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INFERENCE FOR A LINEAR HYPOTHESIS MODEL USING TWO PRELIMINARY TESTS OF SIGNIFICANCE

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§ 1. Introduction.

1.1. Related Papers and Object of the Present Investigation.

Paull (1948, 1950) studied the effect on size and power of a test procedure using the estimator obtained by the pooling procedure of Bancroft (1944). His investigations were concerned with the component of variance model (Model II). Bechhofer (1951a, b) made a similar investigation for a linear hypothesis model (Model I). Later Bozivich, Bancroft and Hartley (1956a, b) extended the work of Paull and Bechhofer to cover all important combinations of even values of degrees of freedom in the analysis of variance. Srivastava and Bozivich (1961) generalised the studies of Bozivich, Bancroft and Hartley (1956a, b) by applying two preliminary tests of significance for pooling mean squares in Model II and studied the power function of a sometimes pool test procedure (SPTP) for testing the hypothesis of no treatment effects. In the present investigation we have studied a sometimes pool test procedure for testing the hypothesis of no treatment effects in a linear hypothesis model.

1.2. Statement of the Problem.

Given four independent mean squares: V_4 (treatment mean square), V_3 (true error mean square), V_2 (doubtful error II mean square) and V_1 (doubtful error I mean square) based on n_4, n_3, n_2 and n_1 degrees of freedom respectively, we are interested in testing the hypothesis $H_0: E(V_4)=E(V_3)$ against the one sided alternative $H_1: E(V_4) > E(V_3)$ when given a priori that $E(V_1)$ and/or $E(V_2) \geqq E(V_3)$. Now one of the ways of testing H_0 against H_1 is to compare V_4 and V_3 by the F-satistic and to reject H_0 if the observed value of the ratio V_4/V_3 turns out to be significant. It is, however, suspected though not known with certainty that $E(V_1)$ and/or $E(V_2)$ might be equal to $E(V_3)$, in which case we can construct other test statistic(s) for testing H_0 by considering appropriate combination(s) of V_1 and/or V_2 with V_3.

1.3. Description of Incompletely Specified Models.

Let us consider production of a certain type of goods in a factory and assume that a number of such units are manufactured with the help of p machines each running on q days for r hours daily. After every hour s units are sampled and the quality characteristic is measured. Let y_{ijkm} denote the observation on m-th unit during the k-th hour of j-th day produced by i-th machine. Then the sample observations may be represented by the model

(1.3.1) $$y_{ijkm}=\mu+\alpha_i+\beta_{j(i)}+\lambda_{k(ij)}+\varepsilon_{ijkm},$$

where $i=1,2,\cdots,p$; $j=1,2,\cdots,q$; $k=1,2,\cdots,r$; $m=1,2,\cdots,s$; $\mu+\alpha_i+\beta_{j(i)}+\lambda_{k(ij)}$ is the population average of the measurements of units produced during the k-th hour of j-th day by i-th machine, $\mu+\alpha_i+\beta_{j(i)}$ is the average of the measurements of units produced on j-th day by the i-th machine such that $\sum_k \lambda_{k(ij)}=0$ for all i and j, $\mu+\alpha_i$ is the average of the measurements of units produced by i-th machine such that $\sum_j \beta_{j(i)}=\sum_{jk}\lambda_{k(ij)}=0$ for all i. Likewise, μ is the average of $\mu+\alpha_i$ such that $\sum_i \alpha_i=0$. Finally, ε_{ijkm} are random variables distributed as $N(0,\sigma^2)$. The analysis of variance resulting from model (1.3.1) is shown in the table given below:

Table 1.1. ANOVA Model I. Nested classification.

Source of Variation	Degrees of Freedom	Mean Square	Expected Mean Square
Between Machines	$n_4=p-1$	V_4	$\sigma_4^2=\sigma^2+qrs\,\sigma_\alpha^2$
Within hours	$n_3=pqr(s-1)$	V_3	$\sigma_3^2=\sigma^2$
Between days within machines	$n_2=p(q-1)$	V_2	$\sigma_2^2=\sigma^2+rs\sigma_\beta^2$
Between hours within machines and days	$n_1=pq(r-1)$	V_1	$\sigma_1^2=\sigma^2+s\sigma_\lambda^2$

where the finite population variances σ_α^2, σ_β^2 and σ_λ^2 are defined by

$$\sigma_\alpha^2=\frac{1}{p-1}\sum_i \alpha_i^2,\quad \sigma_\beta^2=\frac{1}{p(q-1)}\sum_{ij}\beta_{j(i)}^2\quad\text{and}\quad \sigma_\lambda^2=\frac{1}{pq(r-1)}\sum_{ijk}\lambda_{k(ij)}^2.$$

Now, if $\sigma_\alpha^2\geqq 0$, $\sigma_\beta^2\geqq 0$, $\sigma_\lambda^2\geqq 0$, then we have from (1.3.1),

(1.3.2) $$y_{ijkm}=\mu+\alpha_i+\beta_{j(i)}+\lambda_{k(ij)}+\varepsilon_{ijkm},\qquad \text{for}\quad \sigma_\beta^2>0,\ \sigma_\lambda^2>0;$$

(1.3.3) $$y_{ijkm}=\mu+\alpha_i+\beta_{j(i)}+\varepsilon_{ijkm},\qquad \text{for}\quad \sigma_\beta^2>0,\ \sigma_\lambda^2=0;$$

(1.3.4) $$y_{ijkm}=\mu+\alpha_i+\lambda_{k(ij)}+\varepsilon_{ijkm},\qquad \text{for}\quad \sigma_\beta^2=0,\ \sigma_\lambda^2>0;$$

(1.3.5) $$y_{ijkm}=\mu+\alpha_i+\varepsilon_{ijkm},\qquad \text{for}\quad \sigma_\beta^2=0,\ \sigma_\lambda^2=0.$$

In these cases (1.3.1) is called an incompletely specified model.

However, if it is known with certainty that

(1.3.6) $$\sigma_\beta^2>0\quad\text{and}\quad\sigma_\lambda^2>0,$$

then the appropriate model is (1.3.2) and we call (1.3.1) a completely specified full model (CSFM). On the other hand, if it is known with certainty that

(1.3.7) $$\sigma_\beta^2=0\quad\text{and}\quad\sigma_\lambda^2=0,$$

then the appropriate model is (1.3.5) and in this situation (1.3.1) is a completely specified model.

1.4. Mathematical Formulation of the Pooling Procedure.

In the terminology of the area of an incompletely specified model the mean squares 'Between days within machines' and 'Between hours within machines and days' are called doubtful error mean squares. The mean square corresponding to 'Within hours' is called the true error mean square and it is possible to write Table 1.1 as follows:

Table 1.2. Analysis of Variance. Model I with $\sigma_\beta^2>0$, $\sigma_\lambda^2>0$.

Source of Variation	Degrees of Freedom	Mean Square	Expected Mean Square
Treatments	n_4	V_4	$\sigma_4^2=\sigma_3^2\left(1+\frac{2\lambda_4}{n_4}\right)$
True Error	n_3	V_3	$\sigma_3^2=\sigma_3^2$
Doubtful Error II	n_2	V_2	$\sigma_2^2=\sigma_3^2\left(1+\frac{2\lambda_2}{n_2}\right)$
Doubtful Error I	n_1	V_1	$\sigma_1^2=\sigma_3^2\left(1+\frac{2\lambda_1}{n_1}\right)$

We assume that the true error mean square is distributed as $\chi_3^2\sigma_3^2/n_3$, where χ_3^2 is the central chi-square statistic based on n_3 degrees of freedom and the remaining three mean squares V_i's $(i=1, 2, 4)$ are distributed as $\chi_i'^2\sigma_3^2/n_i$, where $\chi_i'^2$ is the non-central chi-square statistic based on n_i degrees of freedom and the noncentrality parameter is $\lambda_i=\frac{1}{2}n_i(\sigma_i^2-\sigma_3^2)/\sigma_3^2$. We are interested in testing the hypothesis $H_0: \sigma_4^2=\sigma_3^2$ (i.e. $\lambda_4=0$) against the one-sided alternative $H_1: \sigma_4^2>\sigma_3^2$ (i.e. $\lambda_4>0$) when there is a possibility that λ_1 or λ_2 or both may be equal to zero.

The sometimes pool test procedure which we have studied for testing H_0 consists of the four non-overlaping critical regions $R_i's$ $(i=1, 2, 3, 4)$, $R_i\cap R_j=\phi$ (null set) for $i\neq j$, given by

$$R_1: \ (V_1/V_3)<F_1, \ (V_2/V_{13})<F_2, \ \text{and} \ (V_4/V_{123})\geqq F_3;$$
$$R_2: \ (V_1/V_3)\geqq F_1, \ (V_2/V_3)\geqq F_4, \ \text{and} \ (V_4/V_3)\geqq F_5;$$
$$R_3: \ (V_1/V_3)\geqq F_1, \ (V_2/V_3)<F_4, \ \text{and} \ (V_4/V_{23})\geqq F_6;$$
$$R_4: \ (V_1/V_3)<F_1, \ (V_2/V_{13})\geqq F_2, \ \text{and} \ (V_4/V_{13})\geqq F_7:$$

where

$$V_{ij}=(n_iV_i+n_jV_j)/n_{ij}, \qquad V_{123}=(n_1V_1+n_2V_2+n_3V_3)/n_{123},$$
$$n_{ij}=n_i+n_j, \qquad n_{123}=n_1+n_2+n_3, \qquad F_1=F(n_1, n_3; \alpha_1),$$
$$F_2=F(n_2, n_{13}; \alpha_2), \quad F_3=F(n_4, n_{123}; \alpha_3), \quad F_4=F(n_2, n_3; \alpha_4),$$
$$F_5=F(n_4, n_3; \alpha_5), \quad F_6=F(n_4, n_{23}; \alpha_6), \quad F_7=F(n_4, n_{13}; \alpha_7),$$

and $F(n_i, n_j; \alpha_k)$ refers to the upper $100\alpha_k\%$ point of the F-distribution with the numerator degrees of freedom n_i and the denominator degrees of freedom n_j. In the present study we have investigated the power and size disturbances of the above sometimes pool test procedure as a test of H_0, and on the basis of theoretical and empirical results we have inferred for the advisability or otherwise of using the proposed test procedure.

It may be remarked here that the completely hierarchical classification is not the only analysis of variance situation to which our results will be applicable. As a matter of fact, they are applicable as well to completely crossed and nested factorial experiments with at least three factors, all factors being fixed.

§ 2. Formulas for Power Using Patnaik's Approximation.

Let $P(R_i)$ $(i=1, 2, 3, 4)$, denote the probability of the event (critical region) R_i. Then the probability P of rejecting H_0: $\lambda_4=0$ i. e. the power of the proposed test is given by

$$P=P\Big(\bigcup_{i=1}^{4} R_i\Big)=\sum_{i=1}^{4} P(R_i)\,, \quad \text{since} \quad R_i \cap R_j=\phi \qquad \text{for} \quad i \neq j\,.$$

It is to be noted here that the probability P (power of the test) is a function of fourteen parameters, namely, 4 degrees of freedom n_i, n_2, n_3, n_4 ; 3 noncentrality parameters λ_1, λ_2, λ_4 ; 7 levels of significance α_1, α_2, α_4 (preliminary levels), α_3, α_5, α_6, α_7 (final levels). In the special case when $\lambda_4=0$, the probability P will be equal to the size of the test.

2.1. Integral Expressions for Power Components.

We shall now derive expressions for power components $P(R_i)=P_i$ $(i=1, 2, 3, 4)$, say. Using Patnaik's (1949) approximation to the noncentral chi-square, we get the joint density of the four independent mean squares as follows;

$$h(V_1, V_2, V_3, V_4)=K^* V_1^{\frac{1}{2}\nu_1-1} V_2^{\frac{1}{2}\nu_2-1} V_3^{\frac{1}{2}n_3-1} V_4^{\frac{1}{2}\nu_4-1} \times \exp\left\{\frac{(-1)}{2\sigma_3^2}\left(\frac{n_1 V_1}{c_1}+\frac{n_2 V_2}{c_2}+n_3 V_3+\frac{n_4 V_4}{c_4}\right)\right\},$$

where K^* is a constant and is independent of V_i's $(i=1, 2, 3, 4)$, $\nu_i=n_i+4\lambda_i^2/(n_i+4\lambda_i)$ and $c_i=1+2\lambda_i/(n_i+2\lambda_i)$.

Introducing the variates

$$u_1=\frac{n_4 V_4}{n_3 V_3 c_4}\,, \quad u_2=\frac{n_2 V_2}{n_3 V_3 c_2}\,, \quad u_3=\frac{n_1 V_1}{n_3 V_3 c_1} \quad \text{and} \quad w=\frac{n_3 V_3}{2\sigma_3^2}\,,$$

and integrating out w from 0 to ∞ we obtain the joint density of the new variates u_1, u_2, u_3 as follows:

$$f(u_1, u_2, u_3)=K\,\frac{u_1^{a_4-1} u_2^{a_2-1} u_3^{a_1-1}}{(1+u_1+u_2+u_3)^{a_{1234}}}\,, \tag{2.1.1}$$

where

$$K=\frac{\Gamma a_{1234}}{\Gamma a_1 \Gamma a_2 \Gamma a_3 \Gamma a_4}\,, \quad a_i=\frac{1}{2}\,\nu_i\ (i=1, 2, 4)\,, \quad a_3=\frac{1}{2}\,n_3$$

and $a_{1234}=a_1+a_2+a_3+a_4$.

The limits of integration of the new variates are as follows:

For P_1

$$0 \leqq u_3 \leqq a, \quad 0 \leqq u_2 \leqq b(1+c_1u_3), \quad c(1+c_1u_3+c_2u_2) \leqq u_1 \leqq \infty ;$$

For P_2

$$a \leqq u_3 \leqq \infty, \quad d \leqq u_2 \leqq \infty, \quad e \leqq u_1 \leqq \infty ;$$

For P_3

$$a \leqq u_3 \leqq \infty, \quad 0 \leqq u_2 \leqq d, \quad f(1+c_2u_2) \leqq u_1 \leqq \infty ;$$

For P_4

$$0 \leqq u_3 \leqq a, \quad b(1-c_1u_3) \leqq u_2 \leqq \infty, \quad g(1+c_1u_3) \leqq u_1 \leqq \infty ;$$

where

$$a = \frac{n_1F_1}{n_3c_1}, \quad b = \frac{n_2F_2}{n_{13}c_2}, \quad c = \frac{n_4F_3}{n_{123}c_4}, \quad d = \frac{n_2F_4}{n_3c_2},$$

$$e = \frac{n_4F_5}{n_3c_4}, \quad f = \frac{n_4F_6}{n_{23}c_4} \quad \text{and} \quad g = \frac{n_4F_7}{n_{13}c_4}.$$

Now, the four power components P_i's $(i=1, 2, 3, 4)$ can be written as follows:

(2.1.2) $$P_1 = \int_{u_3=0}^{a} \int_{u_2=0}^{b(1-c_1u_3)} \int_{u_1=c(1+c_1u_3+c_2u_2)}^{\infty} f(u_1, u_2, u_3)\, du_3 du_2 du_1 ;$$

(2.1.3) $$P_2 = \int_{u_3=a}^{\infty} \int_{u_2=d}^{\infty} \int_{u_1=e}^{\infty} f(u_1, u_2, u_3)\, du_3 du_2 du_1 ;$$

(2.1.4) $$P_3 = \int_{u_3=a}^{\infty} \int_{u_2=0}^{d} \int_{u_1=f(1+c_2u_2)}^{\infty} f(u_1, u_2, u_3)\, du_3 du_2 du_1 ;$$

(2.1.5) $$P_4 = \int_{u_3=0}^{a} \int_{u_2=b(1-c_1u_3)}^{\infty} \int_{u_1=g(1+c_1u_3)}^{\infty} f(u_1, u_2, u_3)\, du_3 du_2 du_1 .$$

2.2. Series Formulas.

In deriving series formulas for P_i's $(i=1, 2, 3, 4)$ we assume that ν_1, ν_2, n_3 and ν_4 are even integers. For convenience we shall first evaluate P_2. From (2.1.3) and (2.1.1) we get

(2.2.1) $$P_2 = K \int_{u_3=a}^{\infty} \int_{u_2=d}^{\infty} \int_{u_1=e}^{\infty} \frac{u_1^{a_4-1} u_2^{a_2-1} u_3^{a_1-1} du_3 du_2 du_1}{(1+u_1+u_2+u_3)^{a_{1234}}} .$$

If we integrate (2.2.1) with respect to u_1 by making use of the transformation $t=(1+u_2+u_3)/(1+u_1+u_2+u_3)$, then we obtain

$$P_2 = KS_i \int_a^{\infty} \int_d^{\infty} \frac{(1+u_2+u_3)^i u_2^{a_2-1} u_3^{a_1-1} du_3 du_2}{(1+e+u_2+u_3)^{a_{123}+i}},$$

where

$$S_i = \sum_{i=0}^{a_4-1} (-1)^i \binom{a_4-1}{i} \Big/ (a_{123}+i) \quad \text{and} \quad a_{123} = \frac{\nu_1+\nu_2+n_3}{2}.$$

We expand $(1+u_2+u_3)^i$ by means of the binominal theorem in terms of $(1+u_3)$ and u_2, make the transformation $z=(1+e+u_3)/(1+e+u_2+u_3)$, integrate out z and obtain

$$P_2 = KS_iS_{jk} \int_a^{\infty} \frac{u_3^{a_1-1}(1+u_3)^{i-j}(1+e+u_3)^k du_3}{(1+d+e+u_3)^{a_{13}+i-j+k}},$$

where

$$S_{jk}=\sum_{j=0}^{i}\binom{i}{j}\sum_{k=0}^{a_2+j-1}\frac{(-1)^k\binom{a_2-j-1}{k}}{a_{13}-i-j+k}\,,\qquad a_{13}=\frac{1}{2}(\nu_1+n_3)\,.$$

Expanding $(1+u_3)^{i-j}$ by binominal theorem we obtain

$$P_2=KS_{ijkm}\int_a^\infty\frac{u_3^{a_1+m-1}(1+e+u_3)^k}{(1+d-e+u_3)^{a_{13}+i-j+k}}\,du_3\,,$$

where

$$S_{ijkm}=S_iS_{jk}\sum_{m=0}^{i-j}\binom{i-j}{m}\,.$$

By making the transformation $r=(1+d+e)/(1+d+e+u_3)$ and simplifying, we obtain

$$P_2=KS_{ijkm}S_p\frac{d^pB(X_2\,;\,a_3+i-j-m+p,\,a_1+m)}{(1+d+e)^{a_3+i-j-m+p}}\,,$$

where

$$S_p=\sum_{p=0}^{k}(-1)^k\binom{k}{p}\,,\qquad X_2=\frac{1+d+e}{1+a+d+e}$$

and

$$B(X\,;\,p,\,q)=\int_0^X y^{p-1}(1-y)^{q-1}dy\,.$$

Using procedures similar to those in deriving P_2 we obtain series formulas for P_1, P_3 and P_4 as follows:

$$P_1=KS_{ijkm}\Big[\frac{1}{(1+cc_2)^{a_2+j}}\Big\{\frac{(-1)^m\{c(c_1-1)\}^mB(X_{11}\,;\,a_1+m,\,a_3)}{(1+c)^{a_3+i-j}(1+cc_1)^{a_1+m}}$$

$$-S_p\frac{(1+c)^{k-p}\{b(c_1-1)(1+cc_2)\}^pB(X_{12}\,;\,a_1+m+p,\,a_3+i-j-m)}{(1+bc_1+cc_1+bcc_1c_2)^{a_1+m+p}(1+b+c+bcc_2)^{a_3+i-j+k-m}}\Big\}\Big]\,;$$

$$P_3=KS_{ijkm}\frac{1}{(1+c_2f)^{a_2+j}}\Big[\frac{(-1)^mf^mB(X_{31}\,;\,a_3+m,\,a_1)}{(1+f)^{a_3+m}}$$

$$-S_p\frac{\{d(1+c_2f)\}^pB(X_{32},\,a_3+i-j-m+p,\,a_1+m)}{(1+d+f+c_2df)^{a_3+i-j-m+p}}\Big]\,;$$

$$P_4=KS_{ijkm}S_p\frac{(1+g)^{k-p}\{b(c_1-1)\}^pB(X_4\,;\,a_1+m+p,\,a_3+i-j-m)}{(1+b+g)^{a_3+i-j+k-m}(1+bc_1+gc_1)^{a_1+m+p}}\,;$$

where

$$X_{11}=\frac{a(1+cc_1)}{1+c+a(1+cc_1)}\,,\qquad X_{12}=\frac{a(1+bc_1+cc_1+bcc_1c_2)}{1+b+c+bcc_2+a(1+bc_1+cc_1+bcc_1c_2)}\,,$$

$$X_{31}=\frac{1+f}{1+a+f}\,,\qquad X_{32}=\frac{1+d+f+c_2df}{1+a+d+f+c_2df}\,,\qquad X_4=\frac{a(1+bc_1+gc_1)}{1+b+g+a(1+bc_1+gc_1)}\,.$$

§ 3. Discussion of Power and Size of the SPTP.

In this section we shall discuss the results of size and power of our sometimes pool test procedure (SPTP). From the numerical calculations that we have made for size, we see that the size of our test statistic constructed, using the preliminary tests, is not fixed. In many cases it is distorted and the amount of distortion can be considerable depending upon the degrees of freedom n_i $(i=1, 2, 3, 4)$, preliminary levels of significance α_1, α_2, α_4 and the variance ratios $\phi_{13}=\sigma_1^2/\sigma_3^2$, $\phi_{23}=\sigma_2^2/\sigma_3^2$. We shall discuss in the following the disturbances in size on the basis of theoretical result obtained in Appendix I and the numerical results assembled in Tables 1-6 (Appendix II).

We have remarked earlier that the power of our test procedure is a function of fourteen parameters, four degrees of freedom n_i's $(i=1, 2, 3, 4)$: seven levels of significance α_1, α_2, α_4 (preliminary), α_3, α_5, α_6, α_7 (final); and three noncentrality parameters λ_1, λ_2, λ_4. An analytical study of the power function for variations of one or more of these parameters is not possible and hence we have resorted to an empirical investigation of the power function. As has been pointed out by Pull (1948) and Bozivich Bancroft and Hartley (1956a, b) that the degrees of freedom are completely determined by the experiment, while the nuisance parameters λ_1, λ_2, λ_4 are, in general, unknown to the experimenter and hence none of teese 7 parameters are at the disposal of the experimeter. In general, the final levels of significance α_3, α_5, α_6, α_7 are all equal and chosen before the experiment is done; hence the only parameters available to the experimenter are the preliminary levels α_1, α_2, α_4. In the present study we have taken the preliminary levels $\alpha_1=\alpha_2=\alpha_4$. The main question is now about a proper choice of preliminary level(s) of significance so that the size of the final test approaches close to the anticipated probability of the error of the first kind. The final levels of significance often chosen in practice are .01 and .05 and hence in our discussion we have chosen these levels of significance $\alpha_3=\alpha_5=\alpha_6=\alpha_7=.05$. The choice of the preliminary levels has been made for .01, .05 and .25. In the choice of these preliminary levels we have kept the following points in mind that

(a) the size of our test procedure should be in the vicinity of the prescribed final level of significance;

(b) for this choice of preliminary level which controls the size, there should be a gain in power of the SPTP over that of the never pool test of the same size.

The numerical calculations for size and power have been made with the help of series formulas developed in Section (2.2) and use has been made of the digital computer, CDC 3600-160A, installed at the Tata Institute of Fundamental Research, Bombay. These numerical results have been assembled in Appendix II.

3.1. Discussion of Size.

In Appendix I we have proved that the lower bound of the size of our test procedure for $\lambda_1=\lambda_2=0$ (i. e. $\phi_{13}=\phi_{23}=1$) is $(1-\alpha_1)(1-\alpha_2)\alpha_3$ and that the upper bound for size for the same values of λ_1 and λ_2 is $(1-\alpha_1)(1-\alpha_2)\alpha_3+\alpha_5+(1-\alpha_4)\alpha_6+(1-\alpha_1)\alpha_7$. In case the finals levels $\alpha_3=\alpha_5=\alpha_6=\alpha_7=\alpha_f$ (say), and the preliminary levels $\alpha_1=\alpha_2=\alpha_4=\alpha_p$ (say), then the lower bound and upper bound for size become $(1-\alpha_p)^2\alpha_f$ and $(2-\alpha_p)^2\alpha_f$ respectively. For $\alpha_f=.05$, and $\alpha_p=.01, .05, .25$ these values turn out to be .049, .045, .028 for lower bound and .198, .190, .153 for upper bound respectively.

The studies of the component of variance models made by Paull (1948), Bozivich, Bancroft and Hartley (1956a, b), Srivastava (1960), Srivastava and Bozivich (1961), with the application of preliminary level at .05 led, in many cases, to a size maximum much greater than .05. On the other hand, the application of a preliminary level at .25 resulted in a size maximum less than .10. These authors found better control on size disturbances using the preliminary level at .25. We have, therefore, applied a preliminary level equal to .25 in some of our Tables 1-6 and we observe from Table 3.1 that the size maximum increases with the preliminary level. For

Table 3.1. Magnitude of Size Maximum for $\lambda_1=\lambda_2=0$ (i.e. $\phi_{13}=\phi_{23}=1$).

Degrees of Freedom				Preliminary Levels		
n_1	n_2	n_3	n_4	.01	.05	.25
2	2	2	2	.0608	.0849	.1108
6	2	2	2	.0590	.0817	.1104
10	2	2	2	.0583	.0802	.1090
20	2	2	2	—	.0789	—
30	2	2	2	—	.0775	—
2	4	2	2	.0611	.0862	—
2	10	2	2	—	.0881	—
6	10	2	2	.0601	.0844	—
6	20	2	2	.0606	.0857	—
10	10	2	2	—	.0822	—
2	2	6	2	.0552	.0654	—
2	2	10	2	.0532	.0595	.0691
2	2	20	2	—	.0547	—
10	10	8	2	—	.0638	—
2	2	2	4	.0625	.0906	—
2	2	2	10	.0639	.0958	—
2	2	2	20	—	.0980	—
2	2	2	30	—	.0988	—
10	2	2	8	—	.0897	—
2	4	2	4	.0629	.0925	.1176
2	6	2	6	—	.0971	—
2	10	2	10	.0648	.1024	—
2	10	2	20	.0653	.1059	—

example, when $n_1=n_2=n_3=n_4=2$ and $\alpha_1=\alpha_2=\alpha_4=.05$, the size maximum is .0849, whereas for the same combination of degrees of freedom it is .0608 for $\alpha_1=\alpha_2=\alpha_4=.01$ and is .1108 for $\alpha_1=\alpha_2=\alpha_4=.25$. We also observe from Tables 1-6 that the position of size maximum in each case is fixed and does not change. These two results are in contrast to the studies of Bozivich, Bancroft and Hartley (1956), Srivastava and Bozivich (1961), Srivastava (1960) and agree with results of the studies made by Gupta (1965). Such a point was also noticed by Lemus (1955) and the reversal in these results is mainly due to the differences in the two types of models considered. The result regarding the location of the size maximum is a very important feature of the test and has reduced our work to a considerable extent. It is on account of this result that we have not attempted any further calculations for the size of our test procedure. Any further computations, if made, have to be for $\lambda_1=\lambda_2=0$ only (see Table 3.1), and fortunately this value is free from the approximation mentioned earlier.

We next consider the effects on the size maximum of our test procedure for variations in the degrees of freedom when the other parameters are kept constant. From Table 3.1 we observe that for fixed values of preliminary and final levels of significance the size maximum

(i) decreases as n_1 increases for fixed values of n_2, n_3 and n_4,
(ii) decreases as n_3 increases for fixed values of n_1, n_2 and n_4,
(iii) increases as n_2 increases for fixed values of n_1, n_3 and n_4,
(iv) increases as n_4 increases for fixed values of n_1, n_2 and n_3.

If we now fix a 'reasonable size tolerance' to be .075 as that of Gupta (1965), then it is possible for us to conjecture the combinations of degrees of freedom which would be satisfactory (i. e. for these the size maximum will not exceed .075). For example, when $n_1=n_2=n_4=2$, $n_3=20$; $\alpha_1=\alpha_2=\alpha_4=.05$, the size maximum is .0547 which is atisfactory. Table 3.2 shows some satisfactory combinations of degrees of

Table 3.2. Satisfactory Combinations of Degree of Freedom.
For $\alpha_1=\alpha_2=\alpha_4=.05$.

n_1	n_2	n_3	n_4	Upper Limit to the Size Maximum
$\geqq 2$	2	20	2	.0547
$\geqq 10$	10	$\geqq 8$	2	.0608
$\geqq 2$	2	$\geqq 10$	2	.0595
$\geqq 2$	2	$\geqq 6$	2	.0654
>30	2	>2	2	.0775

freedom when the preliminary levels are taken at .05. Since the size maximum decreases as n_1 and/or n_3 increases; hence if we increase them either separately or together, we should get a control on size maximum. We also know that pooling is effective for n_3 small and n_1, n_2 large, hence the combinations shown in Table 3.2 would be quite satisfactory. For $n_1=n_2=n_3=2$, $n_4=4$, the size maximum is .0906 for $\alpha_1=\alpha_2=\alpha_4=.05$, hence in this case to get a control over size maximum, either we have to increase n_1 and/or n_3 suitably or we have to apply $\alpha_1=\alpha_2=\alpha_4=.01$. From Table 3.1, we observe that the preliminary level at .01 controls the size to the reasonable tolerance.

Table 3.3. Unsatisfactory Combinations of Degrees of Freedom.
For $\alpha_1=\alpha_2=\alpha_4=.05$.

n_1	n_2	n_3	n_4	Lower Limit to the Size Maximum
20	$\geqq 2$	2	$\geqq 2$	.0789
10	$\geqq 10$	2	$\geqq 2$	.0822
2	$\geqq 2$	2	$\geqq 4$	.0906
10	$\geqq 2$	2	$\geqq 8$	.0897
2	$\geqq 10$	2	$\geqq 20$	.1059

Next, we conjecture about the unsatisfactory combinations of degrees of freedom. For example, when $n_1=n_3=2$, $n_2=10$, $n_4=20$ the size maximum is .1059 for $\alpha_1=\alpha_2=\alpha_4=.05$. This value is, therefore, unsatisfactory. Table 3.3 gives some of the combinations of degrees of freedom which would be unsatisfactory at $\alpha_1=\alpha_2=\alpha_4=.05$.

Finally, we know that as $\lambda_1\to\infty$, $\lambda_2\to\infty$ (i.e. $\phi_{13}\to\infty$, $\phi_{23}\to\infty$), the probability of pooling of doubtful error mean squares with the true error mean square will tend to zero and the sometimes pool test would approach the never pool test of the size equal to the final level of significance.

3.2. Discussion of Power.

In this section we shall compare the power of our SPTP with that of the never pool test (NPT) procedure. For making such a comparison it is necessary that these two test procedures should have the same size. It is, therefore, decided to compute the power of the NPT procedure corresponding to the same size of the SPTP for different values of ϕ_{43}. More precisely, for comparing the power of the two test procedures we have adopted the following line of approach:

(i) For fixed values of ϕ_{13} and ϕ_{23}, compute size of the SPTP.

(ii) For this value of size, compute power of the two test procedures for specified values of ϕ_{43}.

Inorder to study the power functions of the two test procedures, we have made computations for 4 sets of degrees of freedom n_i's ($i=1, 2, 3, 4$) and the preliminary levels $\alpha_1=\alpha_2=\alpha_4=.05$. These computations have been assembled in Appendix II (Tables 7-12). From these tables we observe that for fixed values of ϕ_{13} and ϕ_{23} the power of both the test procedures is a monotone increasing function of ϕ_{43}. We also notice from these tables that for $\phi_{13}=\phi_{23}=1.00$, the SPTP is always more powerful than the NPT procedure.

For $n_1=10$, $n_2=n_3=n_4=2$; $\phi_{13}\leqq 2.58$, $\phi_{23}\leqq 9.47$, we notice from Table 8 that the power of the SPTP is always greater than that of the NPT procedure. For the same set of degrees of freedom but for $\phi_{13}=3.41$, $\phi_{23}=1.00$, the SPTP is less powerful than the NPT procedure for $\phi_{43}=3.41$, but as ϕ_{43} increases the SPTP again becomes more and more powerful than the NPT procedure. For $\phi_{13}\geqq 3.41$, $\phi_{23}>1.00$, there is a transition in power gain of the SPTP over that of the NPT procedure, and in this region the SPTP is always less powerful than the corresponding NPT procedure.

For $n_1=n_2=10$, $n_3=n_4=2$ (see Table 10), the SPTP is always more powerful than the NPT procedure for $\phi_{13}\leqq 3.00$ and $\phi_{23}\leqq 2.15$. For $\phi_{13}\geqq 5.45$, $\phi_{23}>2.15$, the situation is reversed and now the NPT procedure becomes more powerful than the SPTP.

For $n_1=n_2=10$, $n_3=8$, $n_4=2$ and for $\phi_{13}\leqq 1.69$, $\phi_{23}\leqq 1.69$, the SPTP has more power than the NPT procedure (see Table 11). As ϕ_{23} increases to 3.00 and for $\phi_{13}=1.69$, the transition from power gain to power loss ocurs at $\phi_{43}=3.41$. For $\phi_{13}\geqq 3.00$ and $\phi_{23}\geqq 3.00$ the NPT is more powerful than the SPT (sometimes pool test). Similarly, for $n_1=10$, $n_2=n_3=2$, $n_4=8$ (see Table 12), the SPT is always more powerful than the NPT for $\phi_{13} \leqq 2.15$ and $\phi_{23} \leqq 9.47$. For values of $\phi_{13} \geqq 5.45$ and $\phi_{23} \geqq 1.00$ the situation is reversed.

All these results illustrate the fact that the SPT is more powerful for small values of ϕ_{13} and ϕ_{23}, and less powerful for large values of ϕ_{13} and ϕ_{23}. We also know that as $\phi_{13} \to \infty$ and $\phi_{23} \to \infty$ the SPT approaches the NPT at the final level of significance, and hence the difference in powers of the two test procedures approaches zero.

When we change preliminary levels from 0.25 to 0.01 (see Tables 7 and 9) the power gains are increased and after certain values of ϕ_{13} and ϕ_{23} we have power losses. These power losses occur for large values of ϕ_{13} and ϕ_{23}, but in all these cases we have a better control on size which is also less than 0.05. This result is reverse to the result of Paull (1948, 1950), who proved that the border line test is always more powerful than the corresponding NPT of the same size. The reason for this is obvious because of the difference in the two types of models considered.

3.3. Conclusions.

From the results on size and power of the proposed test we are led to conclude that an experimenter using this procedure should take n_1 large keeping n_2 and n_4 moderately small. In case the treatment degrees of freedom n_4 are large, there is a gain in power but there will also be large disturbances in size maximum.

As we have already remarked that the noncentrality parameters λ's are unknown to the experimenter being functions of the variance ratios ϕ_{13}, ϕ_{23} and ϕ_{43}. It is, therefore, suggested that he should not make an indiscriminate use of 'always pool' or 'never pool' test procedures. However, if the experimenter suspects that 'small' values of ϕ_{13} and ϕ_{23} can be envisaged as a possibility, he is advised to use the SPT procedure with 5% preliminary levels except in the situations described under Table 3.3 (section 3.1), where he should apply $\alpha_1 = \alpha_2 = \alpha_4 = 0.01$. On the other hand, if the experimenter can make no such assumptions about ϕ_{13} and ϕ_{23}, he is advised to use $\alpha_1 = \alpha_2 = \alpha_4 = 0.01$ or even smaller to guard himself against the possiblity of power losses. But, in this case, he must realise that

(i) for ϕ_{13} and ϕ_{23} large, the power gain would be very small,

(ii) for small values of ϕ_{13} and ϕ_{23}, the test procedure may have a size much smaller than 0.05, and a test which is more powerful than the never pool test of size 0.05.

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Appendix I. Theoretical result.

For $\lambda_1 = \lambda_2 = \lambda_4 = 0$, (2.1.1) becomes

(1) $$f(u_1, u_2, u_3)=\frac{G u_1^{\frac{1}{2}n_4-1} u_2^{\frac{1}{2}n_2-1} u_3^{\frac{1}{2}n_1-1}}{(1+u_1+u_2+u_3)^{\frac{1}{2}n_{1234}}},$$

where

(2) $$G=\frac{\Gamma\left(\frac{1}{2}n_{1234}\right)}{\Gamma\left(\frac{1}{2}n_1\right)\Gamma\left(\frac{1}{2}n_2\right)\Gamma\left(\frac{1}{2}n_3\right)\Gamma\left(\frac{1}{2}n_4\right)}, \qquad n_{1234}=\sum_{i=1}^{4} n_i .$$

Let S_i's denote the corresponding P_i's and $S=\sum_{i=1}^{4} S_i$, size of the test (for this particular case). Then we have

RESULTS 1. $L \leqq S \leqq L+\alpha_5+(1-\alpha_4)\alpha_6+(1-\alpha_1)\alpha_7$, where $L=(1-\alpha_1)(1-\alpha_2)(1-\alpha_3)$.

PROOF. The expressions for S_i's will become as follows:

(3) $$S_1=\int_{u_3=0}^{u_1^0}\int_{u_2=0}^{u_2^0(1+u_3)}\int_{u_1=u_3^0(1+u_2+u_3)}^{\infty} f(u_1, u_2, u_3)\, du_3 du_2 du_1 ,$$

(4) $$S_2=\int_{u_3=u_1^0}^{\infty}\int_{u_2=u_4^0}^{\infty}\int_{u_1=u_5^0}^{\infty} f(u_1, u_2, u_3)\, du_3 du_2 du_1 ,$$

(5) $$S_3=\int_{u_3=u_1^0}^{\infty}\int_{u_2=0}^{u_4^0}\int_{u_1=u_6^0(1+u_2)}^{\infty} f(u_1, u_2, u_3)\, du_3 du_2 du_1 ,$$

(6) $$S_4=\int_{u_3=0}^{u_1^0}\int_{u_2=u_2^0(1+u_3)}^{\infty}\int_{u_1=u_7^0(1+u_3)}^{\infty} f(u_1, u_2, u_3)\, du_3 du_2 du_1 ,$$

where

$$u_1^0=(n_1F_1)/n_3 , \qquad u_2^0=(n_2F_2)/n_{13} , \qquad u_3^0=(n_4F_3)/n_{123} ,$$
$$u_4^0=(n_2F_4)/n_3 , \qquad u_5^0=(n_4F_5)/n_3 , \qquad u_6^0=(n_4F_6)/n_{23} \qquad \text{and}$$
$$u_7^0=(n_4F_7)/n_{13} .$$

To evaluate S_1, we make the transformation

$$u_3=x_3 , \qquad u_2=(1+x_3)x_2 , \qquad u_1=(1+x_2)(1+x_3)x_1$$

and obtain

$$S_1=G\int_0^{u_1^0}\frac{x_1^{\frac{1}{2}n_1-1}dx_3}{(1+x_3)^{\frac{1}{2}n_{13}}}\int_0^{u_2^0}\frac{x_2^{\frac{1}{2}n_2-1}dx_2}{(1+x_2)^{\frac{1}{2}n_{123}}}\int_{u_3^0}^{\infty}\frac{x_1^{\frac{1}{2}n_4-1}dx_1}{(1+x_1)^{\frac{1}{2}n_{1234}}},$$

where

$$n_{123}=n_1+n_2+n_3 .$$

or

(7) $$S_1=(1-\alpha_1)(1-\alpha_2)\alpha_3=L ,$$

by using the relation between the F-distribution and incomplete Beta function. Hence,

$$S=L+S_2+S_3+S_4 .$$

As S_2, S_3 and S_4, being probabilities, are non-negative quantities and, therefore, we have

(8) $$S \geqq L .$$

We can write (4) as

$$S_2 \leqq \int_{u_3=0}^{\infty} \int_{u_2=0}^{\infty} \int_{u_1=u_5^0}^{\infty} f(u_1, u_2, u_3)\, du_3 du_2 du_1 .$$

Integrating out u_2 and u_3, and using the relation between F-distribution and incomplete Beta function we obtain

(9) $$S_2 \leqq \alpha_5 .$$

Using preceding methods, with suitable transformations, we can easily obtain

(10) $$S_3 \leqq (1-\alpha_4)\alpha_6 ,$$

(11) $$S_4 \leqq (1-\alpha_1)\alpha_7 .$$

Combining (7), (9), (10) and (11) we obtain

(12) $$S \leqq L + \alpha_5 + (1-\alpha_4)\alpha_6 + (1-\alpha_1)\alpha_7 .$$

Hence our result follows from (8) and (12).

Appendix II. Numerical results.

Size of the Sometimes Pool Test Procedure. (Tables 1-6).

Table 1. $\alpha_1 = \alpha_2 = \alpha_4 = .01,\ n_1 = 10,\ n_2 = n_3 = n_4 = 2.$

	ϕ_{23}			
ϕ_{13}	1. 00	3. 41	5. 45	9. 47
1. 00	. 0583	. 0446	. 0466	. 0544
2. 58	. 0156	. 0179	. 0197	. 0210
3. 41	. 0153	. 0192	. 0223	. 0242
5. 45	. 0187	. 0220	. 0272	. 0316

Table 2. $\alpha_1 = \alpha_2 = \alpha_4 = .05,\ n_1 = 10,\ n_2 = n_3 = n_4 = 2.$

	ϕ_{23}			
ϕ_{13}	1. 00	3. 41	5. 45	9. 47
1. 00	. 0802	. 0731	. 0750	. 0774
2. 58	. 0472	. 0441	. 0457	. 0466
3. 41	. 0501	. 0408	. 0467	. 0477
5. 45	. 0582	. 0463	. 0481	. 0494

Table 3. $\alpha_1 = \alpha_2 = \alpha_4 = .25,\ n_1 = 10,\ n_2 = n_3 = n_4 = 2.$

	ϕ_{23}			
ϕ_{13}	1. 00	3. 41	5. 45	9. 47
1. 00	. 1090	. 0880	. 0866	. 0865
2. 58	. 0798	. 0531	. 0513	. 0511
3. 41	. 0801	. 0522	. 0504	. 0502
5. 45	. 0808	. 0522	. 0502	. 0500

Table 4. $\alpha_1 = \alpha_2 = \alpha_4 = .05,\ n_1 = n_2 = 10,\ n_3 = n_4 = 2.$

ϕ_{13}	ϕ_{23} 1.00	1.69	2.15	3.00	5.45
1.00	.0822	.0710	.0741	.0741	.0777
1.69	.0545	.0480	.0474	.0483	.0505
2.15	.0472	.0442	.0444	.0454	.0469
3.00	.0429	.0424	.0436	.0456	.0469
5.45	.0464	.0429	.0441	.0467	.0670

Table 5. $\alpha_1 = \alpha_2 = \alpha_4 = .05,\ n_1 = n_2 = 10,\ n_3 = 8,\ n_4 = 2.$

ϕ_{13}	ϕ_{23} 1.00	1.69	2.15	3.00	5.45
1.00	.0638	.0541	.0543	.0574	.0606
1.69	.0475	.0394	.0391	.0410	.0438
2.15	.0456	.0380	.0380	.0402	.0427
3.00	.0486	.0393	.0397	.0425	.0456
5.45	.0584	.0431	.0424	.0454	.0478

Table 6. $\alpha_1 = \alpha_2 = \alpha_4 = .05,\ n_1 = 10,\ n_2 = n_3 = 2,\ n_4 = 8.$

ϕ_{13}	ϕ_{23} 1.00	3.41	5.45	9.47	19.49
1.00	.0897	.0804	.0826	.0846	.0848
1.69	.0547	.0504	.0516	.0523	.0523
2.15	.0531	.0480	.0491	.0496	.0496
3.00	.0576	.0482	.0492	.0496	.0497
5.45	.0691	.0490	.0496	.0500	.0500
9.47	.0758	.0493	.0496	.0500	.0500

* Powers of the Sometimes Pool Test Procedure and of the Never Pool Test Procedure of the Same Size (Tables 7–12).

Table 7. $n_1 = 10,\ n_2 = n_3 = n_4 = 2,\ \alpha_1 = \alpha_2 = \alpha_4 = .01.$

ϕ_{13}	ϕ_{23}	ϕ_{43} 1.00	3.41	5.45	9.47
1.00	1.00	.0583 .0583	.4054 .1820	.6698 .2736	.9292 .4256
	3.41	.0446 .0446	.3228 .1421	.5705 .2167	.8741 .3451
	5.45	.0466 .0466	.3126 .1480	.5467 .2253	.8488 .3574
	9.47	.0544 .0544	.3477 .1889	.5808 .2678	.8543 .3914
2.58	1.00	.0156 .0156	.1101 .0519	.2604 .0816	.6086 .1372

	3.41	.0179	.0794	.1922	.5070
		.0179	.0595	.0939	.1560
	5.45	.0197	.0672	.1574	.4383
		.0197	.0653	.1019	.1703
	9.47	.0210	.0641	.1375	.3608
		.0210	.0693	.1083	.1803
3.41	1.00	.0153	.0638	.1542	.4376
		.0153	.0509	.0799	.1350
	3.41	.0192	.0503	.1141	.3516
		.0192	.0635	.0993	.1664
	5.45	.0223	.0463	.0940	.2937
		.0223	.0736	.1147	.1904
	9.47	.0242	.0442	.0768	.2231
		.0242	.0795	.1238	.2050
5.45	1.00	.0187	.0404	.0654	.1790
		.0187	.0620	.0968	.1621
	3.41	.0220	.0366	.0530	.1402
		.0220	.0726	.1133	.1884
	5.45	.0272	.0419	.0529	.1191
		.0272	.0891	.1380	.2274
	9.47	.0316	.0489	.0553	.0939
		.0316	.1028	.1587	.2589

* In Tables 7–12 first horizontal line refers to the power of the SPT procedure whereas the second horizontal line refers to the power of the NPT procedure of the same size.

Table 8. $n_1 = 10,\ n_2 = n_3 = n_4 = 2,\ \alpha_1 = \alpha_2 = \alpha_4 = .05.$

		ϕ_{43}			
ϕ_{13}	ϕ_{23}	1.00	3.41	5.45	9.47
1.00	1.00	.0802	.4324	.6881	.9339
		.0802	.2424	.3339	.5344
	3.41	.0731	.3894	.6363	.9054
		.0731	.2231	.3307	.5016
	5.45	.0750	.3920	.6357	.9014
		.0750	.2284	.3385	.5104
	9.47	.0774	.4074	.6543	.9116
		.0774	.2349	.3464	.5215
2.58	1.00	.0472	.1742	.3248	.6482
		.0472	.1499	.2278	.3612
	3.41	.0441	.1487	.2717	.5693
		.0441	.1406	.2144	.3423
	5.45	.0457	.1466	.2581	.5344
		.0457	.1455	.2213	.3521
	9.47	.0466	.1521	.2642	.5289
		.0466	.1480	.2253	.3578
3.41	1.00	.0501	.1453	.2456	.5090
		.0501	.1584	.2399	.3787
	3.41	.0488	.1264	.2072	.4358
		.0488	.1427	.2175	.3465
	5.45	.0467	.1283	.1999	.4012
		.0467	.1484	.2256	.3581

	9.47	.0477 .0477	.1332 .1534	.2031 .2299	.3854 .3641
5.45	1.00	.0582 .0582	.1522 .1817	.2082 .2736	.3315 .4249
	3.41	.0463 .0463	.1234 .1468	.1728 .2233	.2822 .3549
	5.45	.0481 .0481	.1300 .1525	.1780 .2316	.2696 .3668
	9.47	.0494 .0494	.1398 .1563	.1915 .2371	.2689 .3746

Table 9. $n_1 = 10,\ n_2 = n_3 = n_4 = 2,\ \alpha_1 = \alpha_2 = \alpha_4 = .25.$

		ϕ_{43}			
ϕ_{13}	ϕ_{23}	1.00	3.41	5.45	9.47
1.00	1.00	.1090 .1090	.4828 .3157	.7251 .4522	.9434 .6612
	3.41	.0880 .0880	.4325 .2628	.6782 .3839	.9234 .5678
	5.45	.0866 .0866	.4265 .2592	.6712 .3791	.9195 .5622
	9.47	.0865 .0865	.4256 .2590	.6700 .3789	.9186 .5617
2.58	1.00	.0798 .0798	.3040 .2413	.4846 .3552	.7633 .5325
	3.41	.0531 .0531	.2098 .1674	.3642 .2523	.6571 .3961
	5.45	.0513 .0513	.1960 .1624	.3404 .2449	.6262 .3857
	9.47	.0511 .0511	.1932 .1619	.3342 .2442	.6151 .3847
3.41	1.00	.0801 .0801	.2934 .2421	.4549 .3563	.7071 .5338
	3.41	.0522 .0522	.1880 .1648	.3132 .2486	.5661 .3911
	5.45	.0504 .0504	.1724 .1600	.2846 .2412	.5225 .3807
	9.47	.0502 .0502	.1691 .1595	.2767 .2404	.5054 .3794
5.45	1.00	.0808 .0808	.2991 .2440	.4567 .3588	.6764 .5371
	3.41	.0522 .0522	.1800 .1648	.2869 .2486	.4858 .3911
	5.45	.0502 .0502	.1625 .1595	.2526 .2404	.4251 .3794
	9.47	.0500 .0500	.1588 .1588	.2430 .2396	.3997 .3783

Table 10. $n_1 = n_2 = 10,\ n_3 = n_4 = 2,\ \alpha_1 = \alpha_2 = \alpha_4 = .05.$

		ϕ_{43}			
ϕ_{13}	ϕ_{23}	1.00	3.41	5.45	9.47
1.00	1.00	.0822 .0822	.4614 .2476	.7262 .3639	.9542 .5429
	1.69	.0710 .0710	.3881 .2175	.6421 .3229	.9161 .4977
	2.15	.0710 .0710	.3719 .2175	.6156 .3229	.8969 .4977
	3.00	.0741 .0741	.3754 .2259	.6092 .3344	.8811 .5063
	5.45	.0777 .0777	.4078 .2356	.6531 .3476	.9057 .5229
1.69	1.00	.0543 .0543	.3334 .1706	.5906 .2572	.8972 .4131
	1.69	.0480 .0480	.2589 .1521	.4858 .2311	.8288 .3662
	2.15	.0474 .0474	.2322 .1505	.4387 .2286	.7859 .3623
	3.00	.0483 .0483	.2141 .1531	.3948 .2324	.7267 .3679
	5.45	.0505 .0505	.2339 .1595	.4177 .2417	.7176 .3810
2.15	1.00	.0472 .0472	.2767 .1499	.5157 .2278	.8520 .3612
	1.69	.0442 .0442	.2134 .1410	.4153 .2149	.7721 .3427
	2.15	.0444 .0444	.1890 .1416	.3674 .2157	.7204 .3438
	3.00	.0454 .0454	.1680 .1445	.3143 .2203	.6414 .3501
	5.45	.0469 .0469	.1760 .1490	.3126 .2265	.5893 .3593
3.00	1.00	.0429 .0429	.2098 .1371	.4068 .2093	.7601 .3347
	1.69	.0424 .0424	.1651 .1357	.3237 .2069	.6711 .3314
	2.15	.0436 .0436	.1494 .1392	.2847 .2143	.6147 .3391
	3.00	.0456 .0456	.1355 .1451	.2383 .2208	.5243 .3515
	5.45	.0469 .0469	.1346 .1490	.2123 .2265	.4122 .3593
5.45	1.00	.0464 .0464	.1614 .1475	.2713 .2244	.5302 .3564
	1.69	.0429 .0429	.1275 .1371	.2154 .2093	.4539 .3347
	2.15	.0441 .0441	.1199 .1406	.1937 .2144	.4091 .3423
	3.00	.0467 .0467	.1204 .1484	.1767 .2256	.3450 .3581
	5.45	.0670 .0670	.1144 .2097	.1700 .3129	.1459 .4782

Table 11. $n_1 = n_2 = 10,\ n_3 = 8,\ n_4 = 2,\ \alpha_1 = \alpha_2 = \alpha_4 = .05.$

ϕ_{13}	ϕ_{23}	ϕ_{43} = 1.00	3.41	5.45	9.47
1.00	1.00	.0638 .0638	.4607 .3962	.7355 .6414	.9604 .9013
	1.69	.0541 .0541	.4058 .3614	.6761 .6032	.9369 .8796
	3.00	.0574 .0574	.4054 .3736	.6658 .6172	.9243 .8876
1.69	1.00	.0475 .0475	.3808 .3356	.6523 .5731	.9289 .8609
	1.69	.0394 .0394	.3195 .3007	.5760 .5300	.8892 .8316
	3.00	.0411 .0411	.3001 .3084	.5342 .5404	.8479 .8383
3.00	1.00	.0486 .0486	.3481 .3399	.5958 .5784	.8873 .8647
	1.69	.0393 .0393	.2854 .3000	.5144 .5300	.8357 .8340
	3.00	.0425 .0425	.2675 .3145	.4642 .5478	.7689 .8438
5.45	1.00	.0584 .0584	.3905 .3778	.6487 .6204	.8972 .8901
	1.69	.0431 .0431	.3128 .3169	.5455 .5507	.8412 .8459
	3.00	.0454 .0454	.2901 .3270	.4915 .5630	.7738 .8540

Table 12. $n_1 = 10,\ n_2 = n_3 = 2,\ n_4 = 8,\ \alpha_1 = \alpha_2 = \alpha_4 = .05.$

ϕ_{13}	ϕ_{23}	ϕ_{43} = 1.00	1.81	3.41	4.44
1.00	1.00	.0897 .0897	.2850 .1560	.7023 .2730	.8619 .3392
	3.41	.0804 .0804	.2430 .1414	.6253 .2474	.7987 .3083
	5.45	.0826 .0826	.2475 .1434	.6228 .2525	.7913 .3149
	9.47	.0846 .0846	.2584 .1474	.6468 .2687	.8119 .3221
1.69	1.00	.0547 .0547	.1346 .0966	.4247 .1743	.6049 .2196
	3.41	.0504 .0504	.1130 .0895	.3427 .1613	.5226 .2045
	5.45	.0516 .0516	.1147 .0914	.3322 .1647	.5003 .2084
	9.47	.0523 .0523	.1198 .0928	.3512 .1674	.5212 .2108
2.15	1.00	.0531 .0531	.1024 .0942	.3016 .1694	.4771 .2140

	3. 41	. 0480 . 0480	. 0889 . 0831	. 2370 . 1533	. 3813 . 1941
	5. 45	. 0491 . 0491	. 0909 . 0869	. 2277 . 1576	. 3573 . 1993
	9. 47	. 0496 . 0496	. 0940 . 0881	. 2405 . 1593	. 3720 . 2013
3. 00	1. 00	. 0576 . 0576	. 0924 . 1020	. 1881 . 1828	. 2936 . 2307
	3. 41	. 0482 . 0482	. 0805 . 9853	. 1535 . 1546	. 2305 . 1957
	5. 45	. 0492 . 0492	. 0834 . 0871	. 1520 . 1580	. 2164 . 1997
	9. 47	. 0496 . 0496	. 0855 . 0881	. 1590 . 1593	. 2233 . 2013
5. 45	1. 00	. 0691 . 0691	. 1172 . 1214	. 1786 . 2163	. 2059 . 2713
	3. 41	. 0490 . 0490	. 0836 . 0867	. 1380 . 1573	. 1652 . 1989
	5. 45	. 0496 . 0496	. 0857 . 0881	. 1437 . 1593	. 1711 . 2013
	9. 47	. 0500 . 0500	. 0880 . 0886	. 1535 . 1607	. 1856 . 2029

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