NUMERICAL TABLES OF OPTIMUM SEQUENTIAL DESIGNS BASED ON MARKOV CHAINS FOR SELECTING ONE OF TWO MEDICAL TREATMENTS

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NUMERICAL TABLES OF OPTIMUM SEQUENTIAL DESIGNS BASED ON MARKOV CHAINS FOR SELECTING ONE OF TWO MEDICAL TREATMENTS

By

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1. Introduction and principle of problem

The present problem is to determine an optimum statistical procedure in choosing one of two populations in the light of observations.

We are now faced to decide a superior medical treatment in two, on a minimum number of patients from an ethical point of view. In such situations, since we cannot give any probability of two kinds of decision errors *a priori*, this is not merely a problem of applying usual sequential tests, but also is to make a new type of sequential test in order to give an optimum plan for medical trials.

The assumptions are prepared throughout this paper as follows.

(1) There are N patients to be treated with one of the two treatments, A or B. The effective proportions of the two treatments are unknown to us and denoted by p_A and p_B. N is fixed and large.

(2) We suppose that the clinical trial is performed sequentially on pairs of two patients drawn from the N patients, administrating the treatment A on a patient and B on the other.

(3) If such sequential clinical trials are terminated, one treatment is decided to be superior to the other, and the selected treatment will be applied on all of the remaining patients.

Let us now define a discrepancy function p between p_A and p_B as follows,

$$p=\frac{p_B(1-p_A)}{p_B(1-p_A)+p_A(1-p_B)}\ ,$$

where p is a binomial parameter of double dichotomies based on odds ratios. Then $p=0.5$ is critical in a sense that if $p<0.5$ then treatment A is selected and otherwise treatment B is selected as a superior one.

Therefore, according to the decision procedure defined above, the expected loss

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is shown as

(1) $$\text{E LOSS} = G\cdot|p-0.5|\cdot[E(n)+\{N-2E(n)\}\cdot P_r(\text{Selecting Inferior})]$$

on the number of patients treated with the inferior treatment in an ethical sense, where G is a proportional constant and $E(n)$ is an expected number of pairs sequentially observed.

Now we assume an a priori distribution of p with a density function $f(p)$.

Then our criterion of optimum sequential plans is to minimize the average risk (or the over-all expected loss) $\overline{\text{E LOSS}}$ over the *a priori* distribution $f(p)$.

In such decision theoretical situation of medical sequential plans, Armitage [1], [2], Colton [5], Choi [4], and Sugimura-Asano [7]~[15] have partially discussed and constructed some types of sequential plans. Furthermore, the authors recently presented the whole formulation and properties in a category proposed by them, which may be published in *Ann. Inst. Stat. Math.*.

The purpose of this paper, therefore, is to give the available tables and figures computed by a FACOM 270-30 and to discuss further details for the practice with some summarized results of the above paper.

2. Definitions, formulations and the results

2.1. Open type.

The open type introduced here is shown in Fig. 1 and is just like in Sugimura-Goto-Asano [16], [17]. Both acceptance regions A and B are given by the outsides of horizontal lines at an assigned integer $\pm c$ of favorable pairs of A or B. The middle region is to be still continued the clinical trial.

For example, a sequence of results of favorable pairs observed, i. e., (B, B, A, B, B, A, B, B) is plotted in Fig. 1. The figure illustrates consequently accepting B as the preference.

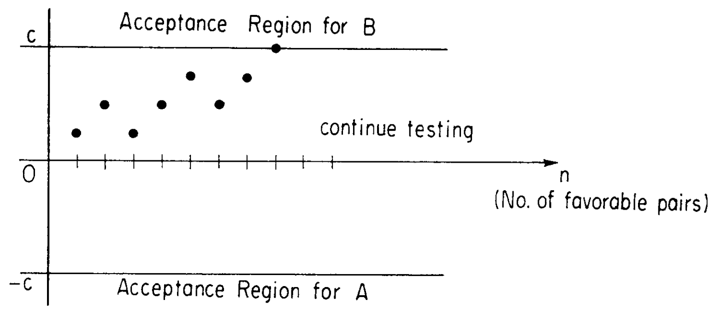

Fig. 1. Sequential procedure for an open type ($c=4$).

Then two kinds of probability of acceptance A or B, $\alpha_1^*(p)$ and $\alpha_3^*(p)$ in this plans are obtained as follows.

(2) $$\alpha_1^*(p) = \lim_{j\to\infty} P_r\begin{Bmatrix}\text{the sequential test terminates and accepts } B \\ \text{at the } (c+2j)\text{th sampling step}\end{Bmatrix}$$

$$=p^c+\left\{\sum_{k=1}^{c-1}\binom{c}{k}p^{c+k}q^kD_{(c-k-1)}\right\}\Big/D_{(c-1)},$$

where $q=1-p$, $\binom{c}{k}$ indicates a possible number of drawing k things from c,

$$(3)\qquad D_{(c-1)}=\left|\mathbf{D}_{(c-1)}\right|=\begin{vmatrix} p^2+q^2 & -q^2 & 0 & \cdots & \cdots & 0\\ -p^2 & p^2+q^2 & -q^2 & \ddots & & \vdots\\ 0 & -p^2 & p^2+q^2 & -q^2 & \ddots & \vdots\\ \vdots & \ddots & \ddots & \ddots & \ddots & 0\\ \vdots & & \ddots & -p^2 & p^2+q^2 & -q^2\\ 0 & \cdots & \cdots & 0 & -p^2 & p^2+q^2 \end{vmatrix},$$

(4) $D_{(0)}\equiv 1$, and

$$D_{(h)}=(p^2+q^2)D_{(h-1)}-p^2q^2D_{(h-2)}\qquad (h=2, 3, \cdots, c-1).$$

In the same manner, we obtain that

$$(5)\qquad \alpha_3^*(p)=\lim_{j\to\infty}P_r.\left\{\begin{matrix}\text{the sequential test terminates and accepts}\\ A \text{ at the } (c+2j)\text{th sampling step}\end{matrix}\right\}$$

$$=q^c+\left\{\sum_{k=1}^{c-1}\binom{c}{k}p^kq^{c+k}D_{(c-k-1)}\right\}\Big/D_{(c-1)}$$

$$=1-\alpha_1^*(p).$$

Furthermore, the average sampling favorable pair number ASN, $E(n|p, c)$ or $E(n)$, is given as follows,

$$(6)\qquad E(n|p, c)=c+\sum_{i=1}^{c-1}\sum_{j=1}^{c-1}s_{i+1}a_{ij},$$

where a_{ij} shows the (i, j) element of an inverse matrix $\boldsymbol{D}^{-1}_{(c-1)}$

$$(7)\qquad \mathbf{D}^{-1}_{(c-1)}=\frac{1}{D_{(c-1)}}\begin{pmatrix} D_{(0)}D_{(c-2)} & D_{(0)}D_{(c-3)}q^{2\cdot1} & D_{(0)}D_{(c-4)}q^{2\cdot2} & \cdots & D_{(0)}q^{2(c-2)}\\ D_{(0)}D_{(c-3)}p^{2\cdot1} & D_{(1)}D_{(c-3)} & D_{(1)}D_{(c-4)}q^{2\cdot1} & \cdots & D_{(1)}q^{2(c-3)}\\ D_{(0)}D_{(c-4)}p^{2\cdot2} & D_{(1)}D_{(c-4)}p^{2\cdot1} & D_{(2)}D_{(c-4)} & \cdots & D_{(2)}q^{2(c-4)}\\ D_{(0)}D_{(c-5)}p^{2\cdot3} & D_{(1)}D_{(c-5)}p^{2\cdot2} & D_{(2)}D_{(c-5)}p^{2\cdot1} & \ddots & D_{(3)}q^{2(c-5)}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ D_{(0)}D_{(1)}p^{2(c-3)} & D_{(1)}D_{(1)}p^{2(c-4)} & D_{(2)}D_{(1)}p^{2(c-5)} & \cdots & D_{(c-3)}q^{2\cdot1}\\ D_{(0)}D_{(0)}p^{2(c-2)} & D_{(1)}D_{(0)}p^{2(c-3)} & D_{(2)}D_{(0)}p^{2(c-4)} & \cdots & D_{(c-2)}D_{(0)} \end{pmatrix}$$

and

(8) $s_{i+1}=\binom{c}{i}p^{c-i}q^{i} \qquad (i=1, 2, \cdots, c-1)$.

By using (2), (5) and (6), the over-all expected loss function is expressed under an *a priori* distribution $f(p)$ for p as follows.

(9) $$\overline{\text{E LOSS}}/\text{NG}=\int_{0.5}^{\Delta_2}(p-0.5)f(p)dp+\int_{\Delta_1}^{\Delta_2}(0.5-p)\cdot\alpha_1^*(p)\cdot f(p)\cdot dp$$
$$+\frac{1}{N}\int_{\Delta_1}^{\Delta_2}(0.5-p)\{1-2\alpha_1^*(p)\}\cdot E(n\,|\,p,c)\cdot f(p)\cdot dp,$$

where the interval (Δ_1, Δ_2) indicates a defined interval of p in *a priori* distribution for p, and indicates $0<\Delta_1\leqq 0.5\leqq\Delta_2<1$.

If $f(p)$ is a discrete type, the above integrations are replaced by adequate summations in the following way.

(10) $$\overline{\text{E LOSS}}/\text{NG}=\sum_{\{i\,|\,p_i>0.5\}}(p_i-0.5)\cdot f(p_i)+\sum_{all\ i}(0.5-p_i)\cdot\alpha_1^*(p_i)\cdot f(p_i)$$
$$+\frac{1}{N}\sum_{all\ i}(0.5-p_i)(1-2\alpha_1^*(p_i))\cdot E(n\,|\,p_i,c)\cdot f(p_i).$$

2.2. Closed and unwedged type of the sequential plans

The closed and unwedged type is shown Fig. 2. The decision rule is to accept one of three hypothesis, $H_1: p>0.5$ (i. e., $p_A<p_B$), $H_3: p<0.5$ (i. e., $p_A>p_B$) and H_2: $p=0.5$ (i. e., $p_A=p_B$), with an assigned n_t pairs and the horizontal lines of both sides.

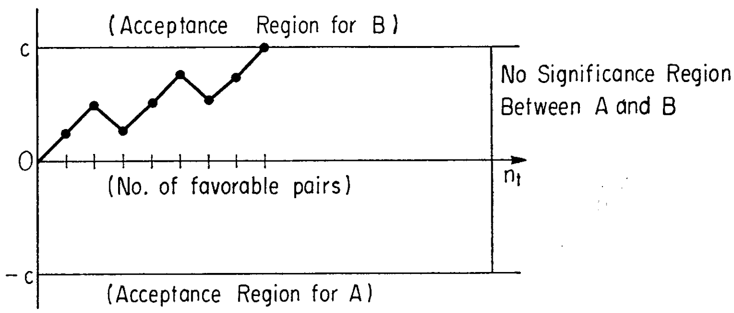

Fig. 2. Sequential procedure for a closed and unwedged type ($c=4$).

That is to say, if the number of favorable pairs attains to the upper, or lower region, we accept the hypothesis H_1 (select B) or hypothesis H_3 (select A) and if the pass attains to the outside of n_t, we accept H_2 (no difference between A and B).

Then the probability $\alpha_i(p)$ is given for accepting a hypothesis H_i, $i=1, 2, 3$, as follows.

(11) $$\alpha_1(p)=S_0(p)\cdot T_2(p)^h\cdot E_1,$$
$$\alpha_3(p)=S_0(q)\cdot T_2(q)^h\cdot E_1,$$
$$\alpha_2(p)=S_0(p)\cdot T_2(p)^h\cdot\sum_{j=2}^{c}E_j=1-\{\alpha_1(p)+\alpha_3(p)\},$$

where $q=1-p$, $h=(n_t-c)/2$, and E_j is an elementary vector with one as the j-th component, $j=1, 2, \cdots, c+1$, and

(12) $$S_0(p)=(s_1, s_2, \cdots, s_{c+1}),$$

(13) $$s_{i+1}=\binom{c}{i}p^{c-i}q^i, \qquad i=0, 1, 2, \cdots, c,$$

(14) $$T_2=\begin{pmatrix} 1 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ p^2 & 2pq & q^2 & 0 & & & \vdots \\ 0 & p^2 & 2pq & q^2 & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & & & & p^2 & 2pq & q^2 \\ 0 & \cdots & \cdots & \cdots & 0 & 0 & 1 \end{pmatrix} \quad ((c+1)\times(c+1) \text{ matrix}),$$

and where if n_t-c is odd, then $\alpha_i(p; c, n_t)=\alpha_i(p; c, n_t-1)$, and therefore we may put n_t-c is even without any loss of generality.

Furthermore, the average sampling number of favorable pair is as follows.

(15) $$E(n\,|\,p, c, n_t)=n_t-2\sum_{j=0}^{h-1}\{S_0(p)\cdot T_2(p)^j+S_0(q)\cdot T_2(q)^j\}\cdot E_1.$$

Now let us assume an additional assumption is that if H_1 or H_3 is accepted, then we take treatment B or A on the remaining $N-2n$ patients, and if H_2 accepted, then we accept half and half B and A on the remaining.

Then the over-all expected loss function is given by

(16) $$\overline{\text{E LOSS}}/\text{NG}=\frac{1}{2}\left\{\int_{0.5}^{\Delta_2}(p-0.5)\cdot f(p)\cdot dp-\int_{\Delta_1}^{0.5}(p-0.5)\cdot f(p)\cdot dp\right\}$$
$$-\frac{1}{2N}\int_{\Delta_1}^{\Delta_2}(p-0.5)\{\alpha_1(p)-\alpha_3(p)\}\{N-2E(n\,|\,p, c, n_t)\}\cdot f(p)\cdot dp,$$

where $\Delta_1<0.5<\Delta_2$. If the *a priori* distribution $f(p)$ is discrete, the $\overline{\text{E LOSS}}/\text{NG}$ of (16) is given by replacing the above integrations by the adequate summations like in the previous section.

2.3. Closed and wedged type of the sequential plans

This is a modification of the unwedged type with a wedge-like boundary with the angle θ, a little bit greater than $\pi/4$, at the non-significance region, like in Fig. 3. The angle θ means that any sample path does not come back the continued region if the path enters once the non-significance region, and that n_t-c+1 pairs are minimumly needed to get the decision of non-significance.

The probabilities to accept H_i, for the triplet of (p, c and n_t) is quite the same as $\alpha_i(p)$ ($i=1, 2, 3$) in section **2.2.**

On the other hand, the average sampling number $E(n|p, c, n_t)$ of favorable pairs is shown in the following way.

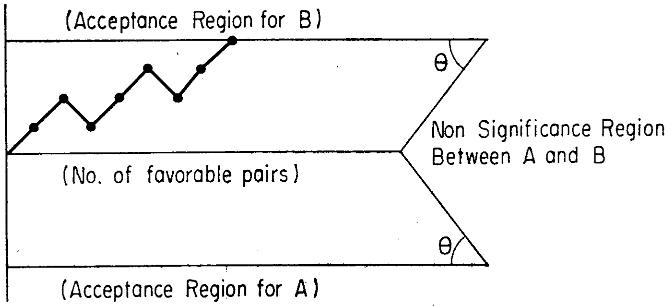

Fig. 3. Boundaries for a closed and wedged type of a two-sided sequential test.

$$(17) \qquad E(n|p, c, n_t) = [n_t\{S_h(p)+S_h(q)\} - 2\sum_{j=0}^{h-1}\{S_j(p)+S_j(q)\}] \cdot E_1 + \sum_{j=3}^{2d}(n_t-2d+j)\{S_{h-d+1}(p)\cdot p^{j-3}\cdot q + S_{h-d+1}(q)\cdot p\cdot q^{j-3}\}\cdot E_d + (1-\delta)(n_t-c+2)\cdot S_{h-d+1}(p)\cdot E_{d+1},$$

where $d=(c+\delta)/2$ and $\delta=0$ for an even c, $\delta=1$ for an odd c.

From the viewpoint of minimum loss for an *a priori* distribution $f(p)$, the expected loss function can be obtained by the same formulation as in section **2.2.** Whence, how to determine the optimum c is game-theoretical.

3. Numerical cases

3.1. Open type

$\alpha_1^*(p)$ and $E(n|p, c)$ are given in Tab. I and II for each pair of (c, p), where $c=5(1)15$ and $p=0.1(0.1)0.9$, and these curves on p's are drawn for $c=5(1)15$ in Fig. I and II, respectively.

The values of $\overline{\text{E LOSS}}$/NG are also available for $c=5(1)15$ at Tab. III and these are shown in Fig. III, for some cases of the *a priori* distributions of p, denoted by L_i, S_i, R_i $(i=1, 2, 3, 1', 2')$ in Tab. 1, in order to illustrate the behavior on c's, where $N=1{,}000$, and where the minimum $\overline{\text{E LOSS}}$/NG is denoted by (*) for each type of *a priori* distributions.

3.2. Cases of closed types

$\alpha_1(p; c, n_t)$'s are given in Tab. IV, for $n_t=10(1)50$, $c=4(1)24$, and $p=0.2(0.1)0.8$, two curves of $\alpha_1(p)$ and $\alpha_3(p)$ are shown for $n_t=30(10)50$ at Fig. IV for $c=4(4)24$, and these illustrate the relations among $\alpha_i(p)$'s $(i=1, 2, 3)$ due to p's.

The curve of $\alpha_i(p)$ $(i=1, 3)$ for the other value of n_t may be also without any difficulty obtained by drawing the similar figure as Fig. IV based on Tab. IV.

Furthermore, the curve of $E(n|p, c, n_t)$ on p's (i.e., $p=0.1(0.1)0.9$) is shown for

Table 1. The *a priori* distributions.

Type	S_1			S_2			S_3			$S_{1'}$			$S_{2'}$		
p	.7	.8	.9	.6	.7	.8	.4	.5	.6	.1	.2	.3	.2	.3	.4
$f(p)$	.25	.50	.25	.52	.50	.25	.25	.50	.25	.25	.50	.25	.25	.50	.25

Type	S_4								
p	.1	.2	.3	.4	.5	.6	.7	.8	.9
$f(p)$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

Type	R_1			R_2			R_3			$R_{1'}$			$R_{2'}$		
p	.7	.8	.9	.6	.7	.8	.4	.5	.6	.1	.2	.3	.2	.3	.4
$f(p)$	.15	.50	.35	.15	.50	.35	.15	.50	.35	.15	.50	.35	.15	.50	.35

Type	L_1			L_2			L_3			$L_{1'}$			$L_{2'}$		
p	.7	.8	.9	.6	.7	.8	.4	.5	.6	.1	.2	.3	.2	.3	.4
$f(p)$	.35	.50	.15	.35	.50	.15	.35	.50	.15	.35	.50	.15	.35	.50	.15

each pair (n_t, c) of $n_t=10(5)50$ and $c=4(2)24$, by Fig. V_1 or V_2, by Fig. V_1 for the unwedged design or by Fig. V_2 for the wedged design.

The behaviors of $\overline{\text{E LOSS}}$/NG on c's (i.e., $c=4(2)24$) are consequently explained for each of $n_t=10(5)50$ by Fig. VI_1 and VI_2, by Fig. VI_1 for the unwedged design and by Fig. VI_2 for the wedged design for each type of *a priori* distributions in Tab. 1.

4. Properties of $\alpha_i(p)$, $E(n)$ and $\overline{\text{E LOSS}}$

4.1. Open type

Firstly, the value of $\alpha_1^*(p)$ for a fixed $c=5(1)15$ increases from 0 to 1 as p increases from 0 to 1, while it increases as c increases from 5 to 15 when $p>0.5$, and decreases as c increases when $p<0.5$.

Secondly, as to $E(n|p, c)$, the value of $E(n|p, c)$ for a fixed $p=0.1(0.1)0.9$ increases as c increases from 5 to 15, and the curve on p's is like a Gaussian shape with the maximum height at $p=0.5$, for any $c=5(1)15$.

Moreover, $E(n|p, c)=E(n|1-p, c)$ for any c., and as for two *a priori* distributions whose probability distributions are mutually symmetric for $p=0.5$ (i.e., types R_i and $L_{i'}$, S_i and $S_{i'}$, L_i and $R_{i'}$ $(i=1, 2)$, R_3 and L_3), the value of $\overline{\text{E LOSS}}$/NG for a type is equal to that of the other, for a fixed c.

Let us consider a shape of the curve of $\overline{\text{E LOSS}}$/NG which is drawn according to c's (i.e., $c=5(1)15$), for each type of *a priori* distributions.

(1) Then the curve is concave for any type of *a priori* distributions.

(2) $\overline{\text{E LOSS}}$/NG increases as the center of *a priori* distribution goes to 0.5(, that is, $R_1\to S_1\to L_1\to R_2\to S_2\to L_2\to R_3\to S_3$) for a fixed c, and then the difference between

both $\overline{\text{E LOSS}}$'s of R_1-type and of S_3-type (i.e., [$\overline{\text{E LOSS}}$ of S_3]−[$\overline{\text{E LOSS}}$ of R_1]) for the same c comes to zero as c increased.

(3) The optimum c^* becomes large little by little for minimizing $\overline{\text{E LOSS}}$/NG as the center of *a priori* distribution goes to 0.5 (i.e., $R_1 \rightarrow S_1 \rightarrow \cdots \rightarrow R_3 \rightarrow S_3$).

4.2. Closed types

Firstly, it is obvious that $\alpha_3(p) = \alpha_1(1-p)$ and $\alpha_2(p) = 1 - \{\alpha_1(p) + \alpha_3(p)\}$.

The followings become clear from Tab. IV and Fig. IV. (1) The value of $\alpha_1(p)$ increases from 0 to 1 as p increases from 0.1 to 0.9, for any pair (n_t, c). (2) The value of $\alpha_1(p)$ decreases for any (p, n_t) as c increases from 4 to 24, and moreover it shows a slight decrease when n_t is not less than a certain value. (3) On the other hand, $\alpha_1(p)$ increases for any (p, c) as n_t increases from 10 to 50, and further it shows a rapid increase when c is larger than a certain value.

Secondly, as to the properties of $E(n|p, c, n_t)$, the following conclusions (1)~(6) may be derived from Fig. V_1 and V_2.

(1) It is clear that $E(n|p, c, n_t) = E(n|1-p, c, n_t)$ for any (c, n_t).

(2) For any n_t and c, the curve of $E(n|p, c, n_t)$ due to p's (i.e., $p = 0.1(0.1)0.9$) is like a Gaussian shape with the maximal height at $p = 0.5$ in the unwedged design. However, in the wedged design, the curve $E(n|p, c, n_t)$ shows the locally minimum at $p = 0.5$ when c is not less than a certain integer (for example, when c is not less than 16 for $n_t = 49$ or 50).

(3) The function $E(n|p, c, n_t)$ of c increases monotonously for any n_t and the increase tends gradually as n_t increases from 10 to 50.

(4) The function $E(n|p, c, n_t)$ of n_t also increases monotonously for any c and it shows rapid increase as c increases from 4 to 24.

(5) The value of $E(n|p, c, n_t)$ of the wedged design is equal to or less than that of the unwedged for any (p, n_t), except the case of $c = 4$, and the difference between both values of $E(n|p, c, n_t)$ of these designs grows gradually large as a value c increase. For example, the former is about 28 percent less than the latter for $p = 0.5$, $n_t = 50$, and $c = 20$.

(6) When p goes to 0.1 or 0.9, the value of $E(n|p, c, n_t)$ of the wedged design also coincides well with that of the unwedged for any (c, n_t).

In conclusion, the following results (1)~(4) may be summerized with respect to the properties of $\overline{\text{E LOSS}}$.

(1) When there are mutually symmetric *a priori* distributions for $p = 0.5$ (i.e., types R_i and $L_{i'}$, S_i and $S_{i'}$, $L_i = R_{i'}$ $(i = 1, 2)$ R_3 and L_3), a value of $\overline{\text{E LOSS}}$/NG equals that of another distribution for any (c, n_t), and also $\overline{\text{E LOSS}}$/NG of type S_3 equals that of type R_3.

(2) The function $\overline{\text{E LOSS}}$/NG of c increases monotonously when n_t is nearly less than 34, and its increase becomes gradually flat as n_t becomes fairly large. On the other hand, when n_t is nearly equal to or larger than 34, $\overline{\text{E LOSS}}$/NG shows concave with a minimum height at $c = 5 \sim 7$.

(3) The range of $\overline{\text{E LOSS}}$/NG, that is, $[\underset{(c,n_t)}{\text{Max.}}\ \overline{\text{E LOSS}}/\text{NG}]-[\underset{(c,n_t)}{\text{Min.}}\ \overline{\text{E LOSS}}/\text{NG}]$ for $c=4(1)24$, and $n_t=10(1)50$ (where $2c<n_t$) increases from 0.025 to 0.048 as *a priori* distribution changes from R_1 to L_2 through a route $R_1 \rightarrow S_1 \rightarrow L_1 \rightarrow R_2 \rightarrow S_2 \rightarrow L_2$. This is summerized in Table 2.

Table 2. Range of $\overline{\text{E LOSS}}$/NG, types R_i, S_i, L_i ($i=1, 2$).

a priori distribution	R_1	S_1	L_1	R_2	S_2	L_2
Max. [$\overline{\text{E LOSS}}$/NG]	.028	.033	.039	.051	.053	.055
Min. [$\overline{\text{E LOSS}}$/NG]	.003	.003	.004	.005	.006	.007
Range	.025	.030	.035	.046	.047	.048

(Maximization and Minimization of $\overline{\text{E LOSS}}$/NG are considered under the conditions $4 \leqq c \leqq 24$, $10 \leqq n \leqq 50$, and $2c \leqq n_t$.)

The range of $\overline{\text{E LOSS}}$/NG of type S_4 is larger than that of types R_3, S_3, and L_3 as follows.

Table 3. Range of $\overline{\text{E LOSS}}$/NG, types R_3, S_3, L_3, and S_4.

a priori distribution	R_3, S_3, L_3	S_4
Max. [$\overline{\text{E LOSS}}$/NG]	.025	.082
Min. [$\overline{\text{E LOSS}}$/NG]	.007	.005
Range	.018	.077

(4) For any type of *a priori* distributions in Tab. 1, the value of $\overline{\text{E LOSS}}$/NG of the wedged design almost coincides with that of the unwedged design for any pair (c, n_t).

Therefore it is worthless for us to consider some differences between a wedged and an unwedged designs in view of minimizing ethycal $\overline{\text{E LOSS}}$/NG.

(5) When n_t may be at least 50 from our physical aspects, the optimum designs, giving the minimum $\overline{\text{E LOSS}}$/NG, are summerized in Tab. 4, for *a priori* distributions and wedged and unwedged designs.

5. Examples

5.1. Open type

Example 1. When p is not less than 0.6, determine a value c by which B-acceptance region may be decided in order to select treatment B as the better with probability 0.95 at least. From Tab. I or Fig. I, the value c must be at least equal to 8.

Example 2. When $c=10$ in *Ex. 1*, find out an average sampling number of favorable pair of patients in case of open type. It is 30 pairs at the most from Tab. II or Fig. II.

Example 3. When an *a priori* distribution $f(p)$ is either of three types L_2, S_2,

Table 4. Optimum Designs.

a priori distribution	Unwedged Design			Wedged Design		
	n_t	c	Min. $[\overline{\text{E LOSS/NG}}]$	n_t	c	Min. $[\overline{\text{E LOSS/NG}}]$
R_1	31	5	0. 003	31	5	0. 003
	40	6		40	6	
S_1	41	5	0. 003	43	5	0. 003
	48	6		50	6	
L_1	31	5		31	5	
	36	6	0. 004	36	6	0. 004
	43	7		43	7	
R_2	50	6	0. 005	50	6	0. 005
S_2				43	5	
	50	6	0. 006	42	6	0. 007
				47	7	
L_2	45	5		47	5	
	44	6	0. 008	44	6	0. 008
	47	7		47	7	
R_3, S_3, L_3	47	5		47	5	
	44	6	0. 007	46	6	0. 007
	49	7		49	7	
S_4	48	6	0. 005	50	6	0. 005

or R_2, and when $c=8$ in *Ex. 1*, it follows from Tab. III that the $\overline{\text{E LOSS}}$/NG is at least about 77.8 per cent more than the minimum $\overline{\text{E LOSS}}$/NG, since the $\overline{\text{E LOSS}}$/NG is 0.00096 for $c=8$ and then the minimum $=0.00054$ (, where $c=12$ or 13 gives the minimum) in the case of type R_2, and the $\overline{\text{E LOSS}}$/NG is 0.00132 for $c=8$ and then the minimum $=0.00057$ in the case of type S_2, and moreover the $\overline{\text{E LOSS}}$/NG is 0.00169 for $c=8$ and then the minimum $=0.00059$ in the case of type L_2.

5.2. Closed types

Example 4. Assume that the effective probability p_A of standard treatment A is known to be 0.60, and p_B of test treatment B estimated to be either of three values 0.86, 0.78, and 0.69. Find out the probabilities which treatment A and B may be selected as the better by a closed type where $n_t=40$ and $c=10$. The answer is given from Tab. IV or Fig. IV, that is, $p \doteqdot 0.80$, 0.70 or 0.60 for $p_B=0.86$, 0.78 or 0.69, respectively. Therefore, the probability for selecting B i.e., $\alpha_1(p)=0.9979$ or 0.9093 or 0.5107, respectively. Also, the probability for selecting A, i. e. $\alpha_3(p)$, is 0, 0.0002 or 0.0089, respectively.

Example 5. Find out the number of the average sampling number of favorable pair of patients for each value of p_B's assumed in *Ex. 4* in cases of the unwedged and the wedged designs.

Fig. V_1 and V_2 give the following answer (Tab. 5).

Table 5. $E(n|p, c, n_t)$ for $n_t=40$, $c=10$.

p_B	p	Unwedged Design	Wedged Design
0. 86	0. 80	17	17
0. 78	0. 70	24	24
0. 60	0. 60	33	31

From Tab. 5, the ASN of the wedged design becomes less than that of the unwedged when $p=0.60$, that is, p is about 0.5 nearer than 0.70 or 0.80.

Example 6. Assume that an *a priori* distribution $f(p)$ is type S_2. Find out the $\overline{\text{E LOSS}}$/NG for $c=10$, $n_t=42$ and the minimum $\overline{\text{E LOSS}}$/NG for $n_t=42$, in each case of both designs.

These values required above are nearly 0.014 and 0.007 in both cases, respectively, from Fig. VI_1 and VI_2, and therefore the former has about twice of the latter, and moreover the optimum value c^* for $n_t=42$ is 6, that is, less than $c=10$.

If we determine B-acceptance region as $c=4$, the $\overline{\text{E LOSS}}$/NG for $c=4$, is about 28.6 per cent more than the minimum $\overline{\text{E LOSS}}$/NG for the optimal $c^*=6$, where $n_t=42$ in the same manner mentioned above.

6. Numerical tables and figures

Table I. $\alpha_1^*(p)$ of Open Type.

c \ p	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
5	0.99998	0.99902	0.98575	0.88364	0.50000	0.11636	0.01425	0.00098	0.00002
6	1.00000	0.99976	0.99384	0.91929	0.50000	0.08071	0.00616	0.00024	0.00000
7	1.00000	0.99994	0.99735	0.94471	0.50000	0.05529	0.00265	0.00006	0.00000
8	1.00000	0.99998	0.99886	0.96245	0.50000	0.03755	0.00114	0.00002	0.00000
9	1.00000	1.00000	0.99951	0.97465	0.50000	0.02535	0.00049	0.00000	0.00000
10	1.00000	1.00000	0.99979	0.98295	0.50000	0.01705	0.00021	0.00000	0.00000
11	1.00000	1.00000	0.99991	0.98857	0.50000	0.01143	0.00009	0.00000	0.00000
12	1.00000	1.00000	0.99996	0.99235	0.50000	0.00765	0.00004	0.00000	0.00000
13	1.00000	1.00000	0.99998	0.99489	0.50000	0.00511	0.00002	0.00000	0.00000
14	1.00000	1.00000	0.99999	0.99659	0.50000	0.00341	0.00001	0.00000	0.00000
15	1.00000	1.00000	1.00000	0.99772	0.50000	0.00228	0.00000	0.00000	0.00000

Table II. The Average Sampling Favorable Pair-Numbers of Open Type.

c \ p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
5	5.62489	6.65854	8.57185	12.09091	15.00000	12.09091	8.57185	6.65854	5.62489
6	6.74999	7.99756	10.40763	15.57881	21.00000	15.57881	10.40763	7.99756	6.74999
7	7.87500	9.33262	12.20365	19.06479	28.00000	19.06479	12.20365	9.33262	7.87500
8	9.00000	10.66646	13.97726	22.49786	36.00000	22.49786	13.97726	10.66646	9.00000
9	10.12500	11.99994	15.73903	25.85912	45.00000	25.85912	15.73903	11.99994	10.12500
10	11.25000	13.33332	17.49477	29.14769	55.00000	29.14769	17.49477	13.33332	11.25000
11	12.37500	14.66666	19.24753	32.37140	66.00000	32.37140	19.24753	14.66666	12.37500
12	13.50000	16.00000	20.99884	35.54108	78.00000	35.54108	20.99884	16.00000	13.50000
13	14.62500	17.33333	22.74945	38.66771	91.00000	38.66771	22.74945	17.33333	14.62500
14	15.75000	18.66666	24.49974	41.76102	105.00000	41.76102	24.49974	18.66666	15.75000
15	16.87498	19.99998	26.24988	44.82910	120.00000	44.82910	26.24988	19.99998	16.87498

Table III. $\overline{\text{E LOSS}}$/NG of Open Type.

c \ Type	R_1, L_1'	S_1, S_1'	L_1, R_1'	R_2, L_2'	S_2, S_2'	L_2, R_2'	R_3, S_3, L_3	S_4
5	0.00078	0.00106	0.00134	0.00345	0.00457	0.00569	0.00587	0.00341
6	0.00046	0.00058	0.00070	0.00206	0.00285	0.00364	0.00410	0.00225
7	0.00038	0.00042	0.00047	0.00134	0.00188	0.00242	0.00285	0.00156
8	0.00036(*)	0.00037(*)	0.00039(*)	0.00096	0.00132	0.00169	0.00198	0.00113
9	0.00038	0.00038	0.00039(*)	0.00075	0.00099	0.00124	0.00139	0.00087
10	0.00042	0.00041	0.00040	0.00063	0.00079	0.00095	0.00099	0.00071
11	0.00045	0.00044	0.00044	0.00057	0.00068	0.00078	0.00073	0.00062
12	0.00049	0.00048	0.00047	0.00054(*)	0.00061	0.00067	0.00056	0.00056
13	0.00053	0.00052	0.00051	0.00054(*)	0.00058	0.00062	0.00045	0.00054(*)
14	0.00057	0.00056	0.00055	0.00055	0.00057(*)	0.00059(*)	0.00038	0.00054(*)
15	0.00061	0.00060	0.00058	0.00057	0.00058	0.00059(*)	0.00034(*)	0.00054(*)

Table IV. $\alpha_1(0.2;c,n_t)$ of Closed Type (continued), $c=4(1)24$, $n_t=10(1)50$.

n_t \ c	4 (5)	6 (7)	8~24 (9)
10	0.0035	0.0002	0.0000
11	0.0008	0.0000	0.0000
12	0.0037	0.0002	0.0000
13	0.0009	0.0000	0.0000
14	0.0038	0.0002	0.0000
15	0.0009	0.0001	0.0000
16	0.0038	0.0002	0.0000
17	0.0009	0.0001	0.0000
18	0.0039	0.0002	0.0000
19	0.0010	0.0001	0.0000
20	0.0039	0.0002	0.0000
21	0.0010	0.0001	0.0000
22	0.0039	0.0002	0.0000
23	0.0010	0.0001	0.0000
24	0.0039	0.0002	0.0000
25	0.0010	0.0001	0.0000
26	0.0039	0.0002	0.0000
27	0.0010	0.0001	0.0000
28	0.0039	0.0002	0.0000
29	0.0010	0.0001	0.0000
30	0.0039	0.0002	0.0000
31	0.0010	0.0001	0.0000
32	0.0039	0.0002	0.0000
33	0.0010	0.0001	0.0000
34	0.0039	0.0002	0.0000
35	0.0010	0.0001	0.0000
36	0.0039	0.0002	0.0000
37	0.0010	0.0001	0.0000
38	0.0039	0.0002	0.0000
39	0.0010	0.0001	0.0000
40	0.0039	0.0002	0.0000
41	0.0010	0.0001	0.0000
42	0.0039	0.0002	0.0000
43	0.0010	0.0001	0.0000
44	0.0039	0.0002	0.0000
45	0.0010	0.0001	0.0000
46	0.0039	0.0002	0.0000
47	0.0010	0.0001	0.0000
48	0.0039	0.0002	0.0000
49	0.0010	0.0001	0.0000
50	0.0039	0.0002	0.0000

Table IV. $\alpha_1(0.3; c, n_t)$ of Closed Type (continued), $c=4(1)24$, $n_t=10(1)50$.

n_t \ c	4 (5)	6 (7)	8 (9)	10 (11)	12~24 (13)
10	0.0235	0.0025	0.0003	0.0000	0.0000
11	0.0088	0.0009	0.0001	0.0000	0.0000
12	0.0261	0.0033	0.0003	0.0000	0.0000
13	0.0101	0.0012	0.0001	0.0000	0.0000
14	0.0279	0.0039	0.0004	0.0000	0.0000
15	0.0111	0.0015	0.0002	0.0000	0.0000
16	0.0293	0.0044	0.0005	0.0001	0.0000
17	0.0119	0.0017	0.0002	0.0000	0.0000
18	0.0302	0.0047	0.0006	0.0001	0.0000
19	0.0124	0.0019	0.0002	0.0000	0.0000
20	0.0309	0.0050	0.0007	0.0001	0.0000
21	0.0129	0.0020	0.0003	0.0000	0.0000
22	0.0314	0.0053	0.0008	0.0001	0.0000
23	0.0132	0.0022	0.0003	0.0000	0.0000
24	0.0317	0.0055	0.0009	0.0001	0.0000
25	0.0315	0.0023	0.0003	0.0000	0.0000
26	0.0320	0.0056	0.0009	0.0001	0.0000
27	0.0136	0.0023	0.0004	0.0001	0.0000
28	0.0322	0.0057	0.0010	0.0002	0.0000
29	0.0138	0.0024	0.0004	0.0001	0.0000
30	0.0323	0.0058	0.0010	0.0002	0.0000
31	0.0139	0.0024	0.0004	0.0001	0.0000
32	0.0324	0.0059	0.0010	0.0002	0.0000
33	0.0140	0.0025	0.0004	0.0001	0.0000
34	0.0325	0.0060	0.0010	0.0002	0.0000
35	0.0141	0.0025	0.0004	0.0001	0.0000
36	0.0325	0.0060	0.0011	0.0002	0.0000
37	0.0141	0.0025	0.0004	0.0001	0.0000
38	0.0326	0.0060	0.0011	0.0002	0.0000
39	0.0141	0.0026	0.0005	0.0001	0.0000
40	0.0326	0.0061	0.0011	0.0002	0.0000
41	0.0142	0.0026	0.0005	0.0001	0.0000
42	0.0326	0.0061	0.0011	0.0002	0.0000
43	0.0142	0.0026	0.0005	0.0001	0.0000
44	0.0326	0.0061	0.0011	0.0002	0.0000
45	0.0142	0.0026	0.0005	0.0001	0.0000
46	0.0326	0.0061	0.0011	0.0002	0.0000
47	0.0142	0.0026	0.0005	0.0001	0.0000
48	0.0326	0.0061	0.0011	0.0002	0.0000
49	0.0142	0.0026	0.0005	0.0001	0.0000
50	0.0326	0.0061	0.0011	0.0002	0.0000

Table IV. $\alpha_1(0.4; c, n_t)$ of Closed Type (continued), $c=4(1)24$, $n_t=10(1)50$.

n_t \ c	4 (5)	6 (7)	8 (9)	10 (11)	12 (13)	14 (15)	16 (17)	18 (19)	20~24 (21)
10	0.0878	0.0164	0.0036	0.0007	0.0002	0.0000	0.0000	0.0000	0.0000
11	0.0449	0.0077	0.0016	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000
12	0.1017	0.0226	0.0036	0.0007	0.0002	0.0000	0.0000	0.0000	0.0000
13	0.0543	0.0112	0.0016	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000
14	0.1132	0.0284	0.0055	0.0007	0.0002	0.0000	0.0000	0.0000	0.0000
15	0.0624	0.0146	0.0026	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000
16	0.1225	0.0338	0.0074	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000
17	0.0695	0.0180	0.0037	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000
18	0.1302	0.0386	0.0094	0.0018	0.0003	0.0000	0.0000	0.0000	0.0000
19	0.0757	0.0211	0.0049	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000
20	0.1365	0.0430	0.0114	0.0025	0.0004	0.0001	0.0000	0.0000	0.0000
21	0.0810	0.0240	0.0060	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000
22	0.1416	0.0469	0.0132	0.0032	0.0006	0.0001	0.0000	0.0000	0.0000
23	0.0857	0.0267	0.0072	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000
24	0.1458	0.0504	0.0150	0.0039	0.0008	0.0002	0.0000	0.0000	0.0000
25	0.0897	0.0292	0.0083	0.0021	0.0004	0.0001	0.0000	0.0000	0.0000
26	0.1493	0.0536	0.0167	0.0046	0.0011	0.0002	0.0000	0.0000	0.0000
27	0.0932	0.0314	0.0094	0.0025	0.0006	0.0001	0.0000	0.0000	0.0000
28	0.1521	0.0564	0.0182	0.0052	0.0013	0.0003	0.0001	0.0000	0.0000
29	0.0963	0.0335	0.0104	0.0029	0.0007	0.0001	0.0000	0.0000	0.0000
30	0.1544	0.0589	0.0197	0.0059	0.0016	0.0004	0.0001	0.0000	0.0000
31	0.0989	0.0354	0.0114	0.0033	0.0009	0.0002	0.0000	0.0000	0.0000
32	0.1563	0.0612	0.0210	0.0066	0.0018	0.0005	0.0001	0.0000	0.0000
33	0.1012	0.0372	0.0123	0.0037	0.0010	0.0002	0.0001	0.0000	0.0000
34	0.1579	0.0632	0.0223	0.0072	0.0021	0.0006	0.0001	0.0000	0.0000
35	0.1032	0.0387	0.0132	0.0041	0.0012	0.0003	0.0001	0.0000	0.0000
36	0.1592	0.0651	0.0234	0.0078	0.0024	0.0006	0.0002	0.0000	0.0000
37	0.1050	0.0402	0.0140	0.0045	0.0013	0.0004	0.0001	0.0000	0.0000
38	0.1602	0.0667	0.0245	0.0083	0.0026	0.0007	0.0002	0.0000	0.0000
39	0.1065	0.0415	0.0148	0.0049	0.0015	0.0004	0.0001	0.0000	0.0000
40	0.1611	0.0682	0.0255	0.0089	0.0028	0.0008	0.0002	0.0001	0.0000
41	0.1078	0.0427	0.0155	0.0053	0.0016	0.0005	0.0001	0.0000	0.0000
42	0.1618	0.0695	0.0264	0.0094	0.0031	0.0009	0.0003	0.0001	0.0000
43	0.1089	0.0438	0.0162	0.0056	0.0018	0.0005	0.0001	0.0000	0.0000
44	0.1623	0.0706	0.0273	0.0098	0.0033	0.0010	0.0003	0.0001	0.0000
45	0.1099	0.0448	0.0168	0.0059	0.0020	0.0006	0.0002	0.0000	0.0000
46	0.1628	0.0717	0.0281	0.0103	0.0035	0.0011	0.0003	0.0001	0.0000
47	0.1107	0.0457	0.0174	0.0062	0.0021	0.0007	0.0002	0.0001	0.0000
48	0.1632	0.0726	0.0288	0.0107	0.0038	0.0012	0.0004	0.0001	0.0000
49	0.1115	0.0466	0.0179	0.0065	0.0022	0.0007	0.0002	0.0001	0.0000
50	0.1635	0.0735	0.0295	0.0111	0.0040	0.0013	0.0004	0.0001	0.0000

Table IV. $\alpha_1(0.5; c, n_t)$ of Closed Type (continued), $c=4(1)24$, $n_t=10(1)50$.

n_t \ c	4 (5)	6 (7)	8 (9)	10 (11)	12 (13)	14 (15)	16 (17)	18 (19)	20 (21)	22 (23)	24
10	0.2266	0.0654	0.0225	0.0074	0.0023	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000
11	0.1460	0.0386	0.0129	0.0042	0.0013	0.0004	0.0001	0.0000	0.0000	0.0000	
12	0.2666	0.0923	0.0225	0.0074	0.0023	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000
13	0.1796	0.0574	0.0129	0.0042	0.0013	0.0004	0.0001	0.0000	0.0000	0.0000	
14	0.3008	0.1185	0.0352	0.0074	0.0023	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000
15	0.2101	0.0768	0.0213	0.0042	0.0013	0.0004	0.0001	0.0000	0.0000	0.0000	
16	0.3300	0.1435	0.0490	0.0127	0.0023	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000
17	0.2377	0.0963	0.0309	0.0075	0.0013	0.0004	0.0001	0.0000	0.0000	0.0000	
18	0.3549	0.1671	0.0636	0.0192	0.0044	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000
19	0.2628	0.1153	0.0414	0.0118	0.0026	0.0004	0.0001	0.0000	0.0000	0.0000	
20	0.3761	0.1892	0.0784	0.0266	0.0072	0.0015	0.0002	0.0001	0.0000	0.0000	0.0000
21	0.2854	0.1338	0.0525	0.0169	0.0043	0.0009	0.0001	0.0000	0.0000	0.0000	
22	0.3945	0.2100	0.0931	0.0347	0.0106	0.0026	0.0005	0.0001	0.0000	0.0000	0.0000
23	0.3059	0.1516	0.0639	0.0227	0.0066	0.0015	0.0003	0.0000	0.0000	0.0000	
24	0.4097	0.2294	0.1078	0.0433	0.0146	0.0041	0.0009	0.0002	0.0000	0.0000	0.0000
25	0.3244	0.1686	0.0755	0.0290	0.0094	0.0025	0.0005	0.0001	0.0000	0.0000	
26	0.4230	0.2475	0.1221	0.0522	0.0192	0.0059	0.0015	0.0003	0.0000	0.0000	0.0000
27	0.3412	0.1849	0.0872	0.0357	0.0125	0.0037	0.0009	0.0002	0.0000	0.0000	
28	0.4342	0.2644	0.1360	0.0614	0.0241	0.0081	0.0023	0.0005	0.0001	0.0000	0.0000
29	0.3564	0.2004	0.0987	0.0428	0.0161	0.0052	0.0014	0.0003	0.0001	0.0000	
30	0.4439	0.2802	0.1496	0.0708	0.0294	0.0107	0.0033	0.0009	0.0002	0.0000	0.0000
31	0.3701	0.2152	0.1102	0.0501	0.0201	0.0070	0.0021	0.0005	0.0001	0.0000	
32	0.4521	0.2949	0.1627	0.0801	0.0351	0.0135	0.0046	0.0013	0.0003	0.0001	0.0000
33	0.3825	0.2293	0.1214	0.0576	0.0243	0.0090	0.0029	0.0008	0.0002	0.0000	
34	0.4591	0.3086	0.1754	0.0895	0.0410	0.0167	0.0060	0.0019	0.0005	0.0001	0.0000
35	0.3937	0.2427	0.1325	0.0652	0.0288	0.0113	0.0039	0.0012	0.0003	0.0001	
36	0.4651	0.3214	0.1877	0.0989	0.0470	0.0201	0.0076	0.0026	0.0008	0.0002	0.0000
37	0.4039	0.2554	0.1433	0.0730	0.0336	0.0139	0.0051	0.0017	0.0005	0.0001	
38	0.4702	0.3334	0.1995	0.1081	0.0533	0.0237	0.0095	0.0034	0.0011	0.0003	0.0001
39	0.4130	0.2675	0.1539	0.0807	0.0385	0.0166	0.0064	0.0022	0.0007	0.0002	
40	0.4746	0.3446	0.2109	0.1173	0.0596	0.0275	0.0115	0.0043	0.0015	0.0004	0.0001
41	0.4213	0.2790	0.1641	0.0884	0.0436	0.0195	0.0079	0.0029	0.0009	0.0003	
42	0.4783	0.3550	0.2219	0.1263	0.0660	0.0315	0.0137	0.0054	0.0019	0.0006	0.0002
43	0.4289	0.2900	0.1741	0.0961	0.0488	0.0226	0.0096	0.0037	0.0013	0.0004	
44	0.4815	0.3647	0.2324	0.1352	0.0725	0.0357	0.0161	0.0066	0.0025	0.0008	0.0002
45	0.4357	0.3004	0.1839	0.1038	0.0541	0.0259	0.0114	0.0045	0.0016	0.0005	
46	0.4842	0.3738	0.2426	0.1439	0.0789	0.0400	0.0186	0.0079	0.0031	0.0011	0.0003
47	0.4418	0.3102	0.1934	0.1114	0.0595	0.0293	0.0133	0.0055	0.0021	0.0007	
48	0.4865	0.3822	0.2524	0.1524	0.0854	0.0444	0.0213	0.0094	0.0038	0.0014	0.0005
49	0.4474	0.3196	0.2025	0.1189	0.0649	0.0328	0.0153	0.0066	0.0026	0.0009	
50	0.4885	0.3901	0.2618	0.1608	0.0919	0.0489	0.0241	0.0110	0.0046	0.0018	0.0006

Table IV. $\alpha_1(0.6; c, n_t)$ of Closed Type (continued), $c=4(1)24$, $n_t=10(1)50$.

n_t \ c	4 (5)	6 (7)	8 (9)	10 (11)	12 (13)	14 (15)	16 (17)	18 (19)	20 (21)	22 (23)	24
10	0.4445	0.1864	0.0916	0.0432	0.0197	0.0088	0.0038	0.0016	0.0007	0.0003	0.0001
11	0.3413	0.1315	0.0632	0.0293	0.0132	0.0058	0.0025	0.0011	0.0005	0.0002	
12	0.5150	0.2573	0.0916	0.0432	0.0197	0.0088	0.0038	0.0016	0.0007	0.0003	0.0001
13	0.4122	0.1911	0.0632	0.0293	0.0132	0.0058	0.0025	0.0011	0.0005	0.0002	
14	0.5728	0.3238	0.1399	0.0432	0.0197	0.0088	0.0038	0.0016	0.0007	0.0003	0.0001
15	0.4741	0.2502	0.1012	0.0293	0.0132	0.0058	0.0025	0.0011	0.0005	0.0002	
16	0.6202	0.3846	0.1906	0.0725	0.0197	0.0088	0.0038	0.0016	0.0007	0.0003	0.0001
17	0.5280	0.3070	0.1434	0.0514	0.0132	0.0058	0.0025	0.0011	0.0005	0.0002	
18	0.6590	0.4398	0.2415	0.1066	0.0361	0.0088	0.0038	0.0016	0.0007	0.0003	0.0001
19	0.5748	0.3605	0.1876	0.0784	0.0252	0.0058	0.0025	0.0011	0.0005	0.0002	
20	0.6908	0.4895	0.2913	0.1439	0.0571	0.0174	0.0038	0.0016	0.0007	0.0003	0.0001
21	0.6155	0.4102	0.2323	0.1092	0.0412	0.0120	0.0025	0.0011	0.0005	0.0002	
22	0.7168	0.5342	0.3391	0.1831	0.0820	0.0295	0.0082	0.0016	0.0007	0.0003	0.0001
23	0.6508	0.4562	0.2767	0.1426	0.0610	0.0210	0.0056	0.0011	0.0005	0.0002	
24	0.7382	0.5743	0.3844	0.2231	0.1100	0.0450	0.0148	0.0038	0.0007	0.0003	0.0001
25	0.6814	0.4983	0.3199	0.1778	0.0840	0.0329	0.0104	0.0025	0.0005	0.0002	
26	0.7557	0.6102	0.4271	0.2631	0.1403	0.0636	0.0239	0.0073	0.0017	0.0003	0.0001
27	0.7081	0.5373	0.3615	0.2140	0.1096	0.0477	0.0173	0.0050	0.0011	0.0002	
28	0.7700	0.6425	0.4670	0.3025	0.1722	0.0848	0.0355	0.0124	0.0035	0.0008	0.0001
29	0.7312	0.5728	0.4012	0.2504	0.1373	0.0651	0.0263	0.0088	0.0024	0.0005	
30	0.7818	0.6713	0.5042	0.3409	0.2051	0.1084	0.0496	0.0193	0.0062	0.0016	0.0003
31	0.7513	0.6053	0.4389	0.2866	0.1664	0.0849	0.0375	0.0141	0.0044	0.0011	
32	0.7914	0.6972	0.5388	0.3780	0.2385	0.1338	0.0659	0.0281	0.0102	0.0031	0.0008
33	0.7687	0.6349	0.4745	0.3222	0.1966	0.1067	0.0508	0.0209	0.0074	0.0022	
34	0.7993	0.7204	0.5709	0.4136	0.2720	0.1606	0.0843	0.0389	0.0155	0.0053	0.0015
35	0.7838	0.6620	0.5080	0.3569	0.2274	0.1301	0.0661	0.0295	0.0114	0.0038	
36	0.8057	0.7411	0.6007	0.4476	0.3051	0.1885	0.1045	0.0515	0.0223	0.0084	0.0027
37	0.7970	0.6867	0.5395	0.3906	0.2585	0.1549	0.0833	0.0398	0.0167	0.0061	
38	0.8110	0.7597	0.6283	0.4800	0.3378	0.2171	0.1263	0.0659	0.0306	0.0125	0.0044
39	0.8084	0.7093	0.5690	0.4231	0.2895	0.1807	0.1021	0.0518	0.0233	0.0092	
40	0.8154	0.7763	0.6537	0.5107	0.3696	0.2460	0.1493	0.0820	0.0404	0.0177	0.0068
41	0.8183	0.7299	0.5966	0.4542	0.3202	0.2073	0.1224	0.0653	0.0313	0.0133	
42	0.8189	0.7913	0.6773	0.5397	0.4006	0.2750	0.1733	0.0996	0.0517	0.0241	0.0100
43	0.8269	0.7487	0.6224	0.4841	0.3504	0.2343	0.1438	0.0804	0.0407	0.0185	
44	0.8218	0.8046	0.6991	0.5671	0.4306	0.3040	0.1981	0.1185	0.0645	0.0318	0.0141
45	0.8344	0.7658	0.6465	0.5125	0.3799	0.2616	0.1663	0.0969	0.0514	0.0247	
46	0.8242	0.8166	0.7192	0.5930	0.4595	0.3326	0.2235	0.1385	0.0787	0.0407	0.0191
47	0.8409	0.7815	0.6690	0.5396	0.4087	0.2890	0.1895	0.1146	0.0635	0.0321	
48	0.8262	0.8273	0.7378	0.6173	0.4872	0.3608	0.2492	0.1595	0.0942	0.0509	0.0251
49	0.8465	0.7958	0.6900	0.5653	0.4367	0.3162	0.2133	0.1334	0.0769	0.0406	
50	0.8278	0.8369	0.7550	0.6402	0.5139	0.3885	0.2750	0.1813	0.1108	0.0624	0.0322

Table IV. $\alpha_1(0.7; c, n_t)$ of Closed Type (continued), $c=4(1)24$, $n_t=10(1)50$.

n_t \ c	4 (5)	6 (7)	8 (9)	10 (11)	12 (13)	14 (15)	16 (17)	18 (19)	20 (21)	22 (23)	24
10	0.6968	0.4060	0.2664	0.1685	0.1037	0.0623	0.0368	0.0214	0.0122	0.0069	0.0039
11	0.6095	0.3305	0.2127	0.1326	0.0806	0.0480	0.0281	0.0162	0.0092	0.0052	
12	0.7733	0.5258	0.2664	0.1685	0.1037	0.0623	0.0368	0.0214	0.0122	0.0069	0.0039
13	0.6994	0.4480	0.2127	0.1326	0.0806	0.0480	0.0281	0.0162	0.0092	0.0052	
14	0.8282	0.6240	0.3774	0.1685	0.1037	0.0623	0.0368	0.0214	0.0122	0.0069	0.0039
15	0.7680	0.5500	0.3147	0.1326	0.0806	0.0480	0.0281	0.0162	0.0092	0.0052	
16	0.8676	0.7027	0.4794	0.2601	0.1037	0.0623	0.0368	0.0214	0.0122	0.0069	0.0039
17	0.8203	0.6357	0.4136	0.2132	0.0806	0.0480	0.0281	0.0162	0.0092	0.0052	
18	0.8958	0.7651	0.5691	0.3535	0.1734	0.0623	0.0368	0.0214	0.0122	0.0069	0.0039
19	0.8600	0.7063	0.5044	0.2995	0.1401	0.0480	0.0281	0.0162	0.0092	0.0052	
20	0.9161	0.8144	0.6457	0.4429	0.2516	0.1124	0.0368	0.0214	0.0122	0.0069	0.0039
21	0.8902	0.7638	0.5849	0.3855	0.2099	0.0897	0.0281	0.0162	0.0092	0.0052	
22	0.9306	0.8531	0.7101	0.5251	0.3328	0.1738	0.0712	0.0214	0.0122	0.0069	0.0039
23	0.9132	0.8103	0.6546	0.4674	0.2851	0.1431	0.0563	0.0162	0.0092	0.0052	
24	0.9410	0.8835	0.7636	0.5985	0.4127	0.2425	0.1170	0.0442	0.0122	0.0069	0.0039
25	0.9306	0.8477	0.7140	0.5427	0.3617	0.2049	0.0952	0.0346	0.0092	0.0052	
26	0.9485	0.9073	0.8076	0.6627	0.4884	0.3146	0.1720	0.0770	0.0270	0.0069	0.0039
27	0.9439	0.8777	0.7641	0.6104	0.4363	0.2718	0.1435	0.0620	0.0210	0.0052	
28	0.9538	0.9260	0.8437	0.7180	0.5582	0.3869	0.2333	0.1190	0.0497	0.0163	0.0039
29	0.9539	0.9017	0.8060	0.6700	0.5068	0.3409	0.1990	0.0982	0.0397	0.0125	
30	0.9576	0.9407	0.8731	0.7652	0.6211	0.4570	0.2983	0.1688	0.0807	0.0315	0.0097
31	0.9616	0.9210	0.8407	0.7220	0.5719	0.4094	0.2595	0.1424	0.0659	0.0250	
32	0.9604	0.9522	0.8970	0.8051	0.6769	0.5231	0.3644	0.2243	0.1194	0.0536	0.0197
33	0.9674	0.9363	0.8694	0.7666	0.6308	0.4755	0.3224	0.1929	0.0997	0.0435	
34	0.9624	0.9612	0.9164	0.8386	0.7259	0.5842	0.4296	0.2835	0.1649	0.0828	0.0351
35	0.9718	0.9486	0.8931	0.8048	0.6834	0.5378	0.3858	0.2479	0.1403	0.0685	
36	0.9638	0.9683	0.9322	0.8666	0.7684	0.6397	0.4922	0.3444	0.2156	0.1188	0.0564
37	0.9751	0.9584	0.9125	0.8371	0.7298	0.5955	0.4479	0.3058	0.1866	0.1001	
38	0.9648	0.9738	0.9449	0.8899	0.8049	0.6895	0.5512	0.4052	0.2700	0.1606	0.0839
39	0.9777	0.9663	0.9285	0.8644	0.7703	0.6480	0.5074	0.3647	0.2372	0.1376	
40	0.9655	0.9781	0.9553	0.9093	0.8362	0.7336	0.6058	0.4646	0.3264	0.2073	0.1173
41	0.9796	0.9725	0.9415	0.8873	0.8054	0.6953	0.5634	0.4231	0.2907	0.1803	
42	0.9660	0.9815	0.9636	0.9253	0.8628	0.7724	0.6557	0.5213	0.3834	0.2576	0.1561
43	0.9811	0.9775	0.9522	0.9065	0.8356	0.7374	0.6154	0.4800	0.3457	0.2271	
44	0.9664	0.9842	0.9704	0.9385	0.8853	0.8062	0.7008	0.5748	0.4397	0.3101	0.1994
45	0.9822	0.9815	0.9609	0.9225	0.8615	0.7745	0.6630	0.5342	0.4008	0.2769	
46	0.9667	0.9863	0.9758	0.9494	0.9042	0.8354	0.7411	0.6244	0.4942	0.3636	0.2461
47	0.9831	0.9847	0.9681	0.9359	0.8836	0.8071	0.7061	0.5853	0.4550	0.3284	
48	0.9669	0.9879	0.9803	0.9584	0.9202	0.8607	0.7768	0.6698	0.5462	0.4170	0.2952
49	0.9837	0.9873	0.9739	0.9470	0.9023	0.8355	0.7447	0.6328	0.5075	0.3805	
50	0.9670	0.9892	0.9838	0.9658	0.9336	0.8823	0.8083	0.7111	0.5952	0.4694	0.3456

Table IV. $\alpha_1(0.8; c, n_t)$ of Closed Type (continued), $c=4(1)24$, $n_t=10(1)50$.

n_t \ c	4 (5)	6 (7)	8 (9)	10 (11)	12 (13)	14 (15)	16 (17)	18 (19)	20 (21)	22 (23)	24
10	0. 8991	0. 6950	0. 5715	0. 4578	0. 3590	0. 2765	0. 2097	0. 1571	0. 1163	0. 0853	0. 0620
11	0. 8583	0. 6325	0. 5130	0. 4064	0. 3157	0. 2412	0. 1818	0. 1353	0. 0997	0. 0728	
12	0. 9431	0. 8131	0. 5715	0. 4578	0. 3590	0. 2765	0. 2097	0. 1571	0. 1163	0. 0853	0. 0620
13	0. 9173	0. 7648	0. 5130	0. 4064	0. 3157	0. 2412	0. 1818	0. 1353	0. 0997	0. 0728	
14	0. 9672	0. 8868	0. 7144	0. 4578	0. 3590	0. 2765	0. 2097	0. 1571	0. 1163	0. 0853	0. 0620
15	0. 9517	0. 8523	0. 6631	0. 4064	0. 3157	0. 2412	0. 1818	0. 1353	0. 0997	0. 0728	
16	0. 9803	0. 9318	0. 8145	0. 6118	0. 3590	0. 2765	0. 2097	0. 1571	0. 1163	0. 0853	0. 0620
17	0. 9716	0. 9084	0. 7739	0. 5612	0. 3157	0. 2412	0. 1818	0. 1353	0. 0997	0. 0728	
18	0. 9875	0. 9590	0. 8815	0. 7314	0. 5121	0. 2765	0. 2097	0. 1571	0. 1163	0. 0853	0. 0620
19	0. 9831	0. 9435	0. 8515	0. 6875	0. 4650	0. 2412	0. 1818	0. 1353	0. 0997	0. 0728	
20	0. 9914	0. 9754	0. 9251	0. 8187	0. 6430	0. 4202	0. 2097	0. 1571	0. 1163	0. 0853	0. 0620
21	0. 9898	0. 9654	0. 9038	0. 7836	0. 5985	0. 3781	0. 1818	0. 1353	0. 0997	0. 0728	
22	0. 9935	0. 9852	0. 9531	0. 8798	0. 7465	0. 5544	0. 3389	0. 1571	0. 1163	0. 0853	0. 0620
23	0. 9937	0. 9788	0. 9384	0. 8531	0. 7081	0. 5113	0. 3025	0. 1353	0. 0997	0. 0728	
24	0. 9947	0. 9911	0. 9707	0. 9213	0. 8241	0. 6687	0. 4695	0. 2691	0. 1163	0. 0853	0. 0620
25	0. 9959	0. 9870	0. 9608	0. 9019	0. 7930	0. 6289	0. 4294	0. 2385	0. 0997	0. 0728	
26	0. 9953	0. 9946	0. 9818	0. 9491	0. 8801	0. 7602	0. 5890	0. 3913	0. 2108	0. 0853	0. 0620
27	0. 9972	0. 9921	0. 9753	0. 9353	0. 8561	0. 7260	0. 5495	0. 3552	0. 1857	0. 0728	
28	0. 9957	0. 9967	0. 9887	0. 9673	0. 9194	0. 8301	0. 6907	0. 5106	0. 3213	0. 1631	0. 0620
29	0. 9380	0. 9951	0. 9844	0. 9577	0. 9016	0. 8022	0. 6547	0. 4727	0. 2896	0. 1429	
30	0. 9959	0. 9979	0. 9930	0. 9791	0. 9465	0. 8817	0. 7727	0. 6183	0. 4361	0. 2603	0. 1249
31	0. 9984	0. 9970	0. 9902	0. 9726	0. 9335	0. 8599	0. 7418	0. 5818	0. 4009	0. 2332	
32	0. 9960	0. 9986	0. 9957	0. 9867	0. 9647	0. 9188	0. 8363	0. 7093	0. 5457	0. 3674	0. 2083
33	0. 9987	0. 9982	0. 9939	0. 9825	0. 9556	0. 9023	0. 8110	0. 6770	0. 5100	0. 3356	
34	0. 9960	0. 9991	0. 9973	0. 9916	0. 9770	0. 9449	0. 8840	0. 7842	0. 6436	0. 4751	0. 3056
35	0. 9988	0. 9989	0. 9962	0. 9886	0. 9707	0. 9327	0. 8641	0. 7561	0. 6099	0. 4412	
36	0. 9961	0. 9994	0. 9983	0. 9947	0. 9850	0. 9630	0. 9190	0. 8425	0. 7268	0. 5762	0. 4085
37	0. 9989	0. 9993	0. 9976	0. 9927	0. 9806	0. 9542	0. 9037	0. 8193	0. 6967	0. 5426	
38	0. 9961	0. 9995	0. 9990	0. 9966	0. 9903	0. 9753	0. 9441	0. 8868	0. 7948	0. 6659	0. 5095
39	0. 9990	0. 9995	0. 9885	0. 9954	0. 9873	0. 9691	0. 9326	0. 8684	0. 7690	0. 6346	
40	0. 9961	0. 9996	0. 9994	0. 9979	0. 9938	0. 9837	0. 9618	0. 9198	0. 8485	0. 7421	0. 6031
41	0. 9990	0. 9997	0. 9991	0. 9971	0. 9917	0. 9793	0. 9534	0. 9055	0. 8273	0. 7143	
42	0. 9961	0. 9997	0. 9996	0. 9987	0. 9960	0. 9893	0. 9741	0. 9438	0. 8899	0. 8047	0. 6858
43	0. 9990	0. 9998	0. 9994	0. 9981	0. 9947	0. 9862	0. 9681	0. 9330	0. 8728	0. 7809	
44	0. 9961	0. 9997	0. 9997	0. 9992	0. 9974	0. 9930	0. 9826	0. 9611	0. 9210	0. 8545	0. 7560
45	0. 9990	0. 9998	0. 9996	0. 9988	0. 9965	0. 9909	0. 9783	0. 9530	0. 9077	0. 8348	
46	0. 9961	0. 9997	0. 9998	0. 9995	0. 9984	0. 9954	0. 9884	0. 9732	0. 9440	0. 8931	0. 8139
47	0. 9990	0. 9998	0. 9998	0. 9993	0. 9978	0. 9940	0. 9853	0. 9674	0. 9338	0. 8773	
48	0. 9961	0. 9997	0. 9999	0. 9997	0. 9990	0. 9970	0. 9923	0. 9818	0. 9607	0. 9225	0. 8602
49	0. 9990	0. 9999	0. 9999	0. 9995	0. 9986	0. 9961	0. 9902	0. 9775	0. 9530	0. 9100	
50	0. 9961	0. 9997	0. 9999	0. 9998	0. 9993	0. 9981	0. 9949	0. 9876	0. 9726	0. 9444	0. 8964

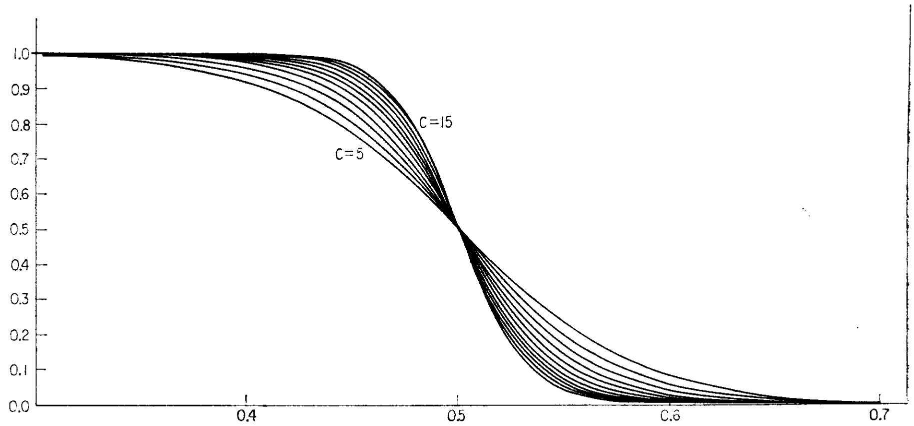

Fig. I. Curves for the probabilities $\alpha_3^*(p)$ of Open Types.

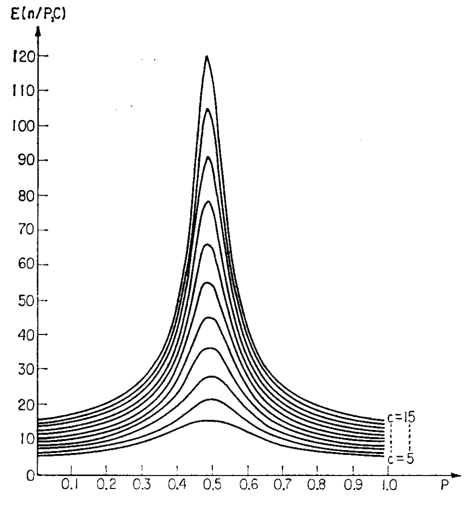

Fig. II. Curves for the average sampling favorable pair numbers of Open Types.

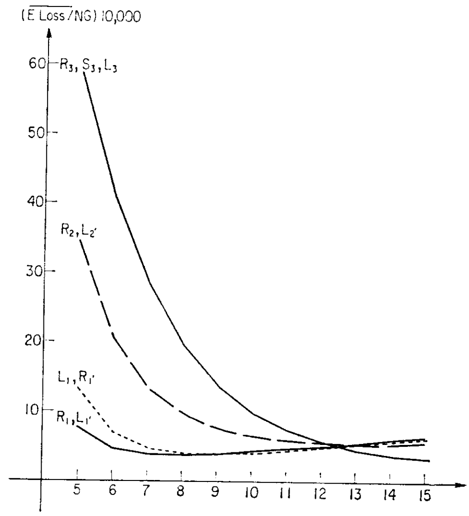

Fig. III. $\overline{\text{E LOSS}}$/NG of Open Types.

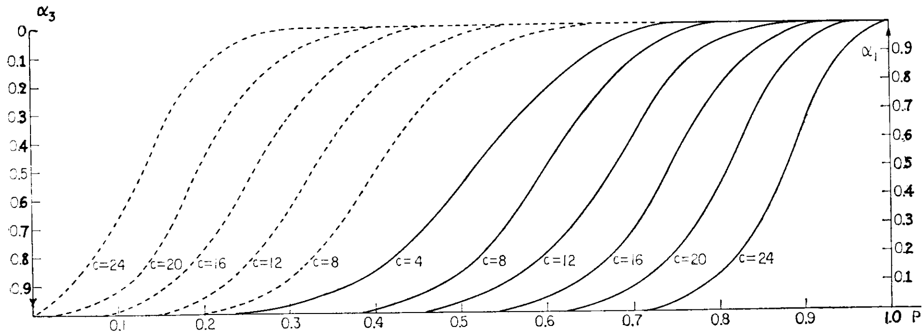

Fig. IV. Monograph of $\alpha_1(p; c, n_t)$ and $\alpha_3(p; c, n_t)$ for $n_t=30$.

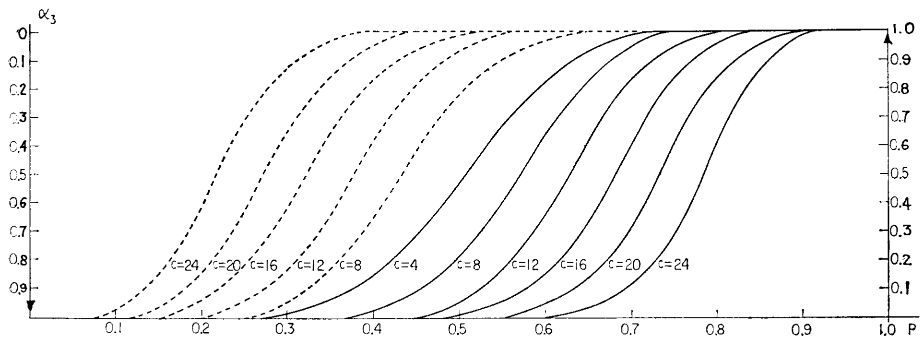

Fig. IV. Monograph of $\alpha_1(p; c, n_t)$ and $\alpha_3(p; c, n_t)$ for $n_t=40$.

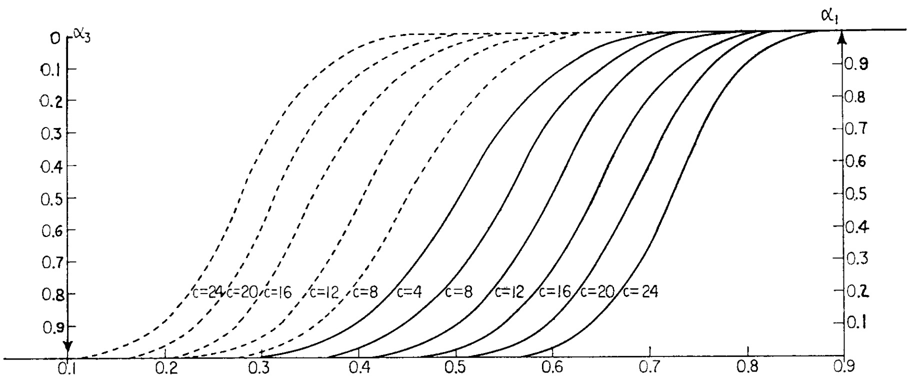

Fig. IV. Monograph of $\alpha_1(p; c, n_t)$ and $\alpha_3(p; c, n_t)$ for $n_t=50$.

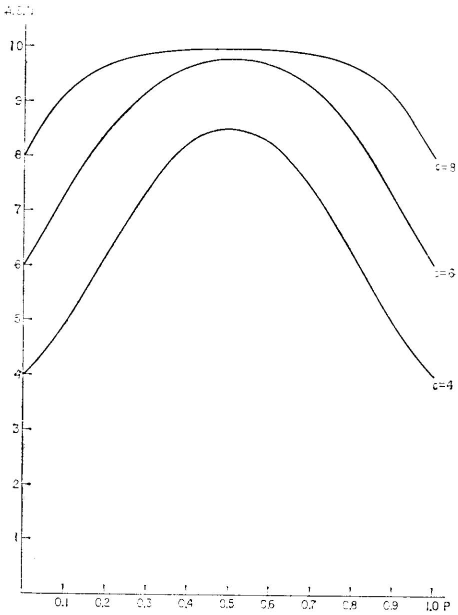

Fig. V_1. Curves for ASN of Closed Type, Unwedged Design (continued), $n_t=10$, $c=4(2)\ 8$.

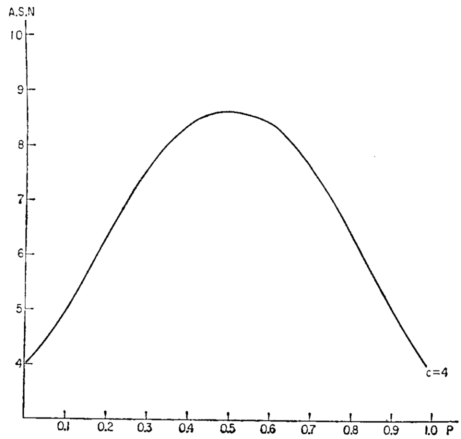

Fig. V_2. Curves for ASN of Closed Type, Wedged Design (continued), $n_t=10$, $c=4$.

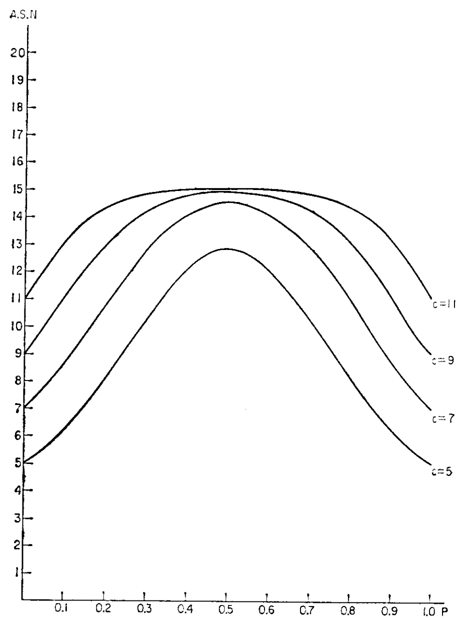

Fig. V_1. Curves for ASN of Closed Type, Unwedged Design (continued), $n_t=15$, $c=5(2)11$.

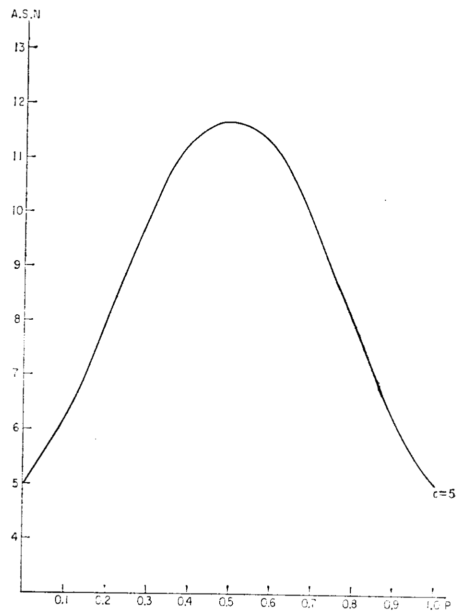

Fig. V_2. Curves for ASN of Closed Type, Wedged Design (continued), $n_t=15$, $c=5$.

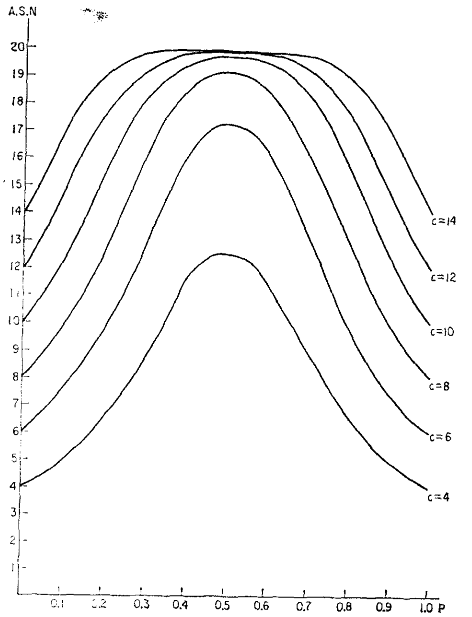

Fig. V_1. Curves for ASN of Closed Type, Unwedged Design (continued), $n_t=20$, $c=4(2)14$.

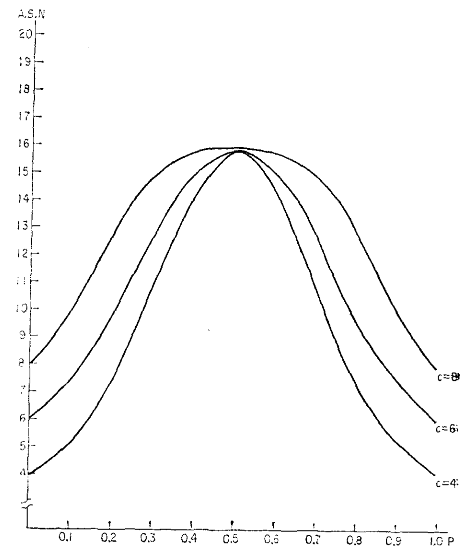

Fig. V_2. Curves for ASN of Closed Type, Wedged Design (continued), $n_t=20$, $c=4(2)8$.

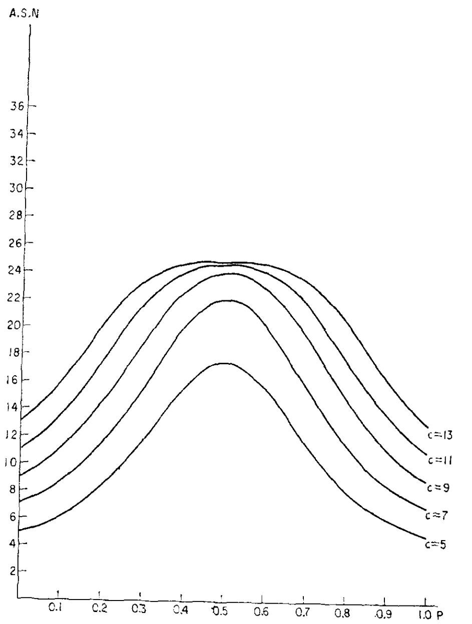

Fig. V_1. Curves for ASN of Closed Type, Unwedged Design (continued), $n_t=25$, $c=5(2)13$.

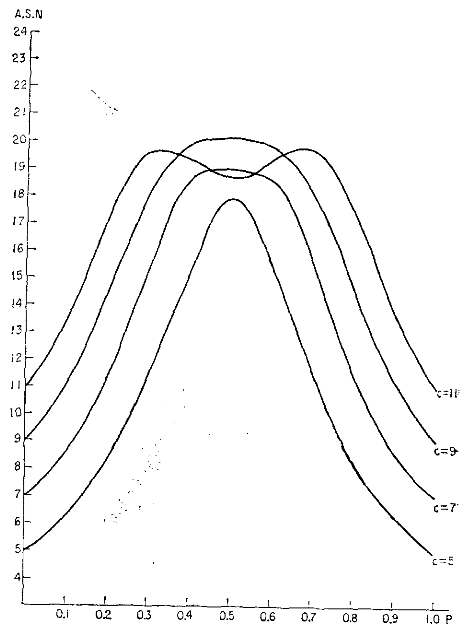

Fig. V_2. Curves for ASN of Closed Type, Wedged Design (continued), $n_t=25$, $c=5(2)11$.

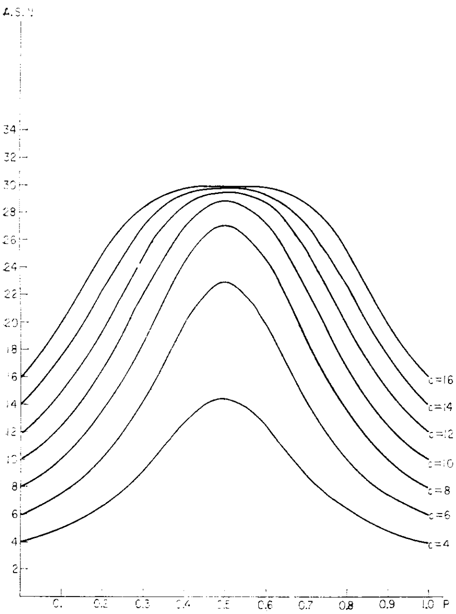

Fig. V_1. Curves for ASN of Closed Type, Unwedged Design (continued), $n_t=30$, $c=4(2)16$.

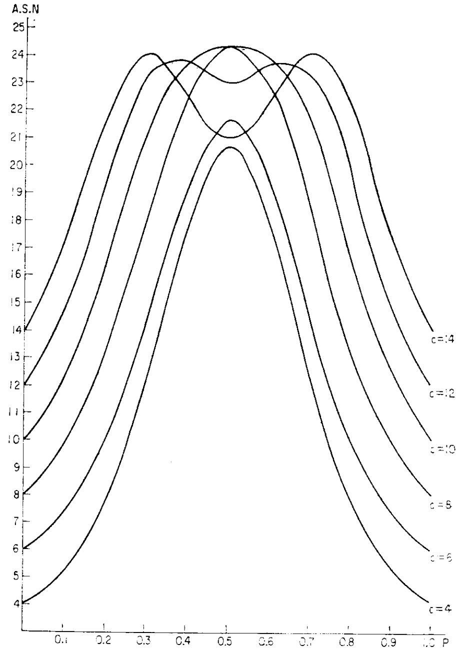

Fig. V_2. Curves for ASN of Closed Type, Wedged Design (continued), $n_t=30$, $c=4(2)14$.

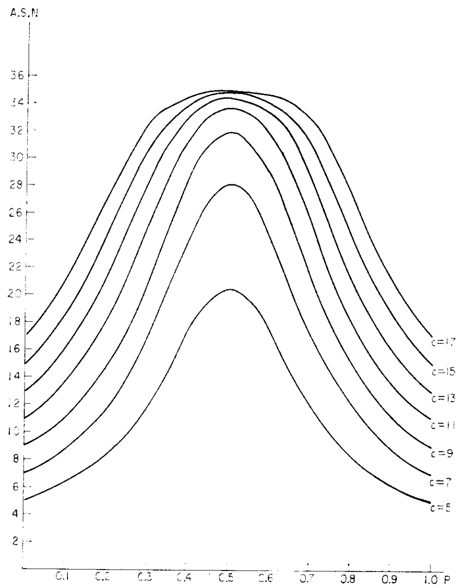

Fig. V_1. Curves for ASN of Closed Type, Unwedged Design (continued), $n_t=35$, $c=5(2)17$.

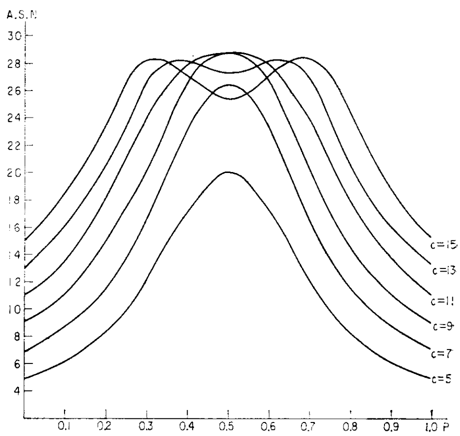

Fig. V_2. Curves for ASN of Closed Type, Wedged Design (continued), $n_t=35$, $c=5(2)15$.

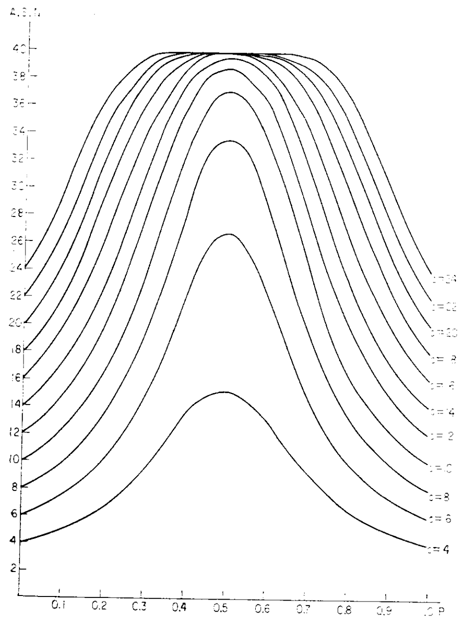

Fig. V_1. Curves for ASN of Closed Type, Unwedged Design (continued), $n_t=40$, $c=4(2)24$.

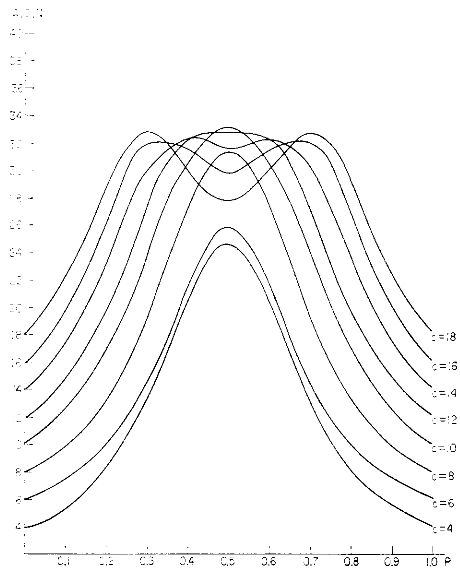

Fig. V_2. Curves for ASN of Closed Type, Wedged Design (continued), $n_t=40$, $c=4(2)18$.

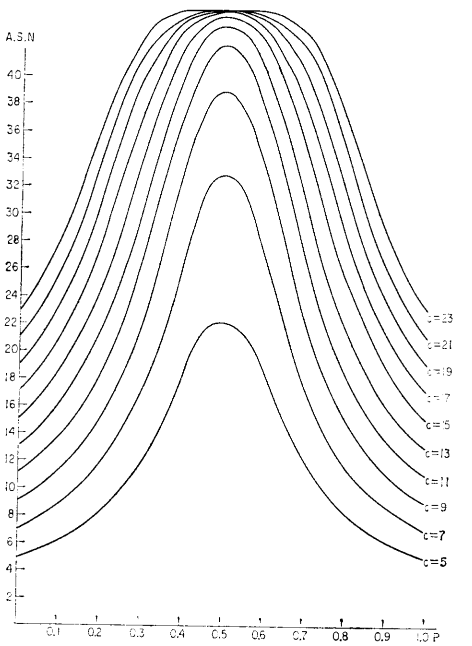

Fig. V_1. Curves for ASN of Closed Type, Unwedged Design (continued), $n_t=45$, $c=5(2)23$.

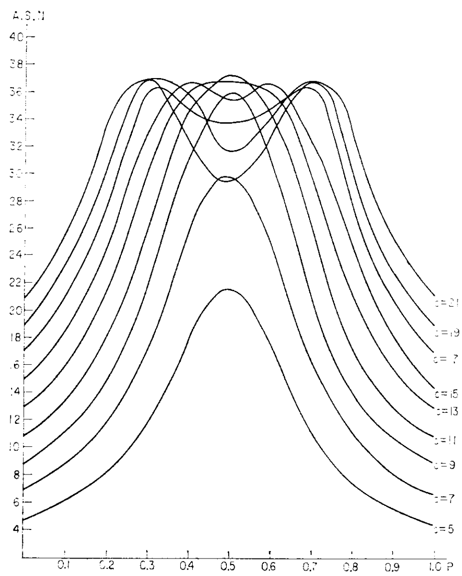

Fig. V_2. Curves for ASN of Closed Type, Wedged Design (continued), $n_t=45$, $c=5(2)21$.

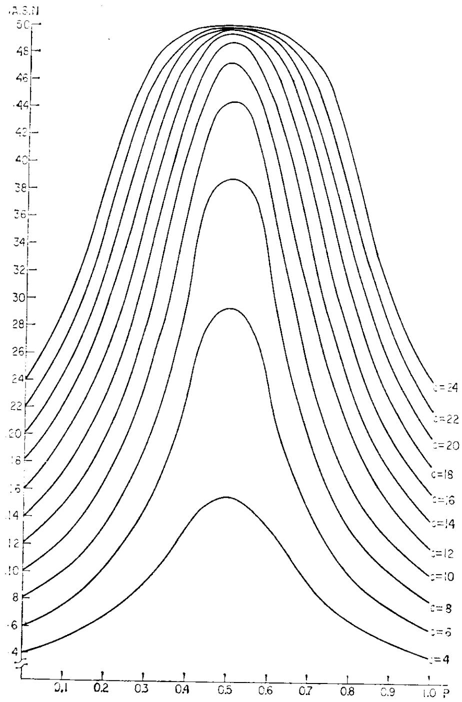

Fig. V_1. Curves for ASN of Closed Type, Unwedged Design (continued), $n_t=50,\ c=4(2)24$.

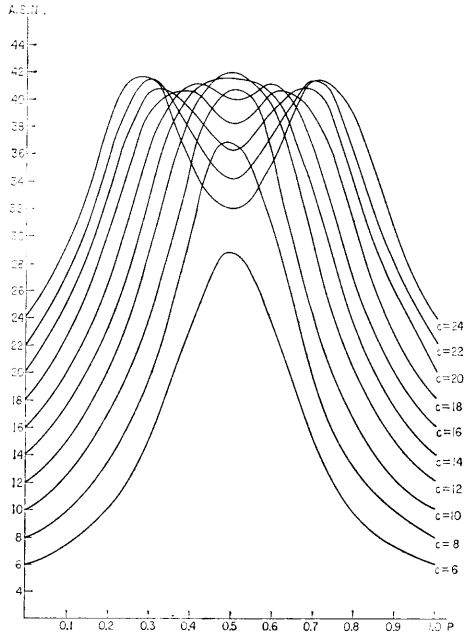

Fig. V_2. Curves for ASN of Closed Type, Wedged Design (continued), $n_t=50,\ c=6(2)24$.

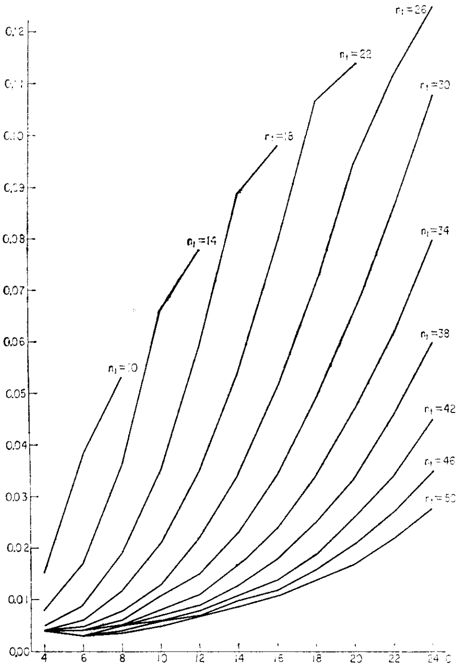

Fig. VI_1. $\overline{\text{E LOSS}}$/NG, Unwedged Design, Type R_1 $(c<n_t)$.

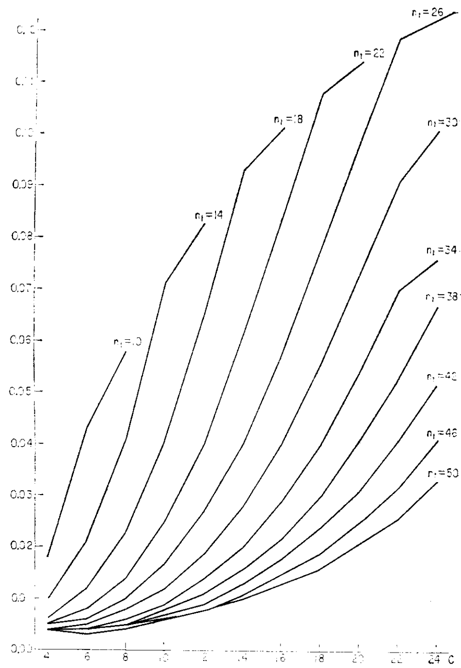

Fig. VI_1. $\overline{\text{E LOSS}}$/NG, Unwedged Design, Type S_1 $(c<n_t)$.

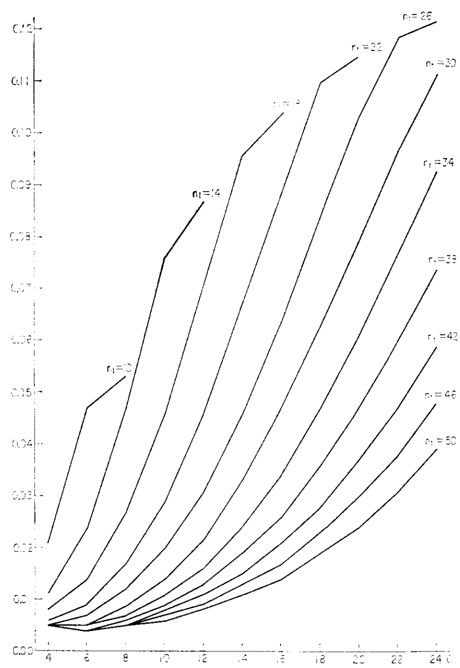

Fig. VI_1. $\overline{\text{E LOSS}}$/NG, Unwedged Design, Type L_1 $(c<n_t)$.

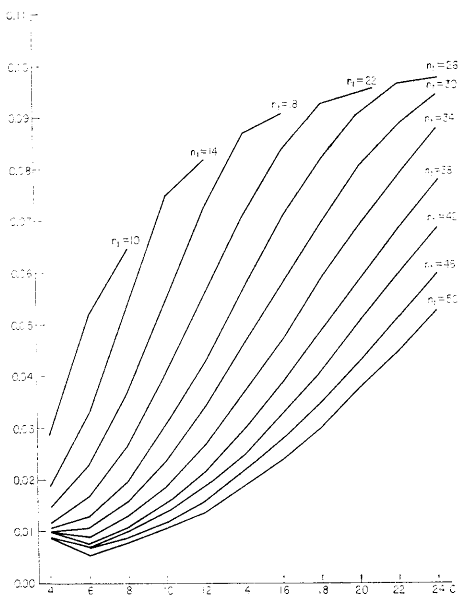

Fig. VI_1. $\overline{\text{E LOSS}}$/NG, Unwedged Design, Type R_2 $(c<n_t)$.

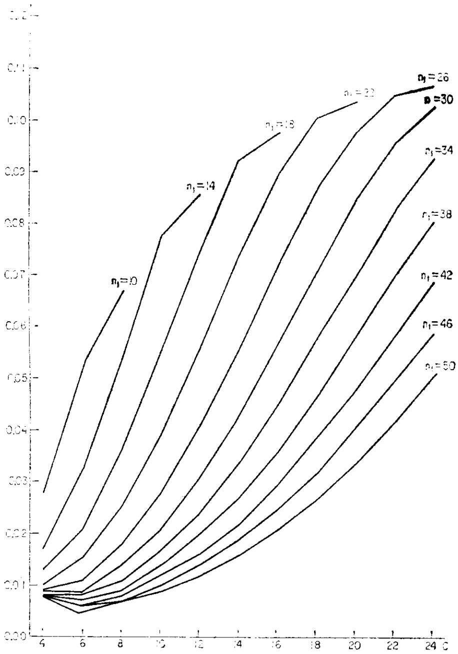

Fig. VI_1. $\overline{\text{E LOSS}}$/NG, Unwedged Design, Type S_2 $(c<n_t)$.

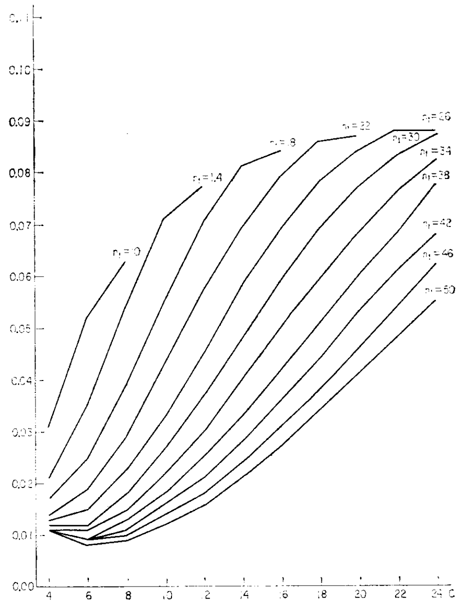

Fig. VI_1. $\overline{\text{E LOSS}}$/NG, Unwedged Design, Type L_2 $(c<n_t)$.

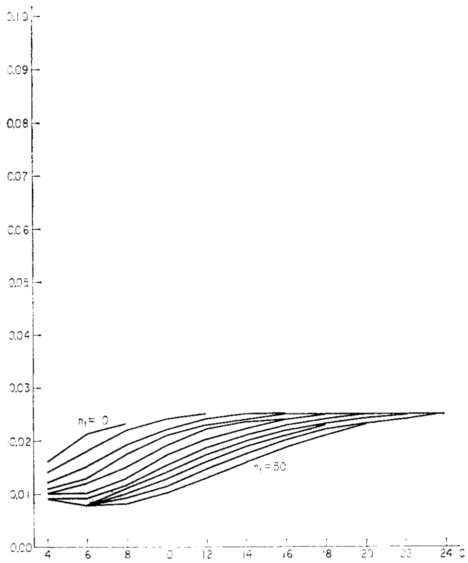

Fig. VI_1. $\overline{\text{E LOSS}}$/NG, Unwedged Design, Type R_3 $(c<n_t)$.

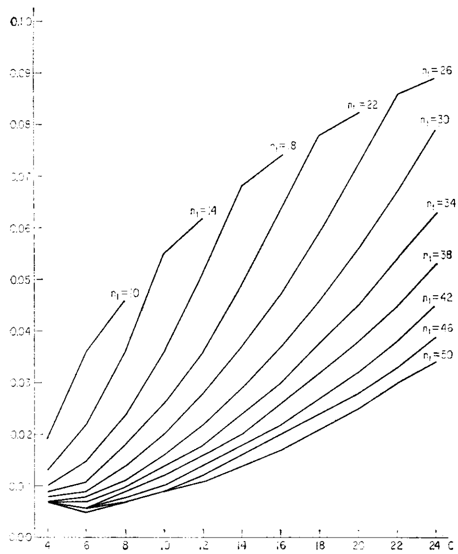

Fig. VI_1. $\overline{\text{E LOSS}}$/NG, Unwedged Design, Type S_4 $(c<n_t)$.

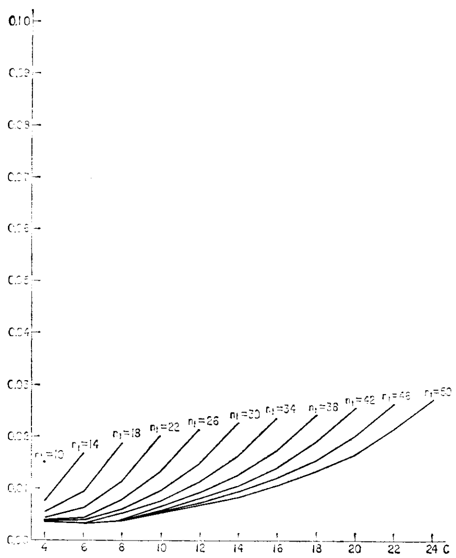

Fig. VI₂. $\overline{\text{E LOSS}}$/NG, Wedged Design, Type R_1 $(2c<n_t)$.

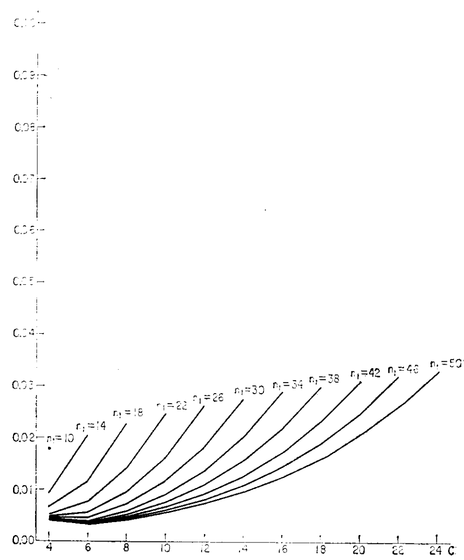

Fig. VI₂. $\overline{\text{E LOSS}}$/NG, Wedged Design, Type S_1 $(2c<n_t)$.

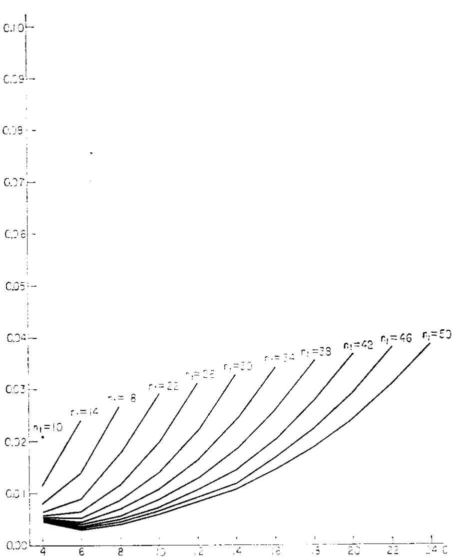

Fig. VI₂. $\overline{\text{E LOSS}}$/NG, Wedged Design, Type L_1 $(2c<n_t)$.

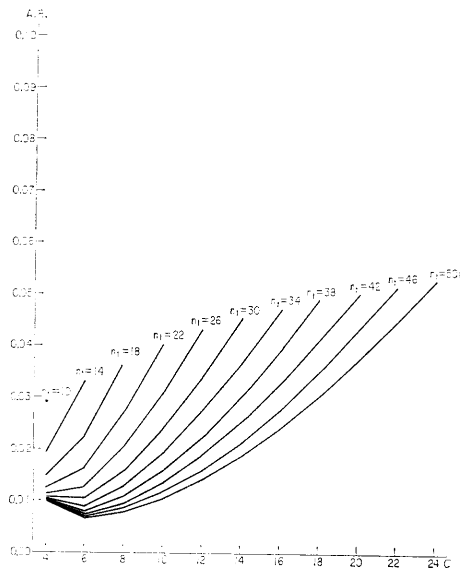

Fig. VI₂. $\overline{\text{E LOSS}}$/NG, Wedged Design, Type R_2 $(2c<n_t)$.

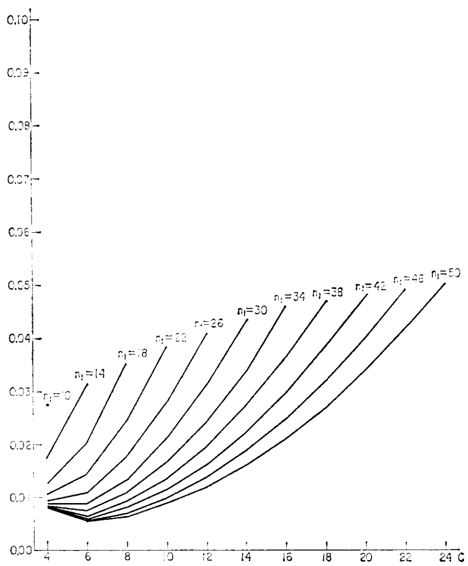

Fig. VI_2. $\overline{\text{E LOSS}}$/NG, Wedged Design, Type S_2 ($2c<n_t$).

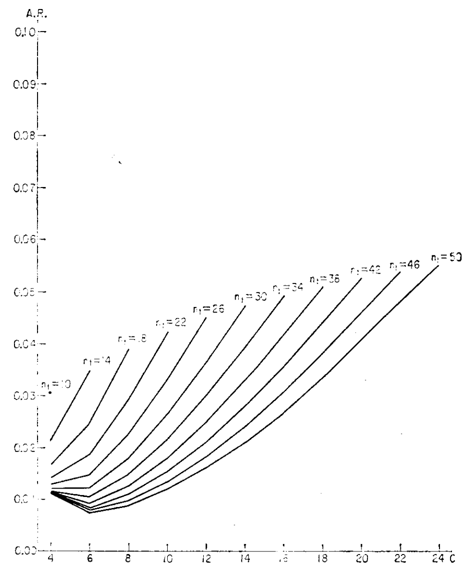

Fig. VI_2. $\overline{\text{E LOSS}}$/NG, Wedged Design, Type L_2 ($2c<n_t$).

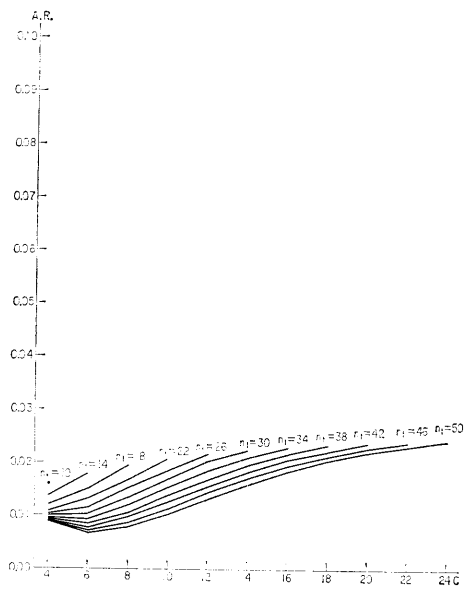

Fig. VI_2. $\overline{\text{E LOSS}}$/NG, Wedged Design, Type R_3 ($2c<n_t$).

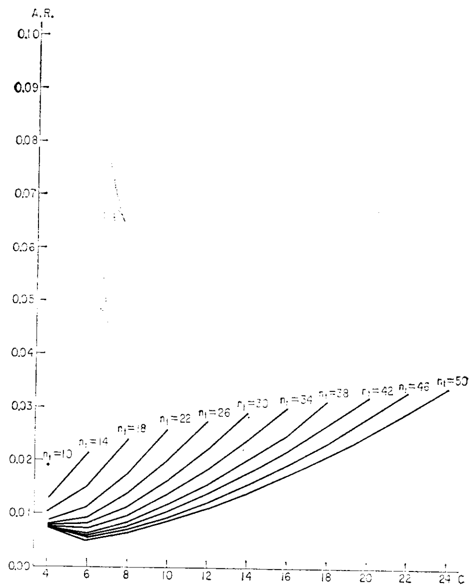

Fig. VI_2. $\overline{\text{E LOSS}}$/NG, Wedged Design, Type S_4 ($2c<n_t$).

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