OSCILLATORY PHENOMENA IN THE CELL SPACE WITH INHIBITION STATES $ PHI $: INFORMATION SCIENCE APPROACH TO BIOMATHEMATICS, X

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OSCILLATORY PHENOMENA IN THE CELL SPACE WITH INHIBITION STATES ϕ

—INFORMATION SCIENCE APPROACH TO BIOMATHEMATICS, X

By

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1. Introduction

In one of the joint papers Kitagawa and Yamaguchi [5], we have introduced a notion of inhibition states ϕ in our cell space approach. In a cell space with unit square cells, the existence of a pair of inhibition states ϕ will secure us a certain feature of oscillatory phenomena by giving some restrictions to the free application of LMT introduced and discussed in a series of our papers [1]~[9]. It is the purpose of the present paper to discuss one type of the fundamental configurations

$$C=\begin{pmatrix} \phi & 0 & 0 & 0 \\ 1 & a & b & 0 \\ 1 & c & d & 0 \\ 1 & 1 & 1 & \phi \end{pmatrix}, \qquad (1.01)$$

where each of a, b, c and d are either 1 or 0.

In fact there are $2^4=16$ configurations in total, and the existence of two inhibition states ϕ in the corner will yield us a certain type of oscillatory transition phenomena among these sixteen configurations, which seems to be interesting in itself as giving a simple example of phenomena which are oscillatory in the interior and fixed unchanged on boundary of a system.

This paper is a completion of some suggestions given in Section 6 of our paper Kitagawa and Yamaguchi [5].

2. Notations

For a moment let us denote by C_i ($i=1, 2, \cdots, 8$) each of the following matrices

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$$
(2.01)\quad \begin{cases} c_1=\begin{pmatrix}0&0\\0&0\end{pmatrix}, & c_2=\begin{pmatrix}0&0\\1&0\end{pmatrix}, & c_3=\begin{pmatrix}1&0\\0&0\end{pmatrix}, \\ c_4=\begin{pmatrix}0&0\\0&1\end{pmatrix}, & c_5=\begin{pmatrix}0&1\\0&0\end{pmatrix}, & c_6=\begin{pmatrix}1&0\\1&0\end{pmatrix}, \\ c_7=\begin{pmatrix}0&0\\1&1\end{pmatrix}, & c_8=\begin{pmatrix}0&1\\1&0\end{pmatrix}. \end{cases}
$$

Then let us denote by $\bar{c}_i$ the matrix whose element is conjugate to each corresponding element of c_i.

$$
(2.02)\quad \begin{cases} \bar{c}_1=\begin{pmatrix}1&1\\1&1\end{pmatrix}, & \bar{c}_2=\begin{pmatrix}1&1\\0&1\end{pmatrix}, & \bar{c}_3=\begin{pmatrix}0&1\\1&1\end{pmatrix}, \\ \bar{c}_4=\begin{pmatrix}1&1\\1&0\end{pmatrix}, & \bar{c}_5=\begin{pmatrix}1&0\\1&1\end{pmatrix}, & \bar{c}_6=\begin{pmatrix}0&1\\0&1\end{pmatrix}, \\ \bar{c}_7=\begin{pmatrix}1&1\\0&0\end{pmatrix}, & \bar{c}_8=\begin{pmatrix}1&0\\0&1\end{pmatrix}. \end{cases}
$$

For each confiuration (1.01), there are nine cases for occurrence of firing points as shown by seven dotted points and two non-firing points denoted by $\times$ including ϕ in their respective 2×2 basic cell spaces:

$$
(2.03)\quad T\begin{pmatrix} \phi & {}_{\times} & 0 & \cdot & 0 & \cdot & 0 \\ 1 & \cdot & a & \cdot & b & \cdot & 0 \\ 1 & \cdot & c & \cdot & d & \times & 0 \\ 1 & & 1 & & 1 & & \phi \end{pmatrix}.
$$

Let us assume, in this paper, that the occurance probability of each of these nine points is equal to each other, hence with probability 1/9. Then an application of LMT, the transformation satisfying the principle of local majority, yields us a 16×16 transition probability matrix among the sixteen configurations given by (1.01). In stead of writing out the 16×16 transition probability matrix, let us start with each of C_i and $\bar{C}_i$ $(i=1, 2, 3, \cdots, 8)$, which are defined by

$$
(2.04)\quad C_i=\begin{pmatrix} \phi & 0 & 0 & 0 \\ 1 & \multicolumn{2}{c}{\boxed{\;c_i\;}} & 0 \\ 1 & & & 0 \\ 1 & 1 & 1 & \phi \end{pmatrix}, \qquad \bar{C}_i=\begin{pmatrix} \phi & 0 & 0 & 0 \\ 1 & \multicolumn{2}{c}{\boxed{\;\bar{c}_i\;}} & 0 \\ 1 & & & 0 \\ 1 & 1 & 1 & \phi \end{pmatrix}.
$$

Then we observe immediately that C_1 is transformed into either of C_1 and C_2 with the probabilities 8/9 and 1/9 respectively. Let us denote the transition by the formula:

$$
(2.05)\quad TC_1=T\begin{pmatrix} \phi & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & \phi \end{pmatrix}=\frac{8}{9}C_1+\frac{1}{9}C_2.
$$

In similar notations, we can and we shall write

(2.06) $$TC_2=T\begin{pmatrix}\phi & 0 & 0 & 0\\ 1 & 0 & 0 & 0\\ 1 & 1 & 0 & 0\\ 1 & 1 & 1 & \phi\end{pmatrix}=\frac{1}{9}C_1+\frac{6}{9}C_2+\frac{1}{9}C_6+\frac{1}{9}C_7$$

(2.07) $$TC_3=T\begin{pmatrix}\phi & 0 & 0 & 0\\ 1 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 1 & 1 & 1 & \phi\end{pmatrix}=\frac{5}{9}C_3+\frac{2}{9}C_1+\frac{2}{9}C_6$$

(2.08) $$TC_4=T\begin{pmatrix}\phi & 0 & 0 & 0\\ 1 & 0 & 0 & 0\\ 1 & 1 & 1 & 0\\ 1 & 1 & 1 & \phi\end{pmatrix}=\frac{5}{9}C_4+\frac{2}{9}C_1+\frac{2}{9}C_7$$

(2.09) $$TC_5=T\begin{pmatrix}\phi & 0 & 0 & 0\\ 1 & 0 & 1 & 0\\ 1 & 0 & 0 & 0\\ 1 & 1 & 1 & \phi\end{pmatrix}=\frac{4}{9}C_5+\frac{4}{9}C_1+\frac{1}{9}C_8$$

(2.10) $$TC_6=T\begin{pmatrix}\phi & 0 & 0 & 0\\ 1 & 1 & 0 & 0\\ 1 & 1 & 0 & 0\\ 1 & 1 & 1 & \phi\end{pmatrix}=\frac{7}{9}C_6+\frac{1}{9}C_2+\frac{1}{9}\bar{C}_5$$

(2.11) $$TC_7=T\begin{pmatrix}\phi & 0 & 0 & 0\\ 1 & 0 & 0 & 0\\ 1 & 1 & 1 & 0\\ 1 & 1 & 1 & \phi\end{pmatrix}=\frac{7}{9}C_7+\frac{1}{9}\bar{C}_5+\frac{1}{9}C_2$$

(2.12) $$TC_8=T\begin{pmatrix}\phi & 0 & 0 & 0\\ 1 & 0 & 1 & 0\\ 1 & 1 & 0 & 0\\ 1 & 1 & 1 & \phi\end{pmatrix}=\frac{4}{9}C_8+\frac{3}{9}C_2+\frac{1}{9}\bar{C}_3+\frac{1}{9}\bar{C}_4.$$

Similarly we shall find the formulas for $T\bar{C}_i$ ($i=1, 2, \cdots, 8$). Before proceeding it is noted that there exists a set of transposition relations S among the sixteen matrices $\{C_i\}$ and $\{\bar{C}_i\}$ such that

(2.13) $$S\bar{C}_1=S\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}=\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}=\bar{C}_1 \qquad \text{(invariant)}$$

$$S\bar{C}_2=S\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}=\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}=\bar{C}_5$$

$$S\bar{C}_3=S\begin{pmatrix}0 & 1\\ 1 & 1\end{pmatrix}=\begin{pmatrix}0 & 1\\ 1 & 1\end{pmatrix}=\bar{C}_3 \qquad \text{(invariant)}$$

$$S\bar{C}_4 = S\begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix} = \begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix} = \bar{C}_4 \qquad \text{(invariant)}$$

$$S\bar{C}_5 = S\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix} = \begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix} = \bar{C}_2$$

$$S\bar{C}_6 = S\begin{pmatrix}0 & 1\\ 0 & 1\end{pmatrix} = \begin{pmatrix}0 & 0\\ 1 & 1\end{pmatrix} = C_7$$

$$S\bar{C}_7 = S\begin{pmatrix}1 & 1\\ 0 & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\ 1 & 0\end{pmatrix} = C_6$$

$$S\bar{C}_8 = S\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = \bar{C}_8 .$$

After this preparation, let us observe

$$T\bar{C}_1 = T\begin{pmatrix}\phi & 0 & 0 & 0\\ 1 & 1 & 1 & 0\\ 1 & 1 & 1 & 0\\ 1 & 1 & 1 & \phi\end{pmatrix} = \frac{8}{9}\bar{C}_1 + \frac{1}{9}\bar{C}_5 \tag{2.14}$$

$$T\bar{C}_2 = T\begin{pmatrix}\phi & 0 & 0 & 0\\ 1 & 1 & 1 & 0\\ 1 & 0 & 1 & 0\\ 1 & 1 & 1 & \phi\end{pmatrix} = \frac{4}{9}\bar{C}_2 + \frac{1}{9}\bar{C}_8 + \frac{4}{9}\bar{C}_1 \tag{2.15}$$

$$T\bar{C}_3 = T\begin{pmatrix}\phi & 0 & 0 & 0\\ 1 & 0 & 1 & 0\\ 1 & 1 & 1 & 0\\ 1 & 1 & 1 & \phi\end{pmatrix} = \frac{5}{9}\bar{C}_3 + \frac{2}{9}C_7 + \frac{2}{9}\bar{C}_1 \tag{2.16}$$

$$T\bar{C}_4 = T\begin{pmatrix}\phi & 0 & 0 & 0\\ 1 & 1 & 1 & 0\\ 1 & 1 & 0 & 0\\ 1 & 1 & 1 & \phi\end{pmatrix} = \frac{5}{9}\bar{C}_4 + \frac{2}{9}C_6 + \frac{2}{9}\bar{C}_1 \tag{2.17}$$

$$T\bar{C}_5 = T\begin{pmatrix}\phi & 0 & 0 & 0\\ 1 & 1 & 0 & 0\\ 1 & 1 & 1 & 0\\ 1 & 1 & 1 & \phi\end{pmatrix} = \frac{6}{9}\bar{C}_5 + \frac{1}{9}C_6 + \frac{1}{9}C_7 + \frac{1}{9}\bar{C}_1 \tag{2.18}$$

$$T\bar{C}_6 = T\begin{pmatrix}\phi & 0 & 0 & 0\\ 1 & 0 & 1 & 0\\ 1 & 0 & 1 & 0\\ 1 & 1 & 1 & \phi\end{pmatrix} = \frac{5}{9}\bar{C}_6 + \frac{2}{9}C_4 + \frac{2}{9}\bar{C}_3 \tag{2.19}$$

$$T\bar{C}_7 = T\begin{pmatrix}\phi & 0 & 0 & 0\\ 1 & 1 & 1 & 0\\ 1 & 0 & 0 & 0\\ 1 & 1 & 1 & \phi\end{pmatrix} = \frac{5}{9}\bar{C}_7 + \frac{2}{9}C_3 + \frac{2}{9}\bar{C}_4 \tag{2.20}$$

$$(2.21) \qquad T\overline{C}_8 = T\begin{pmatrix} \phi & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & \phi \end{pmatrix} = \frac{4}{9}\overline{C}_8 + \frac{3}{9}\overline{C}_5 + \frac{1}{9}C_4 + \frac{1}{9}C_3 .$$

3. Matrix of transition probabilities P

Now in order to summarize the transition relations given by (2.05)$\sim$(2.12) and (2.14)$\sim$(2.21), there must be a caution for choosing the order of sixteen configurations by which an insight can be secured into the structure of transition matrices. Such a caution will yield us the 16×16 matrix of the transition probabilities given by the following T. P., where we write simply j to denote the probability $j/9$ ($j=1, 2, \cdots, 8$).

$$\begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ \overline{C}_6 \\ C_8 \\ C_6 \\ C_7 \\ \overline{C}_8 \\ \overline{C}_7 \\ \overline{C}_2 \\ \overline{C}_4 \\ \overline{C}_3 \\ \overline{C}_5 \\ \overline{C}_1 \end{array} \begin{array}{c} \begin{array}{cccccccccccccccc} C_1 & C_2 & C_3 & C_4 & C_5 & \overline{C}_6 & C_8 & C_6 & C_7 & \overline{C}_8 & \overline{C}_7 & \overline{C}_2 & \overline{C}_4 & \overline{C}_3 & \overline{C}_5 & \overline{C}_1 \end{array} \\ \begin{pmatrix} 8 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 6 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 5 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 4 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 4 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 5 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 6 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 8 \end{pmatrix} \end{array}$$

4. Subroutes imbedded in the transition routes of P

An observation gives us a set of subroutes imbedded in the transition routes shown by the transition matrix P.

(I) SUBROUTE A

This consists of 6 configurations C_1, C_2, C_6, C_7, $\overline{C}_5$ and $\overline{C}_1$ among which a closed subroute A shown in Figure 1 exists, where the number j associated with each path denotes the transition probability $j/9$, for each j.

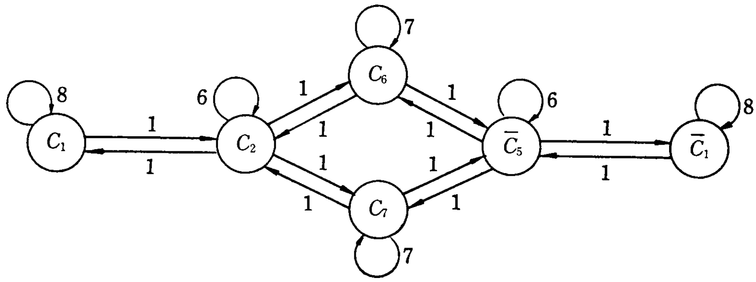

Fig. 1. Subroute A.

(II) SUBROUTE B

This consists of 4 configurations C_3, C_4, $\bar{C}_3$ and $\bar{C}_4$ among which paths with their respective probabilities are given so as to connect with configurations belonging to subroute A, as shown in Figure 2.

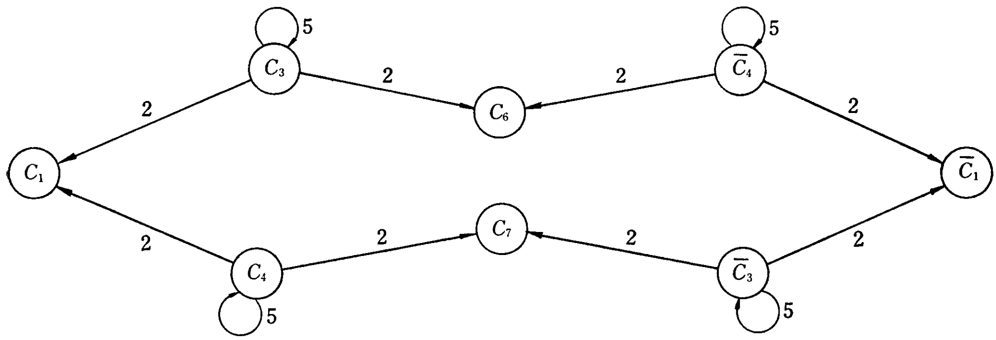

Fig. 2. Subroute B.

(III) SUBROUTE C

This consists of two configurations $\bar{C}_6$ and $\bar{C}_7$ for each of which there exists a set of paths connecting with some configurations belonging to subroute B, as shown in Figure 3.

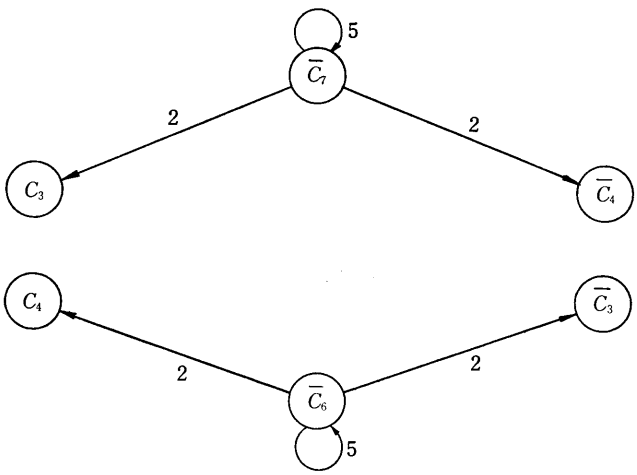

Fig. 3. Subroute C.

(IV) SUBROUTE *D*

This consists of two configurations C_8 and $\bar{C}_8$ for each of which there exists a set of paths connecting with some configurations belonging to either of subroutes *A* and *B*, as shown in Figure 4.

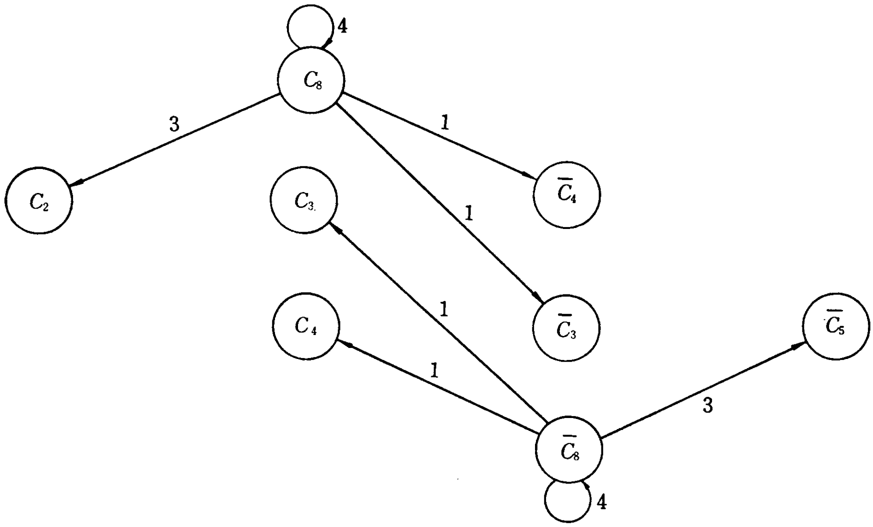

Fig. 4. Subroute *D*.

(V) SUBROUTE *E*

This consists of two configurations C_5 and $\bar{C}_2$ for each of which there exists a set of paths connecting with other subroutes already defined, as shown in Figure 5.

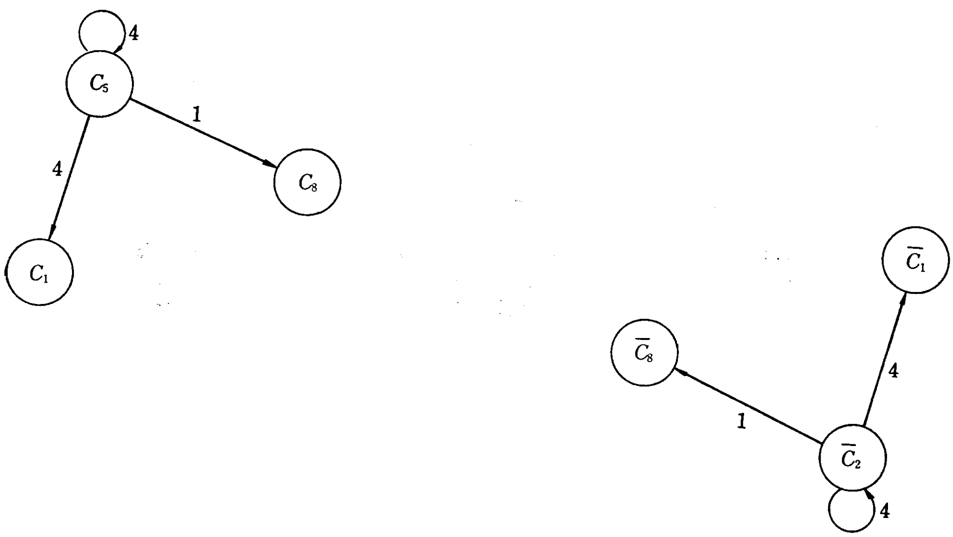

Fig. 5. Subroute *E*.

In view of these five subroutes, it will be of some use to introduce the notations,

(4.01) $\bar{C}_5 = C_2^*,\quad C_7 = C_6^*,\quad \bar{C}_7 = \bar{C}_6^*,\quad \bar{C}_2 = C_5^*,$

in order to make clear the correspondence between each pair of configurations.

The amalgamations of subroutes A, B and C can be also fairly distinctly given by denoting each configuration by the symbol of subroute to which it belongs, as shown in Figure 6.

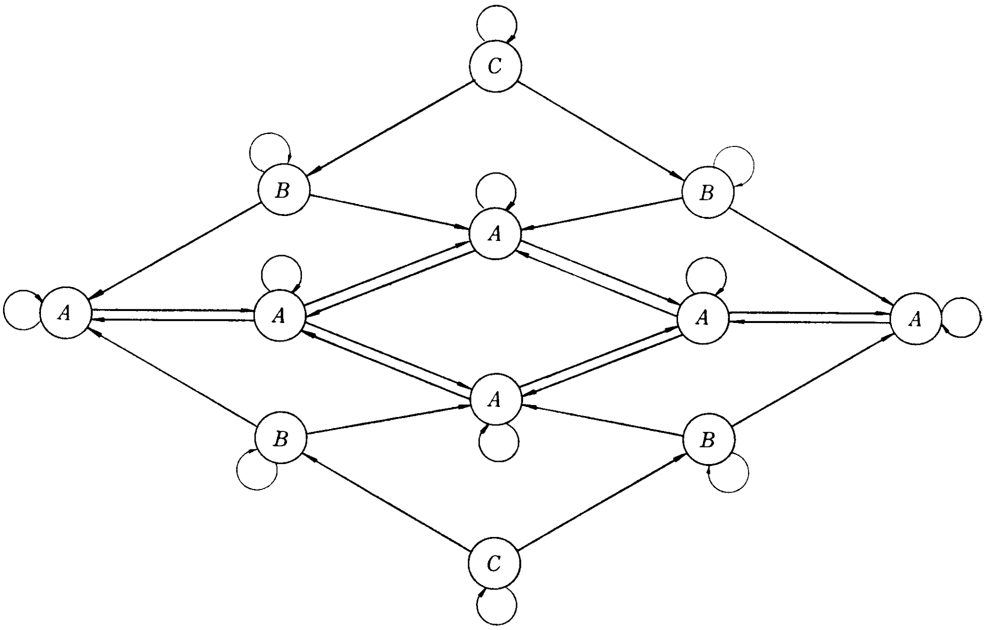

Fig. 6. Coexistence of subroutes A, B and C.

The whole route digraph which can be obtained by adding to Figure 6 the subroutes D and E can be easily given.

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