STABLE CONFIGURATION UNDER LOCAL MAJORITY TRANSFORMATIONS ON CELL SPACE : INFORMATION SCIENCE APPROACH TO BIOMATHEMATICS, V

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STABLE CONFIGURATION UNDER LOCAL MAJORITY TRANSFORMATIONS ON CELL SPACE —INFORMATION SCIENCE APPROACH TO BIOMATHEMATICS, V

By

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§ 1. Introduction

In his recent paper [1], Kitagawa has suggested an urgent need for building up a new mathematical model which should reflect some essential features of biomathematics. As the first step consideration to reply to this need, we have introduced in our previous paper Kitagawa and Yamaguchi [2] several notions such as cell space, local mapping transformation satisfying the principle of local majority, stable configurations, inhibitation state ϕ and etc. The cell space defined there is a finite, two-demensional rectangular array of cell automata, where each cell is assumed to be an identical square cell. These notions in such a cell space formulation have been generalized and deeply discussed in Kitagawa [4]. The author of the present paper introduced in her paper Yamaguchi [3] the stability index for stable configurations defined in our paper [2].

The purpose of this paper is to investigate several characteristic structural properties of stable configurations in a cell space consisting of triangle unit cells which has been introduced by Kitagawa [4]. We shall call such a cell space a triangular cell space. The basic definitions and notations introduced in the previous papers [2] and [4] will be used here also.

In SECTION 2 we shall give some basic definitions in our triangular cell space such as triangular basic cell space and local majority transformation in that space.

In SECTION 3 we are concerned with construction of stable configurations under our local majority transformation in our triangular cell space. For this purpose we shall introduce a notion of determinative cell subspace. This notion is shown to be crucial for our construction of any stable configuration as well as for our decision whether any given configuration in a triangular cell space is stable or not. We have incidentally reached this notion through our proof of our THEOREM 1 which asserts that the number of all the possible stable configurations in a $\Delta^{(n)}$ cell space is equal to $2^{3(n-1)}$.

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In SECTION 4 we shall give several examples in order to illustrate our main result obrained in SECTION 3. These examples yields us a preparatory consideration to the proof of THEOREM 2 to be given in SECTION 5.

The purpose of SECTION 5 is to discuss a structural feature of stable configurations in a $\Delta^{(n)}$ cell space. THEOREM 2 amounts to assert that any stable configuration in a $\Delta^{(n)}$ cell space can be decomposed into a superposition of certain elementary stable configurations each of which corresponds to their respective simple configuration assigned in a certain common determinative subspace.

It is also possible to define and discuss a stability index of stable configuration in our triangular cell space quite similarly as in our previous paper [3].

Fourthermore it is interesting to consider stochastic transition phenomena of configurations in our triangular cell space by introducing a certain probability scheme of firing points, as we have done in our joint paper [2].

§ 2. Definitions

Kitagawa [4] generalized the notion of stable configurations to cell space whose unit cells are either triangles or hexagons. In the present paper we shall confine ourselves with cell space consisting of triangular unit cells, which is called a triangular cell space. In what follows let us consider a triangular cell space which has the form of large regular triangle in itself with n cells in each boundary side. We shall denote it by $\Delta^{(n)}$ (triangular) cell space. A notation for illustrating the location of each unit triangular cell in such a $\Delta^{(n)}$ cell space is indispensable for our consideration. A set of the locations of $2(n-i)+1$ cells in the i-th row from the bottom in $\Delta^{(n)}$ cell space is represented by the vector

$$((i, 1), (i, 2), (i, 3), \cdots, (i, 2(n-i)+1)) \tag{2.1}$$

for $i=1, 2, \cdots, n$. We shall call this a cell coordination in $\Delta^{(n)}$ cell space.

EXAMPLE 2.1. A cell coordination in $\Delta^{(4)}$ is shown in the following Figure 2.1.

In our previous paper [2] dealing with an $m\times n$ square cell space, a basic square

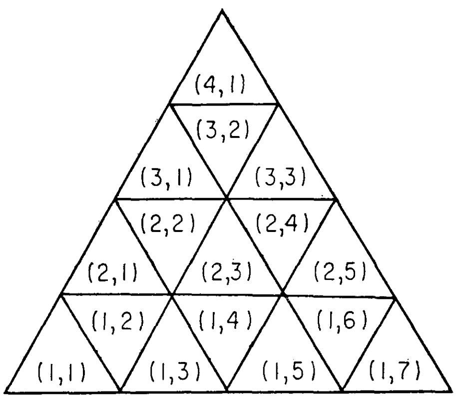

Fig. 2.1. Cell coordination in $\Delta^{(4)}$.

cell space was defined as a 2×2 cell space. Now in our present $\Delta^{(n)}$ triangular cell space, we shall define every $\Delta^{(2)}$ cell space as a basic cell space. In what follows we shall call it a $\Delta^{(2)}$ basic (cell) space. A few examples in $\Delta^{(4)}$ are shown in Figure 2.2 (a), (b), (c) and (d). Generally speaking there are two kinds of $\Delta^{(2)}$ basic cell

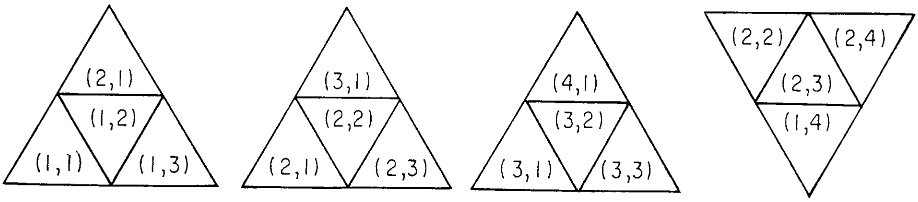

Fig. 2.2. Examples of $\Delta^{(2)}$ basic space in $\Delta^{(4)}$.

spaces with reference to our consideration in the $\Delta^{(n)}$ triangular cell space as shown in Figure 2.3, (a) and (b). However in the following discussions in this paper there is seldom any need for distingushing one of these two $\Delta^{(2)}$ basic cell spaces with each other, and we shall denote both of them simply by $\Delta^{(2)}_{i,j}$.

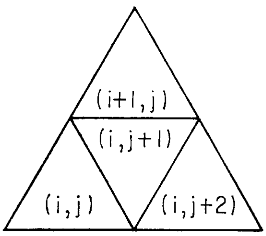

Fig. 2.3, (a). $\overline{\Delta}^{(2)}_{ij}$.

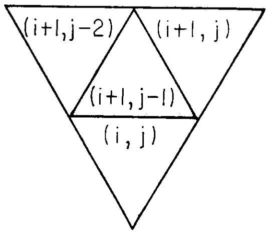

Fig. 2.3, (b). $\underline{\Delta}^{(2)}_{ij}$.

Now regarding the possible states of each cell in $\Delta^{(n)}$ cell space we shall assume that there are exactly two alternatives, which are denoted by 1 and 0 respectively. An assignment of each state x_{ij} to each (i, j) cell in $\Delta^{(n)}$ cell space is called a configuration in the $\Delta^{(n)}$ cell space, and will be denoted by $\Delta^{(n)}(X)$. Specially when all x_{ij} in $\Delta^{(n)}(X)$ are equal to 1, we shall denote the configuration by $\mathrm{I}^{(n)}$. Similary when all x_{ij} in $\Delta^{(n)}(X)$ are equal to 0, we shall denote by $\mathrm{O}^{(n)}$.

Now let us consider $\Delta^{(2)}(X)$ and $\Delta^{(2)}(Y)$ which are defined over the same $\Delta^{(2)}$ basic space. Now let us define a local majority transformation (LMT) in our $\Delta^{(n)}$ cell space as we have done for an $m\times n$ square cell space in our previous paper [2].

DEFINITION 2.1. *A local mapping transformation defined for a $\Delta^{(2)}$ basic space* LT : $\Delta^{(2)}(X)\rightarrow\Delta^{(2)}(Y)$ *is said to satisfy the principle of local majority and will be simply called a local majority transformation* (LMT), *if*

$$\text{(2.2)}\qquad \mathrm{LMT}:\Delta^{(2)}_{ij}(X)\longrightarrow\Delta^{(2)}_{ij}(Y)=\begin{cases}\mathrm{I}^{(2)} & \text{if } S_{ij}(X)>2\\ \Delta_{ij}(X) & \text{if } S_{ij}(X)=2\\ \mathrm{O}^{(2)} & \text{if } S_{ij}(X)<2,\end{cases}$$

where we have put, for $\bar{\Delta}^{(2)}_{ij}(X)$, (2.3) $$S_{ij}(X)\equiv x_{ij}+x_{i,j+1}+x_{i,j+2}+x_{i+1,j}$$

and, for $\Delta_{ij}^{(2)}(X)$

(2.4) $$S_{ij}(X)\equiv x_{ij}+x_{i+1,j-2}+x_{i+1,j-1}+x_{i+1,j}\,.$$

Now let us introduce

DEFINITION 2.2. *A configuration* $\Delta^{(n)}(X)$ *in a* $\Delta^{(n)}$ *cell space is said to be stable if it is invariant under any application of* LMT *with reference to any* $\Delta^{(2)}$ *basic cell space located in the* $\Delta^{(n)}$ *cell space.*

§ 3. Construction of stable configurations

Before we shall give a systematic proceduce for constructing stable configurations, let us solve the problem how many stable configurations are possible in a $\Delta^{(n)}$ cell space. By induction we observe the following fundamental

THEOREM 1. *The number of all the possible stable configurations under* LMT *in a* $\Delta^{(n)}$ *cell space is* $2^{3(n-1)}$ *for* $n\geqq 2$.

PROOF. The proof is given by induction regarding n. For $n=2$, the cell space $\Delta^{(2)}$ is equivalent to a basic cell space consisting of four triangle cells. In order that $\Delta^{(2)}$ cell space is stable, the states of any three cells in $\Delta^{(2)}$ can be arbitrarily given and then the state of one remaining cell has be be uniquely determined. Hence for $n=2$, we have just 2^3 possible stable configurations, which shows the validity of the assertion to Theorem because $2^3=2^{3(2-1)}$.

Now let us assume that our assertion is valid for all $n\leqq m$. Let us construct a stable configuration in a $\Delta^{(m+1)}$ cell space by adding to a fixed configuration in $\Delta^{(m)}$ cell space a configuration in the cell subspace defined as an assemble of cells

(3.1) $$\begin{gathered}\{(1,2m+1),(1,2m),(2,2m-1),(2,2m-2),\\ \cdots,(i,2(m+1-i)+1),(i,2(m+1-i)),\cdots\\ (m-1,5),(m-1,4),(m,3),(m,2),(m+1,1)\}\,,\end{gathered}$$

according to our coordination introduced in SECTION 2. The situation is illustrated in Figure 3.1.

First of all it is noted that any stable configuration in $\Delta^{(m+1)}$ should be a stable configuration in $\Delta^{(m)}$. We are now searching for all the possible stable configurations in $\Delta^{(m+1)}$ by adding the assemble states of (3.1) to each of different stable configurations in the $\Delta^{(m)}$ cell space whose total number is $2^{3(m-1)}$ according to our assumption of induction. Secondly the state of the cell $(2,2m-2)$ is uniquely determined by considering the $\Delta^{(2)}$ basic cell space consisting of the cells $(1,2m-2)$, $(2,2m-3)$, $(2,2m-4)$ and our new cell $(2,2\mathrm{m}-2)$, because it should be stable. Similarly each state of all the cells $(i,2(m+1-i)$ $(=2,3,\cdots,m-1)$ is uniquely determined. Now let each one state of two cells $(1,2m)$ and $(1,2m+1)$ be arbitrarily chosen. Then all the states in $(i,2(m+1-i)+1)$ $(i=2,3,\cdots,m)$ are uniquely determined, and there are two cells whose states ars still undetermined, that is, the cells $(m,2)$ and $(m+1,1)$. By choosing arbitrarily a state of the cell $(m+1,1)$, for instance, the remaining state

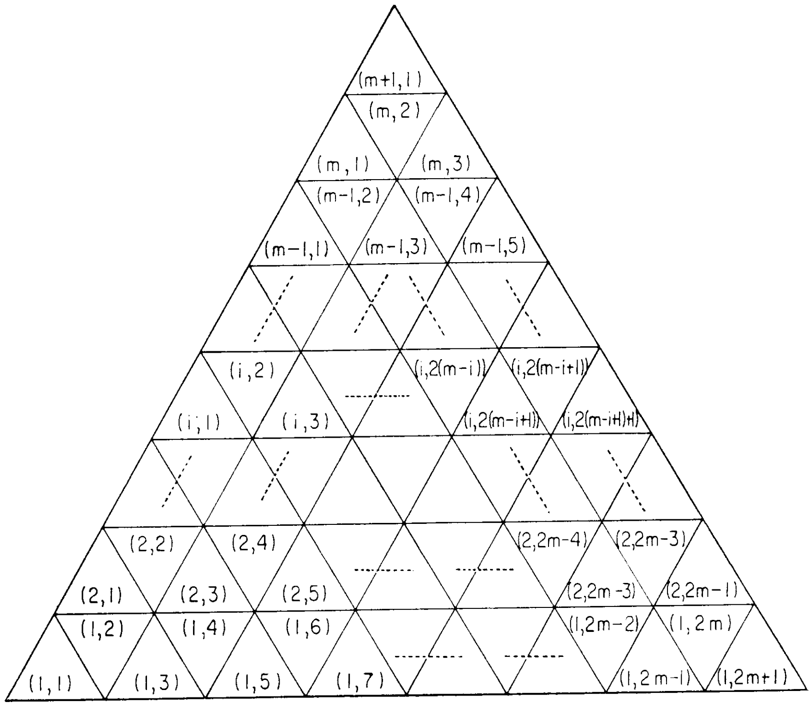

Fig. 3.1. An extension of $\Delta^{(m)}$ cell spacə into $\Delta^{(m+1)}$ cell space by adding the subspace (3.1).

in the cell $(m, 2)$ is now uniquely determined. Therefore we find there is just 2^3 stable configurations in $\Delta^{(m+1)}$ cell space to each assigned stable configuration in $\Delta^{(m)}$ cell space. Consequently the number of all the possible stable configurations in $\Delta^{(m+1)}$ cell space is equal to $2^{3(m-1)}\times 2^3 = 2^{3((m+1)-1)}$, which are all different with each other, completing our proof by induction.

Incidently the proof to THEOREM 1 has shown that any stable configuration in a $\Delta^{(n)}$ cell space can be obtained in a uniquely way by assigning the states of suitably chosen $3(n-1)$ triangular cells contained in the $\Delta^{(n)}$ cell space. In this connection it is crucial to introduce

DEFINITION 3.1. *A set* D *consisting of* $3(n-1)$ *triangular cells in a* $\Delta^{(n)}$ *cell space is said to be determinative cell subspace in a* $\Delta^{(n)}$ *cell space if it satisfies the following two conditions:*

(1°) *To every configuration in the set* D *there corresponds one and only one stable configuration in the* $\Delta^{(n)}$ *cell space whose restriction in the set* D *is coincident with the given configuration in the set* D.

(2°) *Any stable configuration in the* $\Delta^{(n)}$ *cell space is uniquely determined by its restriction in the set* D.

There are many examples of determinative cell subspaces. We shall explain some of them in connection with boundary value problems in stable configurations.

EXAMPLE 3.1. Let us unsider the set $D_0^{(n)}$ in a $\Delta^{(n)}$ cell space defined by

$$\mathrm{D}_0^{(n)}=\begin{pmatrix}(1,1),(1,2),(1,3),\cdots,(1,2n-2),(1,2n-1)\\(2,2),(3,2),(4,2),\cdots,(n-1,2)\end{pmatrix}$$

as shown with hatching in Figure 3.2. It can be readily observed that $\mathrm{D}_0^{(n)}$ is a determinative cell subspace.

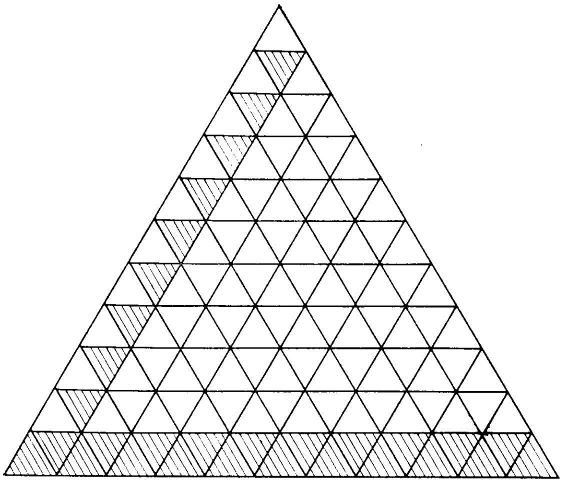

Fig. 3.2. $\mathrm{D}_0^{(n)}$ as a determinative subspace.

EXAMPLE 3.2. In his recent contribution Kitagawa [4] discussed determinism and nondeterminism in a $\varDelta^{(n)}$ cell space, (see Kitagawa [4] 4. 5. 3). He explained his idea with reference to an extension of stable configuration in $\varDelta^{(7)}$ cell space into stable configuration in $\varDelta^{(10)}$. His idea is applicable to an extension of stable configuration in $\varDelta^{(n)}$ cell space into stable configuration in $\varDelta^{(n+3)}$. We can explain his idea with reference to our coordinate system introduced in this paper, and we shall get a determinative cell subspace which is characteristic in our triangular cell space.

§ 4. Examples of stable configuration in $\varDelta^{(n)}$ cell space with reference to configurations in the determinative subspace $\mathrm{D}_0^{(n)}$

Let us denote any configuration in the determinative subspace $\mathrm{D}_0^{(n)}$ by

(4.1) $$\mathrm{D}_0^{(n)}(X)=\begin{pmatrix}x_{11}x_{12}x_{13}\cdots x_{1,2n-2}x_{1,2n-1}\\x_{22}x_{32}x_{42}\cdots x_{n-1,2}\end{pmatrix}.$$

The following examples are special cases of $\mathrm{D}_0^{(n)}(X)$. Since the set $\mathrm{D}_0^{(n)}$ is a determinative subspace in $\varDelta^{(n)}$, there exists one and only stable configuration in the $\varDelta^{(n)}$ space whose restriction in the subspace $\mathrm{D}_0^{(n)}$ is coincident with $\mathrm{D}_0^{(n)}(X)$.

EXAMPLE 4.1. Let us consider the configuration $\mathrm{D}_0^{(n)}(X)$ in which all x's are equal to 0, namely

(4.1) $$\mathrm{D}_0^{(n)}(X)=\begin{pmatrix}0&0&0\cdots0&0\\0&0&0\cdots0&\end{pmatrix}.$$

Then it is evident that the stable configuration determined by (4.1) is $O^{(n)}$, that is, the configuration in which all cells in the $\Delta^{(n)}$ cell space have the state 0.

EXAMPLE 4.2. Let us consider the configuration $D_0^{(n)}(X)$ in which $x_{11}=1$ and all other x's are equal to 0, namely

$$D_0^{(n)}(X)=\begin{pmatrix} 1 & 0 & 0\cdots 0 & 0 \\ 0 & 0 & 0\cdots 0 & \end{pmatrix} \tag{4.2}$$

$$= D_0^{(n)}(1, 1), \text{ say}.$$

The stable configuration $\{y_{ij}\}$ determined by (4.2) is given

$$y_{ij}=\begin{cases} 1 & \text{if } (i, j)\in E_0^{(n)}(1, 1) \\ 0 & \text{if } (i, j)\notin E_0^{(n)}(1, 1), \end{cases} \tag{4.3}$$

where the set of cells $E_0^{(n)}(1, 1)$ is defined by

$$E_0^{(n)}(1, 1)=\{(i, 1)\,;\ i=1, 2, \cdots, n\}. \tag{4.4}$$

Let us denots by $C(E_0^{(n)}(1, 1))$ this stable configuration. See the configuration $C(E_0^{(13)}(1, 1))$ in Figure 4.1, (a).

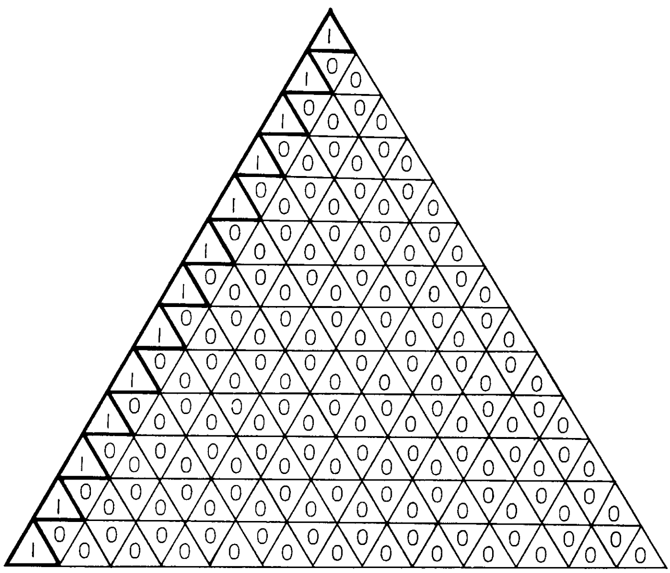

Fig. 4.1, (a). Elementary stable configuration $C(E_0^{(13)}(1, 1))$.

EXAMPLE 4.3. Let us consider the configuration $D_0^{(n)}(X)$ in which $x_{12}=1$ and all other x's are equal to 0, namely

$$D_0^{(n)}(X)=\begin{pmatrix} 0 & 1 & 0\cdots 0 & 0 \\ 0 & 0 & 0\cdots 0 & \end{pmatrix} \tag{4.5}$$

$$= D_0^{(n)}(1, 2), \text{ say}.$$

The stable configuration $\{y_{ij}\}$ determined by (4.5) is given by

$$y_{ij} = \begin{cases} 1 & \text{if } (i, j) \in \mathrm{E}_0^{(n)}(1, 2) \\ 0 & \text{if } (\mathrm{i}, i) \notin \mathrm{E}_0^{(n)}(1, 2), \end{cases}$$

where the set of cells $\mathrm{E}_0^{(n)}(1, 2)$ is defined by

(4.7) $$\mathrm{E}_0^{(n)}(1, 2) = \{(i, 1)\,;\ i = 2, 3, \cdots, n\}\,.$$

Let us denote by $C(\mathrm{E}_0^{(n)}(1, 2))$ this stable configuration. See the configuration $C(\mathrm{E}_0^{(13)}(1, 2))$ in Figure 4.1, (b).

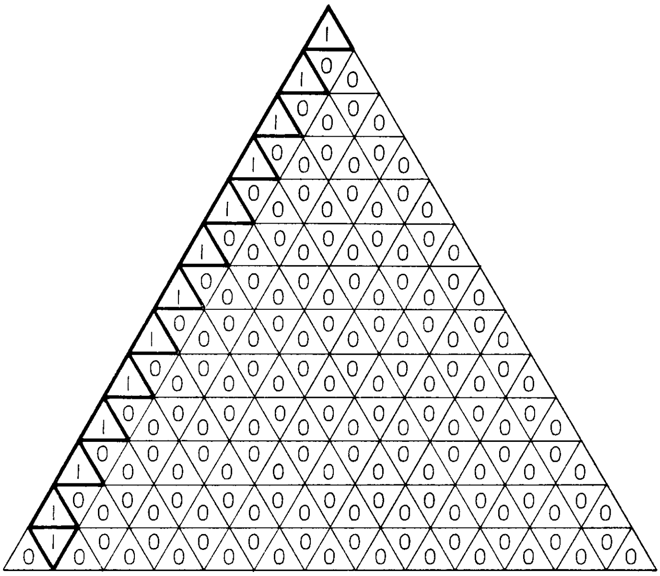

Fig. 4.1, (b). Elementary stable configuration $C(\mathrm{E}_0^{(13)}(1, 2))$.

EXAMPLE 4.4. Let us consider the configuration $\mathrm{D}_0^{(n)}(X)$ in which $x_{l2} = 1$ $(1 < l < n)$ and all other x's are equal to 0, namely

(4.8) $$\mathrm{D}_0^{(n)}(X) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 \cdots 0 & \underset{(l)}{1} & 0 \cdots 0 \end{pmatrix}$$

$$= \mathrm{D}_0^{(n)}(l, 2), \text{ say}\,.$$

The stable configuration $\{y_{ij}\}$ determined by (4.8) is given by

(4.9) $$y_{ij} = \begin{cases} 1 & \text{if } (i, j) \in \mathrm{E}_0^{(n)}(l, 2) \\ 0 & \text{if } (i, j) \notin \mathrm{E}_0^{(n)}(l, 2), \end{cases}$$

where the set of cells $\mathrm{E}_0^{(n)}(l, 2)$ is defined by

(4.10) $$\mathrm{E}_0^{(n)}(l, 2) = \{(l, 2j)\,;\ j = 1, 2, 3, \cdots, n-l\}$$
$$\cup \{(l+1, 2j+1)\,;\ j = 0, 1, 2, \cdots, n-(l+1)\}\,.$$

Let us denote by $C(\mathrm{E}_0^{(n)}(l, 2))$ this stable configuration. See the configuration

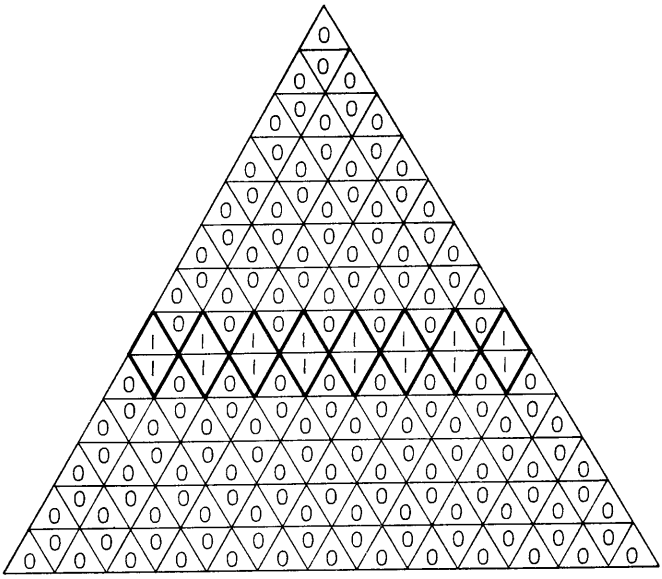

Fig. 4.1, (c). Elementary stable configuration $C(\mathrm{E}_0^{(13)}(5, 2))$.

$C(\mathrm{E}_0^{(13)}(5, 2))$ in Figure 4.1, (c).

EXAMPLE 4.5. Let us consider the configuration $\mathrm{D}^{(0)}(X)$ in which $x_{1l}=1 (2<l \leqq 2n-1)$ with a certain odd niteger and all other x's are equal to 0, namely

$$\mathrm{D}_0^{(n)}(X)=\begin{pmatrix} 0 & 0\cdots 0 & \overset{(l)}{1} & 0 & 0\cdots 0 \\ 0 & 0\cdots 0 & & & \end{pmatrix} \tag{4.11}$$

$$=\mathrm{D}_0^{(n)}(1, l), \text{ say}.$$

The stable configuration $\{y_{i,j}\}$ determined by (4.11) is given by

$$y_{ij}=\begin{cases} 1 & \text{if } (i, j)\in \mathrm{E}_0^{(n)}(1, l) \\ 0 & \text{if } (i, j)\notin \mathrm{E}_0^{(n)}(1, l), \end{cases} \tag{4.12}$$

where the set of cells $\mathrm{E}_0^{(n)}(1, l)$ is defined by

$$\begin{aligned} \mathrm{E}_0^{(n)}(1, l)= &\Big\{(i, j)\,;\, i=1, 2, \cdots, \frac{l-1}{2}, j=l-2(i-1), l-2(i-1)+1, \cdots, l\Big\} \\ &\cup\Big\{\Big(\frac{l+1}{2}, 2j+1\Big)\,;\, j=0, 1, 2, \cdots, \frac{l-1}{2}\Big\} \\ &\cup\Big\{(i, j)\,;\, i=\frac{l+3}{2}, \frac{l+3}{2}+1, \cdots, n-\frac{l+1}{2}, \\ &\qquad j=l+1, l+2, \cdots, 2(n-i)+1\Big\} \\ &\cup\Big\{\Big(\frac{l+1}{2}, 2j\Big)\,;\, j=\frac{l+1}{2}, \frac{l+1}{2}+1, \cdots, n-\frac{l+1}{2}\Big\}. \end{aligned} \tag{4.13}$$

Let us denote by $C(\mathrm{E}_0^{(n)}(1, l))$ this stable configuration. See the configuration

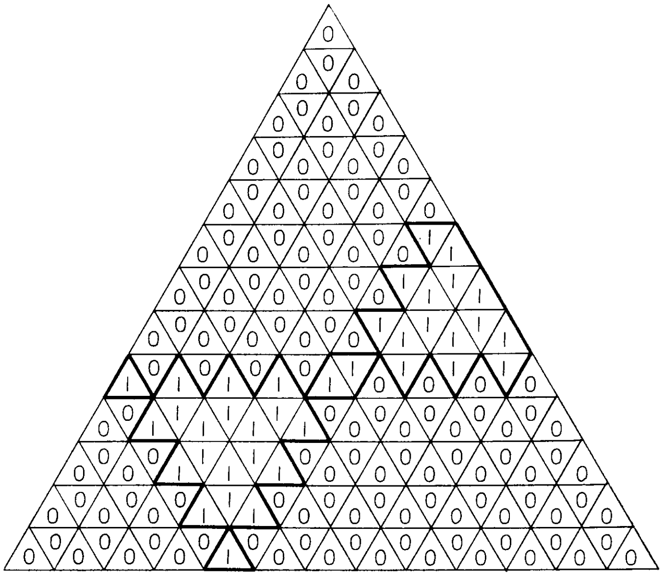

Fig. 4.1, (d). Elementary stable configuration $C(\mathrm{E}_0^{(13)}(1, 9))$.

$C(\mathrm{E}_0^{(13)}(1, 9)$ in Figure 4.1, (d).

EXAMPLE 4.6. Let us consider the configuration $\mathrm{D}_0^{(n)}(X)$ in which $x_{1l}=1$ $(2<l\leqq 2n)$ with a certain even integer l and other x's are equal to 0, namely

$$\mathrm{D}_0^{(n)}(X)=\begin{pmatrix} 0 & 0\cdots 0 & 1 & 0\cdots 0 & 0 \\ 0 & 0\cdots 0 & & & \end{pmatrix} \tag{4.14}$$

$$=\mathrm{D}_0^{(n)}(1, l), \text{ say}.$$

The stable configuration $\{y_{ij}\}$ determined by (4.14) is given by

$$y_{ij}=\begin{cases} 1 & \text{if } (i, j)\in \mathrm{E}_0^{(n)}(1, l) \\ 0 & \text{if } (i, j)\notin \mathrm{E}_0^{(n)}(1, l), \end{cases} \tag{4.15}$$

where the set of cells $\mathrm{E}_0^{(n)}(1, l)$ is defined by

$$\begin{aligned} \mathrm{E}_0^{(n)}(1, l)= &\Big\{(i, j)\,;\, i=2, 3, \cdots, \frac{l}{2}, \ j=l-2i+3,\ l-2i+4, \cdots, l-1\Big\} \\ &\cup\Big\{\Big(\frac{l}{2}-1, 2j+1\Big): j=0, 1, 2, \cdots, \frac{l-2}{2}\Big\} \\ &\cup\Big\{(i, j)\,;\ i=\frac{l}{2}+2, \frac{l}{2}+3, \cdots, n-1-\frac{l}{2}, \\ &\qquad j=l+2, l+3, \cdots, 2(n-i)+1\Big\} \\ &\cup\Big\{\Big(\frac{l}{2}-1, 2j\Big)\,;\ j=\frac{l}{2}, \frac{l}{2}+1, \cdots, n-1-\frac{l}{2}\Big\}. \end{aligned} \tag{4.16}$$

Let us denote by $C(\mathrm{E}_0^{(n)}(1, l))$ this stable configuration. See the configuration

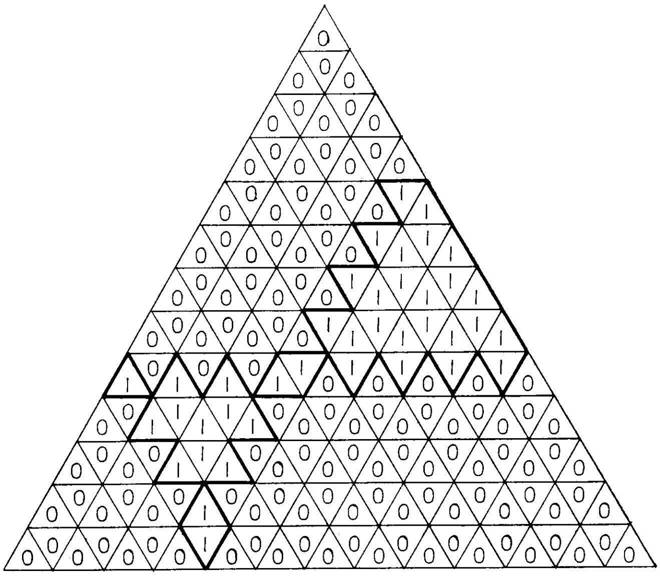

Fig. 4.1, (e). Elementary stable configuration $C(\mathrm{E}_0^{(13)}(1, 8))$.

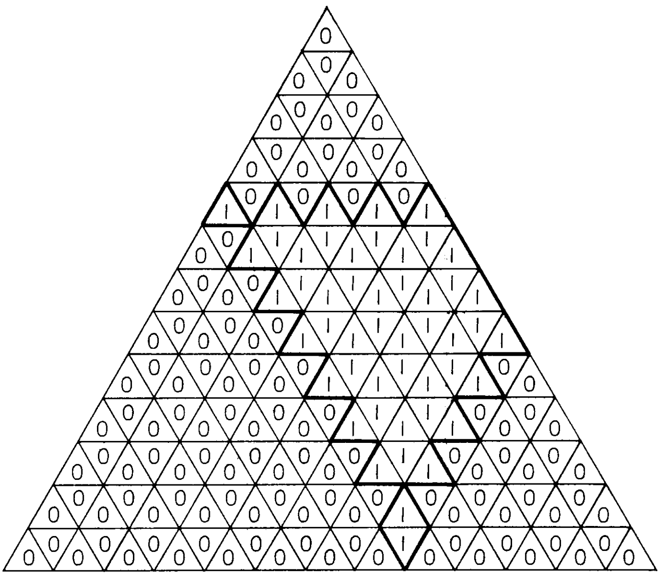

Fig. 4.1, (f). Elementary stable configuration $C(\mathrm{E}_0^{(13)}(1, 16))$.

$C(\mathrm{E}_0^{(13)}(1, 8))$ in Figure 4.1, (e) and the configuration $C(\mathrm{E}_0^{(13)}(1, 16))$ in Figure 4.1, (f).

§ 5. Structure of stable configurations in $\Delta^{(n)}$ cell space

In SECTION 4 we have introduced a set of the configurations $\{\mathrm{D}_0^{(n)}(1, i)\}$ $(i=1, 2, \cdots, 2n-1)$ and $\{\mathrm{D}_0^{(n)}(j, 2)\}$ $(j=2, 3, \cdots, n-1)$ in the determinative subspace $\mathrm{D}_0^{(n)}$. Now it can be readily observed that any configuration $\mathrm{D}_0^{(n)}(X)$ can be uniquely expressed as a boolean sum of these configurations. In fact we have

LEMMA 5.1. *We have*

$$\mathrm{D}_0^{(n)}(X)=\sum_{i=1}^{2n-1} x_{1,i}\mathrm{D}_0^{(n)}(1, i)+\sum_{j=2}^{n-1} x_{j,2}\mathrm{D}_0^{(n)}(j, 2), \tag{5.1}$$

where the additions in the right hand side are given according to Boolean algebra.

In SECTION 4 we have introduced a set of stable configurations $\{C(\mathrm{E}_0^{(n)}(1, i))\}$ $(i=1, 2, \cdots, 2n-1)$ and $\{C(\mathrm{E}_0^{(n)}(j, 2))\}$ $(j=2, 3, \cdots, n-1)$ in $\Delta^{(n)}$ cell space where each $C(\mathrm{E}_0^{(n)}(1, i))$ and each $C(\mathrm{E}_0^{(n)}(j, 2))$ are uniquely determined by each $\mathrm{D}_0^{(n)}(1, i)$ and each $\mathrm{D}_0^{(n)}(j, 2)$ respectively. Let us call these stable configurations as elementary stable configurations. By use of these elementary stable configurations we shall show that any stable configuration $C(X)$ can be expressed as a superposition of these elementary configurations. In fact we have the following fundamental

THEOREM 2. *Any stable configuration $C(X)$ in a $\Delta^{(n)}$ cell space can be expressed as a superposition of the elementary stable configurations to the effect that*

$$C(X)=\sum_{i=1}^{2n-1} x_{1,i}C(\mathrm{E}_0^{(n)}(1, i))+\sum_{j=1}^{n-1} x_{j,2}C(\mathrm{E}_0^{(n)}(j, 2)), \tag{5.2}$$

where the additions in the right hand side are given according to Boolean algebra. Conversely, for each assigned configuration $\mathrm{D}_0^{(n)}(X)$ in the determinative subspace $\mathrm{D}_0^{(n)}$, the right hand side of (5.2) *gives us a stable configuration in $\Delta^{(n)}$ cell space.*

In order to prove THEOREM 2, let us rewrite (5.1) and (5.2) in a form which is more suited to mathematical induction by introducing a new notation system to the effect that, for $i=1, 2, 3, \cdots, 2n-1$,

$$\begin{cases} u_i=x_{1,i} \\ \mathrm{F}_n(i)=\mathrm{E}_0^{(n)}(1, i) \\ \mathrm{G}_n(i)=\mathrm{D}_0^{(n)}(1, i) \end{cases} \tag{5.3}$$

and, for $i=2n+(j-2)$, $j=2, 3, \cdots, n-1$,

$$\begin{cases} u_i=x_{j,2} \\ \mathrm{F}_n(i)=\mathrm{E}_0^{(n)}(j, 2) \\ \mathrm{G}_n(i)=\mathrm{D}_0^{(n)}(j, 2). \end{cases} \tag{5.4}$$

These notation yield us

$$C(X)=\sum_{i=1}^{3(n-1)} u_i\mathrm{F}_n(i) \tag{5.5}$$

$$\mathrm{D}^{(n)}(X)=\sum_{i=1}^{3(n-1)} u_i\mathrm{G}_n(i). \tag{5.6}$$

Then the content of THEOREM 2 is equivalent to assert

LEMMA 5.2. (1°) *Any stable configuration whose restriction is given by* (5.6) *can be expressed as* (5.5).

(2°) *For any assigned vector $(u_1, u_2, \cdots, u_{3(n-1)})$ where each u_i may be either* 1 *or* 0, *the righthand side of* (5.5) *gives us a stable configuration in $\Delta^{(n)}$ cell space.*

PROOF OF LEMMA 5.2. Let us write for a moment

$$C(\Delta^{(n)};r)=\sum_{i=1}^{r}u_i\mathrm{F}_n(i) \tag{5.7}$$

$$\mathrm{D}_0^{(n)}(\Delta^{(n)};r)=\sum_{i=1}^{r}u_i\mathrm{G}_n(i) \tag{5.8}$$

for $r=1, 2, \cdots, 2(n-1)$.

Let us prove LEMMA 5.2 by induction regarding r. For $r=1$, the assertion of LEMMA 5.2 is evident.

Now let us assume that the assertion of LEMMA 5.2 is valid for $r\leqq m$. For the moment let us denote by $x_{ij}(C)$ a state of the (i, j) cell in the $\Delta^{(n)}$ cell space which is associated with a stable configuration C. Then what we have to prove is to show the Boolean equation

$$x_{ij}(C(\Delta^{(n)};r+1))=x_{ij}(C(\Delta^{(n)};r))+u_{r+1}x_{ij}(\mathrm{F}_n(r+1)), \tag{5.9}$$

where the addition in the righthand side is given by Boolean algebra.

The proof of (5.9) can be obtained by direct construction of $C(\Delta^{(n)};r+1)$ by considering the effect of $u_{r+1}\mathrm{G}_n(r+1)$ from the bottom of $C(\Delta^{(n)};r)$ to the top in a sequential way. Let us write, for the moment,

$$\begin{cases} x_{ij}(C(\Delta^{(n)};r)=x_{i,j}^{(r)} \\ x_{i,j}(C(\Delta^{(n)};r+1))=x_{i,j}^{(r+1)} \\ x_{ij}(\mathrm{F}_n(r+1))=y_{i,j}^{(r+1)}. \end{cases} \tag{5.10}$$

Let us denote the bottom situation by using these notations as in Figure 5.1, (a) and (b) which is corresponding to the case of $\mathrm{D}_0^{(n)}(1, i)$ for some even integer i.

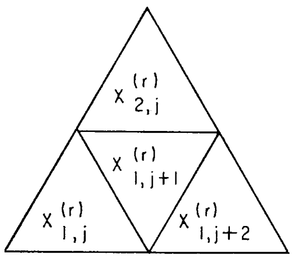

Fig. 5.1, (a). $\Delta_{i,j}^{(2)}(X)$.

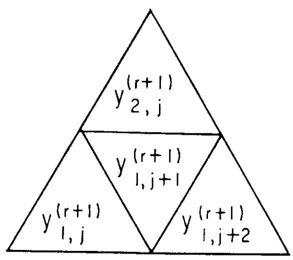

Fig. 5.1, (b). $\Delta_{i,j}^{(2)}(Y)$.

Since in this case $y_{1,j}^{(r+1)}=y_{1,j+2}^{(r+1)}=0$ and $y_{1,j+1}^{(r+1)}=1$, we have

$$\begin{cases} x_{1,j}^{(r+1)}=x_{1,j}^{(r)}, \qquad x_{1,j+2}^{(r+1)}=x_{1,j+2}^{(r)} \\ x_{1,j+1}^{(r+1)}=x_{1,j+1}^{(r)}=1-x_{1,j+1}^{(r)}, \end{cases} \tag{5.11}$$

In order that this basic cell space is stable, it thrns out

$$x_{2,j}^{(r+1)}=x_{1,j}^{(r)}=1-x_{1,j}^{(r)}. \tag{5.12}$$

The equalities given in (5.11) and (5.12) amounts to assert the validity of (5.9) for this particular basic cell. Similar observations can be applied to all the cells having the coordinate $(1, j)$ $(1\leqq j\leqq 2n-1)$. Then we can proceed to the unique determination of the state of cell having the coordinate $(2, j)$, $1 \leqq j \leqq 2n-3$, which will again show the validity of (5.9). We can proceed from the bottom of $\Delta^{(n)}$ to the top of $\Delta^{(n)}$ step by step in a sequential way. In each stage we can confirm the validity of (5.9) and we can prohongate its validity to the next higher stage by induction. This completes the proof of (5.9) for the case when $G_n(i)$ is one of $D_0^{(n)}(1, l)$ with an odd integer l. In the case when $G_n(i)$ is one of $D_0^{(n)}(j, 2)$ our determination of the stable configuration will be done step by step in a sequential way with the sole difference that it goes from the left to the right now. This completes the proof of LEMMA 5.2 and hence that of THEOREM 2.

§ 6. Acknowledgement

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Literature

[1] KITAGAWA, T.: *A Contribution to the Methodology of Biomathematics—Information Science Approach to Biomathematics*, I, Research Report of the Research Institute of the Fundamental Information Science, Fac. Sci., Kyushu Univ., No. 9 (1970).

[2] KITAGAWA, T. and YAMAGUCHI, M.: *Local Majority Transformations on Cell Space—Information Science Approach to Biomathematics*, II, Research Report of the Research Institute of the Fundamental Information Science, Fac. Sci., Kyushu Univ., No. 10 (1970). Bull. Math. Statisti, Vol. 14, No. 3/4 (1971), 61-82.

[3] YAMAGUCHI, M.: *The Stability Index of Stable Configurations under Local Mojority Transformation on Cell Space—Information Science Approach to Biomathematics*, III, Research Report of the Research Institute of the Fundamental Information Science, Fac. Sci. Kyushu Univ., No. 11 (1970). Bull. Math. Statist., Vol. 14, No. 3/4 (1971), 83-91.

[4] KITAGAWA, T.: *Prolegomena to Cell Space Approaches—Information Science Approach to Biomathematics*, IV, Research Report of the Research Institute of the Fundamental Information Science, Fac. Sci., Kyushu Univ., No. 12 (1970).