

The Combined Use of Runs in Statistical Quality Controls II

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<https://doi.org/10.5109/12977>

出版情報 : 統計数理研究. 7 (3/4), pp.53-72, 1957-03. Research Association of Statistical Sciences

バージョン :

権利関係 :



THE COMBINED USE OF RUNS IN STATISTICAL QUALITY CONTROLS II

By

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(Received Jan. 28, 1957)

§ 1. **Introduction and summary.** This paper is a continuation of the one published by KITAGAWA and SEGUCHI [1], and gives numerical results which may be useful for practical applications. In the previous paper we discussed the combined uses of runs in statistical quality controls on the basis of recurrent probabilistic events in the sense of FELLER [2], where some functional equations and notion of mutually exclusive events play fundamental roles. The applications of our theoretical results were however mainly concerned with the reconsiderations of the efficiencies of the limits given by H. WEILER [3] which may be adopted in the sense of combined uses. The relative comparison among the statistical control procedures $C(1)[3.000]$, $C(1, 2)[3.205, 2.068]$, $C^{(\pm)}(1, 2)[3.205, 1.927]$, $C(2)[1.932]$ and $C^{(\pm)}(2)[1.785]$ was given there in term of the expected value, or its reciprocal number, of numbers of samples attaining the first occurrence of significance for each of these statistical controlling procedure.

In this paper we shall restrict ourselves with the case when $C(1)[a_1]$, $C(1, 2)[a_1, a_2]$, $C^{(\pm)}(1, 2)[a_1, a_2]$, $C(2)[a_2]$ and $C^{(\pm)}(2)[a_2]$ and consequently with the controlling powers $[T_k(1)]^{-1}$, $[T_k(1, 2)]^{-1}$, $[T_k^{(\pm)}(1, 2)]^{-1}$, $[T_k(2)]^{-1}$ and $[T_k^{(\pm)}(2)]^{-1}$ and also with $[T_l^*(1)]^{-1}$, $[T_l^*(1, 2)]^{-1}$, $[T_l^{*(\pm)}(1, 2)]^{-1}$, $[T_l^*(2)]^{-1}$ and $[T_l^{*(\pm)}(2)]^{-1}$ respectively. The problems discussed in this paper can be summarized as follows:

(1°) For assigned common values of $\alpha = \beta_0 = T_0^{-1}$, to compare the powers of $[T_k(1)]^{-1}$, $[T_k(1, 2)]^{-1}$, $[T_k^{(\pm)}(1, 2)]^{-1}$, $[T_k(2)]^{-1}$ and $[T_k^{(\pm)}(2)]^{-1}$ as functions of k .

(2°) For assigned common values of $\alpha = \beta_l^* = T_l^{*-1}$, to compare the powers of $[T_l^*(1)]^{-1}$, $[T_l^*(1, 2)]^{-1}$, $[T_l^{*(\pm)}(1, 2)]^{-1}$, $[T_l^*(2)]^{-1}$ and $[T_l^{*(\pm)}(2)]^{-1}$ as functions of l .

(3°) To find out the effect of choices of a_1 and a_2 on the power curves.

The reciprocal number of the expected value of numbers of samples attaining the first occurrence of significance for each of these statistical controlling procedures may be considered as the *power of the controlling procedure C*, and is a function $[T_k(C)]^{-1}$ of the shift k when the hypothesis H_k that our parent population is $N(\xi + k\sigma, \sigma^2)$ holds true or a function $[T_l^*(C)]^{-1}$ of the parameter l when the hypothesis H_l^* that our parent population is $N(\xi, l^2\sigma^2)$ holds true.

The results of this paper can be summarized as follows. Let us consider the two cases when $\alpha=0.010$ and $\alpha=0.001$. For the procedures $C(1)[a_1]$, $C(2)[a_1]$ and $C^{(\pm)}(2)[a_1]$, the value of α will determine each of their single control limits respectively. On the other hand there remain infinite possibilities of choosing a pair of two control limits a_1 and a_2 for each of the procedures $C(1, 2)[a_1, a_2]$ and $C^{(\pm)}(1, 2)[a_1, a_2]$. In order to discuss the problem (3°), let us notice that

$$(1.01) \quad \frac{1}{T_0(1, 2)} = \frac{1}{T_0(1)} + \frac{1}{T_0(2)},$$

$$(1.02) \quad \frac{1}{T_0^{(\pm)}(1, 2)} = \frac{1}{T_0^{(\pm)}(1)} + \frac{1}{T_0^{(\pm)}(2)}$$

and let us assign a value of the ratio ρ defined by $\rho = T_0(1)/T_0(2)$ or by $\rho = T_0^{(\pm)}(1)/T_0^{(\pm)}(2)$ respectively. Let us consider the three levels of $\rho = 0.5$, 1.0 and 2.0. For each assigned value of these ρ levels we can determine the fundamental probabilities $P_{+1} = P_{+1}(0)$ and $P_{+2} = P_{+2}(0)$ under the hypothesis H_0 that the population is $N(\xi, \sigma^2)$, as will be enunciated in § 2, and consequently the controlling limits a_1 and a_2 for each of controlling procedures, which lead us to the complete specification of our statistical controlling procedures. The values of controlling limits are given in Table I (1) ~ (2), the fundamental probabilities in Table II (1) ~ (6) for the values of k from 0.0 to 3.0 (0.2) and in Table II (7) ~ (12) for the values of l from 1.00 to 3.00 (0.25), and finally the reciprocal numbers of the expected value of numbers of samples attaining the first occurrence of significance for each of these statistical controlling procedures for the values of k under the hypothesis H_k , and also for the values of l under the hypothesis H_l^* .

In the first place, the following observations can be made from Tables III, (1) ~ (4) for the value of k in $0.0 < k \leq 3.0$, and also for the value of l in $1.00 < l \leq 3.00$:

(a) For the shift of population mean under the hypothesis H_k , the controlling powers of $C^{(\pm)}(1, 2)$ and $C^{(\pm)}(2)$ are stronger than that of $C(1)$. The values of $[T_k^{(\pm)}(1, 2)]^{-1}/[T_k(1)]^{-1}$ and $[T_k^{(\pm)}(2)]^{-1}/[T_k(1)]^{-1}$ are given in Table IV.

(b) In the cases of $C(1, 2)$ and $C^{(\pm)}(1, 2)$, when ρ varies from 0.5 to 2.0, we can not see any remarkable difference between the powers of $C(1, 2)$ and $C^{(\pm)}(1, 2)$ for the variation of ratio ρ .

(c) For the change of the population variance under the hypothesis H_l^* , the controlling powers of $C(1, 2)$, $C^{(\pm)}(1, 2)$ and $C(1)$ are not so remarkable different, but those of $C^{(\pm)}(2)$ and $C(2)$ are very weak.

Hence, in order to compare the characters of controlling methods $C(1)$, $C(1, 2)$, $C^{(\pm)}(1, 2)$, $C(2)$ and $C^{(\pm)}(2)$ more seriously, we shall show Tables III, (1) ~ (4) by the graphes of Figures I, (1) ~ (2), however restricting with the cases of $C(1)$, $C^{(\pm)}(1, 2)$ and $C^{(\pm)}(2)$, and $\rho = 1.0$.

Secondly, the following be observed among others from these Tables and graphes:

(d) The power of $C(1, 2)$ is almost equal to that of $C(2)$, for small k , and to that of $C(1)$ for large k . The power of $C^{(\pm)}(1, 2)$ is almost equal to that of $C^{(\pm)}(2)$ for small k , and to that of $C(1)$ for large k .

(e) The combination of the four observations (a) \sim (d) leads us to the conclusion to the effect that controlling procedure $C^{(\pm)}(1, 2)$ is more powerful than $C(1)$, that is, the type of the ordinary control chart method, so far as the shift of the population mean under the hypothesis H_k , and the change of the population variance under the hypothesis H_i^* are concerned.

In summary our recommendation is that "*the use of the statistical quality control procedure $C^{(\pm)}(1, 2)$, that is, the combined use of runs of lengths 1 and 2, should be considered, under certain condition, in place of the usual control chart method with the single limit of the significance.*"

§ 2. The calculation of controlling power. We proceed in the following manner.

(1°) For each assigned value of α for $C(1)$ and $C(2)$ and for each assigned pair of values of α and ρ for $C(1, 2)$ and $C^{(\pm)}(1, 2)$, we can determine $T_0(1)$ and / or $T_0(2)$ as follows:

(a) For the case $C(1)$, we have $T_0(1) = \alpha^{-1}$.

(b) For the case $C(2)$, we have $T_0(2) = \alpha^{-1}$.

(c) For the case $C(1, 2)$, we have $T_0(1) = (1 + \rho)/\alpha$ and $T_0(2) = (1 + \rho)/\rho\alpha$.

and similarly for the case $C^{(\pm)}(1, 2)$.

(2°) The next step is to find out the values of P_{+1} and P_{+2} under the null hypothesis H_0 (or H_i^*), that is, that the population is $N(\xi, \sigma^2)$. The values of P_{+1} , P_{-1} , P_{+2} , P_{-2} and so on under the null hypothesis H_0 ($\equiv H_i^*$) will be denoted more accurately by $P_{+1}(0)$ (or $P_{+1}^*(1)$), $P_{-1}(0)$ (or $P_{-1}^*(1)$), $P_{+2}(0)$ (or $P_{+2}^*(1)$), $P_{-2}(0)$ (or $P_{-2}^*(1)$) and so on.

It is to be noted that $P_{+1}(0) = P_{-1}(0) = P_{+1}^*(1) = P_{-1}^*(1)$ and $P_{+2}(0) = P_{-2}(0) = P_{+2}^*(1) = P_{-2}^*(1)$.

(a) $P_{+1} = P_{-1} = [2T_0(1)]^{-1}$.

(b) For the cases $C(1, 2)$ and $C(2)$, we have $P_2 = P_{+2} + P_{-2} = 2P_{+2}$ and $\theta = [t_0(2)]^{-1} = P_2^2(1 + P_2)^{-1(1)}$. Consequently we have

$$(2.01) \quad P_{+2} = P_{-2} = \frac{1}{2}P_2 = \frac{\theta + (\theta^2 + 4\theta)^{1/2}}{4}.$$

(c) For the cases $C^{(\pm)}(1, 2)$ and $C^{(\pm)}(2)$, we have $T_0^{(+)}(2) = T^{(-)}(2)$ and $\theta = [T_0(2)]^{-1} = 2[T_0^{(+)}(2)]^{-1} = 2P_{+2}^2(1 + P_{+2})^{-1}$. Consequently we have

(1) For $C(2)$, $\theta = \alpha$.

$$(2.02) \quad P_{+2} = P_{-1} = \frac{\theta + (\theta^2 + 8\theta)^{1/2}}{4}.$$

(3°) The third step is to find out the controlling limits. Here we notice that, under the null hypothesis

$$(2.03) \quad P_{+1} = \Phi(\infty) - \Phi(a_1),$$

$$(2.04) \quad P_{+2} = \Phi(a_1) - \Phi(a_2),$$

where we have

$$(2.05) \quad \Phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^u \exp\{-t^2/2\} dt,$$

which give us the values of a_1 and / or a_2 , since we have already determined the values of P_{+1} and P_{+2} under the null hypothesis for each of the statistical control procedures $C(1)$, $C(2)$, $C^{\pm}(2)$, $C(1, 2)$ and $C^{(\pm)}(1, 2)$.

(4°) The fourth step is to give the values of the fundamental probabilities P_{+1} , P_{-1} , P_{+2} and P_{-2} under the hypothesis H_k and also those of P_{+1}^* , P_{-1}^* , P_{+2}^* and P_{-2}^* under the hypothesis H_l^* which can be written as

$$(2.06) \quad P_{+1}(k) = \Phi(\infty) - \Phi(a_1 - k), \quad P_{-1}(k) = \Phi(-a_1 - k) - \Phi(-\infty),$$

$$(2.07) \quad P_{+2}(k) = \Phi(a_1 - k) - \Phi(a_2 - k), \quad P_{-2}(k) = \Phi(-a_2 - k) - \Phi(-a_1 - k),$$

under the hypothesis H_k , and

$$(2.08) \quad P_{+1}^*(l) = \Phi(\infty) - \Phi\left(\frac{a_1}{l}\right), \quad P_{-1}^*(l) = \Phi\left(\frac{a_1}{l}\right) - \Phi(-\infty),$$

$$(2.09) \quad P_{+2}^*(l) = \Phi\left(\frac{a_1}{l}\right) - \Phi\left(\frac{a_2}{l}\right), \quad P_{-2}^*(l) = \Phi\left(-\frac{a_2}{l}\right) - \Phi\left(-\frac{a_1}{l}\right),$$

under the hypothesis H_l^* .

(5°) The fifth step is to calculate the reciprocal number of the expected value of numbers of samples for the first occurrence of significance for each of our statistical control procedures.

(a) For the procedure $C(1)$, we have $[T_k(1)]^{-1} = P_1(k)$ and $[T_l^*(1)]^{-1} = P_1^*(l)$.

(b) For the procedure $C(2)$, we have $[T_k(2)]^{-1} = (1 + P_2(k))P_2(k)^{-2}$ and $[T_l^*(2)]^{-1} = (1 + P_2^*(l))P_2^*(l)^{-2}$.

(c) For the procedure $C(1, 2)$, we have

$$(2.10) \quad [T_k(1, 2)]^{-1} = P_1(k) + \frac{P_2(k)^2}{1 + P_2(k)}$$

and

$$(2.11) \quad [T_l^*(1, 2)]^{-1} = P_1^*(l) + \frac{P_2^*(l)^2}{1 + P_2^*(l)}.$$

(d) For the procedure $C^{(\pm)}(1, 2)$ we have

$$(2.12) \quad [T_k^{(\pm)}(1, 2)]^{-1} = P_1(k) + \frac{P_{+2}(k)^2}{1 + P_{+2}(k)} + \frac{P_{-2}(k)^2}{1 + P_{-2}(k)}$$

and

$$(2.13) \quad [T_l^*(1, 2)]^{-1} = P_1^*(l) + \frac{P_{+2}^*(l)^2}{1 + P_{+2}^*(l)} + \frac{P_{-2}^*(l)^2}{1 + P_{-2}^*(l)}.$$

Remark. *The effects of the ratio ρ .*

(1) The ratio ρ can vary from 0 to ∞ . It has the following implications, that is; there are the following relations between the controlling procedures $C(1)$, $C(1, 2)$, $C^{(\pm)}(1, 2)$, $C(2)$ and $C^{(\pm)}(2)$:

(i) $C(2)$ is the special type of $C(1, 2)$, when the ratio $\rho = T_0(1)/T_0(2) = 0$; (ii) $C^{(\pm)}(2)$ is the special type of $C^{(\pm)}(1, 2)$, when the ratio $\rho = T_0^{(\pm)}(1)/T_0^{(\pm)}(2) = 0$; (iii) $C(1)$ is the special type of $C(1, 2)$ or $C^{(\pm)}(1, 2)$, when the ratio $\rho = \infty$.

(2) The power of $C(1, 2)$ (or $C^{(\pm)}(1, 2)$), when we hold $\alpha = \text{const.}$, varies from that of $C(1)$ to that of $C(2)$ (or $C^{(\pm)}(2)$), as ρ varies from ∞ to 0.

(3) We may find the optimum value of ρ among $0 < \rho < \infty$, for some criterion of optimality. This was not treated in this paper. However we can observe that the value of α can be practically in the range of $0.001 \leq \alpha \leq 0.010$, and consequently in view of Fig. I, (1) and (2), optimum value of ρ is supposed to belong to the range $0.5 \leq \rho \leq 2.0$.

(4) The another effect of the ratio ρ is that, for smaller α , the power is stronger for smaller ρ , while for larger α , the power is stronger for larger ρ . This means that the use of length 2 is more effective for the case of α is comparatively small.

The authors express their hearty thanks for the financial aid given by the Mainichi Newspaper Company under the 1956 grant of the Mainichi Scientific Encouragement to which the authors owed much in preparing our numerical calculations published in this paper.

Table I (1). The controlling limits for the controlling procedures **C**(1), **C**(2) and **C**^(±)(2) for assigned values of α .

α a C	0.010		0.001	
	a_1	a_2	a_1	a_2
C (1)	2.576	—	3.290	—
C (2)	—	1.621	—	2.143
C ^(±) (2)	—	1.452	—	2.002

Table II (1). The fundamental probabilities falling into the domains limited by control charts. $\alpha = 0.010$

\mathbf{C} P k	C (1)		C (2)		C ^(±) (2)	
	$P_{+1}^*(k)$	$P_{-1}^*(k)$	$P_{+2}^*(k)$	$P_{-2}^*(k)$	$P_{+2}^*(k)$	$P_{-2}^*(k)$
0.0	0.00500	0.00500	0.05251	0.05251	0.07325	0.07325
0.2	0.00875	0.00275	0.07766	0.03430	0.10529	0.04927
0.4	0.01478	0.00146	0.11104	0.02164	0.14640	0.03201
0.6	0.02408	0.00075	0.15363	0.01318	0.19711	0.02009
0.8	0.03787	0.00037	0.20582	0.00774	0.25720	0.01216
1.0	0.05751	0.00017	0.26730	0.00438	0.32563	0.00710
1.2	0.08441	0.00008	0.33688	0.00239	0.40052	0.00400
1.4	0.11980	0.00004	0.41255	0.00126	0.47926	0.00217
1.6	0.16453	0.00002	0.49162	0.00064	0.55883	0.00114
1.8	0.21887	0.00001	0.57103	0.00031	0.63608	0.00057
2.0	0.28231	0.00000	0.64766	0.00015	0.70815	0.00028
2.2	0.35346	0.00000	0.71871	0.00007	0.77277	0.00013
2.4	0.43015	0.00000	0.78201	0.00003	0.82844	0.00006
2.6	0.50957	0.00000	0.83621	0.00001	0.87452	0.00003
2.8	0.58862	0.00000	0.88080	0.00001	0.91117	0.00001
3.0	0.66422	0.00000	0.91605	0.00000	0.93919	0.00000

Table I (2). The controlling limits for the controlling procedures $C(1, 2)$ and $C^{(\pm)}(1, 2)$ for assigned values of α and ρ .

α	ρ		0.5		1.0		2.0	
	C	a						
			a_1	a_2	a_1	a_2	a_1	a_2
0.010	$C(1, 2)$		2.935	1.704	2.807	1.760	2.713	1.836
	$C^{(\pm)}(1, 2)$		2.935	1.546	2.807	1.609	2.713	1.695
0.001	$C(1, 2)$		3.588	2.219	3.481	2.272	3.403	2.344
	$C^{(\pm)}(1, 2)$		3.588	2.084	3.481	2.140	3.403	2.216

Table II (2). The fundamental probabilities falling into the domains limited by control charts. $\alpha = 0.001$

$\begin{matrix} C \\ P \\ k \end{matrix}$	$C(1)$		$C(2)$		$C^{(\pm)}(2)$	
	$P_{+1}^*(k)$	$P_{-1}^*(k)$	$P_{+2}^*(k)$	$P_{-2}^*(k)$	$P_{+2}^*(k)$	$P_{-2}^*(k)$
0.0	0.00050	0.00050	0.01606	0.01606	0.02264	0.02264
0.2	0.00100	0.00024	0.02601	0.00957	0.03577	0.01383
0.4	0.00193	0.00011	0.04067	0.00550	0.05458	0.00815
0.6	0.00357	0.00005	0.06142	0.00304	0.08046	0.00463
0.8	0.00639	0.00002	0.08964	0.00163	0.11468	0.00254
1.0	0.01101	0.00001	0.12652	0.00084	0.15817	0.00134
1.2	0.01831	0.00000	0.17284	0.00041	0.21128	0.00068
1.4	0.02938	0.00000	0.22874	0.00020	0.27359	0.00033
1.6	0.04551	0.00000	0.29357	0.00009	0.34384	0.00016
1.8	0.06811	0.00000	0.36580	0.00004	0.41996	0.00007
2.0	0.09853	0.00000	0.44315	0.00002	0.49920	0.00003
2.2	0.13786	0.00000	0.52273	0.00001	0.57848	0.00001
2.4	0.18673	0.00000	0.60141	0.00000	0.65469	0.00001
2.6	0.24510	0.00000	0.67616	0.00000	0.72508	0.00000
2.8	0.31207	0.00000	0.74441	0.00000	0.78757	0.00000
3.0	0.38591	0.00000	0.80428	0.00000	0.84086	0.00000

Table II (3). The fundamental probabilities falling into the domains limited by control charts.**C**(1, 2) $\alpha = 0.010$

ρ k	0.5				1.0				2.0			
	$P_{+1}^*(k)$	$P_{-1}^*(k)$	$P_{+2}^*(k)$	$P_{-2}^*(k)$	$P_{+1}^*(k)$	$P_{-1}^*(k)$	$P_{+2}^*(k)$	$P_{-2}^*(k)$	$P_{+1}^*(k)$	$P_{-1}^*(k)$	$P_{+2}^*(k)$	$P_{-2}^*(k)$
0.0	0.00167	0.00167	0.04252	0.04252	0.00250	0.00250	0.03670	0.03670	0.00333	0.00333	0.02985	0.02985
0.2	0.00312	0.00086	0.06317	0.02760	0.00457	0.00132	0.05481	0.02368	0.00599	0.00179	0.04494	0.01909
0.4	0.00562	0.00043	0.09050	0.01726	0.00804	0.00067	0.07887	0.01472	0.01036	0.00093	0.06514	0.01175
0.6	0.00977	0.00020	0.12503	0.01041	0.01366	0.00033	0.10937	0.00881	0.01730	0.00046	0.09093	0.00696
0.8	0.01638	0.00009	0.16662	0.00605	0.02238	0.00016	0.14615	0.00508	0.02787	0.00022	0.12223	0.00397
1.0	0.02650	0.00004	0.21422	0.00339	0.03538	0.00007	0.18825	0.00282	0.04336	0.00010	0.15822	0.00218
1.2	0.04137	0.00002	0.26576	0.00182	0.05403	0.00003	0.23371	0.00151	0.06514	0.00005	0.19725	0.00115
1.4	0.06239	0.00001	0.31817	0.00095	0.07971	0.00001	0.27971	0.00078	0.09459	0.00002	0.23683	0.00059
1.6	0.09094	0.00000	0.36765	0.00048	0.11372	0.00001	0.32272	0.00039	0.13285	0.00001	0.27386	0.00029
1.8	0.12819	0.00000	0.41005	0.00023	0.15697	0.00000	0.35899	0.00018	0.18062	0.00000	0.30502	0.00014
2.0	0.17490	0.00000	0.44149	0.00011	0.20983	0.00000	0.38500	0.00008	0.23792	0.00000	0.32721	0.00006
2.2	0.23117	0.00000	0.45888	0.00005	0.27193	0.00000	0.39811	0.00004	0.30398	0.00000	0.33810	0.00003
2.4	0.29633	0.00000	0.46046	0.00002	0.34200	0.00000	0.39691	0.00002	0.37714	0.00000	0.33648	0.00001
2.6	0.36881	0.00000	0.44606	0.00001	0.41801	0.00000	0.38154	0.00001	0.45502	0.00000	0.32255	0.00001
2.8	0.44631	0.00000	0.41716	0.00000	0.49721	0.00000	0.35362	0.00000	0.53467	0.00000	0.29781	0.00000
3.0	0.52591	0.00000	0.37660	0.00000	0.57652	0.00000	0.31599	0.00000	0.61294	0.00000	0.26484	0.00000

Table II (4). The fundamental probabilities falling into the domains limited by control charts.

$C(1, 2) \quad \alpha = 0.001$

ρ k	0.5				1.0				2.0			
	$P_{+1}^*(k)$	$P_{-1}^*(k)$	$P_{+2}^*(k)$	$P_{-2}^*(k)$	$P_{+1}^*(k)$	$P_{-1}^*(k)$	$P_{+2}^*(k)$	$P_{-2}^*(k)$	$P_{+1}^*(k)$	$P_{-1}^*(k)$	$P_{+2}^*(k)$	$P_{-2}^*(k)$
0.0	0.00017	0.00017	0.01308	0.01308	0.00025	0.00025	0.01129	0.01129	0.00033	0.00033	0.00921	0.00921
0.2	0.00035	0.00008	0.02139	0.00771	0.00052	0.00012	0.01862	0.00660	0.00068	0.00016	0.01534	0.00532
0.4	0.00072	0.00003	0.03374	0.00438	0.00103	0.00005	0.02957	0.00372	0.00134	0.00007	0.02461	0.00296
0.6	0.00140	0.00001	0.05132	0.00239	0.00198	0.00002	0.04528	0.00202	0.00253	0.00003	0.03805	0.00159
0.8	0.00265	0.00001	0.07530	0.00126	0.00367	0.00001	0.06684	0.00105	0.00462	0.00001	0.05667	0.00082
1.0	0.00483	0.00000	0.10660	0.00064	0.00655	0.00000	0.09514	0.00053	0.00913	0.00001	0.08034	0.00041
1.2	0.00847	0.00000	0.14563	0.00031	0.01127	0.00000	0.13059	0.00026	0.01380	0.00000	0.11252	0.00020
1.4	0.01434	0.00000	0.19206	0.00015	0.01872	0.00000	0.17289	0.00012	0.02259	0.00000	0.15000	0.00009
1.6	0.02341	0.00000	0.24455	0.00007	0.02999	0.00000	0.22081	0.00005	0.03569	0.00000	0.19274	0.00004
1.8	0.03689	0.00000	0.30072	0.00003	0.04598	0.00000	0.27248	0.00002	0.05447	0.00000	0.23875	0.00002
2.0	0.05614	0.00000	0.35718	0.00001	0.06930	0.00000	0.32351	0.00001	0.08031	0.00000	0.28512	0.00001
2.2	0.08257	0.00000	0.40985	0.00001	0.10010	0.00000	0.37120	0.00000	0.11449	0.00000	0.32826	0.00000
2.4	0.11742	0.00000	0.45440	0.00000	0.13985	0.00000	0.41108	0.00000	0.15793	0.00000	0.36440	0.00000
2.6	0.16158	0.00000	0.48682	0.00000	0.18916	0.00000	0.43939	0.00000	0.21099	0.00000	0.39004	0.00000
2.8	0.21535	0.00000	0.50403	0.00000	0.24794	0.00000	0.45331	0.00000	0.27326	0.00000	0.40255	0.00000
3.0	0.27827	0.00000	0.50433	0.00000	0.31526	0.00000	0.45143	0.00000	0.34347	0.00000	0.40061	0.00000

Table II (5). The fundamental probabilities falling into the domains limited by control charts. $C^{(\pm)}(1, 2) \quad \alpha = 0.010$

ρ k	0.5				1.0				2.0			
	$P_{+1}^*(k)$	$P_{-1}^*(k)$	$P_{+2}^*(k)$	$P_{-2}^*(k)$	$P_{+1}^*(k)$	$P_{-1}^*(k)$	$P_{+2}^*(k)$	$P_{-2}^*(k)$	$P_{+1}^*(k)$	$P_{-1}^*(k)$	$P_{+2}^*(k)$	$P_{-2}^*(k)$
0.0	0.00167	0.00167	0.05939	0.05939	0.00250	0.00250	0.05131	0.05131	0.00333	0.00333	0.04170	0.04170
0.2	0.00312	0.00086	0.08603	0.03955	0.00457	0.00132	0.07485	0.03391	0.00599	0.00179	0.06147	0.02726
0.4	0.00562	0.00043	0.12028	0.02540	0.00804	0.00067	0.10529	0.02160	0.01036	0.00093	0.08730	0.01716
0.6	0.00977	0.00020	0.16230	0.01573	0.01366	0.00033	0.14283	0.01326	0.01730	0.00046	0.11946	0.01041
0.8	0.01638	0.00009	0.21145	0.00940	0.02238	0.00016	0.18688	0.00784	0.02787	0.00022	0.15752	0.00608
1.0	0.02650	0.00004	0.26604	0.00541	0.03538	0.00007	0.23588	0.00447	0.04336	0.00010	0.20017	0.00342
1.2	0.04137	0.00002	0.32330	0.00300	0.05403	0.00003	0.28724	0.00245	0.06514	0.00005	0.24516	0.00185
1.4	0.06239	0.00001	0.37957	0.00160	0.07971	0.00001	0.33751	0.00130	0.09459	0.00002	0.28941	0.00096
1.6	0.09094	0.00000	0.43059	0.00083	0.11372	0.00001	0.38269	0.00066	0.13285	0.00001	0.32930	0.00048
1.8	0.12819	0.00000	0.47206	0.00041	0.15697	0.00000	0.41877	0.00032	0.18062	0.00000	0.36119	0.00023
2.0	0.17490	0.00000	0.50019	0.00020	0.20983	0.00000	0.44227	0.00015	0.23792	0.00000	0.38189	0.00011
2.2	0.23117	0.00000	0.51227	0.00009	0.27193	0.00000	0.45081	0.00007	0.30398	0.00000	0.38924	0.00005
2.4	0.29633	0.00000	0.50712	0.00004	0.34200	0.00000	0.44352	0.00003	0.37714	0.00000	0.38246	0.00002
2.6	0.36881	0.00000	0.48525	0.00002	0.41801	0.00000	0.42115	0.00001	0.45502	0.00000	0.36225	0.00001
2.8	0.44631	0.00000	0.44877	0.00001	0.49721	0.00000	0.38597	0.00001	0.53467	0.00000	0.33076	0.00000
3.0	0.52591	0.00000	0.40111	0.00000	0.57652	0.00000	0.34137	0.00000	0.61294	0.00000	0.29111	0.00000

Table II (6). The fundametnal probabilities falling into the domains limited by control charts.

$C^{(\pm)}(1, 2) \quad \alpha = 0.001$

ρ		0.5				1.0				2.0			
k	P	$P_{+1}^*(k)$	$P_{-1}^*(k)$	$P_{+2}^*(k)$	$P_{-2}^*(k)$	$P_{+1}^*(k)$	$P_{-1}^*(k)$	$P_{+2}^*(k)$	$P_{-2}^*(k)$	$P_{+1}^*(k)$	$P_{-1}^*(k)$	$P_{+2}^*(k)$	$P_{-2}^*(k)$
0.0		0.00017	0.00017	0.01841	0.01841	0.00025	0.00025	0.01593	0.01593	0.00033	0.00033	0.01301	0.01301
0.2		0.00035	0.00008	0.02943	0.01111	0.00052	0.00012	0.02567	0.00953	0.00068	0.00016	0.02122	0.00769
0.4		0.00072	0.00003	0.04538	0.00646	0.00103	0.00005	0.03990	0.00549	0.00134	0.00007	0.03335	0.00438
0.6		0.00140	0.00001	0.06750	0.00362	0.00198	0.00002	0.05980	0.00305	0.00253	0.00003	0.05052	0.00240
0.8		0.00265	0.00001	0.09692	0.00196	0.00367	0.00001	0.08645	0.00163	0.00462	0.00001	0.07377	0.00127
1.0		0.00483	0.00000	0.13436	0.00108	0.00655	0.00000	0.12059	0.00084	0.00913	0.00001	0.10286	0.00065
1.2		0.00847	0.00000	0.17988	0.00051	0.01127	0.00000	0.16234	0.00042	0.01380	0.00000	0.14102	0.00032
1.4		0.01434	0.00000	0.23265	0.00025	0.01872	0.00000	0.21093	0.00020	0.02259	0.00000	0.18466	0.00015
1.6		0.02341	0.00000	0.29079	0.00012	0.02999	0.00000	0.26461	0.00009	0.03569	0.00000	0.23325	0.00007
1.8		0.03689	0.00000	0.35132	0.00005	0.04598	0.00000	0.32095	0.00004	0.05447	0.00000	0.28424	0.00003
2.0		0.05614	0.00000	0.41039	0.00002	0.06930	0.00000	0.37503	0.00002	0.08031	0.00000	0.33419	0.00001
2.2		0.08257	0.00000	0.46361	0.00001	0.10010	0.00000	0.42383	0.00001	0.11449	0.00000	0.37913	0.00001
2.4		0.11742	0.00000	0.50658	0.00000	0.13985	0.00000	0.46272	0.00000	0.15793	0.00000	0.41506	0.00000
2.6		0.16158	0.00000	0.53550	0.00000	0.18916	0.00000	0.48808	0.00000	0.21099	0.00000	0.43852	0.00000
2.8		0.21535	0.00000	0.54766	0.00000	0.24794	0.00000	0.49744	0.00000	0.27326	0.00000	0.44714	0.00000
3.0		0.27827	0.00000	0.54190	0.00000	0.31526	0.00000	0.48985	0.00000	0.34347	0.00000	0.44001	0.00000

Table II (7). The fundamental probabilities falling into the domains limited by control charts.
 $\alpha = 0.010$

$\begin{matrix} C \\ P \\ l \end{matrix}$		C(1)	C(2)	C(\pm)(2)
		$P_{+1}^*(l)$	$P_{+2}^*(l)$	$P_{+2}^*(l)$
1.00		0.00500	0.05251	0.07325
1.25		0.01965	0.09732	0.12262
1.50		0.04299	0.13985	0.16652
1.75		0.07051	0.17722	0.20327
2.00		0.09887	0.20868	0.23392
2.25		0.12611	0.23576	0.25946
2.50		0.15151	0.25849	0.28062
2.75		0.17412	0.27793	0.29875
3.00		0.19517	0.29460	0.31419

Table II (8). The fundamental probabilities falling into the domains limited by control charts.
 $\alpha = 0.001$

$\begin{matrix} C \\ P \\ l \end{matrix}$		C(1)	C(2)	C(\pm)(2)
		$P_{+1}^*(l)$	$P_{+2}^*(l)$	$P_{+2}^*(l)$
1.00		0.00050	0.01606	0.02264
1.25		0.00424	0.04326	0.05458
1.50		0.01415	0.07650	0.09094
1.75		0.03005	0.11029	0.12631
2.00		0.04999	0.14186	0.15841
2.25		0.07187	0.17055	0.18673
2.50		0.09409	0.19572	0.21157
2.75		0.11585	0.21799	0.23331
3.00		0.13632	0.23761	0.25239

Table II (9). The fundamental probabilities falling into the domains limited by control charts. C(1, 2) $\alpha = 0.010$

$\begin{matrix} \rho \\ P \\ l \end{matrix}$		0.5		1.0		2.0	
		$P_{+2}^*(l)$	$P_{+1}^*(l)$	$P_{+1}^*(l)$	$P_{+2}^*(l)$	$P_{+1}^*(l)$	$P_{+2}^*(l)$
1.00		0.00167	0.04252	0.00250	0.03670	0.00333	0.02985
1.25		0.00944	0.07701	0.01235	0.06721	0.01500	0.05591
1.50		0.02517	0.10280	0.03067	0.08973	0.03523	0.07525
1.75		0.04677	0.11826	0.05436	0.10285	0.06057	0.08652
2.00		0.07105	0.12606	0.08016	0.10927	0.08739	0.09192
2.25		0.09612	0.12841	0.10602	0.11109	0.11391	0.09334
2.50		0.12020	0.12742	0.13072	0.11000	0.13896	0.09251
2.75		0.14299	0.12464	0.15363	0.10746	0.16182	0.09025
3.00		0.16404	0.12098	0.17464	0.10397	0.18300	0.08727

Table II (10). The fundamental probabilities falling into the domains limited by control charts. $C(1, 2) \quad \alpha = 0.001$

ρ l	P	0.5		1.0		2.0	
		$P_{+1}^*(l)$	$P_{+2}^*(l)$	$P_{+1}^*(l)$	$P_{+2}^*(l)$	$P_{+1}^*(l)$	$P_{+2}^*(l)$
1.00		0.00017	0.01308	0.00025	0.01129	0.00033	0.00921
1.25		0.00205	0.03590	0.00268	0.03186	0.00324	0.02715
1.50		0.00838	0.06119	0.01014	0.05474	0.01163	0.04739
1.75		0.02013	0.08227	0.02335	0.07379	0.02589	0.06440
2.00		0.03641	0.09709	0.04084	0.08714	0.04438	0.07622
2.25		0.05536	0.10671	0.06093	0.09532	0.06527	0.08344
2.50		0.07564	0.11163	0.08196	0.09971	0.08676	0.08737
2.75		0.09595	0.11389	0.10276	0.10165	0.10804	0.08906
3.00		0.11585	0.11380	0.12302	0.10150	0.12840	0.08900

Table II (11). The fundamental probabilities falling into the domains limited by control charts. $C^{(\pm)}(1, 2) \quad \alpha = 0.010$

ρ l	P	0.5		1.0		2.0	
		$P_{+1}^*(l)$	$P_{+2}^*(l)$	$P_{+1}^*(l)$	$P_{+2}^*(l)$	$P_{+1}^*(l)$	$P_{+2}^*(l)$
1.00		0.07167	0.05939	0.00250	0.05131	0.00333	0.04170
1.25		0.00944	0.09861	0.01235	0.08670	0.01500	0.07255
1.50		0.02517	0.12610	0.03367	0.11096	0.03523	0.09401
1.75		0.04677	0.14185	0.05436	0.12469	0.06057	0.10570
2.00		0.07105	0.14871	0.08016	0.13025	0.08739	0.11083
2.25		0.09612	0.14992	0.10602	0.13129	0.11391	0.11182
2.50		0.12020	0.14809	0.13072	0.12907	0.13896	0.10993
2.75		0.14299	0.14407	0.15363	0.12565	0.16182	0.10713
3.00		0.16404	0.13924	0.17464	0.12134	0.18300	0.10304

Table II (12). The fundamental probabilities falling into the domains limited by control charts. $C^{(\pm)}(1, 2) \quad \alpha = 0.001$

ρ l	P	0.5		1.0		2.0	
		$P_{+1}^*(l)$	$P_{+2}^*(l)$	$P_{+1}^*(l)$	$P_{+2}^*(l)$	$P_{+1}^*(l)$	$P_{+2}^*(l)$
1.00		0.00017	0.01841	0.00025	0.01593	0.00033	0.01301
1.25		0.00205	0.04571	0.00268	0.04077	0.00324	0.03487
1.50		0.00838	0.07404	0.01014	0.06665	0.01163	0.05820
1.75		0.02013	0.09669	0.02335	0.08731	0.02589	0.07687
2.00		0.03641	0.11230	0.04084	0.10147	0.04438	0.08955
2.25		0.05536	0.12187	0.06093	0.10987	0.06527	0.09705
2.50		0.07564	0.12650	0.08196	0.11404	0.08676	0.10105
2.75		0.09595	0.12828	0.10276	0.11553	0.10804	0.10208
3.00		0.11585	0.12768	0.12302	0.11490	0.12840	0.10156

Table III (1). $T_k(\mathbf{C})^{-1}$ The reciprocal number of the expected value of numbers of trials for attaining the first occurrence of the event in the control method \mathbf{C} under the hypothesis H_k .

$$\alpha = 0.010$$

\mathbf{C} ρ k	$\mathbf{C}(1, 2)$			$\mathbf{C}^{(\pm)}(1, 2)$			$\mathbf{C}(1)$	$\mathbf{C}(2)$	$\mathbf{C}^{(\pm)}(2)$
	0.5	1.0	2.0	0.5	1.0	2.0			
0.0	0.01000	0.01002	0.01001	0.00999	0.01001	0.01001	0.01000	0.00998	0.01000
0.2	0.01153	0.01160	0.01163	0.01230	0.01221	0.01206	0.01150	0.01127	0.01234
0.4	0.01653	0.01672	0.01678	0.01959	0.01920	0.01859	0.01624	0.01554	0.01969
0.6	0.02613	0.02648	0.02649	0.03288	0.03201	0.03062	0.02482	0.02385	0.03285
0.8	0.04190	0.04240	0.04224	0.05347	0.05202	0.04957	0.03823	0.03758	0.05276
1.0	0.06543	0.06610	0.06563	0.08247	0.08049	0.07686	0.05769	0.05804	0.08004
1.2	0.09788	0.09885	0.09803	0.12039	0.11816	0.11346	0.08449	0.08595	0.11456
1.4	0.13960	0.14056	0.14016	0.16684	0.16490	0.15957	0.11983	0.12112	0.15528
1.6	0.18999	0.19230	0.19185	0.22055	0.21964	0.21444	0.16455	0.16239	0.20034
1.8	0.24755	0.25188	0.25197	0.27957	0.28058	0.27647	0.21888	0.20774	0.24730
2.0	0.31017	0.31682	0.31862	0.34167	0.34545	0.34346	0.28231	0.25467	0.29358
2.2	0.37553	0.38530	0.38942	0.40470	0.41201	0.41304	0.35346	0.30058	0.33686
2.4	0.44151	0.45479	0.46186	0.46696	0.47828	0.48295	0.43015	0.34319	0.37535
2.6	0.50641	0.52338	0.53368	0.52735	0.54281	0.55135	0.50957	0.38082	0.40799
2.8	0.56910	0.58959	0.60301	0.58532	0.60469	0.61687	0.58862	0.41249	0.43441
3.0	0.62894	0.65240	0.66840	0.64074	0.66340	0.67858	0.66422	0.43796	0.45487

Table III (2). $T_k(\mathbf{C})^{-1}$ The reciprocal number of the expected value of numbers of trials for attaining the first occurrence of the event in the control method \mathbf{C} under the hypothesis H_k .

$$\alpha = 0.001$$

\mathbf{C} $k \backslash \rho$	$\mathbf{C}(1, 2)$			$\mathbf{C}^{\pm}(1, 2)$			$\mathbf{C}(1)$	$\mathbf{C}(2)$	$\mathbf{C}^{\pm}(2)$
	0.5	1.0	2.0	0.5	1.0	2.0			
0.0	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100
0.2	0.00125	0.00125	0.00126	0.00139	0.00137	0.00134	0.00124	0.00122	0.00143
0.4	0.00215	0.00216	0.00215	0.00276	0.00264	0.00250	0.00204	0.00204	0.00289
0.6	0.00416	0.00414	0.00407	0.00570	0.00539	0.00500	0.00362	0.00390	0.00601
0.8	0.00810	0.00800	0.00776	0.01123	0.01056	0.00970	0.00641	0.00763	0.01181
1.0	0.01522	0.01491	0.01517	0.02074	0.01953	0.01873	0.01102	0.01439	0.02160
1.2	0.02706	0.02641	0.02522	0.03589	0.03395	0.03123	0.01831	0.02558	0.03685
1.4	0.04532	0.04424	0.04218	0.05825	0.05546	0.05137	0.02938	0.04265	0.05877
1.6	0.07148	0.06994	0.06685	0.08391	0.08536	0.07981	0.04551	0.06666	0.08798
1.8	0.10643	0.10434	0.10049	0.12822	0.12396	0.11738	0.06811	0.09799	0.12420
2.0	0.15015	0.14838	0.14357	0.17555	0.17159	0.16402	0.09853	0.13609	0.16622
2.2	0.20172	0.20059	0.19562	0.22942	0.22626	0.21871	0.13786	0.17945	0.21200
2.4	0.25939	0.25961	0.25525	0.28775	0.28623	0.27968	0.18673	0.22586	0.25903
2.6	0.32097	0.32329	0.32043	0.34833	0.34925	0.34467	0.24510	0.27277	0.30476
2.8	0.38426	0.38933	0.38879	0.40914	0.41318	0.41141	0.31207	0.31767	0.34699
3.0	0.44735	0.45567	0.45806	0.46872	0.47632	0.47792	0.38591	0.35852	0.38409

Table III (3). $T_l^*(C)$ The reciprocal number of the expected value of numbers of trials for attaining the first occurrence of the event in the control method **C** under the hypothesis H_l^* .
 $\alpha = 0.010$

$\begin{matrix} \text{C} \\ \rho \\ l \end{matrix}$	C (1, 2)			C ^(±) (1, 2)			C (1)	C (2)	C ^(±) (2)
	0.5	1.0	2.0	0.5	1.0	2.0			
1.00	0.01000	0.01002	0.01001	0.00399	0.01001	0.01001	0.01000	0.00998	0.01000
1.25	0.03943	0.04063	0.04125	0.03657	0.03854	0.03982	0.03930	0.03171	0.02679
1.50	0.08541	0.08865	0.09014	0.07859	0.08351	0.08661	0.08558	0.06113	0.04754
1.75	0.13878	0.14381	0.14667	0.12878	0.13636	0.14135	0.14102	0.09276	0.06868
2.00	0.19287	0.19951	0.20333	0.18061	0.19034	0.19690	0.19775	0.12290	0.08869
2.25	0.24471	0.25242	0.25719	0.23133	0.24251	0.25031	0.25221	0.15109	0.10691
2.50	0.29215	0.30111	0.30681	0.27860	0.29095	0.29970	0.30301	0.17619	0.12298
2.75	0.33572	0.34527	0.35124	0.32226	0.33530	0.34437	0.34825	0.19859	0.13744
3.00	0.37521	0.38507	0.39193	0.36211	0.37554	0.38525	0.39034	0.21845	0.15023

Table III (4). $T_l^*(C)$ The reciprocal number of the expected value of numbers of trials for attaining the first occurrence of the event in the control method **C** under the hypothesis H_l^* .
 $\alpha = 0.001$

$\begin{matrix} \text{C} \\ \rho \\ l \end{matrix}$	C (1, 2)			C ^(±) (1, 2)			C (1)	C (2)	C ^(±) (2)
	0.5	1.0	2.0	0.5	1.0	2.0			
1.00	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100	0.00100
1.25	0.00891	0.00917	0.00929	0.00810	0.00855	0.00884	0.00849	0.00689	0.00565
1.50	0.03010	0.03109	0.03148	0.02696	0.02861	0.02967	0.02831	0.02030	0.01516
1.75	0.06351	0.06568	0.06647	0.05732	0.06073	0.06275	0.06011	0.03986	0.02833
2.00	0.10439	0.10755	0.10892	0.09549	0.10038	0.10348	0.09997	0.06271	0.04333
2.25	0.14825	0.15239	0.15440	0.13719	0.14362	0.14770	0.14374	0.08675	0.05877
2.50	0.19203	0.19708	0.19950	0.17970	0.18727	0.19206	0.18817	0.11012	0.07389
2.75	0.23415	0.23986	0.24302	0.22106	0.22944	0.23500	0.23170	0.13237	0.08827
3.00	0.27390	0.28030	0.28369	0.26061	0.26973	0.27552	0.27264	0.15309	0.10172

Table IV (1). $T_k(\mathbf{C})^{-1}/T_k(\mathbf{C}(1))^{-1}$ The relative reciprocal number of the expected value of numbers of trials for attaing the first occurance of the event in the control methods in comparison with the usual method $\mathbf{C}(1)$ under the hypothesis H_k .

$$\alpha = 0.010$$

\mathbf{C}		$\mathbf{C}(1, 2)$			$\mathbf{C}^{\pm}(1, 2)$			$\mathbf{C}(2)$	$\mathbf{C}^{\pm}(2)$	$T_k(\mathbf{C}[1])$
k	ρ	0.5	1.0	2.0	0.5	1.0	2.0			
0.0		1.000	1.003	1.001	1.000	1.001	1.001	0.998	1.000	100.0400
0.2		1.002	1.008	1.011	1.069	1.062	1.048	0.980	1.073	86.9338
0.4		1.018	1.030	1.033	1.206	1.182	1.145	0.957	1.213	61.5839
0.6		1.053	1.067	1.067	1.325	1.289	1.233	0.961	1.323	40.2836
0.8		1.036	1.109	1.105	1.398	1.360	1.296	0.983	1.380	26.1547
1.0		1.134	1.146	1.138	1.430	1.395	1.332	1.006	1.387	17.3349
1.2		1.158	1.170	1.160	1.425	1.399	1.343	1.017	1.356	11.8356
1.4		1.165	1.173	1.170	1.392	1.376	1.332	1.011	1.296	8.3450
1.6		1.155	1.169	1.166	1.340	1.335	1.303	0.987	1.218	6.0773
1.8		1.131	1.151	1.151	1.277	1.282	1.263	0.949	1.130	4.5687
2.0		1.099	1.122	1.129	1.210	1.224	1.217	0.902	1.040	3.5422
2.2		1.062	1.090	1.102	1.145	1.166	1.169	0.850	0.953	2.8292
2.4		1.026	1.057	1.074	1.086	1.112	1.123	0.798	0.873	2.3248
2.6		0.994	1.027	1.047	1.035	1.065	1.082	0.747	0.801	1.9624
2.8		0.967	1.002	1.024	0.994	1.027	1.048	0.701	0.738	1.6989
3.0		0.947	0.982	1.006	0.965	0.999	1.022	0.659	0.685	1.5055

Table IV (2). $T_k(\mathbf{C})^{-1}/T_k(\mathbf{C}(1))^{-1}$ The relative reciprocal number of the expected value of numbers of trials for attaining the first occurrence of the event in the control methods in comparison with the usual method $\mathbf{C}(1)$ under the hypothesis H_k .

$$\alpha = 0.001$$

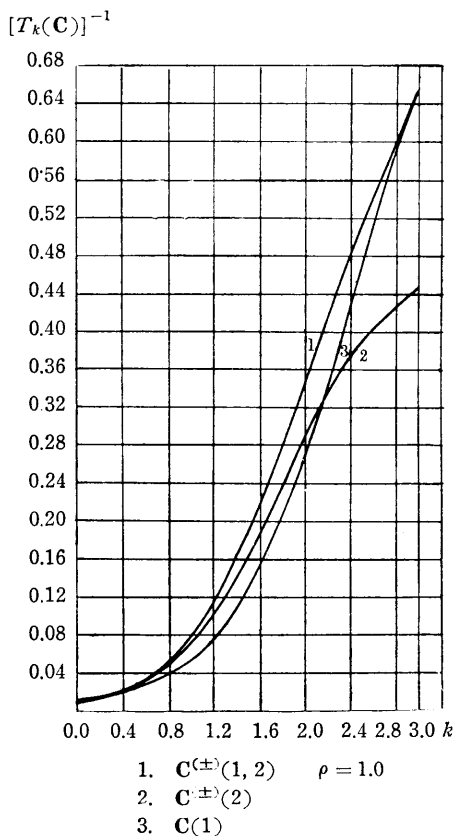
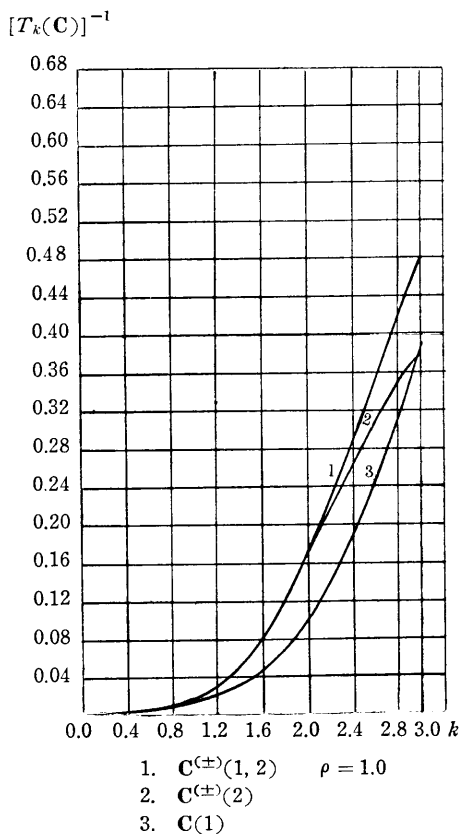
\mathbf{C}		$\mathbf{C}(1, 2)$			$\mathbf{C}^{(\pm)}(1, 2)$			$\mathbf{C}(2)$	$\mathbf{C}^{(\pm)}(2)$	$T_k(\mathbf{C}[1])$
k	ρ	0.5	1.0	2.0	0.5	1.0	2.0			
0.0		0.999	0.997	0.997	0.998	0.998	0.998	0.997	1.000	998.0040
0.2		1.007	1.009	1.010	1.120	1.100	1.076	0.984	1.147	805.1530
0.4		1.054	1.058	1.053	1.355	1.297	1.228	1.000	1.419	490.6771
0.6		1.147	1.143	1.125	1.573	1.488	1.380	1.078	1.660	276.0906
0.8		1.265	1.248	1.211	1.752	1.648	1.514	1.191	1.842	156.0549
1.0		1.381	1.353	1.376	1.882	1.773	1.700	1.306	1.960	90.7441
1.2		1.478	1.442	1.377	1.960	1.854	1.705	1.397	2.012	54.6060
1.4		1.543	1.506	1.436	1.983	1.888	1.749	1.452	2.000	34.0368
1.6		1.571	1.537	1.469	1.954	1.875	1.754	1.465	1.933	21.9713
1.8		1.563	1.532	1.475	1.883	1.820	1.723	1.439	1.824	14.6817
2.0		1.524	1.506	1.457	1.782	1.742	1.665	1.381	1.687	10.1496
2.2		1.463	1.455	1.419	1.664	1.641	1.587	1.302	1.538	7.2539
2.4		1.389	1.390	1.367	1.541	1.533	1.498	1.210	1.387	5.3552
2.6		1.310	1.319	1.307	1.421	1.425	1.406	1.113	1.243	4.0800
2.8		1.231	1.248	1.246	1.311	1.324	1.318	1.018	1.112	3.2044
3.0		1.159	1.181	1.187	1.215	1.234	1.238	0.929	0.995	2.5913

Table IV (3). $T_l^*(\mathbf{C})^{-1}/T_l^*(\mathbf{C}(1))^{-1}$ The relative reciprocal number of the expected value of numbers of trials for attaining the first occurrence of the event in the control methods in comparison with the usual method $\mathbf{C}(1)$ under the hypothesis H_l^* .
 $\alpha = 0.010$

\mathbf{C}		$\mathbf{C}(1, 2)$			$\mathbf{C}^{(\pm)}(1, 2)$			$\mathbf{C}(2)$	$\mathbf{C}^{(\pm)}(2)$	$T_l^*(\mathbf{C}[1])$
l	ρ	0.5	1.0	2.0	0.5	1.0	2.0			
1.00		1.000	1.003	1.001	1.000	1.001	1.001	0.998	1.000	100.0400
1.25		1.003	1.034	1.050	0.931	0.981	1.013	0.807	0.682	25.4440
1.50		0.993	1.031	1.048	0.914	0.971	1.007	0.711	0.553	11.6306
1.75		0.984	1.020	1.040	0.913	0.967	1.002	0.658	0.487	7.0912
2.00		0.975	1.009	1.028	0.913	0.963	0.996	0.622	0.449	5.0570
2.25		0.970	1.001	1.020	0.917	0.962	0.992	0.599	0.424	3.9649
2.50		0.964	0.994	1.013	0.919	0.960	0.989	0.581	0.406	3.3002
2.75		0.964	0.991	1.009	0.925	0.963	0.989	0.570	0.395	2.8715
3.00		0.961	0.986	1.004	0.928	0.962	0.987	0.560	0.385	2.5619

Table IV (4). $T_l^*(\mathbf{C})^{-1}/T_l^*(\mathbf{C}(1))^{-1}$ The relative reciprocal number of the expected value of numbers of trials for attaining the first occurrence of the event in the control methods in comparison with the usual method $\mathbf{C}(1)$ under the hypothesis H_l^* .
 $\alpha = 0.001$

\mathbf{C}		$\mathbf{C}(1, 2)$			$\mathbf{C}^{(\pm)}(1, 2)$			$\mathbf{C}(2)$	$\mathbf{C}^{(\pm)}(2)$	$T_l^*(\mathbf{C}[1])$
l	ρ	0.5	1.0	2.0	0.5	1.0	2.0			
1.00		0.999	0.997	0.997	0.998	0.998	0.998	0.997	1.000	998.0040
1.25		1.050	1.080	1.094	0.954	1.007	1.041	0.812	0.666	117.8134
1.50		1.063	1.098	1.112	0.953	1.011	1.048	0.717	0.536	35.3257
1.75		1.057	1.093	1.106	0.954	1.010	1.044	0.663	0.471	16.6367
2.00		1.044	1.076	1.090	0.955	1.004	1.035	0.627	0.433	10.0030
2.25		1.031	1.060	1.074	0.954	0.999	1.028	0.604	0.409	6.9569
2.50		1.021	1.047	1.060	0.955	0.995	1.021	0.585	0.393	5.3142
2.75		1.011	1.035	1.049	0.954	0.990	1.014	0.571	0.381	4.3160
3.00		1.005	1.028	1.041	0.956	0.989	1.011	0.562	0.373	3.6678

Figure I (1) ($\alpha = 0.010$)Figure I (2) ($\alpha = 0.001$)

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