Environment and Innovation in the Knowledge-based Economy

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Environment and Innovation in the Knowledge-based Economy *

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Abstract

As described in this paper, we consider problems related to the environment and economic growth in a knowledge-based economy. We formulate a theoretical framework of endogenous growth model with human capital and innovation to reflect a knowledge-based economy. In the context of endogenous growth models, innovation and human capital have been regarded in the literature (see Lucas 1988; Romer 1990; Grossman and Helpman 1991; Aghion and Howitt 1992) as forces for promoting economic growth. In those papers, however, innovation and human capital are dealt with separately in a growth context. As described herein, we examine a theoretical framework in which innovation and human capital are consolidated. On the other hand, several papers deal with the relation between the environment and economic growth (see Gradus and

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Smulders (1993), Stokey (1998), Brock and Taylor (2005)). These papers, however, do not properly reflect the knowledge-based economy. Therefore, we consider the problems of environment and economic growth in the context of a theoretical framework in which innovation and human capital are consolidated. The growth rate of per capita income becomes positive in the long run because productivity increases over time by technological progress and human capital accumulation. We will also show that pollution declines in the steady state if and only if the level of abatement technology is sufficiently high.

Key words: Environment, Innovation, Human Capital, Knowledge-based Economy

JEL Classification: O40, Q20

1 Introduction

According to the OECD (1996), "The OECD economies are increasingly based on knowledge and information. Knowledge is now recognised as the driver of productivity and economic growth, leading to a new focus on the role of information, technology and learning in economic performance. The term "knowledge-based economy" stems from this fuller recognition of the place of knowledge and technology in modern OECD economies." (p.3)

Many countries have changed greatly and have become knowledge-based economies in which knowledge engenders the development of a high value-added economy. Knowledge is more important than ever before, although it is not a new idea that knowledge plays an important role in economic growth: even Adam Smith argued for the contribution of knowledge to production.

We construct a growth model that incorporates human capital to reflect this point in a theoretical model. The roles of knowledge have been stressed often by human-capital-based growth models (see Uzawa 1965; Lucas 1988). In such models, knowledge, know-how, and skills embodied in labor (which is a definition of human capital) rather than raw labor, becomes the primary factor of production. Human capital can be accumulated by introducing some resources to the education sector. Although typical traditional growth theory has specifically examined the incentive for physical capital accumulation, it is insufficient to attain a positive growth rate in the steady state. Productivity improvement is necessary for sustainable growth. Human-capital-based growth models subsume that productivity growth emerges from human capital accumulation. We will also show that human capital accumulation is one determinant of sustainable growth.

Furthermore, unlike typical human-capital-based growth models, we consider the roles of human capital in the research lab. Many highly educated researchers and engineers work in the research sector to develop new technology, to invent new products, and to find new processes of production: human capital is an important input in the research lab to achieve innovation. Many R&D-based growth models emphasize the importance of innovation (see Romer 1990; Grossman and Helpman 1991; Aghion and Howitt 1998; Jones (1995); Barro and Sala-i-Martin 2003). Theoretically, technological progress and innovation can shore up the diminishing returns of capital and labor, thereby inducing everlasting productivity improvement. Therefore, per-capita income also increases in the long run without bound. However, prior models have not regarded human capital as the input in the research sector. In such models, physical capital, final goods, or raw labor are used to improve technology or productivity. They might be the important inputs in the research lab because large-scale computer systems, the latest lab equipment and research devices are necessary to implement innovative activities. However, researchers and engineers also play important roles in the R&D sector. These facts imply that the relation between R&D and human capital must be analyzed. Therefore, our model considers human capital and R&D in a single, systematized model.

Although arguments related to productivity are necessary to examine sustainable development, environmental constraints must be discussed. Negative impact on the environment is negligible if the economic scale is small and the ability to absorb pollution exceeds the amount of its emissions. However, various production activities are expanding drastically and our economy, moving in fast-forward, brings out many serious environmental problems such as global warming, acid rain, land contamination, pollution-triggered disease, and so on. We must consider how to balance environmental concerns with economic growth because various environmental problems are becoming more serious and environmental constraints are becoming more severe.

Incorporation of the environmental impact into a growth model is challenged by many researchers (see Gradus and Smulders 1993; Bovenberg and Smulders 1995, 1996; Stokey 1998; Aghion and Howitt 1998, Ch. 5; Brock and Taylor 2005). However, those papers do not reflect the knowledge-based economy properly. Therein, R&D, human capital, and environmental problems are analyzed separately, although they are deeply and mutually correlated.

Therefore, we construct a model that includes not only problems related to economic growth or productivity improvement under knowledge-based economy but also environmental problems induced by various economic activities. R&D, human capital, and environmental pollution are considered in a single and a simple framework of an endogenous growth model.

We will show that the rate innovation and per capita growth rate become positive in the long run if and only if productivity of the education sector (in which human capital accumulates) is high compared to the discount rate. It is noteworthy that the productivity of the education sector is more important than that of the research sector. The dynamic behavior of pollution depends on the productivity of the abatement sector. The amount of emissions decreases in the steady state if abatement technology is high. In that case, it is possible to attain a positive growth rate without imposing a heavy burden on the environment: growth is sustainable.

The conditions for sustainable growth differ from those derived or asserted by Stokey (1998), Aghion and Howitt (1998, Ch. 5), and Ikazaki (2006). Sustainable growth is attainable even if the intertemporal substitution of consumption is elastic. In the models of Stokey (1998), Aghion and Howitt (1998, Ch. 5), and Ikazaki (2006), a necessary condition for sustainable growth is that the intertemporal substitution of consumption must be inelastic.

Brock and Taylor (2005) also proposed a model in which sustainable growth is attainable even if the intertemporal substitution of consumption is elastic. However, the reason underlying this conclusion differs from that of our model. Brock and Taylor assumed that substitution between physical capital and pollution (or energy) in the production sector is facile, which enables the economy to grow perpetually, even if the intertemporal substitution of consumption is elastic (see also Smulders, 2006). [1]

The remainder of this paper is organized as follows. In section 2, we formulate the endogenous growth model with innovation, human capital, and the environment. In section 3, we consider the equilibrium growth path. In section 4, we discuss the results of our model.

2 Growth Model

2.1 Final Goods

First, we will analyze the final good sector. The market for the final good is assumed to be perfectly competitive. Many firms manufacture homogeneous final goods subject to the same technology given as

$$Y(t) = A_Y K(t)^{\beta} D(t)^{\eta} H_F(t)^{1-\beta-\eta}, \tag{2.1}$$

where A_Y is an index of productivity, $Y(t)$ [2] is the quantity of final goods, $K(t)$ is the quantity of physical capital, $D(t)$ is the index of the intermediate goods and $H_F(t)$ denotes

[1] Our results resemble those of Gradus and Smulders (1993). However, they did not examine the role of R&D.

[2] The (t) notation signifies the level of time t throughout this paper.

the quantity of human capital used as input in this sector. Furthermore, we assume that $\beta > 0$, $\eta > 0$, and that $0 < \beta + \eta < 1$. We take the price of final goods as numeraire. Therefore, the profit of final goods producer, $\Pi(t)$ is represented as follows.

$$\Pi(t) = A_Y K(t)^{\beta} D(t)^{\eta} H_F(t)^{1-\beta-\eta} - p_K(t)K(t) - p_D(t)D(t) - w(t)H_F(t),$$

In that equation, $p_K(t)$ is the rental rate of physical capital, $p_D(t)$ is the price of intermediate goods, $D(t)$; $w(t)$ is the wage rate. Firms maximize their profits at each date, taking $p_K(t)$, $p_D(t)$ and $w(t)$ as given. From the firms' profit maximization, we can obtain

$$H_F(t) = (1-\beta-\eta)\frac{Y(t)}{w(t)}, \tag{2.2}$$

$$K(t) = \frac{\beta Y(t)}{p_K(t)}, \tag{2.3}$$

$$D(t) = \frac{\eta Y(t)}{p_D(t)}. \tag{2.4}$$

2.2 Pollution and Abatement

We assume that pollution results only from production processes. For simplicity, pollution that originates in consumption or other activities is not considered. Pollution (denoted by $P(t)$) is represented as

$$P(t) = \frac{K(t)^{\theta_1} X(t)^{\theta_2}}{A_b(t)^{\theta_3}}, \tag{2.5}$$

where $A_b(t)$ denotes abatement input and $\theta_i > 0$ $(i = 1, 2, 3)$. Furthermore, $X(t)$ is specified as

$$X(t) = \left[\int_0^{M(t)} x_i(t)di\right].$$

The definitions of $M(t)$ and $x_i(t)$ will be given in the following subsection.

We assume that a government can impose a tax on consumption. Tax revenue becomes abatement input. More precisely, $A_b(t)$ is given as

$$A_b(t) = mC(t),$$

where m is the consumption tax rate and $C(t)$ represents aggregate consumption in the economy.

2.3 Sector D

In this subsection, we consider the production process of the intermediate goods D. The market for D is assumed to be perfectly competitive. We assume that D is produced from inputs of the differentiated goods. Following Dixit and Stiglitz (1977), we specify the production function of D as

$$D(t) = \left[\int_0^{M(t)} x_i(t)^\alpha di \right]^{\frac{1}{\alpha}},$$

where $M(t)$ denotes the measure (number) of the available intermediate goods, and $x_i(t)(i \in [0, n(t)])$ represents the quantities of ith intermediate goods used in production activities. We also assume that $0 < \alpha < 1$.

Let us consider the behavior of firms in this sector. We denote a subjective equilibrium quantity of an i-th intermediate good as $\hat{x}_i(t)(i \in [0, M(t)])$: $(\hat{x}_i)$ will be the solution of the following problem.

$$\begin{aligned} \text{minimize} \int_0^{M(t)} p_i(t)x_i(t)di, \\ \text{subject to} \int_0^{M(t)} x_i(t)^\alpha di = \hat{D}(t)^\alpha, \end{aligned} \tag{2.6}$$

where $\hat{D}(t) \equiv \left[\int_0^{M(t)} \hat{x}_i(t)^\alpha di \right]^{\frac{1}{\alpha}}$. This is an isoperimetric problem of the calculus of variations. Then we obtain [3]

$$\hat{x}_i(t) = \frac{\hat{D}(t) p_i(t)^{\frac{1}{\alpha-1}}}{\left[\int_0^{M(t)} (p_{i'}(t))^{\frac{\alpha}{\alpha-1}} di'\right]^{\frac{1}{\alpha}}}. \tag{2.7}$$

Because the profit of each firm in sector D is zero, we can show that

$$p_D(t) \left[\int_0^{M(t)} x_i(t)^{\alpha} di\right]^{\frac{1}{\alpha}} = \int_0^{M(t)} p_i(t) x_i(t) di. \tag{2.8}$$

Therefore, we can obtain $p_D(t)$ as follows:

$$\begin{aligned} p_D(t) &= \int_0^{M(t)} p_i(t)^{\frac{-\alpha}{1-\alpha}} \left(\int_0^{M(t)} p_{i'}(t)^{\frac{-\alpha}{1-\alpha}} di'\right)^{-\frac{1}{\alpha}} di \\ &= \int_0^{M(t)} p_i(t)^{\frac{-\alpha}{1-\alpha}} di \cdot \left(\int_0^{M(t)} p_{i'}(t)^{\frac{-\alpha}{1-\alpha}} di'\right)^{-\frac{1}{\alpha}} \\ &= \left(\int_0^{M(t)} p_i(t)^{\frac{-\alpha}{1-\alpha}} di\right)^{\frac{\alpha-1}{\alpha}}. \end{aligned} \tag{2.9}$$

2.4 Sector x

In the intermediate goods sector (sector x), firms produce goods using blueprints that they created in the R&D sector. We assume that, for any $i (i \in [0, M(t)])$, γ units of Y are necessary to produce one unit of an intermediate good. The profit function of firm i (denoted as $q_i(t)$) is given as

$$q_i(t) = [p_i(t) x_i(t) - \gamma] x_i(t).$$

The relation between $p_i(t)$ and $x_i(t)$ is given as (2.7). Thereby, we can obtain

$$p_i(t) = \frac{\gamma}{\alpha} \equiv p. \tag{2.10}$$

Next, we will derive the quantity of each intermediate good. From (2.9) and (2.10), we can obtain

$$p_D(t) = \left(\frac{\gamma}{\alpha}\right) M(t)^{\frac{\alpha-1}{\alpha}}. \tag{2.11}$$

[3] For derivation of (2.7), see the Appendix.

From (2.4), (2.8), (2.9), (2.10), and (2.11), we obtain

$$D(t) = \frac{\eta\alpha}{\gamma} Y(t) M(t)^{\frac{1-\alpha}{\alpha}}, \tag{2.12}$$

$$x_i(t) = \eta \frac{Y(t)}{M(t)} \frac{\alpha}{\gamma} \equiv x(t). \tag{2.13}$$

The profit of the ith firm is represented as

$$q_i(t) = \eta(1-\alpha) \frac{Y(t)}{M(t)} \equiv q(t).$$

2.5 R&D Sector

Firms are allowed to enter freely into the R&D sector. They finance that cost by issuing equity and using human capital to obtain blueprints for new intermediate goods. They become able to produce monotonically over time if they invent successfully. Therefore, we assume that the inventor of an intermediate good of line j retains a perpetual monopoly right over the production and sale of jth intermediate good. We specify the production function in this sector as

$$\dot{M}(t) = \zeta H_R(t), \tag{2.14}$$

where the dot represents differentiation with respect to time such as $\dot{M} \equiv \frac{dM}{dt}$, $\zeta (> 0)$ is the productivity parameter for research and development, and $H_R(t)$ is the quantity of human capital used for R&D. We consider the value of each R&D project. A firm that succeeds in research activities at a particular time can subsequently earn profits by supplying the intermediate good x monotonically to sector D. Consequently, the value of each R&D project can be expressed as

$$Q(t) = \int_t^{\infty} e^{-\int_t^{\tau} r(s)ds} q(\tau) d\tau.$$

Then the profit of the R&D sector is represented as

$$V = \left[Q(t) - \frac{w(t)}{\zeta} \right] \dot{M}(t).$$

Now presume that $\dot{M}(t) > 0$ at the equilibrium. Then,

$$Q(t) = \frac{w(t)}{\zeta}.$$

2.6 Household Sector

Next, we will consider households. We assume that every household is identical and infinitely lived. The total population is normalized to 1. The notation u denotes the fraction of human capital used in production. The corresponding fraction of human capital devoted to education is $1 - u$. Here we designate the assets of a household as $A(t)$. Consequently, the budget constraint is given as

$$\dot{A}(t) = w(t)u(t)H(t) + r(t)A(t) - C(t) - mC(t), \tag{2.15}$$

where $H(t)$ denotes a household's accumulated human capital. We specify the production function in the education sector as

$$\dot{H}(t) = \phi[1 - u(t)]H(t) - \delta_h H(t). \tag{2.16}$$

Here, ϕ is the productivity parameter and δ_h is the depreciation rate of human capital. We specify the objective functional of the representative household as

$$U = \int_0^\infty \left[\frac{[C(t)P(t)^{-\theta}]^{1-\sigma} - 1}{1 - \sigma} \right] e^{-\rho t}\, dt,$$

where $\rho(> 0)$ is the rate of time preference, $\sigma(> 0)$ is the inverse of the inter-temporal elasticity of substitution, and $\theta(> 0)$ denotes environmental consciousness. They decide how much they will consume and save, and how much they will work or accumulate human capital to maximize their utilities over an infinite horizon. They maximize utility subject to their constraints, taking the path of $r(t), w(t)$ as given. More precisely, the representative household tries to solve the following problem.

$$\text{maximize} \int_0^\infty \left[\frac{[C(t)P(t)^{-\theta}]^{1-\sigma}-1}{1-\sigma}\right] e^{-\rho t}\, dt,$$

subject to

$$\dot{A}(t) = w(t)u(t)H(t) + r(t)A(t) - (1+m)C(t),$$

$$\dot{H}(t) = \phi[1-u(t)]H(t) - \delta_h H(t),$$

$$A(0) = A_0 > 0,$$

$$H(0) = H_0 > 0.$$

To solve the optimization problem of the representative household, we formulate the following current value Hamiltonian:

$$\begin{aligned} J(C,u,A,H,\psi_1,\psi_2,t) &= \frac{[CP(t)^{-\theta}]^{1-\sigma}-1}{1-\sigma} + \psi_1[w(t)uH + r(t)A - (1+m)C] \\ &\quad + \psi_2[(\phi(1-u)H - \delta_h H]. \end{aligned}$$

Here, ψ_1 and ψ_2 are the costate variables. Then the necessary conditions are given as

$$\frac{1}{C(t)^\sigma}[P(t)^{-\theta}]^{1-\sigma} = (1+m)\psi_1(t), \tag{2.17}$$

$$w(t)\psi_1(t) = \phi\psi_2(t), \tag{2.18}$$

$$\dot{\psi}_1(t) = [\rho - r(t)]\psi_1(t), \tag{2.19}$$

$$\dot{\psi}_2(t) = \{\rho + \delta_h - \phi[1 - u(t)]\}\psi_2(t) - w(t)u(t)\psi_1(t). \tag{2.20}$$

Transversality conditions are given as

$$\lim_{t\to\infty} e^{-\rho t}\psi_1(t)A(t) = 0, \tag{2.21}$$

and

$$\lim_{t\to\infty} e^{-\rho t}\psi_2(t)H(t) = 0. \tag{2.22}$$

From (2.17) and (2.19), we obtain

$$\sigma\frac{\dot{C}(t)}{C(t)} = r(t) - \rho + \theta(\sigma - 1)\frac{\dot{P}(t)}{P(t)}. \tag{2.23}$$

Actually,

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\sigma}(r(t) - \rho) \equiv g_0$$

if $\theta = 0$. However, pollution affects the utility level in our economy. In this case, $\frac{\dot{C}(t)}{C(t)} > g_0$ if $(\sigma - 1)\frac{\dot{P}(t)}{P(t)} > 0$ and $\frac{\dot{C}(t)}{C(t)} < g_0$ if $(\sigma - 1)\frac{\dot{P}(t)}{P(t)} < 0$. Therefore, pollution does not necessarily reduce growth rate. See Gradus and Smulders (1993) for a similar discussion.

3 Steady Growth Equilibrium Path

In this section, based on the results obtained above, we will specifically examine the steady growth path. The steady growth path is defined as the path such that the general equilibrium holds and the growth rates of all variables in the model are constant. We denote the growth rate of a variable z by g_z. A detailed discussion is presented in the Appendix. In the main text, we will describe the main results only. The growth rate of human capital is given as

$$\frac{\dot{H}(t)}{H(t)} = \phi[1 - u(t)] - \delta_h.$$

Consequently, $u(t)$ remains constant over time on the steady equilibrium growth path. Therefore, we represent $u(t) = u$. We can also show that H_R/H and H_F/H are also constant on a steady equilibrium growth path. That is,

$$g_H = g_{H_F} = g_{H_R}. \tag{3.1}$$

From (2.14), we can show that

$$g_M = \zeta \frac{H_R(t)}{M(t)}.$$

In the steady state, g_M is constant. Therefore, $H_R(t)/M(t)$ must be constant. This fact and (3.1) imply that

$$g_H = g_{H_F} = g_{H_R} = g_M. \tag{3.2}$$

In the appendix, we show that

$$g_Y = g_K = g_C. \tag{3.3}$$

From equations (2.1), (3.2), and (3.3), we can get

$$g_Y^* = \mu g_M^*, \tag{3.4}$$

where

$$\mu \equiv \frac{1}{(1-\beta-\eta)} \left((1-\beta-\eta) + \eta \left(\frac{1-\alpha}{\alpha} \right) \right).$$

Actually, $g_Y^* > g_M^*$ because $\mu > 1$. The growth rate of GDP is higher than that of innovation. We use the asterisk here to denote the steady state level. Using a simple calculation, the innovation rate is derived as

$$g_M^* = \frac{\phi - \delta_h - \rho}{\mu(\sigma-1) + 1 - \mu\theta(\sigma-1)(\theta_1 + \theta_2 - \theta_3)}. \tag{3.5}$$

Here, we assume that

$$\mu(\sigma-1)+1-\mu\theta(\sigma-1)(\theta_1+\theta_2-\theta_3)>0, \tag{3.6}$$

and

$$\phi-\delta_h-\rho>0. \tag{3.7}$$

If we see the dynamic behavior of pollution, we obtain

$$g_P^*=(\theta_1+\theta_2-\theta_3)g_Y^*. \tag{3.8}$$

If $\theta_1+\theta_2-\theta_3>(<)0$, then pollution increases (decreases) over time in the long run.

4 Implications and Concluding Remarks

4.1 Growth Rate, Education Sector, and R&D Sector

First, we specifically examine the growth rate. The productivity of the education sector, ϕ, must be sufficiently high for a positive growth rate. [4] The higher the productivity of the education sector, the higher the growth rate. Interestingly, unlike the models of Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992), the productivity of the R&D sector, ζ, does not affect the growth rate. In their models, the higher the productivity in the R&D sector, the higher the growth rate. This is because our model considers not only R&D but also human capital and the main engine of long run growth is human capital. The R&D sector can accelerate growth (note that $\mu>1$), but R&D is not an engine of growth. To confirm this point, presume that $\alpha=1$. If $\alpha=1$, no R&D takes place because the intermediate goods are perfectly substitutable for one another; a

[4] See Uzawa (1965) and Lucas (1988).

firm cannot earn a profit by obtaining a new blueprint of the $(M+1)$th intermediate good. In this case, $M(t) = M(0)$ for all t. However, the growth rate of per capita income is still positive ($g_Y^* = g_M^* > 0$) if (3.6) and (3.7) are satisfied.

4.2 Growth Rate, Dynamic Behavior of Environment, and Environmental Care

Next, we consider the relation between the growth rate and dynamic behavior of pollution. Presume that $\sigma > 1$: the intertemporal substitution of consumption is not so elastic. Presume also that $\theta_1 + \theta_2 - \theta_3 > 0$. Therefore, pollution increases over time in the long run. Then the more the economy cares for the environment (higher θ), the higher the growth rate. On the other hand, presuming that $\theta_1 + \theta_2 - \theta_3 < 0$ (pollution decreases over time in the long run), the more the economy cares for the environment (higher θ), the lower the growth rate.

Next, presuming that $\sigma < 1$, the intertemporal substitution of consumption is elastic. In this case, $\theta_1 + \theta_2 - \theta_3 > 0$ and higher values of θ correspond to the low growth rate. On the other hand, when $\theta_1 + \theta_2 - \theta_3 < 0$ and pollution decreases over time, the more the economy cares for the environment (higher θ), the higher the growth rate. These conclusions resemble those of Gradus and Smulders (1993), although they did not refer to R&D. The effect of environmental care on the growth rate does not change so much even if we consider not only environment but also R&D or human capital.

4.3 Sustainable Growth and the Environmental Constraint

Pollution increases without bound if $\theta_1 + \theta_2 - \theta_3 > 0$. However, if we consider an environmental constraint (e.g., the pollution absorbing ability of nature is limited), unbounded pollution might not be consistent with sustainable growth. The environmental constraint does not enable our economy to grow in the long run when $\theta_1 + \theta_2 - \theta_3 > 0$.

Presuming that $P(t) < \bar{P}$ must be satisfied for all t, where $\bar{P}$ is positive constant. If this constraint is added to our model, then growth peters out in the long run if $\theta_1 + \theta_2 - \theta_3 > 0$. After $P(t)$ comes at $\bar{P}$, the growth rate of each variable becomes zero (each variable remains constant over time) and the economy does not grow any more. We must consider some ways to increase θ_3 or to decrease θ_1 or θ_2 to avoid this situation. As described in this paper, we assumed that each θ_i $(i = 1, 2, 3)$ is constant. However, θ_i might change in practice in the long run. If so, we should construct an alternative model in which tax revenue is used not only for abatement activity but also for environmental R&D, which affects the values of θ_i.

4.4 Tax Rate and Growth Rate

The government imposes a consumption tax. Tax revenue is used to reduce pollution. We assume that tax rate is constant. The tax rate m does not affect the economic growth rate. However, it does not affect the growth rate of each variable to increase the burden of taxes, which will reduce the level of consumption or pollution. Moreover, the tax rate m is given exogenously. If we consider optimal environmental policy, then, the tax rate will be determined endogenously. However, this change does not affect the main results of our model.

As described in this paper, we investigated, theoretically, long-run equilibrium growth in the context of an endogenous growth model with innovation, human capital, and pollution. From this analysis, we can draw some interesting conclusions listed above. However, the results presented herein suggest several areas for additional research. For example, it would be interesting to analyze the effect of environmental R & D to reduce pollution. Our paper includes R & D. However, environmental R&D is not discussed despite the fact that it is necessary to reserve the environmental level.

Our model specifically examines a single economy. However, environmental problems must be discussed from an international perspective because both resources and pollution are expected to move across regions or countries.

A Appendix

A.1 Demand Function of x_i

Let us consider the problem posed by eq. (2.6). For $\lambda \in R$, we formulate the following Lagrangian.

$$\begin{aligned} \mathcal{L} &= \int_0^M p_i x_i di - \lambda \left[\int_0^M {x_i}^{\alpha} di - \hat{D}^{\alpha} \right] \\ &= \int_0^M [p_i x_i - \lambda {x_i}^{\alpha}] di + \lambda \hat{D}^{\alpha}. \end{aligned}$$

Therefore, according to the Euler equations for this problem, we can show that

$$p_i = \lambda \alpha \hat{x}_i^{\alpha - 1}.$$

Consequently,

$$\hat{x}_i = \left(\frac{p_i}{\lambda \alpha} \right)^{\frac{1}{\alpha - 1}}. \tag{A.1}$$

By substituting (A.1) into the constraint of (2.6), we obtain

$$\int_0^M \left(\frac{p_i}{\lambda \alpha} \right)^{\frac{\alpha}{\alpha - 1}} di = \hat{D}^{\alpha}.$$

Therefore,

$$\frac{1}{\lambda \alpha} = \frac{\hat{D}^{\alpha - 1}}{\left[\int_0^M p_{i'}{}^{\frac{\alpha}{\alpha - 1}} di' \right]^{\frac{\alpha - 1}{\alpha}}}. \tag{A.2}$$

Finally, the following pertain:

$$\begin{aligned}\hat{x}_i &= \left[\frac{\hat{D}^{\alpha-1}}{\left[\int_0^M p_{i'}{}^{\frac{\alpha}{\alpha-1}} di'\right]^{\frac{\alpha-1}{\alpha}}} \cdot p_i\right]^{\frac{1}{\alpha-1}} \\ &= \frac{\hat{D} p_i{}^{\frac{1}{\alpha-1}}}{\left[\int_0^M p_{i'}{}^{\frac{\alpha}{\alpha-1}} di'\right]^{\frac{1}{\alpha}}}. \end{aligned} \tag{A.3}$$

A.2 Growth Rate

Here, we will derive the growth rate of each variable in the steady state. The growth rate of human capital is given as

$$\frac{\dot{H}(t)}{H(t)} = \phi[1 - u(t)] - \delta_h.$$

Consequently, $u(t) = u$ remains constant over time on the steady equilibrium growth path: $uH(t) = H_F(t) + H_R(t)$. Therefore, we obtain the following expression.

$$\begin{aligned} g_H &= g_{H_F}\frac{H_F(t)}{H(t)} + g_{H_R}\frac{H_R(t)}{H(t)} \\ &= g_{H_F}\frac{H_F(t)}{H(t)} + g_{H_R(t)}\left[u - \frac{H_F(t)}{H(t)}\right] \\ &= (g_{H_F} - g_{H_R})\frac{H_F(t)}{H(t)} + u g_{H_R}. \end{aligned}$$

Thereby, $H_F(t)/H(t)$ becomes constant. Actually, $H_R(t)/H(t)$ becomes constant because

$$\frac{H_R(t)}{H(t)} = u - \frac{H_F(t)}{H(t)}.$$

Then we have

$$g_H = g_{H_F} = g_{H_R}.$$

On the other hand, from (2.14), we have the following equation:

$$g_M = \zeta\frac{H_R(t)}{M(t)}.$$

Furthermore, because g_M is constant, $H_R(t)/M(t)$ must be constant. Therefore,

$$g_M = g_{H_R},$$

and

$$g_H = g_{H_F} = g_{H_R} = g_M.$$

From equations (2.1) and (2.12), we can show that

$$Y(t)^{1-\eta} = \left[A_Y \eta^\eta \left(\frac{\alpha}{\gamma}\right)^\eta\right] K(t)^\beta M(t)^{\eta(\frac{1-\alpha}{\alpha})} H_F(t)^{1-\beta-\eta}.$$

This implies

$$(1-\eta)g_Y = \beta g_K + \eta\Big(\frac{1-\alpha}{\alpha}\Big)g_M + (1-\beta-\eta)g_{H_F}. \tag{A.4}$$

Because $r(t)$ is constant on the steady equilibrium growth path and because $p_K(t) = r(t)$, we can show that

$$g_K = g_Y.$$

Therefore, we obtain the following relation between the growth rate and the innovation rate.

$$g_Y = \frac{1}{(1-\beta-\eta)}\left[(1-\beta-\eta) + \eta\Big(\frac{1-\alpha}{\alpha}\Big)\right] g_M. \tag{A.5}$$

We define μ as

$$\mu \equiv \frac{1}{(1-\beta-\eta)}\left((1-\beta-\eta) + \eta\left(\frac{1-\alpha}{\alpha}\right)\right).$$

In fact, $g_Y = \mu g_M$ and $\mu > 1$.

The economy must satisfy the following resource constraint.

$$Y(t) = (1+m)C(t) + \gamma M(t)x(t) + \dot{K}(t) + \delta_k K(t),$$

where $\dot{K}(t) + \delta_k K(t)$ is the gross investment of the physical capital stock. Therefore,

$$\frac{Y(t)}{K(t)} = \frac{(1+m)C(t)}{K(t)} + \frac{\gamma M(t)x(t)}{K(t)} + \frac{\dot{K}(t)}{K(t)} + \delta_k.$$

Because $\gamma M(t)x(t) = \alpha\eta Y(t)$, we can obtain

$$(1-\alpha\eta)\frac{Y(t)}{K(t)} = \frac{(1+m)C(t)}{K(t)} + \frac{\dot{K}(t)}{K(t)} + \delta_k. \tag{A.6}$$

$\dot{K}(t)/K(t)$ remains constant on the steady equilibrium growth path and $Y(t)/K(t)$ is constant. Therefore, $C(t)/K(t)$ is constant. Consequently,

$$g_C = g_K = g_Y.$$

From (2.4), we can get

$$\begin{aligned} g_w &= g_Y - g_{H_F} \\ &= (\mu - 1)g_M. \end{aligned} \tag{A.7}$$

Furthermore, from (2.18), we can obtain

$$\frac{\dot{w}(t)}{w(t)} + \frac{\dot{\psi}_1(t)}{\psi_1(t)} = \frac{\dot{\psi}_2(t)}{\psi_2(t)}. \tag{A.8}$$

From (2.17), we have

$$\begin{aligned} \frac{\dot{\psi}_1}{\psi_1} &= -\sigma\frac{\dot{C}(t)}{C(t)} + (\sigma\theta - \theta)\frac{\dot{P}(t)}{P(t)} \\ &= -\sigma g_Y + (\sigma\theta - \theta)\frac{\dot{P}(t)}{P(t)}. \end{aligned} \tag{A.9}$$

From (2.5), we can show

$$\begin{aligned} \frac{\dot{P}(t)}{P(t)} &= \theta_1\frac{\dot{K}(t)}{K(t)} + \theta_2\frac{\dot{X}(t)}{X(t)} - \theta_3\frac{\dot{A}_b(t)}{A_b(t)} \\ &= (\theta_1 + \theta_2 - \theta_3)g_Y \end{aligned} \tag{A.10}$$

because

$$\frac{\dot{A}_b(t)}{A_b(t)} = g_C = g_X = g_Y.$$

From (A.8) and (A.9), we can get

$$\begin{aligned}\frac{\dot{\psi}_2(t)}{\psi_2(t)} &= (\mu-1)g_M - \sigma\mu g_M - (1-\sigma)\theta(\theta_1+\theta_2-\theta_3)\mu g_M \\ &= [\mu - 1 - \sigma\mu - \mu(1-\sigma)\theta(\theta_1+\theta_2-\theta_3)]g_M. \qquad \text{(A.11)}\end{aligned}$$

Furthermore, from eqs. (2.18) and (2.20), we can get

$$\frac{\dot{\psi}_2(t)}{\psi_2(t)} = \rho + \delta_h - \phi. \qquad \text{(A.12)}$$

Consequently, the rate of innovation is represented as

$$g_M^* = \frac{\phi - \delta_h - \rho}{\mu(\sigma-1) + 1 - \mu\theta(\sigma-1)(\theta_1+\theta_2-\theta_3)}. \qquad \text{(A.13)}$$

Here, we assume that

$$\mu(\sigma-1) + 1 - \mu\theta(\sigma-1)(\theta_1+\theta_2-\theta_3) > 0$$

and

$$\phi - \delta_h - \rho > 0$$

so that $g_M^* > 0$. In the steady state, the growth rates of Y and P (denoted as g_Y^* and g_P^*) are given as

$$g_Y^* = \mu g_M^*,$$

and

$$g_P^* = (\theta_1+\theta_2-\theta_3)g_Y$$

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References

[1] Aghion, P. and P. Howitt (1998) *Endogenous Growth Theory*, MIT Press.

[2] Barro, R. J. and X. Sala-i-Martin (2003) *Economic Growth, 2nd edition*, The MIT Press.

[3] Bovenberg, A. L. and S. Smulders (1995) "Environmental Quality and Pollution-augmenting Technological Change in a Two-sector Endogenous Growth Model," *Journal of Public Economics,* vol. 57, 369-391.

[4] Bovenberg, A. L. and S. Smulders (1996) "Transitional Impacts of Environmental Policy in an Endogenous Growth Model," *International Economic Review,* vol. 37, 861-893.

[5] Brock, W. and W. S. Taylor (2005) "Economic Growth Theory and the Environment : A Review of Theory and Empirics," in P. Aghion and S. Durlauf (eds.) *Handbook of Economic Growth (vol.1B),* Elsevier, 1749-1821.

[6] Dixit, A. K. and J. E. Stiglitz (1977) "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, vol. 67, 297-308.

[7] Gradus, R. and S. Smulders (1993) "The Trade-off Between Environmental Care and Long-term Growth-Pollution in Three Prototype Growth Models," *Journal of Economics,* vol. 58, 25-51.

[8] Grossman, G. M. and E. Helpman (1991) *Innovation and Growth in the Global Economy*, MIT Press, Cambridge.

[9] Ikazaki, D. (2006) "R&D, Human Capital, and Environmental Externality in an Endogenous Growth Model," *International Journal of Global Environmental Issues,* vol. 6, no. 1, pp.29-46.

[10] Jones, C. I. (1995) "R&D Based Models of Economic Growth," *Journal of Political Economy*, vol. 103, 759-784.

[11] Lucas, R. E. Jr. (1988) "On the Mechanics of Economic Development," *Journal of Monetary Economics,* vol. 22, 3-42.

[12] Ministry of Education, Culture, Sports, Science and Technology (2003) *White Papers on Science and Technology* (*in Japanese, Kagaku Gijyutsu Hakusho*), Tokyo.

[13] National Institute of Science and Technology Policy (2000) *Science and Technology Indicators* (*in Japanese, Kagaku Gijyutsu Shihyo*), Tokyo.

[14] OECD (1996) *The Knowledge-based Economy*, (http://www.oecd.org/dataoecd/51/8/1913021.pdf)

[15] Romer, P. M. (1990) "Endogenous Technological Change," *Journal of Political Economy*, vol. 98, 71-102.

[16] Smulders, S. (2006) "Growth and Environment: On U-curves without U-turns," in C. de Miguel, X. Labandeira, B. Manzano (eds.) *Economic Modelling of Climate Change and Energy Policies,* Edward Elgar Publishing, 9-23.

[17] Stokey, N. L. (1998) "Are There Limits to Growth," *International Economic Review*, vol. 39, 1-31.

[18] Uzawa, H. (1965) "Optimal Technical Change in an Aggregative Model of Economic Growth," *International Economic Review*, vol. 6, 18-31.

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