $\mathbf{r}_t dt$. Thus, applying Kalman filtering theory to (3.17) and (3.21), we have equations (3.18).

4. Analyses of physiological data

Applying the method stated in the preceding sections, we try to analyze physiological impulse data [5] which were obtained from the monkey in sexual behavior at Oomura Laboratory of Physiology, Kyushu University, in January 1981. See Fig. 1.

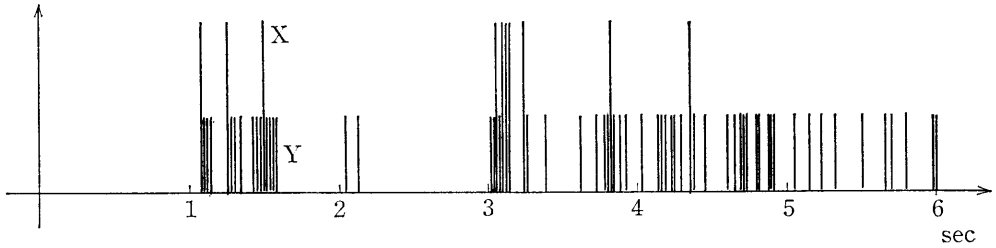


Fig. 1. Impulse sequence in physiological experiment. See [5].

As shown in Fig. 1, two impulse sequences, large and small ones, called X and Y respectively, were simultaneously recorded from a neuron in preoptic area by a micro-electrode during 5 minutes. In the figure, impulse sequence for first six seconds is shown. The large impulses were action potentials of the neuron inserted by the electrode and the small impulses corresponded to action potentials of neighboring neuron. We expected that there were some synaptic connections between the neighboring two neurons. However it was difficult to display any connection organically through broken neurons.

Then, following two connections were assumed between the neighboring two neurons firing large impulse sequence X and small impulse sequence Y , respectively.

- (1) Y is input point process $\{M_t\}$ and X is output point process $\{N_t\}$.
- (2) X is input point process $\{M_t\}$ and Y is output point process $\{N_t\}$.

In each case, estimation of parameters in states equation is performed and then using the estimated parameters and output data $\{N_t\}$, we obtain estimates $\{\pi_t^*\}$.

Lastly, regarding each X and Y as $\{N_t\}$ with uniformly distributed independent process $\{\lambda_t\}$, we obtain estimates $\{\lambda_t^*\}$ in terms of $\{N_t\}$. Then $\{\lambda_t^*\}$ are compared with impulse frequency (impulse numbers per second) of $\{N_t\}$.

4.1. The case with $X = \{N_t\}$ and $Y = \{M_t\}$.

Assertion 2 is applied for estimation of parameters. Equations (3.10) and (3.11) are reduced to

$$\begin{aligned} d\pi_t &= a_t \pi_t dt + b_t dt + u_t dt, \\ \tilde{\pi}_t &= \pi_t + w_t. \end{aligned}$$

Variances of u_t and w_t are selected as

$$V_u(t)=1, \quad V_w(t)=\frac{1}{4}.$$

Observed value $\tilde{\pi}_t$ is considered as impulse frequency of Y .

Then, we obtained estimates a_t^* and b_t^* giving perturbation to error covariance matrix Σ_t by resetting their diagonal elements at 1.0 and non-diagonal elements at 0.0 every 10 seconds (100 steps). Estimates a_t^* and b_t^* are shown in Table 1.

Table 1. Estimates a_t^* and b_t^* of parameters a_t and b_t in terms of Y .

t (sec)	30	60	90	120	150	180	210	240	270	300
a_t^*	-0.295	-0.845	-0.117	-0.761	-0.762	-0.461	-0.875	-0.723	-0.292	-0.192
b_t^*	1.497	1.935	2.542	2.923	2.911	3.140	2.974	3.039	2.856	2.793

Next, fixing the estimates a_t^* and b_t^* , we perform estimation of π_t in terms of X by Assertion 1 in Section 3.1. Equations (3.1) and (3.2) are given by

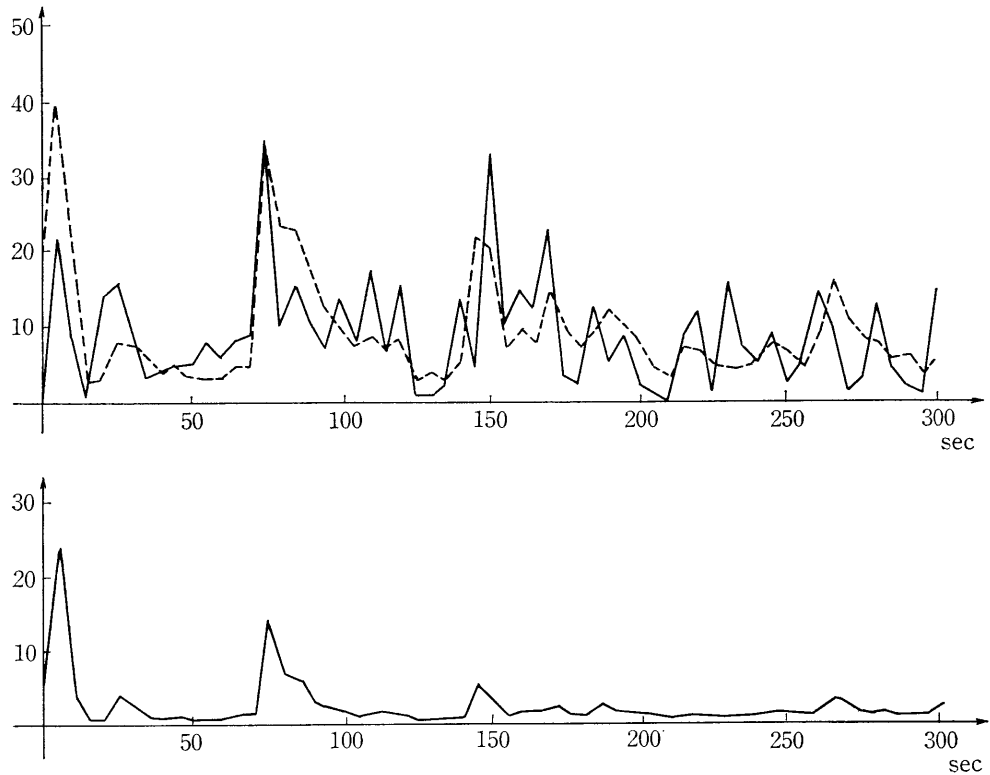


Fig. 2. Estimate π_t^* compared with impulse frequency of Y .
Upper: solid line shows impulse frequency of Y and broken line shows estimate π_t^* .
Lower: mean-square error Σ_t of π_t^* .

$$d\pi_t = a_t^* \pi_t dt + b_t^* dt + u_t dt, \\ \lambda_t = 0.2\pi_t.$$

As the observation of λ_t , we use impulse frequency of X . Estimate π_t^* is calculated and the graph is shown in Fig. 2.

4.2. The case with $X = \{M_t\}$ and $Y = \{N_t\}$.

In this case, numerical calculation is carried out by the same way as Section 4.1, except the exchange, $X = \{M_t\}$, $Y = \{N_t\}$, and $\lambda_t = 5.0\pi_t$. Estimates a_t^* and b_t^* are shown in Table 2. Estimate π_t^* is shown in Fig. 3.

Table 2. Estimates a_t^* and b_t^* of parameters a_t and b_t in terms of X .

t (sec)	30	60	90	120	150	180	210	240	270	300
a_t^*	-0.521	-0.637	-0.467	-0.682	-0.540	-0.389	-0.462	-0.597	-0.643	-0.609
b_t^*	0.532	0.633	0.610	0.684	0.753	0.838	0.806	0.824	0.577	0.353

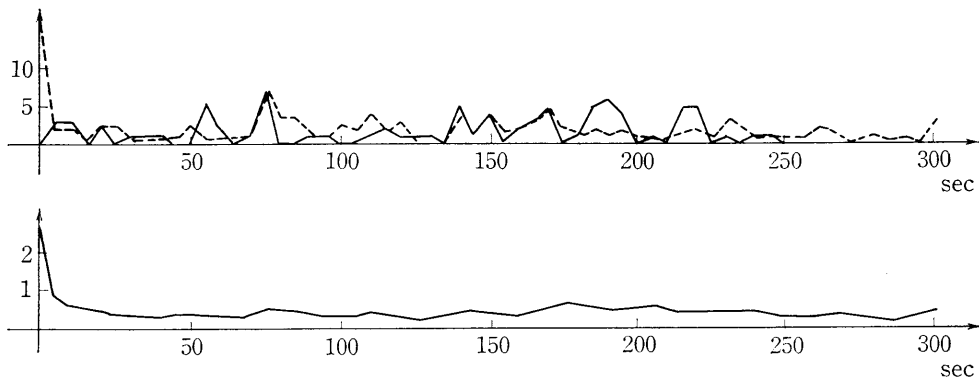


Fig. 3. Estimate π_t^* compared with impulse frequency of X .

Upper: solid line shows impulse frequency of X and broken line shows estimate π_t^* .
Lower: mean-square error Σ_t of π_t^* .

As compared Fig. 3 with Fig. 2, a question arise: which is better model 4.1 or 4.2 to explain the physiological data in Fig. 1? To see some significant differences between two models, we need more experimental facts and analytical study. But, since intensity processes $\{\pi_t\}$ satisfying the stochastic differential equation (3.1) are Markov processes, models of nervous systems given in 4.1 and 4.2 may be called Markov nervous systems.

4.3. Estimation of λ_t in terms of $\{N_t\}$.

We apply Assertion 3 in Section 3.3 for each X and Y which is considered as $\{N_t\}$.

In this case, Kalman filtering theory is applied only for a method of estimating λ_t in terms of $\{N_t\}$ without assuming any structure of nervous system. Then, stochastic process $\{\lambda_t\}$ is considered as independent stochastic process. This section was selected as a simple control model for the modeling of nervous systems in Section 4.1 and 4.2.

At first, in the case of $X = \{N_t\}$, λ_t is given as follows.

$$\lambda_t = 0.9\alpha_t,$$

where $\{\alpha_t\}$ is independent stochastic process and uniformly distributed in $[0, 8]$. Then, we have

$$E(\lambda_t) = 3.6$$

and

$$K_\lambda(t, u) = \begin{cases} 4.32, & t = u \\ 0, & t \neq u. \end{cases}$$

Coefficients a_t and b_t in applied equation (3.17) and c_t are calculated as

$$a_t = 2.0, \quad b_t = 1.0, \quad c_t = \frac{7.2}{\sqrt{3}}.$$

Then estimate λ_t^* is obtained and shown in Fig. 4.

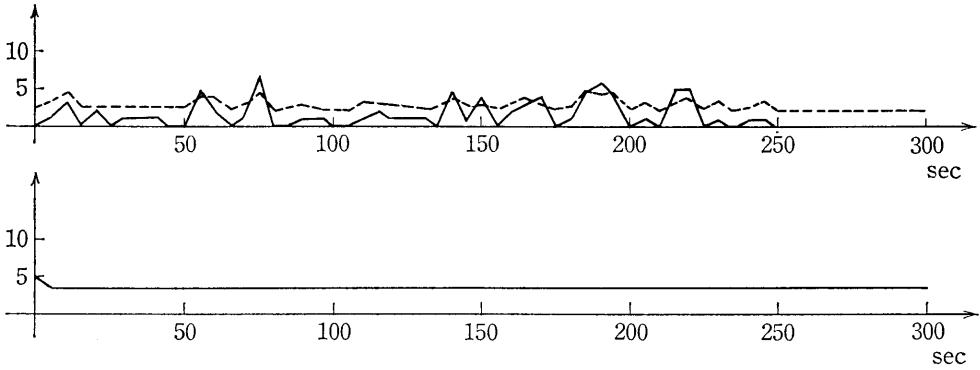


Fig. 4. Estimate λ_t^* obtained in terms of X and compared with impulse frequency of X .

Upper: solid line shows impulse frequency of X and broken line shows estimate λ_t^* .

Lower: mean-square error Σ_t of λ_t^* .

In the case of $Y = \{N_t\}$, λ_t^* is obtained similarly except that

$$\lambda_t = 5.0\alpha_t,$$

$$E(\lambda_t) = 20$$

$$K_\lambda(t, u) = \begin{cases} \frac{400}{3}, & t = u \\ 0, & t \neq u, \end{cases}$$

and coefficients

$$a_t = 2.0, \quad b_t = 1.0, \quad c_t = \frac{40}{\sqrt{3}}.$$

The result is shown in Fig. 5.

Comparing Fig. 4 and Fig. 5 with Fig. 3 and Fig. 2, respectively, the models in

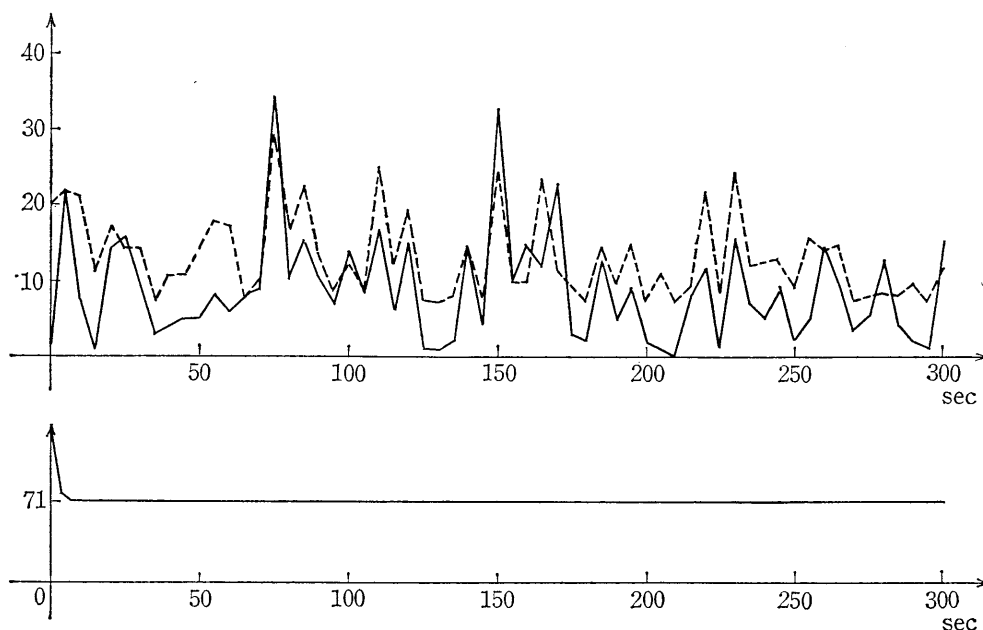


Fig. 5. Estimate λ_t^* obtained in terms of Y and compared with impulse frequency of Y .

Upper: solid line shows impulse frequency of Y and broken line shows estimate λ_t^* .

Lower: mean-square error Σ_t of λ_t^* .

4.1 and 4.2 are better than the models in 4.3 to describe the physiological data in Fig. 1, as we expected.

5. Results and Problems

In this paper, we gave some methods of mathematical modeling and analyses of nervous systems by following two lines.

- (1) Neuron impulse sequences are represented by doubly stochastic Poisson processes.
- (2) The structure of nervous system is described by stochastic differential equations in which random variables are intensity processes.

Then, experimental data recorded from a neuron in the nervous system, preoptic area of monkey, were analyzed by given methods. The system equations were selected to obtain minimum error variance. Good fitting of the estimates to data showed effective modeling. However, when we try to form a model accounting some functions of a nervous system, following questions arise.

- (1) Why are a great number of neurons, of 10^3 or 10^4 , needed for an area which controls single function of a brain? How are these neurons mutually related?
- (2) How are many control areas in brain mutually related and integrated to a function?

Basic principle of the two interrelations may be identical each other, that is, neural network within an area may operate similarly to that between many areas. Methods of multi-dimensional doubly stochastic Poisson processes and of simultaneous stochastic

differential equations, both stated in this paper, may be effective for each study of the two interrelations.

There are some problems in the study.

- (1) More than three impulse sequences which are simultaneously recorded in physiological experiments are needed.
- (2) A method in which some basic networks between two or three neurons are connected for increasing the functions of their networks, must be attacked.
- (3) Modeling of nervous systems is to be studied in connection with changes of neuron impulse patterns caused by animal behavior, that is, taking food, learning and so on.

Experiments designed on the solutions of these problems may give us interesting data forming multi-dimensional neuron impulse sequence. We believe that the method given in this paper becomes powerful tool to analyze these data and to explain advanced functions of nervous systems.

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