# THREE COORDINATE SYSTEMS FOR INFORMATION SCIENCE APPROACHES

Kitagawa, Toshio Research Institute of the Fundamental Information Science, Faculty of Science, Kyushu

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# THREE COORDINATE SYSTEMS FOR INFORMATION SCIENCE APPROACHES

By

# Tosio Kitagawa\*

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# 1. Introduction

We are now entering into the stage of cybernetical era after passing through two previous eras, namely agricultural and industrial eras. The influences of cybernetical techniques have been becoming so profound that sciences and techniques, productions, industries, economics, societies, politics and educations are all, more or less, being subject to crucial transcient states of revolutions. Human beings have now reached to the stage when they can and have to study large systems in various scientific, technological and social fields, because of the forthcoming of cybernetical era.

Now it is the fundamental aspect of our recognition for cybernetical era that information, as well as matter and energy, is becoming one of the fundamental notions in describing our objective world.

System can be recognised as a scheme of black box in which inputs are given and are transformed into outputs. These inputs and outputs are always matter and energy carrying information in some sense. In what follows we shall refer to the information aspects of system in order to concentrate to the topics with which we are particularly concerned. This does imply by no means that matter-energy aspects could be neglected in system approach.

So far as system approach contains block box component in its formulation, it can not but concern ifself with control problem.

In comparison with classical science theory in which description of the object matter should be given before any prediction can be performed and in which control of the object matter should be based upon prediction, it is crucial to note that this classical procedure can be neither necessary nor sufficient for our general system approach. There are three crucial aspects of our approach to general system. In the first place we shall not adhere to descriptive theory, but control aspect will be our primary concern. Secondly we can not confine ourselves to deterministic model but we shold appeal also to probabilistic model. These two aspects may lead us to cybernetical formulation in the sense of Wiener [23]. However these two aspects are not sufficient enough for discussing any system and any large system

<sup>\*</sup> Research Institute of the Fundamental Information science, Faculty of Science, Kyushu University.

in particular, unless we shall employ other principal aspects. In what follows we shall explain what we mean by other principal aspects. Our attitude is deeply connected with information science aspect as a whole, which is somewhat broader than Wiener [23] tried to establish in his formulation of cybernetics. We shall give two coordinations in discussing our information science approach. These two coordinations will lead us to the third coordination. It is the opinion of the author that the cooperative studies in combining these three coordination principles will be indispensable in dealing with any system and any large system in particular from the standpoint information science.

## 2. The 1st Coordination

This coordination is concerned with the following three logical aspects (a) objectivity, (b) subjectivity and (c) practices.

[1] Ad (a). This category is concerned with how to describe our objective world. There are three fundamental principal notions, namely,  $(a_1)$  pattern,  $(a_2)$  chaos and  $(a_3)$  transformation. These three notions can be and should be formulated in terms of mathematical apparatus in the following sense.

(a<sub>1</sub>) Pattern implies algebra, graph, algorithm, automata and so on.

 $(a_2)$  Chaos implies set, measure, probablity and so on.

(a<sub>3</sub>) Transformation implies mapping, function and so on.

Incidentally we should remark that modern mathematics are now getting out of the classical classification of mathematics, such as algebra, geometry and analysis, because of the urgent needs of reforming mathematics for the purpose of aiming at mathematics in the 21st century which should be cultivated in connection with information science.

The classical examples of our formulation emphasising  $(a_1)$ ,  $(a_2)$  and  $(a_3)$  can be seen in the Wiener theory of cybernetics and also in the Fisher theory of statistics ([7], [8] and [9]) in dealing with analysis of variance model.

[2] Ad (b). This category is concerned with some aspects upon which our subjective attitudes to be adopted will be prescribed. That is to say, we are going to prepare an answer to the following problem. Given an objective world before us, what we should now provide with ourselves? There are three fundamental notions;  $(b_1)$  operation,  $(b_2)$  adaptation and  $(b_3)$  strategy.

The meaning of  $(b_1)$  operation should be specified so as to satisfy the three conditions:

(i) Operations are described objectively.

(ii) Effects of operations are measurable.

(iii) Operations are repeatable.

More precise definitions and specifications are given in the definition of operations research by various authors such as Morse-Kimball [16], Churchman-Ackoff-Arnold [5], and Beer [1].

In contrast with  $(b_1)$  operation, adaptation is another principal attitude in face with the objective world in which no sufficient pattern recognition is given to us. Under the influence of the theory of evolution in biology due to Charles Darwin, the British school of empiricism has succeeded in establishing an elaborated theory of statistics for dealing with chaos, as can be seen along the developments given by F. Galton, K. Pearson [19] and R.A. Fisher [9]. In the more concise and more explict formulation, there is a well known procedure of quality control based upon the principle of evolution which is entitled as EVOP, evolutionary operation program, by the works of G. E. P. Box [4].

The notion of adaptation is formulated in modern theory of adaptive control. It is currently understood that adaptive control is concerned with two problems, namely, optimalization and identification problems. The reason why identification problem is required for us to resolve in adaptive control is due to the fact that we are in a situation of having no complete information of the subjective world, and by getting more information on our object world, our control proceduce may be changed.

Besides  $(b_1)$  operation and  $(b_2)$  adaptation, we should add another one notion, namely, that of  $(b_3)$  strategy. Strategy must be introduced primarily for dealing with antagonistic situations. So far as scientific approaches are concerned, it is due to J. von Neumann and O. Morgenstern [19] that we can succeed to formulate explicitly strategies to be adopted in some of antagonistic situations.

It is also to be noted that strategic aspects are indispensable in dealing with the situations in which our information are incomplete and in which our concerns to establish our subjective attitude are so keen that nature, which is substantially to be concieved to be neutral, may turn out to look as if our opponent. This is in fact a mimicry which has been employed in statistical decision function theory of A. Wald [21].

[3] Ad (c). Once our subjective attitudes are settled in front of our objective world, then some principles of practices should be cleared which guide us in dealing with real circumstances. Here are three notions. The first notion is  $(c_1)$  optimalition principle which can be observed in various theories belonging to mathematical programming such as in linear programming by von Dantzig [6] and in dynamic programming by R. Bellman [2] and so on.

The second notion is  $(c_2)$  stability principle which comes from biological behaviour. This principle has its one characteristic aspect in the sense that we can not rely on optimalization principle because of incomplete information and of incomplete operation in dealing with objective world. In stead we are subject to the guide of the external world in some sense.

The third notion in guiding us in practice is  $(c_3)$  learning principle. Again here comes the development of modern control theory and their associated techniques, as can be seen in learning control developed by various authors.

# 3. The second accordination

The second coordination is concerned with three functional uses of information, namely ( $\alpha$ ) cognition, ( $\beta$ ) direction and ( $\gamma$ ) evaluation.

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[1] Ad ( $\alpha$ ). Regarding the methodological aspects of cognition, we should mention three notions of locical thinkings, namely, ( $\alpha_1$ ) deduction, ( $\alpha_2$ ) induction and ( $\alpha_3$ ) abduction. These three methodologies of logics contribute to yield us patterns formulation of our informations.

Deductive method can be observed in mathematical reasonings in its purest form and in logico-mathematical approaches in any theoretical formulation in general. Their foundations have been discussed by schools of mathematicians and logicians as can be observed in the work of Whitehead-Russel [22] and Gödel [11] and so on. These contributions seem to us to be preparing a new foundation for automation theories which are urgently reguired to be established in the realm of information science.

Turning to inductive logics, it is also one of the most remarkable features of this century that statistical science has become one of the key sciences influencing all scientific and technical approaches in the present days. It was in the last half-century when estimation theory and testing hypothesis theory were settled down in subtle form due to the remarkable contributions of R. A. Fisher  $[7]\sim[9]$ , J. Neyman [18], J. Neyman and E. S. Pearson [17] and A. Wald [21] and so on.

Now regarding to the area of abductive approaches, the picture becomes suddenly to be different from the two former areas just mentioned. Indeed we have to enter into uncultivated land from rich and fruitful land of deduction and induction. We are now so much conscious of the fact that the third principle  $(\alpha_3)$  have not yet developed in contrast with the former two  $(\alpha_1)$  and  $(\alpha_2)$ . Because of this fact it is now urgently being required to have a systematic theory of abduction, in order to be well prepared with heuristics.

[2] Ad ( $\beta$ ). The meaning of direction is so broad that it contains three notions of ( $\beta_1$ ) control, ( $\beta_2$ ) eizon, and ( $\beta_3$ ) creation, because each of all these three notions refer to some aspect of doing actions on objective world. The characteristic aspect of ( $\beta_1$ ) control is that it aims to attain at its prescribed goal by performing an action chosen from a set of actions.

The characteristic aspect of  $(\beta_3)$  creation comes from our subjective attitude in which we shall creat a new set of actions in view of informations obtained and which naturally may require a new organization and a new coordinate principle of informations.

Besides these two fundamental notions of  $(\beta_1)$  direction and  $(\beta_3)$  creation there comes another one coined as "eizon" which seems to be indispensable in formulating all the essential aspects of direction. The terminology "eizon" is a new Japanese word which was first introduced by the author in his monograph [15], 1969, October. The word "eizon" is a combination of two Chinease characters used in Japanese pronounciation, "ei" and "son (=zon)", where "ei" comes from the current Japanese word "keiei" corresponding to "management" in English, while "zon" comes from the sophiscated notion "jitsuzon" in Japanese which has been introduced to translate existence in the sense of existentialims in the French school of philosophy. The reason why we introduce the direction aspect of "eizon" comes from the real situations where our direction aspect can not and should not be formulated by either of control and creation aspects. In the first place in these cases we can and should have neither any definitely prescribed single aim nor a prescribed set of actions from which we can choose a suitable subset and/or a subsequence of actions for our direction. This shows a sharp distinction with the current control aspect. Secondly in these cases we can not and we should not be so positive in our direction aspect as to creat any set of actions which will suit our situations. This again shows a clear distinction with the so called creation aspect. In fact we are essetially concerned with the feasibility of the existence, and we can not and we should not be adhering to any unchangeable aim for our direction all of which are essentially subordinate to the feasibility of the existence itself. The supreme coordination principle for management should be also concerned with the feasibility of existence.

[3] Ad ( $\gamma$ ). It is one of the essential aspects of information in virtue of which evaluation will be performed so that data can be reorganized and rearranged and hence some actions can be recommended while other ones may or must be rejected. There are three notions of evaluations. First of all the notion of ( $\gamma_1$ ) efficiency can be introduced in some instances in terms of numerical evaluations of candidate actions which can be enunciated objectively and by which actions with higher efficiencies can be recommended. The second notion ( $\gamma_2$ ) reliability is different from that of efficiency because the former has a broader meaning which is not necessarily numerical notion.

It is different from efficiency which is referred with respect to an evaluation of operations having a certain definite goal of actions. In the case of reliability it is our concern to maintain an existence of the system as a whole, not so particularly being concerned with the performance efficiency of specific operation.

Finally the notion of  $(\gamma_3)$  plasticity refers to a principle of subjective attitude according to which we shall and we can develope creativeness and which is essential to promotion of creativeness.

Our explanation of these nine notions  $(\alpha_i)$ ,  $(\beta_j)$  and  $(\gamma_k)$  (i=1, 2, 3, j=1, 2, 3 and k=1, 2, 3) may not be entirely clear to the reader. It seems indeed to the author that these nine notions should be put into the third coordinate system in order to have a clearer description of their situations. We are appealing to some sort of relativistic logic of mutual specification which the author [13] proposed in 1963.

# 4. The third coordination

The object of this section is to show the third coordination by picking up and rearranging the notions which we have already explained in the first and the second coordinations. In the third coordination we can abduct three principal aspects of our subjective attitudes:  $(III_1)$  control aspect;  $(III_2)$  evolution aspect;  $(III_3)$  creation aspect. Let us explain how they work.

In what follows we use the abbreviations:

 $(I_1)$  O = Objectivity;  $(I_2)$  S = Subjectivity;  $(I_3)$  P = Practices.

 $(II_1) C = Cognition;$   $(II_2) D = Direction;$   $(II_3) E = Evaluation.$ 

Ad (III<sub>1</sub>) CONTROL ASPECT.

This aspect can be described by means of six notions chosen from two coordinations in the following way.

 $\begin{array}{c|cccc} (I_1) & O \longrightarrow & (a_1) \text{ pattern} & (\alpha_1) \text{ deduction} & \longrightarrow C & (II_1) \\ (I_2) & S \longrightarrow & (b_1) \text{ operation} & (\alpha_2) \text{ control} & \longrightarrow D & (II_2) \\ (I_3) & P \longrightarrow & (c_1) \text{ optimalization} & (\alpha_3) \text{ efficiency} & \longrightarrow E & (II_3) \end{array}$ 

Control aspect in the present implication is somewhat narrower than it is now being used in the modern control theory. Here we adhere to somewhat more restricted sense. The examples of the areas in which control aspect is emphasised can be given from those connected with some of operation research, mathematical programming, and certain econometric approaches in economical planning and so on. Throughout these examples we can concieve that the existence of pattern, operation and criterium is essential for control aspect under which control based upon deductive recognition with reference to efficiency notion can be formulated. Our guiding principle is indeed an optimalization.

Ad (III<sub>2</sub>) EIZON ASPECT.

This aspect can be described by means of six notions chosen from two coordinations in the following way.

| (III <sub>2</sub> ) Eizon Asp | ) E | izon . | Aspect |
|-------------------------------|-----|--------|--------|
|-------------------------------|-----|--------|--------|

| $(I_1) \longrightarrow$   | (a <sub>2</sub> ) chaos | $(\alpha_2)$ induction   | ← C (II <sub>1</sub> )            |
|---------------------------|-------------------------|--------------------------|-----------------------------------|
|                           | $(b_2)$ adaptation      | $(\beta_2)$ eizon        | $\leftarrow$ D (II <sub>2</sub> ) |
| $(I_3) P \longrightarrow$ | $(c_2)$ stability       | $(\gamma_2)$ reliability | ← E (II <sub>3</sub> )            |

In order to understand eizon aspect it is indispensable to consider the implications of evolutions in biological existences. Evolutions in biological existences are neither results which have been obtained by any control, nor consequences which have been gained by any design. It is to be noted that, contrary to control, evolution has been realised, without appealing to any choice of pattern criteria and without using any set of optimalization principles. This fact is refleced in the original theory of evolution due to Darwin.

Indeed it is not accidental that statistics has been developed along the line of evolution theory in which we can trace the links connecting an outstanding scholar group of Darwin, Gaton, Pearson and Fisher.

Ad (III<sub>3</sub>) CREATION ASPECT.

Similar approach to those of  $(III_1)$  and  $(III_2)$  can be applied in the last aspect.

| $(I_1) \rightarrow$         | $(a_3)$ trnsformation      | $(\alpha_{s})$ abduction | $\leftarrow$ C (II <sub>1</sub> ) |
|-----------------------------|----------------------------|--------------------------|-----------------------------------|
| $(I_2) S \longrightarrow$   | (b <sub>3</sub> ) strategy | $(\beta_3)$ creation     | $\leftarrow$ D (II <sub>2</sub> ) |
| $(I_{a}) P \longrightarrow$ | (c <sub>3</sub> ) learning | $(\gamma_3)$ plasticity  | ← E (II₃)                         |

#### (III<sub>3</sub>) Creation Aspect

The creation aspect is the aspect in which our subjective attitude of creativeness is emphasised. Such an attitude is useful when we are placed in the situations to which control aspect can not be fully applied, because of the lack of information, and when we want to get out of mercy of violence and disturbances of external world.

The subjective attitude is emphasised in this creation approach, because we shall appeal to transformation and/or strategy in our action, because we shall have to try to learn by our experiences. In performing these it will be indispensable to appeal to a new valuation system and to abduction, which implies heuristical approach for leading us to a new organization of information.

Creation aspect should be distinguished with control aspect as well as with eizon aspect. It is to be pointed out here that control aspect is not sufficient enough for dealing with large system.

The following four figures are given to illustrate a general feature of the relations among the three coordinations.

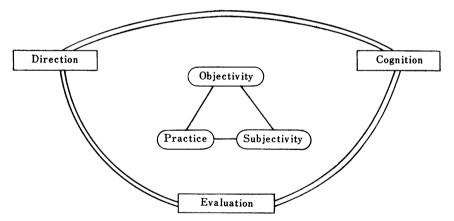


Fig. 1. The first and the second coordinate systems.

The following Table 1 is given to illustrate the constituent items of each coordinate system and the mutual relationship among three coordinations.

Since the implications of three coordinate systems may be now clearer to the reader in view of our illustrations given above, it will be adequate and convinient to coin in some way each of these three coordinate systems in comparison with other systems. In this sense let us call each of the first, the second and the third coordinate systems to be structure, function and feasibility coordinate systems re-

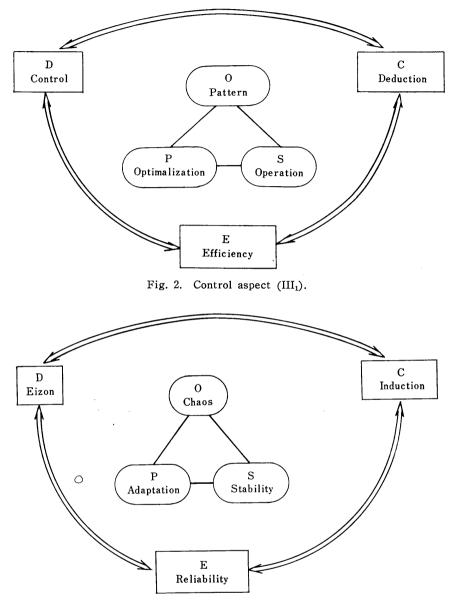


Fig. 3. Eizon aspect (III<sub>2</sub>).

spectively.

In the first place three aspects of objectivity, subjectivity and practices are concerned with a description of structure feature of the subject matters to be dealt with in information science. Therefore the first coordinate system can and will be called to be a structure coordinate system. Secondly three aspects of cognition, direction and evaluation are giving us a decomposition of various function features of information, and hence the second coordinate system can and will be called to

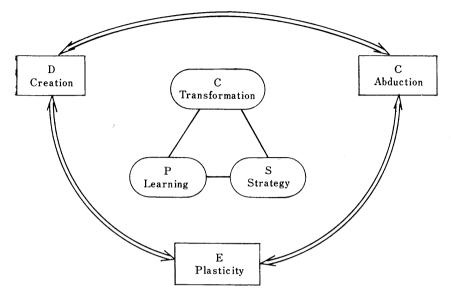


Fig. 4. Creation aspect (III<sub>3</sub>).

Table 1. THREE COORDINATE SYSTEMS

|                               | (III <sub>1</sub> ) Control      | (III <sub>2</sub> ) Eizon     | $(III_3)$ Creation               |
|-------------------------------|----------------------------------|-------------------------------|----------------------------------|
| (I1) Objectivity              | (a <sub>1</sub> ) Pattern        | (a <sub>2</sub> ) Chaos       | (a <sub>3</sub> ) Transformation |
| $(I_2)$ Subjectivity          | (b <sub>1</sub> ) Operation      | (b <sub>2</sub> ) Adaptation  | (b <sub>3</sub> ) Strategy       |
| $(I_3)$ Practices             | (c <sub>1</sub> ) Optimalization | (c <sub>2</sub> ) Stability   | (c <sub>3</sub> ) Learning       |
| (II <sub>1</sub> ) Cognition  | $(\alpha_1)$ Deduction           | $(\alpha_2)$ Induction        | $(\alpha_3)$ Abduction           |
| (II <sub>2</sub> ) Direction  | $(\beta_1)$ Control              | (β <sub>2</sub> ) Eizon       | $(\beta_3)$ Creation             |
| (II <sub>3</sub> ) Evaluation | (71) Efficiency                  | (γ <sub>2</sub> ) Reliability | $(\gamma_3)$ Plasticity          |

be a function coordinate system. Lastly the third coordinate systems, which are divided into three aspects of control, eizon and creation, are concerned with both of structure and function aspects. This implies that the third coordinate system are more fundamentally concerned with our subjective and strategic attitudes based upon our information circumstances in front of objective worlds.

Control, eizon and creation aspects in this sense are concerned with the feasibility of our existences, and hence the author prefers to call the third coordinate system as a feasibility coordination system.

#### 5. Information science in cybernetical era

So far we have referred to three coordinations by means of which our information science approach can be developed. These are more or less concerned with theoretical formulation aspects of our information science. For better understanding

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of our theoretical standpoints it will be adequate and in fact should be required to add the following remarks.

[1] THE RÔLE AND THE SITUATION OF INFORMATION SCIENCE

Information science aims at being a unified scientific approach integrating various phenomena connected with information; that is, production of information, transmission of information, storage of information, reduction of information, pattern recognition due to information, retrieval of information, various operational uses of information and so on.

These information phnomena can be found in biological existences, social lives of human beings, as well as in machines manufactured by human beings.

In view of recent advances in theoretical achievements and experimental works as well as in technical innovations, the following Figure 5 illustrating the rôle of information science in connecting various sciences and humanities by mutual relations of inputs and outputs can be readily understood as being justified as a matter of fact and as a predictor showing us suggestions for future development areas. which can be realised by strengthing their connections and mutual dependences.

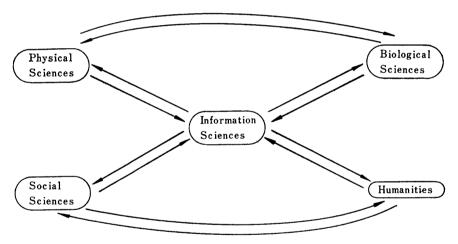


Fig. 5. Interrelationship among sciences and humanities.

#### [2] THE COOPERATION OF THREE APPROACHES IN INFORMATION SCIENCES

Here we should emphasise that it is indispensable for development of information science to have an intimate cooperation among three different approaches, namely, theoretical, experimental and technological approaches. In current methodology of sciences a majority of scholars are prepared to point out the needs for establishing cooperation between theory and experiment because of the facts that their mutual interdependences are real pictures of developments of science. Any theory can be actually invented with ultimate reference to experiment, and any experiment can be fruitfully performed under the quidance of theory. This is of course true. However it ought to be added that, in order to perform experiment, we are relying upon technological development and that, in order to have model building in theory, we are also due to the feasibility of technologies to realise model in actual existences at least approximately.

Biological existences had been with human beings before any civilization were being established. However it is quite a recent phenomenon that human beings can deal with biological information phenomena from the standpoint of information science, thanks to the technological developments of computers and information processing apparatas and systems in the last ten years. Before these technological innovations none could endeavour to have a suitable set of hypotheses to be tested experimentally. Not only for studies of biological existences but also for those of social sciences and humanities, the developments of computers are now showing the possibilities of being capable to have a systimatic approach to these areas, by inventing their adequate mechanical languages as suggested by such authors as Gorn [10].

[3] THE IMPLICATION OF SYSTEM APPROACHES

Besides logical features of system approaches based upon information science which we have explained in the previous four sections, it may be suggestive to note that there are three stages of approaches to each of which information science approach will play an important rôle. These three stages are (i) operation, (ii) system and (iii) organization stages. In discussing biological existences and medical aspects of human beings these three stages ought to be considered. In the third organization stage it is indispensable to take into our formulation human effects in dealing with our problems.

It is also to be noted that it seems to be indispensable to discuss large system before we can establish any scientific formulation of organization.

[4] CYBERNATION AND HUMANISM

In traditional physical science approaches in which matter and energy have been their chief concern without any reference to information, personification aspects have been entirely rejected. Now regarding our formulation of information sciences in which three different coordination including twenty seven concepts are employed, the situations seem to be different from physical science approaches, because some of these twenty seven concepts are so much engaged and being involved with behavioural and human features of living existences and with human features.

Neverthless it should be most clearly claimed that any scientific approach can not rely upon any vague notion of personification. Although these twenty seven concepts originate from human concept formulations, it is indipensable that each one of these concepts should be formulated to satisfy the following three conditions in order to be our object in information science approach. The three conditions are as follows:

(1°) Its model should be described in objective formulation.

(2°) Any scientific hypothesis regarding it can be tested by experiments.

 $(3^{\circ})$  Any theoretical model can be realized by means of technological achievements of the present times.

These three coorditions are strict enough to establish any scientific approach.

but it is also to be observed that the scope and the depth of areas satisfying these conditions are not fixed forever but they are subject to developments of theories, experimental techniques and techological innovations. For example, let us take a notion of learning. It is rather a recent event that human beings can have any theory of learning, thanks to the developments in mathematical psychology and in learning control theory. However we ought to recognize that these recent developments do not cover all aspects of learning which we have in mind. It is something like a mathematical limit to which we can approach as near as possible but at which we shall never attain at any actual circumstance. Therefore the notion of learning is a guiding principle without which any theory can neither be formulated nor developed.

#### Literature

- BEER, S.: Cybernetics and Operational Research, Progress in Operations Research, Vol.
  John Wiley, 1961.
- [2] BELLMAN, R.: Dynamic Programming, Princeton University Press, 1957.
- [3] BELLMAN, R.: Adaptive Control Processes, A Guided Tour, Princeton University Press, 1961.
- [4] Box, G. E. P.: Evolutionary operations: a method for increasing industrial productivity, Appl. Statist., 6 (1957), 3-23.
- [5] CHURCHMAN, C. W., ACKOFF, R. L. and ARNOFF, E. L.: Introduction to Operations Research, John Wiley, 1957.
- [6] DANTZIG, VON G.: Linear Programming and Extensions, Princeton University Press, 1963.
- [7] FISHER, R.A.: The Design of Experiments, Oliver and Boyd, (1st ed.) 1935.
- [8] FISHER, R.A.: Statistical method and Scientific Inferences, London, Olives and Boyd, 1959.
- [9] FISHER, R.A.: Statistical method and science induction, J. Roy. Statist. Soc. Ser. B, 17 (1955), 69-78.
- [10] GORN, S.: The computer and information sciences and the community of disciplines, Behavioral Science, 2 (1967), 433-452.
- [11] GÖDEL, K.: Uber formal unentscheidbare Satze der Principia Mathematica und verwandter System, I, Monatsh. f. Math. Phys., 38 (1931), 173-198.
- [12] KITAGAWA, T.: The Logical aspects of successive processes of statistical inferences and controls, Bull. Inst. Internat. Statist., 38 (1961), 151-164.
- [13] KITAGAWA, T.: The relativistic logic of mutual specificiation in statistics, Mem. Fac. Sci. Kyushu Univ., Ser. A, 17 (1963), 76-105.
- KITAGAWA, T.: Information science and its connection with statistics, Proc. of the Fifth. Berkeley Symposium on Math. Statistics and Probability, 1 (1967), 491-530.
- [15] KIHAGAWA, T.: Logic of information science (in Japanese), Kôdansha Contemporary Series 200, Kôdansha, 1969.
- [16] MORSE, P. M. and KIMBALL, G. E.: Methods of Operations Research, M. I. T., Technology Press and John Wiley, 1951.
- [17] NEYMAN, J. aud PEARSON, E.S.: The testing of statistical hypotheses in relation to probabilities a priore, Proc. Cambridge Philos. Soc., 29 (1938), 492-510.
- [18] NEYMAN, J.: On the problem of the most efficient test of statistical hypotheses, Philos. Trans. Roy. Soc. Ser., A, 23 (1933), 289-337.
- [19] NEUMANN, VON J. and MORGENSTERN, O.: Theory of games and economic behavior, Princeton Univ. Press (1953).

- [20] PEARSON, K.: The Grammar of Science, Oxford University Press, 1896.
- [21] WALD, A.: Statistical Decision Functions, New York, Wiley, 1950.
- [22] WHITEHEAD, A. N. and RUSSEL, B.: Principia mathematica, Cambridge University Press (1st ed.) 1910-13; (2nd ed.) 1925-27.
- [23] WIENER, N.: Cybernetics—Control and Communication in the Animal and the Machine, New York, Wiley, 1948 (1st ed.); New York, M.I.T. Press, 1961 (2nd ed.).