ON CERTAIN APPROXIMATIONS OF POWER OF A TEST PROCEDURE USING TWO PRELIMINARY TESTS IN A MIXED MODEL

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ON CERTAIN APPROXIMATIONS OF POWER OF A TEST PROCEDURE USING TWO PRELIMINARY TESTS IN A MIXED MODEL

By

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Abstract

The present paper is concerned with the derivation of approximate formulae for power components of a sometimes pool test procedure applied to a mixed model experiment. A comparison of the values of power components evaluated by these formulae with those calculated using series formulae has been made.

I. Introduction.

In making inferences from the experimental design models, at times there may arise some doubt regarding the inclusion or not of some of the parameters in the model. For example, in a factorial experiment or an experiment with crossed classification the experimenter may be uncertain as to whether interaction parameter(s) should appear in the model. This uncertainty in the model specification may be due to the lack of knowledge, either theoretical or from past experience, in regard to the interaction effect(s) at issue. Such situations of uncertainty lead to conditional specification of the model and are to be resolved first before making final inferences.

The present study has been made for a mixed model split-plot in time experiment involving conditional specification. We are here mainly interested in making inferences regarding the split-plot treatments (split by time). The uncertainty concerning the inclusion or not of the interactions in this model has been resolved by preliminary tests of significance.

1.1. Related Papers and Objective of the Study.

Bozivich, Bancroft and Hartley (1956) have derived approximate formulae and exact formulae for power components of a test procedure in a component of variance model. Derivation of approximate formulae for power in a mixed model has been given by

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Tailor and Saxena (1974). In both of these studies, the experiments are hierarchal in nature. In the present investigation we have considered a mixed model split-plot in time experiment. This design has frequent use in experiments on forage crops [Steel and Torrie (1960)]. The object of the present study is to derive expressions for the approximate formulae of the power components of the test used and to examine the degree of accuracy of these approximations.

1.2. Statement of the Problem

An experiment on a certain forage crop is conducted to investigate the cutting effects with a split-plot in time layout involving 'r' blocks, 's' varieties and 't' cuttings. Here varieties and cuttings are fixed effects and blocks are random eflects. Let the four independent mean squares comprising cuttings be represented by V_1, V_2, V_3 and V_4 based on n_1, n_2, n_3 and n_4 degrees of freedom respectively. We are interested in testing the null hypothesis $H_0: E(V_4)=E(V_3)$ against the alternative $H_1: E(V_4)>E(V_3)$ when it is doubtful that V_3 and/or V_2 may have the same expectation as V_1. If it is certain that $E(V_3)$ and/or $E(V_2)$ is not equal to $E(V_1)$, the usual procedure for testing H_0 is to compare V_4 with V_3 by the F-statistic and to reject H_0 whenever the observed F-value turns out to be significant. However, if the uncertainty exists in which case $E(V_3)$ and/or $E(V_2)\geqq E(V_1)$, different test statistic(s) for testing H_0 may then be used by considering appropriate combination(s) of V_3 and/or V_2 with V_1. The appropriateness of these combinations may be decided by making preliminary tests for the said uncertainties.

Before we describe the proposed sometimes pool test procedure for testing H_0 and present derivation of approximate formulae for power components of this test, we state the problem precisely in the next section.

1.3. A Precise Formulation of the Problem.

Let us consider the following mixed model for a split-plot in time experiment

$$Y_{ijk}=\mu+\alpha_i+\beta_j+\delta_{ij}+\gamma_k+(\alpha\gamma)_{ik}+(\beta\gamma)_{jk}+\varepsilon_{ijk}\,, \qquad (1.3.1)$$

$$i=1,2,\cdots,r \quad j=1,2,\cdots,s \quad k=1,2,\cdots,t$$

where

Y_{ijk}=yield in the i-th block on the j-th veriety with k-th cutting,
μ=true mean effect,
α_i=true effect of the i-th block,
β_j=true effect of the j-th variety,
δ_{ij}=true effect of the experimental unit in the i-th block subjected to the j-th variety,
γ_k=true effect of the k-th cutting,
$(\alpha\gamma)_{ik}$=true effect of the interaction between the i-th block and the k-th cutting,
$(\beta\gamma)_{jk}$=true effect of the interaction between the j-th variety and the k-th cutting,
ε_{ijk}=true effect of the k-th cutting subjected to the (ij)-th treatment combination;

and also

α_i are NID$(0, \sigma_\alpha^2)$,

$$\sum_j \beta_j=0, \quad \sum_k \gamma_k=0, \quad \sum_j (\beta\gamma)_{jk}=\sum_k (\beta\gamma)_{jk}=0,$$

$(\alpha\gamma)_{ik}$ are NID$(0, \sigma^2_{\alpha\gamma})$, $\sum_k (\alpha\gamma)_{ik}=0$, $\sum_i (\alpha\gamma)_{ik}\neq 0$,

δ_{ij} are NID$(0, \sigma^2_\delta)$,

ε_{ijk} are NID$(0, \sigma^2_\varepsilon)$.

The resulting 'anova' corresponding to (1.3.1) is given as follows:.

Table 1.1. Mixed Model ANOVA for a Split-plot in Time Experiment

Source of variation	Degrees of Freedom	Expected Mean Square
Blocks	$r-1$	$\sigma^2_\varepsilon+st\sigma^2_\alpha$
Varieties	$s-1$	$\sigma^2_\varepsilon+t\sigma^2_\delta+rt\left[\sigma^2_\beta\right]$
Error (a)	$(r-1)(s-1)$	$\sigma^2_\varepsilon+t\sigma^2_\delta$
Cuttings	$t-1$	$\sigma^2_\varepsilon+s\sigma^2_{\alpha\gamma}+rs\left[\sigma^2_\gamma\right]$
Cuttings × blocks	$(r-1)(t-1)$	$\sigma^2_\varepsilon+s\sigma^2_{\alpha\gamma}$
Cuttings × varieties	$(s-1)(t-1)$	$\sigma^2_\varepsilon+r\left[\sigma^2_{\beta\gamma}\right]$
Error (b)	$(r-1)(s-1)(t-1)$	σ^2_ε

where σ^2_β, σ^2_γ and $\sigma^2_{\beta\gamma}$ enclosed within parenthesis refer to finite population variances and equal $\sum_j \beta^2_{j/s-1}$, $\sum_k \gamma^2_{k/t-1}$ and

$$\sum_j \sum_k (\beta\gamma)^2_{jk}/(s-1)(t-1)$$

respectively.

Our main interest is in whether the cuttings have any effect. We, therefore, test the hypothesis concerning γ_k and confine to an abridged 'anova' as shown in Table 1.2

Table 1.2. Mixed Model Abridged ANOVA

Source of variation	Degrees of Freedom	Mean Square	Expected Mean Square
Cuttings	$t-1 \quad =n_4$	V_4	$\sigma^2_\varepsilon+s\sigma^2_{\alpha\gamma}+rs\left[\sigma^2_\gamma\right]=\sigma^2_4$
Cuttings × blocks	$(r-1)(t-1) \quad =n_3$	V_3	$\sigma^2_\varepsilon+s\sigma^2_{\alpha\gamma} \quad =\sigma^2_3$
Cuttings × varieties	$(s-1)(t-1) \quad =n_2$	V_2	$\sigma^2_\varepsilon+r\left[\sigma^2_{\beta\gamma}\right] \quad =\sigma^2_2$
Error	$(r-1)(s-1)(t-1)=n_1$	V_1	$\sigma^2_\varepsilon \quad =\sigma^2_1$

In the above Table 1.2 V_i's $(i=1, 3)$ are distribured as $\chi^2_i\sigma^2_i/n_i$, where χ^2_i is a central chi-square with n_i degrees of freedom. On the other hand V_2 and V_4 are distributed respectively as $\chi'^2_2\sigma^2_1/n_2$ and $\chi'^2_4\sigma^2_3/n_4$ where χ'^2_2 and χ'^2_4 are the non-central chi-squares with n_2 and n_4 degrees of freedom and the non-centrality parameters $n_2(\sigma^2_2-\sigma^2_1)/2\sigma^2_1$ and $n_4(\sigma^2_4-\sigma^2_3)/2\sigma^2_3$.

We are interested in testing the hypothesis $H_0: \sigma^2_4=\sigma^2_3$ against the alternative $H_1: \sigma^2_4>\sigma^2_3$. From Table 1.2, it is clear that the appropriate test of H_0 is to calculate $F=V_4/V_3$ and reject H_0 if the observed value of F comes out to be significant. However, uncertainty might exist in whether $\sigma^2_{\alpha\gamma}$ and/or $\sigma^2_{\beta\gamma}$ is zero. In that case, we first resolve these uncertainties by making preliminary tests of significance on $\sigma^2_{\alpha\gamma}$ and $\sigma^2_{\beta\gamma}$. Hence, we test the preliminary hypotheses $H_{01}: \sigma^2_{\alpha\gamma}=0$ and $H_{02}: \sigma^2_{\beta\gamma}=0$ in succession against their corresponding alternatives $H_{11}: \sigma^2_{\alpha\gamma}>0$ and H_{12} $\sigma^2_{\beta\gamma}>0$. The outcome of these tests are then used in devising appropriate test for H_0. It is also possible to test $\sigma^2_{\beta\gamma}=0$ first and then $\sigma^2_{\alpha\gamma}=0$. This may give rise to another sometimes pool test procedure which we have not studied.

The sometimes pool test procedure which we have proposed for testing H_0 is as follows:

Reject H_0 if any one of the following occurs:

$$\left.\begin{array}{lll}
\text{(i)} & \dfrac{V_3}{V_1}\geqq F(n_3,\ n_1;\alpha_1), & \dfrac{V_4}{V_3}\geqq F(n_4,\ n_3;\alpha_2);\\[2ex]
\text{(ii)} & \dfrac{V_3}{V_1}<F(n_3,\ n_1;\alpha_1), & \dfrac{V_2}{V_{13}}\geqq F(n_2,\ n_1+n_3;\alpha_3),\\[2ex]
 & & \dfrac{V_4}{V_{13}}\geqq F(n_4,\ n_1+n_3;\alpha_4);\\[2ex]
\text{(iii)} & \dfrac{V_3}{V_1}<F(n_3,\ n_1;\alpha_1), & \dfrac{V_2}{V_{13}}<F(n_2,\ n_1+n_3;\alpha_3),\\[2ex]
 & & \dfrac{V_4}{V_{123}}\geqq F(n_4,\ n_1+n_2+n_3;\alpha_5);
\end{array}\right\} \tag{1.3.1}$$

where

$$V_{13}=\frac{n_1V_1+n_3V_3}{n_1+n_3}, \qquad V_{123}=\frac{n_1V_1+n_2V_2+n_3V_3}{n_1+n_2+n_3}$$

and $F(n_i,\ n_j;\alpha_k)$ refers to the upper 100 $\alpha_k\%$ point of the F-distribution with $(n_i,\ n_j)$ degrees of freedom.

2. The Power of the Proposed Sometimes Pool Test Procedure.

The power of the sometimes pool test procedure is the sum of the probabilities associated with three mutually exclusive events given in (1.3.1). We denote these probabilities by P_1, P_2 and P_3, which are nothing but the power components of the test.

The derivation of series formulae for power components has heen desoribed elsewhere. The expressions for these components are as follows:

$$P_1=A_2\sum_{I=0}^{\nu_{4/2-1}}\frac{(-1)^I\binom{\nu_{4/2-1}}{I}}{n_{1/2}+n_{3/2+I}}\sum_{J=0}^{I}\binom{I}{J}\frac{B_{x_1}(n_{1/2+I-J},\ n_{3/2+J})}{(1+b)^{n_{3/2+J}}}, \tag{2.1}$$

$$P_2=A_1\sum_{I=0}^{\nu_{4/2-1}}\frac{(-1)^I\binom{\nu_{4/2-1}}{I}}{n_{1/2}+\nu_{2/2}+n_{3/2+I}}\sum_{J=0}^{I}\binom{I}{J}\sum_{K=0}^{\nu_{2/2+J-1}}\frac{(-1)^K\binom{\nu_{2/2+J-1}}{K}}{n_{1/2}+n_{3/2+I-J+K}}\sum_{L=0}^{I-J}\binom{I-J}{L}$$

$$\sum_{M=0}^{K}\binom{K}{M}\frac{(1+d)^{K-M}(1+g)^MBx_2(n_{3/2+L+M},\ n_{1/2+I-J+K-L-M})}{(1+c+d)^{n_{1/2+I-J+K-L-M}}(1+f+g)^{n_{3/2+L+M}}}, \tag{2.2}$$

$$P_3 = A_1 \sum_{I=0}^{\nu_{4/2-1}} \frac{(-1)^I \binom{\nu_{4/2-1}}{I}}{n_{1/2}+\nu_{2/2}+n_{3/2+I}} \sum_{J=0}^{I} \binom{I}{J}$$

$$\sum_{K=0}^{\nu_{2/2-J-1}} \frac{(-1)^K \binom{\nu_{2/2+J-1}}{K}}{n_{1/2}+n_{3/2+I-J+K}} \sum_{L=0}^{I-J} \binom{I-J}{L}$$

$$\cdot \frac{1}{(1+h)^{\nu_{2/2+J}}} \left[\frac{B_{x_{31}}(n_{3/2+L},\ n_{1/2+I-J-L})}{(1+e)^{n_{1/2+I-J-L}}(1+m)^{n_{3/2+L}}} \right.$$

$$\left. - \sum_{M=0}^{K} \binom{K}{M} \frac{(1+e)^{K-M}(1+m)^M B_{x_{32}}(n_{3/2+L+M},\ n_{1/2+I-J+K-L-M})}{(1+c+e+ch)^{n_{1/2+I-J+K-L-M}}(1+f+m+fh)^{n_{3/2+L+M}}} \right], \tag{2.3}$$

where

$$A_1 = \frac{\Gamma(n_{1/2}+\nu_{2/2}+n_{3/2}+\nu_{4/2})}{\Gamma(n_{1/2})\Gamma(\nu_{2/2})\Gamma(n_{3/2})\Gamma(\nu_{4/2})}, \qquad A_2 = \frac{\Gamma(n_{1/2}+n_{3/2}+\nu_{4/2})}{\Gamma(n_{1/2})\Gamma(n_{3/2})\Gamma(\nu_{4/2})},$$

$$a = \frac{n_3}{\theta_{31} n_1} F(n_3, n_1; \alpha_1), \qquad b = \frac{n_4}{c_4 n_3} F(n_4, n_3; \alpha_2),$$

$$c = \frac{n_2}{c_2(n_1+n_3)} F(n_2, n_1+n_3; \alpha_3), \qquad d = \frac{n_4}{c_4 \theta_{31}(n_1+n_3)} F(n_4, n_1+n_3; \alpha_4),$$

$$e = \frac{n_4}{c_4 \theta_{31}(n_1+n_2+n_3)} F(n_4, n_1+n_2+n_3; \alpha_5), \qquad f = \frac{\theta_{31} n_2}{c_2(n_1+n_3)} F(n_2, n_1+n_3; \alpha_3),$$

$$g = \frac{n_4}{c_4(n_1+n_3)} F(n_4, n_1+n_3; \alpha_4), \qquad h = \frac{c_2 n_4}{c_4 \theta_{31}(n_1+n_2+n_3)} F(n_4, n_1+n_2+n_3; \alpha_5),$$

$$m = \frac{n_4}{c_4(n_1+n_2+n_3)} F(n_4, n_1+n_2+n_3; \alpha_5), \qquad \theta_{31} = \frac{\sigma_3^2}{\sigma_1^2};$$

$$x_1 = \frac{1}{1+a(1+b)}, \qquad x_2 = \frac{a(1+f+g)}{1+c+d+a(1+f+g)},$$

$$x_{31} = \frac{a(1+m)}{1+e+a(1+m)}, \qquad x_{32} = \frac{a(1+f+m+fh)}{1+c+e+ch+a(1+f+m+fh)},$$

We now evaluate the power components using approximations:

2.1. Derivation of Approximate Formulae.

Let

$$\left.\begin{aligned} F_1 &= F(n_3, n_1; \alpha_1) \\ F_2 &= F(n_4, n_3; \alpha_2) \\ F_3 &= F(n_2, n_1+n_3; \alpha_3) \\ F_4 &= F(n_4, n_1+n_3; \alpha_4) \\ F_5 &= F(n_4, n_1+n_2+n_3; \alpha_5) \end{aligned}\right\} \tag{2.1.a}$$

Then the power components P_1, P_2, P_3 can be written as

$$P_1 : \text{Prob.}\left\{\frac{V_3}{V_1} \geqq F_1, \quad \frac{V_4}{V_3} \geqq F_2\right\}, \tag{2.1.b}$$

$$P_2 : \text{Prob.}\left\{\frac{V_3}{V_1} < F_1, \quad \frac{V_2}{V_{13}} \geqq F_3, \quad \frac{V_4}{V_{13}} \geqq F_4\right\}, \tag{2.1.c}$$

$$P_3 : \text{Prob.}\left\{\frac{V_3}{V_1} < F_1, \quad \frac{V_2}{V_{13}} < F_3, \quad \frac{V_4}{V_{123}} \geqq F_5\right\}. \tag{2.1.d}$$

2.1.1. Approximate Formula for P_1.

To evaluate (2.1.b), we use Patnaik's (1949) approximation to non-central chi-square as well as normal approximation to log V_i suggested by Bartlett and Kendall (1946). The latter approximation holds good provided n_i is not too small.

If we write

$$\begin{aligned} y_1 &= \log V_3 - \log V_1 \\ y_2 &= \log V_4 - \log V_3 \end{aligned} \tag{2.1.1.1}$$

then it can be shown that the joint density of y_1 and y_2 is bivariate normal with means:

$$\begin{aligned} \bar{y}_1 &= \log \sigma_3^2 - \log \sigma_1^2 = \log \theta_{31} \\ \bar{y}_2 &= \log \sigma_4^2 - \log \sigma_3^2 = \log\left(1 + \frac{2\lambda_4}{n_4}\right), \end{aligned} \tag{2.1.1.2}$$

variances:

$$\begin{aligned} \text{Var}\,(y_1) &= \frac{2}{n_3 - 1} + \frac{2}{n_1 - 1} \\ \text{Var}\,(y_2) &= \frac{2}{\nu_4 - 1} + \frac{2}{n_3 - 1}, \end{aligned} \tag{2.1.1.3}$$

and the coefficient of correlation:

$$\rho_{y_1 y_2} = -\left\{\frac{(\nu_4 - 1)(n_1 - 1)}{(n_3 + n_1 - 2)(\nu_4 + n_3 - 2)}\right\}^{1/2}. \tag{2.1.1.4}$$

Therefore, P_1 can be written as

$$P_1 \cong \text{Prob.}\ \{x \geqq u,\ y \geqq v,\ \rho_{y_1 y_2}\}, \tag{2.1.1.5}$$

where x and y are standard normal variates and the argument u and v are given by

$$u = \frac{2Z(n_3, n_1; \alpha_1) - \log \theta_{31}}{\sqrt{\dfrac{2}{n_3 - 1} + \dfrac{2}{n_1 - 1}}}$$

$$v = \frac{2Z(n_4, n_3; \alpha_2) - \log\left(1 + \dfrac{2\lambda_4}{n_4}\right)}{\sqrt{\dfrac{2}{\nu_4 - 1} + \dfrac{2}{n_3 - 1}}}.$$

$Z(n_i, n_j; \alpha_k)$ denotes the upper 100 α_k% point of Fisher's Z-distribution with (n_i, n_j) degrees of freedom.

The probability as in (2.1.1.5) can now be evaluated by employing the table of probability integral of a bivariate normal surface [Published by National Bureau of Standards (1959)].

2.1.2. Approximate Formula for P_2.

With regard to P_2, we observe that, in the limit, the three ratios $\frac{V_3}{V_1}$, $\frac{V_2}{V_{13}}$ and $\frac{V_4}{V_{13}}$ are independently distributed since for finite n_3 as $n_1 \to \infty$, (i. e. $s \to \infty$), V_1 and V_{13} both tend to σ_1^2. Hence, (2.1.c) can be written as

$$P_2 \cong \text{Prob.}\left\{\frac{V_3}{V_1} < F_1\right\} \text{Prob.}\left\{\frac{V_2}{V_{13}} \geqq F_3\right\} \text{Prob.}\left\{\frac{V_4}{V_{13}} \geqq F_4\right\} \tag{2.1.2.1}$$

From Patnaik's (1949) approximation we find that $n_2 V_2$ and $n_4 V_4$ are distributed as $\chi_{\nu_2}^2 c_2 \sigma_1^2$ and $\chi_{\nu_4}^2 c_4 \sigma_3^2$, where $\chi_{\nu_2}^2$ and $\chi_{\nu_4}^2$ are the central chi-squares with degrees of freedom

$$\nu_2 = n_2 + \frac{4\lambda_2^2}{n_2 + 4\lambda_2}, \qquad \nu_4 = n_4 + \frac{4\lambda_4^2}{n_4 + 4\lambda_4} \tag{2.1.2.2}$$

and scalar constants

$$c_2 = 1 + \frac{2\lambda_2}{n_2 + 2\lambda_2}, \qquad c_4 = 1 + \frac{2\lambda_4}{n_4 + 2\lambda_4} \tag{2.1.2.3}$$

Also $n_i V_i$ $(i=1, 3)$ is distributed as $\chi_i^2 \sigma_i^2$ where χ_i^2 is a central chi-square with n_i degrees of freedom and $(n_1+n_3)^2 V_{13}$ is approximately distributed as $(n_1\sigma_1^2 + n_3\sigma_3^2)\chi_{13}^2$, where χ_{13}^2 is a central chi-square with n_1+n_3 degrees of freedom.

Making use of these approximations, we can write (2.1.2.1) as

$$P_2 = \text{Prob}\left\{F(n_3, n_1 > \frac{F_1}{\theta_{31}}\right\} \text{Prob.}\left\{F(\nu_2, n_1+n_3) \geqq \frac{n_2(n_1+n_3\theta_{31})F_3}{(n_1+n_3)(n_2+2\lambda_2)}\right\}$$

$$\text{Prob}\left\{F(\nu_4, n_1+n_3) \geqq \frac{n_4(n_1\theta_{31}+n_3)F_4}{(n_1+n_3)(n_4+2\lambda_4)}\right\}, \tag{2.1.2.4}$$

where $F(p, q)$ is the central F-statistic based on (p, q) degrees of freedom.

If we now use the relation

$$\text{Prob.}\ \{F(p, q) \leqq F_0\} = I_x\left(\frac{p}{2}, \frac{q}{2}\right), \tag{2.1.2.5}$$

where $x = pF_0/(q+pF_0)$, between F-integral and normalized incomplete Beta function, we obtain

$$P_2 \cong I_{x_{21}}\left(\frac{n_3}{2}, \frac{n_1}{2}\right)\left[1 - I x_{22}\left(\frac{\nu_2}{2}, \frac{n_1+n_3}{2}\right)\right]\left[1 - I x_{23}\left(\frac{\nu_4}{2}, \frac{n_1+n_3}{2}\right)\right], \tag{2.1.2.6}$$

where

$$x_{21} = \frac{n_3 F_1}{n_1\theta_{31} + n_3 F_1}, \tag{2.1.2.7}$$

$$x_{22} = \frac{\nu_2 n_2(n_1+n_3\theta_{31})F_3}{(n_1+n_3)^2(n_2+2\lambda_2) + \nu_2 n_2(n_1+n_3\theta_{31})F_3} \tag{2.1.2.8}$$

and

$$x_{23}=\frac{\nu_4 n_4(n_1\theta_{31}+n_3)F_4}{(n_1+n_3)^2(n_4+2\lambda_4)+\nu_4 n_4(n_1\theta_{31}+n_3)F_4}\,. \tag{2.1.2.9}$$

To evaluate the probability P_2 given by (2.1.2.6) we can use the Tables of Incomplete Beta Function edited by Pearson (1968).

2.1.3. Approximate Fomula for P_3.

To evaluate P_3, we observe that for finite n_3 if $n_1\to\infty$ (i.e., $s\to\infty$) both V_1 and V_{13} tend to σ_1^2 and V_{123} tends to $\frac{(r-1)\sigma_1^2}{r}+\frac{\sigma_2^2}{r}$ and therefore, in the limit the three ratios $\frac{V_3}{V_1}$, $\frac{V_2}{V_{13}}$ and $\frac{V_4}{V_{123}}$ are independently distributed. Hence, we may write (2.1.d) as

$$P_3\cong\text{Prob.}\left\{\frac{V_3}{V_1}<F_1\right\}\text{Prob.}\left\{\frac{V_2}{V_{13}}<F_3\right\}\text{Prob.}\left\{\frac{V_4}{V_{123}}\geqq F_5\right\} \tag{2.1.3.1}$$

Using Patnaik's approximation we find that $(n_1+n_2+n_3)V_{123}$ is distributed as $c'\chi^2_{\nu'}\sigma_1^2$, where $\chi^2_{\nu'}$ is a central chisquare with degrees of freedom

$$\nu'=n_1+n_2+n_3+\frac{4\lambda_2^2}{n_1+n_2+n_3+4\lambda_2} \tag{2.1.3.2}$$

and the scalar constant

$$c'=1+\frac{2\lambda_2}{n_1+n_2+n_3+2\lambda_2} \tag{2.1.3.3}$$

If we use this approximation and follow the same method as in section 2.1.2, we obtain

$$P_3\cong\text{Prob.}\left\{F(n_3,\ n_1)<\frac{F_1}{\theta_{31}}\right\}\text{Prob.}\left\{F(\nu_2,\ n_1+n_3)<\frac{n_2(n_1+n_3\theta_{31})F_3}{(n_1+n_3)(n_2+2\lambda_2)}\right\}$$

$$\text{Prob.}\left\{F(\nu_4,\ \nu')\geqq\frac{n_4(n_1+n_2+n_3+2\lambda_2)F_5}{(n_4+2\lambda_4)(n_1+n_2+n_3)\theta_{31}}\right\} \tag{2.1.3.4}$$

Making use of (2.1.2.5), we have

$$P_3\cong I_{x_{21}}\left(\frac{n_3}{2},\ \frac{n_1}{2}\right)I_{x_{22}}\left(\frac{\nu_2}{2},\ \frac{n_1+n_3}{2}\right)\left[1-I_{x_{33}}\left(\frac{\nu_4}{2},\ \frac{\nu'}{2}\right)\right], \tag{2.1.3.5}$$

where x_{21} and x_{22} are given by (2.1.2.7) and (2.1.2.8) respectively and

$$x_{33}=\frac{n_4(n_1+n_2+n_3+2\lambda_2)F_5}{\nu'(n_1+n_2+n_3)\theta_{31}+n_4(n_1+n_2+n_3+2\lambda_2)F_5}\,. \tag{2.1.3.6}$$

The probability P_3 given by (2.1.3.5) can easily be computed by using Pearson's (1968) Table.

3. Results and Discussion.

In this section, we attempt to examine how the proposed approximations work. Using approximate formulae developed in section 2.1 we have made numerical calculations for power components. We have also computed values for power components by applying series formulae given by (2.1), (2.2) and (2.3). In order to see how these approximations work, we have considered the values for P_2 and P_3 only obtained by both series and approximate formulae. These values along with their differences have been presented in Tables 3.1-3.4 for $\alpha_p=.50$ and $\alpha_f=.05$, where α_p and α_f are respectively the preliminary and final levels of significance. Tables 3.1 and 3.2 show the values of P_2 computed using series and approximate formulae, alongwith differences in these values for $c_2=1.0000$ and $c_2=1.7071$ respectively. Tables 3.3 and 3.4 show similar values for P_3. In all these tables we have taken $\lambda_4=0$.

It is observed from Tables 3.1 and 3.2 that for a given set of degrees of freedom and for $\theta_{31}=1.0$, the difference between the series and approximate values of P_2 decreases as c_2 increases from 1.0000 to 1.7071. This difierence decreases rapidly as we increase n_1 singly or in combination with n_2 or with n_2 and n_3 and becomes nearly zero (.00008) when n_1, n_2 and n_3 each equals 10. For $\theta_{31}>1.0$, this tendency is reversed. For a fixed value of degrees of freedom combination, the difference between the two values increases as θ_{31} increases and then decteases for large values of θ_{31}.

From Tables 3.3 and 3.4, it can be seen that for $\theta_{31}=1.0$ and $c_2=1.0000$, the approximate formula yields thesame value for P_3 as the series formula for all the sets of degrees of freedom considered. For $n_1=n_2=n_3=n_4=2$ when c_2 exceeds unity, the difference in values obtained by the two formulae differs very slightly from zero. For large values of n_1, n_2 and n_3, this diherence approaches zero. For a fixed value of c_2 and given degrees of freedom combination, the difference in the two values of P_3

Table 3.1. Nature of Approximation of P_2 for $n_4=2$, $\alpha_p=.50$, $\alpha_f=.05$, $\lambda_4=0$, $c_2=1.0000$

n_3	n_2	θ_{31}	$n_1=2$			$n_1=10$		
			Series	approx.	diff.	Series	approx.	diff.
2	2	1.0	.02094	.01250	.00844	.01626	.01250	.00376
		1.5	.02509	.00609	.01900	.02556	.00359	.02197
		2.0	.02736	.00329	.02407	.03102	.00124	.02978
		3.0	.02896	.00118	.02778	.03494	.00021	.03473
		5.0	.02776	.00025	.02751	.03310	.00001	.03309
		8.0	.02397	.00005	.02392	.02720	.00000	.02720
2	10	1.0				.01967	.01250	.00717
		1.5				.02987	.00340	.02647
		2.0				.03522	.00110	.03412
		3.0				.03812	.00017	.03795
		5.0				.03458	.00001	.03457
		8.0				.02758	.00000	.02758
10	10	1.0				.01751	.01250	.00501
		1.5				.01397	.00244	.01153
		2.0				.00944	.00049	.00895
		3.0				.00402	.00002	.00400
		5.0				.00089	.00000	.00089
		8.0				.00016	.00000	.00016

Table 3.2. Nature of Approximation of P_2 for $n_4=2$, $\alpha_p=.50$, $\alpha_f=.05$, $\lambda_4=0$, $c_2=1.7071$

n_3	n_2	θ_{31}	$n_1=2$			$n_1=10$		
			Series	approx.	diff.	Series	approx.	diff.
2	2	1.0	.02481	.02251	.00230	.02420	.02312	.00108
		1.5	.03150	.01210	.01940	.04035	.00693	.03342
		2.0	.03600	.00713	.02887	.05079	.00249	.04830
		3.0	.04094	.00297	.03797	.05975	.00046	.05929
		5.0	.04310	.00079	.04231	.05896	.00004	.05892
		8.0	.04033	.00019	.04014	.04973	.00000	.04973
2	10	1.0				.02499	.02482	.00017
		1.5				.04207	.00750	.03457
		2.0				.05330	.00271	.05059
		3.0				.06320	.00051	.06269
		5.0				.06286	.00004	.06282
		8.0				.05328	.00000	.05328
10	10	1.0				.02499	.02491	.00008
		2.0				.01968	.00208	.01760
		5.0				.00315	.00000	.00315
		8.0				.00070	.00000	.00070

Table 3.3. Nature of Approximation of P_3 for $n_4=2$, $\alpha_p=.50$, $\alpha_f=.05$, $\lambda_4=0$, $c_2=1.0000$

n_3	n_2	θ_{31}	$n_1=2$			$n_1=10$		
			Series	approx.	diff.	Series	approx.	diff.
2	2	1.0	.01250	.01250	.00000	.01250	.01250	.00000
		1.5	.01881	.02300	−.00419	.02182	.02351	−.00169
		2.0	.02353	.03224	−.00871	.02803	.03168	−.00365
		3.0	.02924	.04515	−.01591	.03355	.04070	−.00715
		5.0	.03266	.05514	−.02248	.03346	.04500	−.01154
		8.0	.03114	.05449	−.02335	.02834	.04232	−.01398
2	10	1.0				.01250	.01250	.00000
		1.5				.02341	.02582	−.00241
		2.0				.03069	.03642	−.00573
		3.0				.03694	.04898	−.01204
		5.0				.03650	.05562	−.01912
		8.0				.03055	.05156	−.02101
10	10	1.0				.01250	.01250	.00000
		1.5				.01636	.02227	−.00591
		2.0				.01497	.02315	−.00818
		3.0				.00912	.01515	−.00603
		5.0				.00287	.00458	−.00171
		8.0				.00065	.00096	−.00031

Table 3.4. Nature of Approximation of P_3 for $n_4=2$, $\alpha_p=.50$, $\alpha_f=.05$, $\lambda_4=0$, $c_2=1.7071$

n_3	n_2	θ_{31}	$n_1=1$			$n_1=10$		
			Series	approx.	diff.	Series	approx.	diff.
2	2	1.0	.00074	.00118	−.00044	.00112	.00103	.00009
		1.5	.00150	.00294	−.00144	.00230	.00242	−.00012
		2.0	.00227	.00521	−.00294	.00323	.00384	−.00061
		3.0	.00361	.01028	−.00667	.00427	.00623	−.00196
		5.0	.00518	.01907	−.01389	.00465	.00941	−.00476
		8.0	.00592	.02635	−.02043	.00416	.01204	−.00788
2	10	1.0				.00002	.00001	.00001
		1.5				.00008	.00006	.00002
		2.0				.00015	.00019	−.00004
		3.0				.00027	.00068	−.00041
		5.0				.00039	.00251	−.00212
		8.0				.00040	.00636	−.00596
10	10	1.0				.00002	.00001	.00001
		2.0				.00011	.00031	−.00020
		5.0				.00008	.00105	−.00097
		8.0				.00003	.00050	−.00047

decreases as θ_{31} incresses and is, in general, negative.

We may, therefore, conclude that the approximate formulae for P_2 and P_3 work out satisfactorily for $\theta_{31}=1.0$ and $c_2>1$ (i.e., the interaction exists between cuttings and varieties). For large θ_{31} (say $\theta_{31}\geqq 8.0$), the proposed approximation would work provided each of n_1, n_2 and n_3 is greater than or equal to 10 irrespective of whether the interaction is present or not (i.e., $c_2\geqq 1$). Under these codditions, the proposed approximate formulae render considerable relief in the computation of power values.

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