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AUXILIARY TABLES FOR THE APPLICATIONS OF n-DIMENSIONAL t-DISTRIBUTIONS TO CERTAIN CLASS OF EMPIRICAL FUNCTIONS

By

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The importance of multi-dimensional t-distributions has been indicated in a series of the papers on successive process of statistical inferences by T. KITAGAWA [1]~[3] and some of numerical considerations have been given by C. W. DUNNETT and M. SOBEL [1] and [2].

In 1953 T. KITAGAWA [4] and [5] gave three different formulations of empirical functions and discussed statistical inferences concerning concurrence-functions and their derivatives.

Among others the uses of n-dimensional t-distributions were indicated in his formulation of the empirical functions of the type AD verifying to be fundamentally important. The object of the present paper is to provide for various auxiliary tables which are indispensable for the uses of n-dimensional t-distributions in this sense. We shall, however, restrict ourselves to the special case when the following three conditions are satisfied: (i) the n points of observations $0 \leqq t_1 \leqq t_2 \leqq \cdots \leqq t_n \leqq 1$ is equi-distributed; (ii) the numbers of observations are equal, i.e., $r_1 = r_2 = \cdots = r_n = r$; (iii) $\{\varphi_\nu(t_i; n)\}$ $(\nu = 0, 1, 2, \cdots, n-1)$ is the orthogonal normalised polynomial system associated with $\{t_i\}$.

In view of Theorems 1.1 and 1.2 in T. KITAGAWA [4] our tasks of constructing auxiliary tables were scheduled in the following steps:

(1°) $$L(t; t_i, m) = \sum_{\nu=0}^{m-1} \varphi_\nu(t_i; n)\varphi_\nu(t; n)$$

(2°) $$a_{ij}(t) = L(t; t_i, m)L(t; t, m) \equiv a_{ij}(t; m)$$

(3°) $$A_{ij} = \int_0^1 L(t; t_i, m)L(t; t_j, m)dt \equiv A_{ij}(m)$$

(4°) $$A_{ij}^{(h)} = (\tau_h - \tau_{h-1})^{-1} \int_{\tau_{h-1}}^{\tau_h} L(t; t_i, m)L(t; t_j, m)dt \equiv A_{ij}^{(h)}(m)$$

(5°) $$D_{ij} = \int_0^1 L'(t; t_i, m)L'(t; t_j, m)dt$$

(6°) $$D_{ij}^{(h)} = (\tau_h - \tau_{h-1})^{-1} \int_{\tau_{h-1}}^{\tau_h} L'(t; t_i, m)L'(t; t_j, m)dt,$$

where $m \leqq n$, $i, j = 1, 2, \cdots, n$. Since there may be various systems of divisions $\{\tau_\nu\}$ our tables give the values of

$$(4^\circ)' \qquad A_{ij}(\theta, m) = \int_0^\theta L(t\,;\, t_i, m) L(t\,;\, t_j, m)\, dt$$

$$(6^\circ)' \qquad D'_{ij}(\theta, m) = \int_0^\theta L'(t\,;\, t_i, m) L'(t\,;\, t_j, m)\, dt$$

instead of (4°) and (6°) respectively.

In this paper we give the case when $1 \leqq m \leqq n \leqq 5$, and we are intending to extend our tables for larger numbers of observation-points n. For the expression of the rational numbers, we give their exact numbers by showing recuring decimals if they are infinite decimals, which yield us somewhat unified expressions from place to place, although they are convenient for practical uses. And we have used the gothic letters in order to show the cyclic parts of them, say 9.**481** implies 9.481481 $\cdots$.

The constructions of these tables were suggested to the author by Prof. T. KITAGAWA. Numerical calculations were due to the aids of Miss. T. NAGAMATSU. The author expresses here her hearty thanks to them. Although we have no available tables for n-dimensional t-distribution for $n \geqq 2$, it is hoped that our auxiliary tables may serve to give us numerical pictures of the AD-type functions.

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Table I-*a*

t_i \ φ_ν	$\varphi_0(t_i;3)$	$\varphi_1(t_i;3)$	$\varphi_2(t_i;3)$
t_1 $(=0)$	$1/\sqrt{3}$	$-1/\sqrt{2}$	$1/\sqrt{6}$
t_2 $(=1/2)$	$1/\sqrt{3}$	0	$-2/\sqrt{6}$
t_3 $(=1)$	$1/\sqrt{3}$	$1/\sqrt{2}$	$1/\sqrt{6}$

$$\varphi_0(t;3)=1/\sqrt{3},$$
$$\varphi_1(t;3)=(-1+2t)/\sqrt{2},$$
$$\varphi_2(t;3)=(1-12t+12t^2)/\sqrt{6}.$$

Table I-*b*-(i)

t_i	$\varphi_0(t_i;3)\,\varphi_0(t;3)$	$\varphi_1(t_i;3)\,\varphi_1(t;3)$	$\varphi_2(t_i;3)\,\varphi_2(t;3)$	$L(t;t_i,3)$
t_1	$1/3$	$(1-2t)/2$	$(1-12t+12t^2)/6$	$1-3t+2t^2$
t_2	$1/3$	0	$(-1+12t-12t^2)/3$	$4t-4t^2$
t_3	$1/3$	$-(1-2t)/2$	$(1-12t+12t^2)/6$	$-t+2t^2$

Table I-c-(i)

$t_i,\ t_j$	$L(t;\ t_i,\ 3)\,L(t;\ t_j,\ 3)$
$t_1,\ t_1$	$1-6t+13t^2-12t^3+4t^4$
$t_1,\ t_2$	$4(t\ -4t^2\ +5t^3-2t^4)$
$t_1,\ t_3$	$-t\ +5t^2\ -8t^3+4t^4$
$t_2,\ t_2$	$16(t^2\ -2t^3\ +t^4)$
$t_2,\ t_3$	$4(-t^2\ +3t^3-2t^4)$
$t_3,\ t_3$	$t^2-\ 4t^3+4t^4$

Table I-d-(i)

$t_i,\ t_j$	$\int_0^\theta L(t;\ t_i,3)\,L(t;\ t_j,3)\,dt$	$\int_0^1 L(t;\ t_i,\ 3)\,L(t;\ t_j,\ 3)\,dt$
$t_1,\ t_1$	$\theta-3\theta^2+4.3\theta^3-3\theta^4+0.8\theta^5$	0.13
$t_1,\ t_2$	$2\theta^2-5.3\theta^3+5\theta^4-1.6\theta^5$	0.06
$t_1,\ t_3$	$-0.5\theta^2+1.6\theta^3-2\theta^4+0.8\theta^5$	−0.03
$t_2,\ t_2$	$5.3\theta^3-8\theta^4+3.2\theta^5$	0.53
$t_2,\ t_3$	$-1.3\theta^3+3\theta^4-1.6\theta^5$	0.06
$t_3,\ t_3$	$0.3\theta^3-\theta^4\ +0.8\theta^5$	0.13

Table I-e-(i)

t_i	$L'(t;\ t_i,\ 3)$
t_1	$-3+4t$
t_2	$4-8t$
t_3	$-1+4t$

Table I-f-(i)

$t_i,\ t_j$	$\int_0^\theta L'(t;\ t_i,3)\,L'(t;\ t_j,3)\,dt$	$\int_0^1 L'(t;\ t_i,3)\,L'(t;\ t_j,3)\,dt$
$t_1,\ t_1$	$9\theta-12\theta^2+\ 5.3\theta^3$	2.3
$t_1,\ t_2$	$-12\theta+20\theta^2-10.6\theta^3$	−2.6
$t_1,\ t_3$	$3\theta-8\theta^2\ +\ 5.3\theta^3$	0.3
$t_2,\ t_2$	$16\theta-32\theta^2+21.3\theta^3$	5.3
$t_2,\ t_3$	$-4\theta+12\theta^2-10.6\theta^3$	−2.6
$t_3,\ t_3$	$\theta-4\theta^2\ +\ 5.3\theta^3$	2.3

Table I-*b*-(ii)

t_i	$L(t\,;\,t_i,\,2)$
t_1	$5/6 - t$
t_2	$1/3$
t_3	$-1/6 + t$

Table I-*c*-(ii)

$t_i,\,t_j$	$L(t\,;\,t_i,\,2)\,L(t\,;\,t_j,\,2)$
$t_1,\,t_1$	$25/36 - 5/3t + t^2$
$t_1,\,t_2$	$5/18 - 1/3t$
$t_1,\,t_3$	$-5/36 \quad + t - t^2$
$t_2,\,t_2$	$1/9$
$t_2,\,t_3$	$-1/18 + 1/3t$
$t_3,\,t_3$	$1/36 - 1/3t + t^2$

Table I-*d*-(ii)

$t_i,\,t_j$	$\int_0^\theta L(t\,;\,t_i,\,2)\,L(t\,;\,t_j,\,2)\,dt$	$\int_0^1 L(t\,;\,t_i,\,2)\,L(t\,;\,t_j,\,2)\,dt$
$t_1,\,t_1$	$0.694\theta - 0.83\theta^2 + 0.3\theta^3$	0.194
$t_1,\,t_2$	0.1θ	0.111
$t_1,\,t_3$	$0.139\theta - 0.6\theta^2$	0.027
$t_2,\,t_2$	$0.27\theta - 0.16\theta^2 + 0.3\theta^3$	0.111
$t_2,\,t_3$	$-0.5\theta + 0.16\theta^2$	0.111
$t_3,\,t_3$	$0.027\theta - 0.16\theta^2 + 0.3\theta^3$	0.194

Table I-*e*-(ii)

t_i	$L'(t\,;\,t_i,\,2)$
t_1	4
t_2	-8
t_3	4

Table I-*f*-(ii)

$t_i,\,t_j$	$\int_0^\theta L'(t\,;\,t_i,\,2)\,L(t\,;\,t_j,\,2)\,dt$	$\int_0^1 L'(t\,;\,t_i,\,2)\,L'(t\,;\,t_j,\,2)\,dt$
$t_1,\,t_1$	16θ	16
$t_1,\,t_2$	-32θ	-32
$t_1,\,t_3$	16θ	16
$t_2,\,t_2$	64θ	64
$t_2,\,t_3$	-32θ	-32
$t_3,\,t_3$	16θ	16

Table II-*a*

t_i \ φ_ν	$\varphi_0(t_i;4)$	$\varphi_1(t_i;4)$	$\varphi_2(t_i;4)$	$\varphi_3(t_i;4)$
$t_1(=0)$	$1/2$	$-3/\sqrt{20}$	$1/2$	$-1/\sqrt{20}$
$t_2(=1/3)$	$1/2$	$-1/\sqrt{20}$	$-1/2$	$3/\sqrt{20}$
$t_3(=2/3)$	$1/2$	$1/\sqrt{20}$	$-1/2$	$-3/\sqrt{20}$
$t_4(=1)$	$1/2$	$3/\sqrt{20}$	$1/2$	$1/\sqrt{20}$

$$\varphi_0(t;4)=1/2$$
$$\varphi_1(t;4)=(-3+6t)/\sqrt{20}$$
$$\varphi_2(t;4)=(1-9t+9t^2)/2$$
$$\varphi_3(t;4)=(-1+47t-135t^2+90t^3)/\sqrt{20}$$

Table II-*b*-(i)

t_i	$\varphi_0(t_i;4)\varphi_0(t;4)$	$\varphi_1(t_i;4)\varphi_1(t;4)$	$\varphi_2(t_i;4)\varphi_2(t;4)$	$\varphi_3(t_i;4)\varphi_3(t;4)$	$L(t;t_i,4)$
t_1	$1/4$	$9(1-2t)/20$	$(1-9t+9t^2)/4$	$(1-47t+135t^2-90t^3)/20$	$1-5.5t+9t^2-4.5t^3$
t_2	$1/4$	$3(1-2t)/20$	$-(1-9t+9t^2)/4$	$-3(1-47t-135t^2-90t^3)/20$	$9t-22.5t^2+13.5t^3$
t_3	$1/4$	$-3(1-2t)/20$	$-(1-9t+9t^2)/4$	$3(1-47t+135t^2-90t^3)/20$	$4.5t+18t^2-13.5t^3$
t_4	$1/4$	$-9(1-2t)/20$	$(1-9t+9t^2)/4$	$-(1-47t+135t^2-90t^3)/20$	$t-4.5t^2+4.5t^3$

Table II-*c*-(i)

$t_i,\ t_j$	$L(t\,;\ t_i,\ 4)\,L(t\,;\ t_j,\ 4)$
$t_1,\ t_1$	$1 - 11t + 48.25t^2 - 108t^3 + 130.5t^4 - 81t^5 + 20.25t^6$
$t_1,\ t_2$	$9t - 72t^2 + 218.25t^3 - 317.25t^4 + 222.75t^5 - 60.75t^6$
$t_1,\ t_3$	$-4.5t + 42.75t^2 - 153t^3 + 256.5t^4 - 202.5t^5 + 60.75t^6$
$t_1,\ t_4$	$t - 10t^2 + 38.25t^3 - 69.75t^4 + 60.75t^5 - 20.25t^6$
$t_2,\ t_2$	$81t^2 - 405t^3 + 749.25t^4 - 607.5t^5 + 182.25t^6$
$t_2,\ t_3$	$-40.5t^2 + 263.25t^3 - 587.25t^4 + 546.75t^5 - 182.25t^6$
$t_2,\ t_4$	$9t^2 - 63t^3 + 155.25t^4 - 162t^5 + 60.75t^6$
$t_3,\ t_3$	$20.25t^2 - 162t^3 + 445.5t^4 - 486t^5 + 182.25t^6$
$t_3,\ t_4$	$-4.5t^2 + 38.25t^3 - 114.75t^4 + 141.75t^5 - 60.75t^6$
$t_4,\ t_4$	$t^2 - 9t^3 + 29.25t^4 - 40.5t^5 + 20.25t^6$

Table II-*d*-(i)

$t_i,\ t_j$	$\int_0^\theta L(t\,;\ t_i,\ 4)\,L(t\,;\ t_j,\ 4)\,dt$	$\int_0^1 L(t;t_i,4)L(t;t_j,4)\,dt$
$t_1,\ t_1$	$\theta - 5.5\theta^2 + 16.083\theta^3 - 27\theta^4 + 26.1\theta^5 - 13.5\theta^6 + 2.892857\theta^7$	0.07618
$t_1,\ t_2$	$4.5\theta^2 - 24\theta^3 + 54.5625\theta^4 - 63.45\theta^5 + 37.125\theta^6 - 8.678571\theta^7$	0.05893
$t_1,\ t_3$	$-2.25\theta^2 + 14.25\theta^3 - 38.25\theta^4 + 51.3\theta^5 - 33.76\theta^6 + 8.678571\theta^7$	−0.02183
$t_1,\ t_4$	$0.5\theta^2 - 3.3\theta^3 + 9.5625\theta^4 - 13.95\theta^5 + 10.125\theta^6 - 2.892857\theta^7$	0.00989
$t_2,\ t_2$	$27\theta^3 - 101.25\theta^4 + 149.85\theta^5 - 101.25\theta^6 + 26.035714\theta^7$	0.38571
$t_2,\ t_3$	$-13.5\theta^3 + 65.8125\theta^4 - 117.45\theta^5 + 91.125\theta^6 - 26.34285\theta^7$	−0.04821
$t_2,\ t_4$	$3\theta^3 - 15.75\theta^4 + 31.05\theta^5 - 27\theta^6 + 8.678571\theta^7$	−0.02143
$t_3,\ t_3$	$6.75\theta^3 - 40.5\theta^4 + 89.1\theta^5 - 81\theta^6 + 26.035714\theta^7$	0.38571
$t_3,\ t_4$	$-1.5\theta^3 + 9.5625\theta^4 - 22.95\theta^5 + 23.625\theta^6 - 8.678571\theta^7$	0.05893
$t_4,\ t_4$	$0.3\theta^3 - 2.25\theta^4 + 5.85\theta^5 - 6.75\theta^6 + 2.892857\theta^7$	0.07618

Table II-*e*-(i)

t_i \ L'	$L'(t\,;\,t_i,\,4)$
t_1	$-5.5+18t-13.5t^2$
t_2	$9-45t+40.5t^2$
t_3	$-4.5+36t-40.5t^2$
t_4	$1-9t+13.5t^2$

Table II-*f*-(i)

$t_i,\,t_j$	$\int_0^\theta L'(t\,;\,t_i,\,4)\,L'(t\,;\,t_j,\,4)\,dt$	$\int_0^1 L'(t\,;\,t_i,\,4)L'(t\,;\,t_j,\,4)\,dt$
$t_1,\,t_1$	$30.25\theta-99\theta^2+157.5\theta^3-121.5\theta^4+36.45\theta^5$	3.7000
$t_1,\,t_2$	$-49.5\theta+93.375\theta^2-141.75\theta^3+197.4375\theta^4-109.35\theta^5$	-9.7865
$t_1,\,t_3$	$24.75\theta-139.5\theta^2+310.5\theta^3-303.75\theta^4+109.35\theta^5$	1.3500
$t_1,\,t_4$	$-5.5\theta+33.5\theta^2-83.25\theta^3+91.125\theta^4-36.45\theta^5$	-0.5750
$t_2,\,t_2$	$81\theta-40.5\theta^2+249.75\theta^3-91.125\theta^4+328.05\theta^5$	527.1750
$t_2,\,t_3$	$-40.5\theta+263.25\theta^2-722.25\theta^3+820.125\theta^4-328.05\theta^5$	-7.4250
$t_2,\,t_4$	$9\theta-63\theta^2+189\theta^3-324\theta^4+109.35\theta^5$	-79.6500
$t_3,\,t_3$	$20.25\theta-182\theta^2+553.5\theta^3-729\theta^4+328.05\theta^5$	-9.2000
$t_3,\,t_4$	$-4.5\theta+38.25\theta^2-141.75\theta^3+212.625\theta^4-109.35\theta^5$	-4.7250
$t_4,\,t_4$	$\theta-9\theta^2+36\theta^3-60.75\theta^4+36.45\theta^5$	3.7000

Table II-*b*-(ii)

t_i	$L(t;\, t_i,\, 3)$
t_1	$(19 - 63t + 45t^2)/20$
t_2	$(3 + 39t - 45t^2)/20$
t_3	$(-3 + 51t - 45t^2)/20$
t_4	$(1 - 27t + 45t^2)/20$

Table II-*c*-(ii)

$t_i,\, t_j$	$L(t;\, t_i,\, 3)\, L(t;\, t_j,\, 3)$
$t_1,\, t_1$	$(361 - 2394t + 5679t^2 - 5670t^3 + 2025t^4)/400$
$t_1,\, t_2$	$(57 + 552t - 3177t^2 + 4590t^3 - 2025t^4)/400$
$t_1,\, t_3$	$(-57 + 1158t - 4203t^2 + 5130t^3 - 2025t^4)/400$
$t_1,\, t_4$	$(19 - 576t + 2601t^2 - 4050t^3 + 2025t^4)/400$
$t_2,\, t_2$	$(9 - 234t + 1791t^2 - 3510t^3 + 2025t^4)/400$
$t_2,\, t_3$	$(-9 + 36t + 1989t^2 - 4050t^3 + 2025t^4)/400$
$t_2,\, t_4$	$(3 - 42t - 9600t^2 + 2970t^3 - 2025t^4)/400$
$t_3,\, t_3$	$(9 - 306t + 2871t^2 - 4590t^3 + 2025t^4)/400$
$t_3,\, t_4$	$(-3 + 132t - 1557t^2 + 3510t^3 - 2025t^4)/400$
$t_4,\, t_4$	$(1 - 54t + 819t^2 - 2430t^3 + 2025t^4)/400$

Table II-*d*-(ii)

$t_i,\, t_j$	$\int_0^\theta L(t;\, t_i,\, 3)\, L(t;\, t_j,\, 3)\, dt$	$\int_0^1 L(t;\, t_i,\, 3)\, L(t;\, t_j,\, 3)\, dt$
$t_1,\, t_1$	$(361\theta - 1297\theta^2 + 1559.6\theta^3 - 1417.5\theta^4 + 405\theta^5)/400$	0.11125
$t_1,\, t_2$	$(57\theta + 276\theta^2 - 1059\theta^3 + 1147.5\theta^4 - 405\theta^5)/400$	0.03875
$t_1,\, t_3$	$(-57\theta + 579\theta^2 - 1401\theta^3 + 1282.5\theta^4 - 405\theta^6)/400$	−0.00375
$t_1,\, t_4$	$(19\theta - 288\theta^2 + 867\theta^3 - 1012.5\theta^4 + 405\theta^5)/400$	−0.02375
$t_2,\, t_2$	$(9\theta - 117\theta^2 + 597\theta^3 - 877.5\theta^4 + 405\theta^5)/400$	0.04125
$t_2,\, t_3$	$(-9\theta + 18\theta^2 + 663\theta^3 - 1012.5\theta^4 + 405\theta^5)/400$	0.16125
$t_2,\, t_4$	$(3\theta - 21\theta^2 - 3200\theta^3 + 742.5\theta^4 - 405\theta^5)/400$	−0.00125
$t_3,\, t_3$	$(9\theta - 153\theta^2 + 957\theta^3 - 1147.5\theta^4 + 405\theta^5)/400$	0.17625
$t_3,\, t_4$	$(-3\theta + 66\theta^2 - 519\theta^3 + 877.5\theta^4 - 405\theta^5)/400$	0.04125
$t_4,\, t_4$	$(\theta - 27\theta^2 + 273\theta^3 - 607.5\theta^4 + 405\theta^5)/400$	0.11125

Table II-*e*-(ii)

t_i \ L'	$L'(t\,;\,t_i,\,3)$
t_1	$-3.15 + 4.5\,t$
t_2	$1.95 - 4.5\,t$
t_3	$2.55 - 4.5\,t$
t_4	$-1.35 + 4.5\,t$

Table II-*f*-(ii)

$t_i,\ t_j$	$\int_0^\theta L'(t;t_i,3)\,L'(t;t_j,3)\,dt$	$\int_0^1 L'(t\,;\,t_i,3)\,L'(t;t_j,3)\,dt$
$t_1,\ t_1$	$9.9225\,\theta - 14.175\,\theta^2 + 6.75\,\theta^3$	2.4975
$t_1,\ t_2$	$-6.1425\,\theta + 11.6375\,\theta^2 - 6.75\,\theta^3$	-1.2550
$t_1,\ t_3$	$-8.0325\,\theta + 12.9825\,\theta^2 - 6.75\,\theta^3$	-1.8000
$t_1,\ t_4$	$4.2525\,\theta - 10.2825\,\theta^2 + 6.75\,\theta^3$	0.7200
$t_2,\ t_2$	$3.8025\,\theta - 8.775\,\theta^2 + 6.75\,\theta^3$	1.7775
$t_2,\ t_3$	$4.9725\,\theta - 10.125\,\theta^2 + 6.75\,\theta^3$	1.5975
$t_2,\ t_4$	$-2.6325\,\theta + 7.425\,\theta^2 - 6.75\,\theta^3$	-1.9575
$t_3,\ t_3$	$6.5025\,\theta - 11.475\,\theta^2 + 6.75\,\theta^3$	1.7775
$t_3,\ t_4$	$-3.4425\,\theta + 8.775\,\theta^2 - 6.75\,\theta^3$	-1.4175
$t_4,\ t_4$	$1.8225\,\theta - 6.75\,\theta^2 + 6.75\,\theta^3$	1.8225

Table II-*b*-(iii)

t_i \ L	$L(t\,;\,t_i,\,2)$
t_1	$(7-9t)/10$
t_2	$(4-3t)/10$
t_3	$(1+3t)/10$
t_4	$(-2+9t)/10$

Table II-*c*-(iii)

$t_i,\,t_j$	$L(t\,;\,t_i,\,2)\,L(t\,;\,t_j,\,2)$
$t_1,\,t_1$	$(49-126t+81t^2)/100$
$t_1,\,t_2$	$(28-57t+27t^2)/100$
$t_1,\,t_3$	$(7+12t-27t^2)/100$
$t_1,\,t_4$	$(-14+81t-81t^2)/100$
$t_2,\,t_2$	$(16-24t+9t^2)/100$
$t_2,\,t_3$	$(4+9t-9t^2)/100$
$t_2,\,t_4$	$(-8+42t-27t^2)/100$
$t_3,\,t_3$	$(1+6t+9t^2)/100$
$t_3,\,t_4$	$(-2+3t+27t^2)/100$
$t_4,\,t_4$	$(4-36t+81t^2)/100$

Table II-*e*-(iii)

t_i	$L'(t\,;\,t_i,\,2)$
t_1	-0.9
t_2	-0.3
t_3	0.3
t_4	0.9

Table II-*d*-(iii)

$t_i,\,t_j$	$\int_0^\theta L(t\,;\,t_i,\,2)\,L(t\,;\,t_j,\,2)\,dt$	$\int_0^1 L(t\,;\,t_i,\,2)\,L(t\,;\,t_j,\,2)\,dt$
$t_1,\,t_1$	$(49\theta-63\theta^2+27\theta^3)/100$	0.265
$t_1,\,t_2$	$(28\theta-28.5\theta^2+9\theta^3)/100$	0.085
$t_1,\,t_3$	$(7\theta+6\theta^2-9\theta^3)/100$	0.040
$t_1,\,t_4$	$(-14\theta+40.5\theta^2-9\theta^3)/100$	-0.005
$t_2,\,t_2$	$(16\theta-12\theta^2+3\theta^3)/100$	0.070
$t_2,\,t_3$	$(4\theta+4.5\theta^2-3\theta^3)/100$	-0.055
$t_2,\,t_4$	$(-8\theta+21\theta^2-9\theta^3)/100$	0.040
$t_3,\,t_3$	$(\theta+3\theta^2+3\theta^3)/100$	0.070
$t_3,\,t_4$	$(-2\theta+1.5\theta^2+9\theta^3)/100$	0.085
$t_4,\,t_4$	$(4\theta-18\theta^2+27\theta^3)/100$	0.265

Table II-*f*-(iii)

$t_i,\,t_j$	$\int_0^\theta L(t\,;\,t_i,\,2)\,L(t\,;\,t_j,\,2)\,dt$
$t_1,\,t_1$	$(49\theta-63\theta^2+27\theta^3)/100$
$t_1,\,t_2$	$(28\theta-28.5\theta^2+9\theta^3)/100$
$t_1,\,t_3$	$(7\theta+6\theta^2-9\theta^3)/100$
$t_1,\,t_4$	$(-14\theta+40.5\theta^2-9\theta^3)/100$
$t_2,\,t_2$	$(16\theta-12\theta^2+3\theta^3)/100$
$t_2,\,t_3$	$(4\theta+4.5\theta^2-3\theta^3)/100$
$t_2,\,t_4$	$(-8\theta+21\theta^2-9\theta^3)/100$
$t_3,\,t_3$	$(\theta+3\theta^2+3\theta^3)/100$
$t_3,\,t_4$	$(-2\theta+1.5\theta^2+9\theta^3)/100$
$t_4,\,t_4$	$(4\theta-18\theta^2+27\theta^3)/100$

Table III-*a*

t_i \ φ_ν	$\varphi_0(t_i;5)$	$\varphi_1(t_i;5)$	$\varphi_2(t_i;5)$	$\varphi_3(t_i;5)$	$\varphi_4(t_i;5)$
$t_1(=0)$	$1/\sqrt{5}$	$-2/\sqrt{10}$	$2/\sqrt{14}$	$-1/\sqrt{10}$	$1/\sqrt{70}$
$t_2(=1/4)$	$1/\sqrt{5}$	$-1/\sqrt{10}$	$-1/\sqrt{14}$	$2/\sqrt{10}$	$-4/\sqrt{70}$
$t_3(=2/4)$	$1/\sqrt{5}$	0	$-2/\sqrt{14}$	0	$6/\sqrt{70}$
$t_4(=3/4)$	$1/\sqrt{5}$	$1/\sqrt{10}$	$-1/\sqrt{14}$	$-2/\sqrt{10}$	$-4/\sqrt{70}$
$t_5(=1)$	$1/\sqrt{5}$	$2/\sqrt{10}$	$2/\sqrt{14}$	$1/\sqrt{10}$	$1/\sqrt{70}$

$$\varphi_0(t;5)=1/\sqrt{5},$$
$$\varphi_1(t;5)=(-2+4t)/\sqrt{10},$$
$$\varphi_2(t;5)=16(1/8-t+t^2)/\sqrt{14},$$
$$\varphi_3(t;5)=(-3+86t-240t^2+160t^3)/3\sqrt{10},$$
$$\varphi_4(t;5)=(3-500t+2740t^2-4480t^3+2240t^3)/3\sqrt{70}.$$

Table III-*b*-(i)

t_i	$\varphi_0(t_i;5)\varphi_0(t;5)$	$\varphi_1(t_i;5)\varphi_1(t;5)$	$\varphi_2(t_i;5)\varphi_2(t;5)$	$\varphi_3(t_i;5)\varphi_3(t;5)$
t_1	$1/5$	$4(1-2t)/10$	$4(1-8t+8t^2)/14$	$(3-86t+240t^2-160t^3)/30$
t_2	$1/5$	$2(1-2t)/10$	$2(-1+8t-8t^2)/14$	$(-3+86t-240t^2+160t^3)/15$
t_3	$1/5$	0	$4(-1+8t-8t^2)/14$	0
t_4	$1/5$	$-2(1-2t)/10$	$2(-1+8t-8t^2)/14$	$(3-86t+240t^2-160t^3)/15$
t_5	$1/5$	$-4(1-2t)/10$	$4(1-8t+8t^2)/14$	$(-3+86t-240t^2+160t^3)/30$

t_i	$\varphi_4(t_i;5)\varphi_4(t;5)$	$L(t;t_i,5)$
t_1	$(3-500t+2740t^2-4480t^3+2240t^4)/210$	$1-25/3t+70/3t^2-80/3t^3+32/3t^4$
t_2	$2(-3+500t-2740t^2+4480t^3-2240t^4)/105$	$16t-208/3t^2+96t^3-128/3t^4$
t_3	$(3-500t+2740t^2-4480t^3+2240t^4)/35$	$-12t+76t^2-128t^3+64t^4$
t_4	$2(-3+500t-2740t^2+4480t^3-2240t^4)/105$	$16/3t-112/3t^2+224/3t^3-128/3t^4$
t_5	$(3-500t+2740t^2-4480t^3+2240t^4)/210$	$-t+22/3t^2-16t^3+32/3t^4$

Table III-c-(i)

t_i, t_j	$L(t\,;\,t_i,\,5)\,L(t\,;\,t_j,\,5)$
$t_1,\ t_1$	$1-16.6t+116.1t^2-442.2t^3+1010.2t^4-1422.2t^5+1208.8t^6-568.8t^7+113.7t^8$
$t_1,\ t_2$	$16t-202.6t^2+1047.1t^3-2887.1t^4+4615.1t^5-4295.1t^6+2161.7t^7-455.1t^8$
$t_1,\ t_3$	$-12t+176t^2-1041.3t^3+3224t^4-5674.6t^5+5717.3t^6-3072t^7+682.6t^8$
$t_1,\ t_4$	$16t-512t^2+1530.6t^3-5034.6t^4+9450.6t^5-10153.6t^6+5802.6t^7-1365.3t^8$
$t_1,\ t_5$	$-t+15.6t^2-100.4t^3+341.7t^4-668.4t^5+753.7t^6-455.1t^7+113.7t^8$
$t_2,\ t_2$	$256t^2-2218.6t^3+7879.1t^4-14677.3t^5+15132.4t^6-8192t^7+1820.4t^8$
$t_2,\ t_3$	$-192t^2+2048t^3-8469.3t^4+17706.6t^5-19968t^6+11605.3t^7-2730.6t^8$
$t_2,\ t_4$	$256t^2-2901.3t^3+12885.3t^4-29013.3t^5+35157.3t^6-21845.3t^7+541.3t^8$
$t_2,\ t_5$	$-16t^2+186.6t^3-860.4t^4+2026.6t^5-2588.4t^6+1706.6t^7-455.1t^8$
$t_3,\ t_3$	$144t^2-1824t^3+8848t^4-20992t^5+26112t^6-16384t^7+4096t^8$
$t_3,\ t_4$	$(-192t^2+2560t^3-13248t^4+3392t^5-45568t^6+30720t^7-8192t^8)/3$
$t_3,\ t_5$	$12t^2-164t^3+877.3t^4-2346.6t^5+3328t^6-2389.3t^7+682.6t^8$
$t_4,\ t_4$	$(256t^2-3584t^3+19712t^4-54272t^5+78848t^6-57344t^7+16384t^8)/9$
$t_4,\ t_5$	$-16t^2+229.3t^3-1301.3t^4+3733.3t^5-5717.3t^6+4437.3t^7-1365.3t^8$
$t_5,\ t_5$	$t^2-14.6t^3+85.7t^4-256t^5+412.4t^6-341.3t^7+113.7t^8$

Table III-*d*-(i)

t_i, t_j	$\int_0^\theta L(t;t_i,5)L(t;t_j,5)\,dt$							$\int_0^1 L(t;t_i,5)L(t;t_j,5)\,dt$
t_1, t_1	0	-8.30^2	$+38.7037 0^3$	-110.50^4	$+202.040^5$	-237.0370^6	$+172.6984120^7$	-71.10^8	$+12.5308640^9$	0.05150
t_1, t_2	80^2	-67.50^3	$+261.70^4$	-577.420^5	$+769.1850^6$	-613.5873020^7	$+270.20^8$	-50.5679010^9	0.05220
t_1, t_3	-60^2	$+58.60^3$	-260.30^4	$+655.80^5$	-945.70^6	$+816.7619040^7$	-3840^8	$+75.8510^9$	-0.03070
t_1, t_4	80^2	-170.60^3	$+382.60^4$	-1006.930^5	$+1575.10^6$	-1450.5238090^7	$+725.30^8$	-151.70^9	0.17250
t_1, t_5	-0.50^2	$+5.20^3$	-25.10^4	$+68.350^5$	-111.4740^6	$+107.6825390^7$	-56.80^8	$+12.6419750^9$	-0.00512
t_2, t_2		85.30^3	-554.60^4	$+1575.820^5$	-2446.20^6	$+2161.70^7$	-10240^8	$+202.2716050^9$	0.31604
t_2, t_3		-640^3	$+5120^4$	-1693.860^5	$+2951.10^6$	-2852.5714280^7	$+1450.60^8$	-303.4070^9	-0.06773
t_2, t_4		85.30^3	-725.30^4	$+2577.060^5$	-4835.50^6	$+5022.4825390^7$	-2730.60^8	$+606.8140^9$	0.13545
t_2, t_5		-5.30^3	$+46.60^4$	-172.080^5	$+337.60^6$	-369.70^7	$+213.30^8$	-50.5679010^9	0.00986
t_3, t_3		480^3	-4560^4	$+1769.60^5$	-3498.60^6	$+3730.285714\,0^7$	-20480^8	$+455.10^9$	0.33016
t_3, t_4		-240^3	$+213.30^4$	-883.20^5	$+188.40^6$	-2169.9047620^7	$+12800^8$	-303.4070^9	-0.06772
t_3, t_5		40^3	-410^4	$+175.460^5$	-391.10^6	$+475.4285710^7$	-298.60^8	$+29.8510^9$	-0.03069
t_4, t_4		9.4810^3	-99.50^4	$+438.40^5$	-1005.0370^6	$+1251.50^7$	-796.40^8	$+22.2716050^9$	0.31604
t_4, t_5		-5.30^3	$+57.30^4$	-260.260^5	$+622.20^6$	-816.7619050^7	$+554.6541^8$	-151.7039^9	0.15662
t_5, t_5		0.30^3	-3.60^4	$+17.150^5$	-42.50^6	$+58.9206350^7$	-42.60^8	$+12.6419750^9$	0.05150

Table III-*e*-(i)

t_i	$L'(t\,;\,t_i,\,5)$
t_1	$-25/3+140/3\,t-80\,t^2+128/3\,t^3$
t_2	$16-416/3\,t+288\,t^2-170.6\,t^3$
t_3	$-12+152\,t-384\,t^2+256\,t^3$
t_4	$16/3-224/3\,t+224\,t^2-512/3\,t^3$
t_5	$-1+44/3\,t-48\,t^2+128/3\,t^3$

Table III-*f*-(i)

$t_i,\,t_j$	$\int_0^\theta L'(t\,;\,t_i,\,5)\,L'(t\,;\,t_j,\,5)\,dt$	$\int_0^1 L'(t\,;\,t_i,\,5)\,L'(t\,;\,t_j,\,5)\,dt$
$t_1,\,t_1$	$69.4\,\theta-388.8\,\theta^2+1170.370\,\theta^3-2014.4\,\theta^4+1678.2\,\theta^5-1137.7\,\theta^6+260.0634914\,\theta^7$	393.01058
$t_1,\,t_2$	$-133.3\,\theta+951.1\,\theta^2-3383.703\,\theta^3+6659.5\,\theta^4-9950.75\,\theta^5+4323.5\,\theta^6-140.2539682\,\theta^7$	-1673.82434
$t_1,\,t_3$	$100\,\theta-913.3\,\theta^2+3751.1\,\theta^3-8181.3\,\theta^4+9830.4\,\theta^5-6144\,\theta^6+1560.3809523\,\theta^7$	-3372.73016
$t_1,\,t_4$	$-44.4\,\theta+435.5\,\theta^2-2259.259\,\theta^3+4518.8\,\theta^4-5814.04\,\theta^5+3868.4\,\theta^6-1040.2539682\,\theta^7$	-335.11323
$t_1,\,t_5$	$8.3\,\theta-253.3\,\theta^2+388.148\,\theta^3-952.8\,\theta^4+1219.5\,\theta^5-910.2\,\theta^6+260.0634914\,\theta^7$	-240.34392
$t_2,\,t_2$	$256\,\theta-2218.6\,\theta^2+12814.814\,\theta^3-21333.3\,\theta^4+25766.4\,\theta^5-16384\,\theta^6+4281.7\,\theta^7$	3172.99259
$t_2,\,t_3$	$-192\,\theta+2048\,\theta^2-10225.7\,\theta^3+25792\,\theta^4-34399.73\,\theta^5+23210.6\,\theta^6-6242.3809523\,\theta^7$	-9.62540
$t_2,\,t_4$	$85.3\,\theta-967.1\,\theta^2+3963.259\,\theta^3-14051.5\,\theta^4+20184.17\,\theta^5-14565.2\,\theta^6+4281.7\,\theta^7$	-5351.11852
$t_2,\,t_5$	$-16\,\theta+186.6\,\theta^2-1029.925\,\theta^3+2933.3\,\theta^4-4448.71\,\theta^5+3413.3\,\theta^6-140.2539682\,\theta^7$	898.44233
$t_3,\,t_3$	$144\,\theta-1824\,\theta^2+10773.3\,\theta^3-30720\,\theta^4+45056\,\theta^5-32768\,\theta^6+9362.5714285\,\theta^7$	23.90496
$t_3,\,t_4$	$-64\,\theta+853.3\,\theta^2-5361.7\,\theta^3+16533.3\,\theta^4-26214.4\,\theta^5+20480\,\theta^6-62415.1428571\,\theta^7$	-56188.65397
$t_3,\,t_5$	$12\,\theta-164\,\theta^2+1063.1\,\theta^3-3424\,\theta^4+5734.4\,\theta^5-4778.6\,\theta^6+1560.3809526\,\theta^7$	3.22540
$t_4,\,t_4$	$28.4\,\theta-398.2\,\theta^2+2654.814\,\theta^3-8817.7\,\theta^4+15132.4\,\theta^5-12743.1\,\theta^6+4281.7\,\theta^7$	138.37037
$t_4,\,t_5$	$-5.3\,\theta+76.4\,\theta^2-525.037\,\theta^3+1431.68\,\theta^4-3288.17\,\theta^5+4171.851\,\theta^6-140.2539682\,\theta^7$	14606.38307
$t_5,\,t_5$	$0-14.6\,\theta^2+103.703\,\theta^3-373.3\,\theta^4+911.1\,\theta^5-227.5\,\theta^6+260.0634920\,\theta^7$	660.32275

Table III-*b*-(ii)

t_i	$L(t\,;\,t_i,\,4)$
t_1	$69/70 - 125/21\,t + 72/7\,t^2 - 16/3\,t^3$
t_2	$2/35 + 136/21\,t - 120/7\,t^2 + 32/3\,t^3$
t_3	$-3/35 + 16/7\,t - 16/7\,t^2$
t_4	$2/35 - 88/21\,t + 104/7\,t^2 - 32/3\,t^3$
t_5	$-1/70 + 29/21\,t - 40/7\,t^2 + 16/3\,t^3$

Table III-*c*-(ii)

$t_i,\ t_j$	$L(t\,;\,t_i,\,4)\,L(t\,;\,t_j,\,4)$
$t_1,\ t_1$	$0.971633 - 11.734625\,t + 55.708389\,t^2 - 132.963265\,t^3 + 169.287982\,t^4 - 109.714286\,t^5 + 28.4\,t^6$
$t_1,\ t_2$	$0.056326 + 6.043537\,t - 54.858965\,t^2 + 178.862585\,t^3 - 274.358277\,t^4 + 201.142857\,t^5 - 56.8\,t^6$
$t_1,\ t_3$	$0.084490 + 2.763265\,t - 16.740136\,t^2 + 37.572989\,t^3 - 35.700680\,t^4 + 12.190476\,t^5$
$t_1,\ t_4$	$0.056327 - 4.470748\,t + 40.175964\,t^2 - 142.356462\,t^3 + 238.657596\,t^4 - 188.952381\,t^5 + 56.8\,t^6$
$t_1,\ t_5$	$-0.014082 + 1.446259\,t - 13.999546\,t^2 + 53.551020\,t^3 - 97.886621\,t^4 + 85.3\,t^5 - 28.4\,t^6$
$t_2,\ t_2$	$0.003265 + 0.740136\,t + 39.981960\,t^2 - 220.821769\,t^3 + 4320.36283\,t^4 - 365.714285\,t^5 + 113.7\,t^6$
$t_2,\ t_3$	$-0.004897 - 0.424490\,t + 1614.1496\,t^2 - 54.900680\,t^3 + 63.564625\,t^4 - 24.380952\,t^5$
$t_2,\ t_4$	$0.003265 + 0.130612\,t - 27.268934\,t^2 + 168.054421\,t^3 - 368.471655\,t^4 + 341.3\,t^5 - 113.7\,t^6$
$t_2,\ t_5$	$-0.000816 - 0.001361\,t + 8.861678\,t^2 - 60.527891\,t^3 + 147.229024\,t^4 - 152.380952\,t^5 - 56.8\,t^6$
$t_3,\ t_3$	$0.007347 - 0.391837\,t - 5.616326\,t^2 - 10.448979\,t^3 + 5.224489\,t^4$
$t_3,\ t_4$	$0.004898 + 0.489796\,t - 10.982313\,t^2 + 44.451701\,t^3 - 58.340135\,t^4 + 24.380952\,t^5$
$t_3,\ t_5$	$0.001224 - 0.151020\,t + 3.678911\,t^2 - 16.674829\,t^3 + 25.251700\,t^4 - 12.190476\,t^5$
$t_4,\ t_4$	$0.003265 - 0.478912\,t + 19.258049\,t^2 - 125.736048\,t^3 + 310.131435\,t^4 - 316.952381\,t^5 + 113.7\,t^6$
$t_4,\ t_5$	$0.000817 + 0.138775\,t - 6.325623\,t^2 + 44.919728\,t^3 - 121.977324\,t^4 + 140.190476\,t^5 - 56.8\,t^6$
$t_5,\ t_5$	$0.000204 - 0.039456\,t + 2.070295\,t^2 - 15.934693\,t^3 + 47.383219\,t^4 - 60.952381\,t^5 + 28.4\,t^6$

Table III-*d*-(ii)

t_i, t_j	$\int_0^\theta L(t;\, t_i,\, 4)\, L(t;\, t_j,\, 4)\, dt$	$\int_0^1 L(t;\, t_i,\, 4)\, L(t;\, t_j,\, 4)\, dt$
$t_1,\ t_1$	$0.971633\,\theta - 5.867313\,\theta^2 + 18.569463\,\theta^3 - 33.240816\,\theta^4 + 33.857596\,\theta^5 - 18.252381\,\theta^6 + 4.063492\,\theta^7$	0.06831
$t_1,\ t_2$	$0.056326\,\theta + 3.021769\,\theta^2 - 18.286318\,\theta^3 + 44.715646\,\theta^4 - 54.871655\,\theta^5 + 33.523761\,\theta^6 - 8.126984\,\theta^7$	0.03255
$t_1,\ t_3$	$0.084489\,\theta + 1.381633\,\theta^2 - 5.580045\,\theta^3 + 9.393247\,\theta^4 - 7.140136\,\theta^5 + 2.031746\,\theta^6$	0.17093
$t_1,\ t_4$	$0.056327\,\theta - 2.235374\,\theta^2 + 13.391988\,\theta^3 - 35.589115\,\theta^4 + 47.731519\,\theta^5 - 31.492064\,\theta^6 + 8.126984\,\theta^7$	0.00974
$t_1,\ t_5$	$-0.014082\,\theta + 0.723125\,\theta^2 - 4.66515\,\theta^3 + 13.387755\,\theta^4 - 19.577320\,\theta^5 + 14.2\,\theta^6 - 4.063492\,\theta^7$	0.00097
$t_2,\ t_2$	$0.003265\,\theta + 0.370068\,\theta^2 + 13.327287\,\theta^3 - 55.205442\,\theta^4 + 86.407257\,\theta^5 - 60.952381\,\theta^6 + 16.253968\,\theta^7$	0.20402
$t_2,\ t_3$	$-0.004897\,\theta - 0.212245\,\theta^2 + 5.380498\,\theta^3 - 13.725170\,\theta^4 + 12.712925\,\theta^5 - 4.063492\,\theta^6$	0.09012
$t_2,\ t_4$	$0.003265\,\theta + 0.065306\,\theta^2 - 9.089644\,\theta^3 + 42.013605\,\theta^4 - 73.694331\,\theta^5 + 56.8\,\theta^6 - 16.282539\,\theta^7$	-0.07232
$t_2,\ t_5$	$-0.000817\,\theta - 0.000680\,\theta^2 + 2.953893\,\theta^3 - 15.131972\,\theta^4 + 29.441805\,\theta^5 - 25.396825\,\theta^6 + 8.126984\,\theta^7$	-0.00761
$t_3,\ t_3$	$0.007347\,\theta - 0.195919\,\theta^2 + 1.872109\,\theta^3 + 2.612245\,\theta^4 + 1.044898\,\theta^5$	0.11619
$t_3,\ t_4$	$0.004898\,\theta + 0.244898\,\theta^2 - 3.660771\,\theta^3 + 11.112925\,\theta^4 - 11.668127\,\theta^5 + 4.063492\,\theta^6$	0.09742
$t_3,\ t_5$	$0.001224\,\theta - 0.075510\,\theta^2 + 1.226304\,\theta^3 - 4.168707\,\theta^4 + 5.050340\,\theta^5 - 2.0317460\,\theta^6$	0.00191
$t_4,\ t_4$	$0.003265\,\theta - 0.239456\,\theta^2 + 6.419349\,\theta^3 - 31.434012\,\theta^4 + 62.026287\,\theta^5 - 52.8253970\,\theta^6 + 16.282539\,\theta^7$	0.23758
$t_4,\ t_5$	$0.000817\,\theta + 0.069387\,\theta^2 - 2.108541\,\theta^3 + 11.229932\,\theta^4 - 24.395464\,\theta^5 + 23.365079\,\theta^6 - 8.126984\,\theta^7$	0 03423
$t_5,\ t_5$	$0.000204\,\theta - 0.019728\,\theta^2 + 0.690098\,\theta^3 - 3.983673\,\theta^4 + 9.476644\,\theta^5 - 10.158730\,\theta^6 + 4.063492\,\theta^7$	0.06831

Table III-*e*-(ii)

t_i	$L'(t\,;\,t_i,\,4)$
t_1	$-\,125/21\,+\,144/7\,t\,-\,16\,t^2$
t_2	$136/21\,-\,240/7\,t\,+\,32\,t^2$
t_3	$16/7\,-\,32/7\,t$
t_4	$-\,88/21\,+\,208/7\,t\,-\,32\,t^2$
t_5	$29/21\,-\,80/7\,t\,+\,16\,t^2$

Table III-*f*-(ii)

$t_i,\ t_j$	$\int_0^\theta L'(t\,;\,t_i,\,4)\,L'(t\,;\,t_j;\,4)\,dt$	$\int_0^1 L'(t\,;\,t_i,\,4)\,L'(t\,;\,t_j,\,4)\,dt$
$t_1,\ t_1$	$35.4307813\,\theta - 122.4489600\,\theta^2 + 205.0608860\,\theta^3 - 164.5714253\,\theta^4 + 51.2\,\theta^5$	4.67128
$t_1,\ t_2$	$-\,38.5486900\,\theta + 115.3629077\,\theta^2 - 333.1336389\,\theta^3 + 301.7142892\,\theta^4 - 102.4\,\theta^5$	45.39487
$t_1,\ t_3$	$-\,13.6054400\,\theta + 37.1156403\,\theta^2 - 234.126403\,\theta^3 + 18.2857139\,\theta^4$	$-\,192.33049$
$t_1,\ t_4$	$24.9432700\,\theta - 131.5373939\,\theta^2 + 290.3295685\,\theta^3 - 283.4285657\,\theta^4 + 102.4\,\theta^5$	2.70688
$t_1,\ t_5$	$-\,8.2199413\,\theta + 48.2176794\,\theta^2 - 117.7604571\,\theta^3 + 127.99999975\,\theta^4 - 51.2\,\theta^5$	$-\,0.96262$
$t_2,\ t_2$	$41.9409747\,\theta - 2.2204013\,\theta^2 + 529.9971371\,\theta^3 - 548.5714560\,\theta^4 + 204.8\,\theta^5$	$-\,383.65375$
$t_2,\ t_3$	$14.8027187\,\theta - 53.9863667\,\theta^2 + 24.9033005\,\theta^3 - 36.5714279\,\theta^4$	$-\,50.55779$
$t_2,\ t_4$	$-\,27.1382778\,\theta + 96.9360368\,\theta^2 - 117.1736996\,\theta^3 + 240.4571382\,\theta^4 - 204.8\,\theta^5$	193.08120
$t_2,\ t_5$	$8.9432961\,\theta - 60.68019286\,\theta^2 + 179.8819904\,\theta^3 - 238.5714336\,\theta^4 + 102.4\,\theta^5$	0.97366
$t_3,\ t_3$	$5.2432896\,\theta - 10.4865792\,\theta^2 + 6.9910526\,\theta^3$	1.74776
$t_3,\ t_4$	$-\,9.5782300\,\theta + 43.6596122\,\theta^2 - 511.9999898\,\theta^3 + 36.5714278\,\theta^4$	$-\,441.34718$
$t_3,\ t_5$	$3.1564621\,\theta - 16.2646861\,\theta^2 + 29.6681079\,\theta^3 - 18.2857139\,\theta^4$	$-\,1.72583$
$t_4,\ t_4$	$17.5600621\,\theta - 124.5169869\,\theta^2 + 384.8323026\,\theta^3 - 475.4285619\,\theta^4 + 204.8\,\theta^5$	7.24682
$t_4,\ t_5$	$-\,5.7868386\,\theta + 44.4625779\,\theta^2 - 150.6839749\,\theta^3 + 210.2857101\,\theta^4 - 102.4\,\theta^5$	$-\,4.12253$
$t_5,\ t_5$	$1.9070264\,\theta - 15.7823104\,\theta^2 + 58.4242396\,\theta^3 - 91.4285696\,\theta^4 + 51.2\,\theta^5$	$-\,46.87961$

Table III-*b*-(iii)

t_i	$L(t\,;\,t_i,\,3)$
t_1	$(31 - 108\,t + 80\,t^2)/35$
t_2	$(9 + 26\,t - 40\,t^2)/35$
t_3	$(-3 + 80\,t - 80\,t^2)/35$
t_4	$(-5 + 54\,t - 40\,t^2)/35$
t_5	$(3 - 52\,t + 80\,t^2)/35$

Table III-*c*-(iii)

$t_i,\ t_j$	$L(t\,;\,t_i,\,3)\,L(t\,;\,t_j,\,3)$
$t_1,\ t_1$	$(961 - 6696\,t + 16624\,t^2 - 17280\,t^3 + 6400\,t^4)/1225$
$t_1,\ t_2$	$(279 - 166\,t - 3268\,t^2 + 6400\,t^3 - 3200\,t^4)/1225$
$t_1,\ t_3$	$(-93 + 2804\,t - 11360\,t^2 + 15040\,t^3 - 6400\,t^4)/1225$
$t_1,\ t_4$	$(-155 + 2214\,t - 7472\,t^2 + 8640\,t^3 - 3200\,t^4)/1225$
$t_1,\ t_5$	$(93 - 1936\,t + 8336\,t^2 - 12800\,t^3 + 6400\,t^4)/1225$
$t_2,\ t_2$	$(81 + 468\,t - 44\,t^2 - 2080\,t^3 + 1600\,t^4)/1225$
$t_2,\ t_3$	$(-27 + 642\,t + 1480\,t^2 - 5280\,t^3 + 3200\,t^4)/1225$
$t_2,\ t_4$	$(-45 + 356\,t + 1244\,t^2 - 3200\,t^3 + 1600\,t^4)/1225$
$t_2,\ t_5$	$(27 - 390\,t - 752\,t^2 + 4160\,t^3 - 3200\,t^4)/1225$
$t_3,\ t_3$	$(9 - 480\,t + 6880\,t^2 - 12800\,t^3 + 6400\,t^4)/1225$
$t_3,\ t_4$	$(15 - 562\,t + 4840\,t^2 - 7520\,t^3 + 3200\,t^4)/1225$
$t_3,\ t_5$	$(-9 + 396\,t - 4640\,t^2 + 10560\,t^3 - 6400\,t^4)/1225$
$t_4,\ t_4$	$(25 - 540\,t + 3316\,t^2 - 4320\,t^3 + 1600\,t^4)/1225$
$t_4,\ t_5$	$(-15 + 422\,t - 3328\,t^2 + 6400\,t^3 - 3200\,t^4)/1225$
$t_5,\ t_5$	$(9 - 312\,t + 3184\,t^2 - 8320\,t^3 + 6400\,t^4)/1225$

Table III-*d*-(iii)

$t_i,\ t_j$	$\int_0^\theta L(t\,;\,t_i,\,3)\,L(t\,;\,t_j,\,3)\,dt$	$\int_0^1 L(t\,;\,t_i,\,3)\,L(t\,;\,t_j,\,3)\,dt$
$t_1,\ t_1$	$(961\theta - 3348\theta^2 + 5541.3\theta^3 - 4320\theta^4 + 1280\theta^5)/1225$	0.09333
$t_1,\ t_2$	$(279\theta - 83\theta^2 - 1089.3\theta^3 + 1600\theta^4 - 640\theta^5)/1225$	0.05441
$t_1,\ t_3$	$(-93\theta + 1402\theta^2 - 3786.6\theta^3 + 3760\theta^4 - 1280\theta^5)/1225$	0.00190
$t_1,\ t_4$	$(-155\theta + 1107\theta^2 - 3736\theta^3 + 2160\theta^4 - 640\theta^5)/1225$	-0.01524
$t_1,\ t_5$	$(93\theta - 968\theta^2 + 2778.6\theta^3 - 3200\theta^4 + 1280\theta^5)/1225$	-0.01333
$t_2,\ t_2$	$(81\theta + 234\theta^2 - 14.6\theta^3 - 520\theta^4 + 320\theta^5)/1225$	0.08190
$t_2,\ t_3$	$(-27\theta + 321\theta^2 + 493.3\theta^3 - 1320\theta^4 + 640\theta^5)/1225$	0.00598
$t_2,\ t_4$	$(-45\theta + 178\theta^2 + 414.6\theta^3 - 800\theta^4 + 320\theta^5)/1225$	0.05524
$t_2,\ t_5$	$(27\theta - 195\theta^2 - 250.6\theta^3 + 1040\theta^4 - 640\theta^5)/1225$	-0.01524
$t_3,\ t_3$	$(9\theta + 240\theta^2 + 3440\theta^3 - 3200\theta^4 + 1280\theta^5)/1225$	0.11619
$t_3,\ t_4$	$(15\theta - 281\theta^2 + 1613.3\theta^3 - 1880\theta^4 + 640\theta^5)/1225$	0.08762
$t_3,\ t_5$	$(-9\theta + 198\theta^2 - 1546.6\theta^3 + 2640\theta^4 - 1280\theta^5)/1225$	0.01659
$t_4,\ t_4$	$(25\theta - 270\theta^2 + 1105.3\theta^3 - 1080\theta^4 + 320\theta^5)/1225$	0.08190
$t_4,\ t_5$	$(-15\theta + 211\theta^2 - 1109.3\theta^3 + 1600\theta^4 - 640\theta^5)/1225$	0.03808
$t_5,\ t_5$	$(9\theta - 156\theta^2 + 1061.3\theta^3 - 2080\theta^4 + 1280\theta^5)/1225$	0.09333

Table III-*e*-(iii)

t_i	$L'(t\,;\,t_i,\,3)$
t_1	$(-108+160t)/35$
t_2	$(26-80t)/35$
t_3	$(80-160t)/35$
t_4	$(54-80t)/35$
t_5	$(-52+160t)/35$

Table III-*f*-(iii)

$t_i,\,t_j$	$\int_0^\theta L'(t\,;\,t_i,\,3)\,L'(t\,;\,t_j,\,3)\,dt$	$\int_0^1 L'(t\,;\,t_i,\,3)\,L'(t\,;\,t_j,\,3)\,dt$
$t_1,\,t_1$	$9.5216731\,\theta-14.1061824\,\theta^2+6.9660160\,\theta^3$	2.38151
$t_1,\,t_2$	$-2.2922546\,\theta+5.2245120\,\theta^2-3.4830080\,\theta^3$	-0.55075
$t_1,\,t_3$	$-7.0530912\,\theta+11.8694382\,\theta^2-6.9660160\,\theta^3$	-2.14966
$t_1,\,t_4$	$-4.7608366\,\theta+7.0530912\,\theta^2-3.4830080\,\theta^3$	-1.19075
$t_1,\,t_5$	$4.5845093\,\theta-10.4490240\,\theta^2+6.9660160\,\theta^3$	1.10150
$t_2,\,t_2$	$0.5518391\,\theta-1.6979664\,\theta^2+1.7415040\,\theta^3$	0.59554
$t_2,\,t_3$	$2.2922546\,\theta-4.3102224\,\theta^2+3.4830080\,\theta^3$	1.46504
$t_2,\,t_4$	$1.1461273\,\theta-2.6122560\,\theta^2+1.7415040\,\theta^3$	0.27538
$t_2,\,t_5$	$-1.1036782\,\theta+3.3959328\,\theta^2-3.4830080\,\theta^3$	-1.19075
$t_3,\,t_3$	$5.2245120\,\theta-10.4490240\,\theta^2+6.9660160\,\theta^3$	1.74150
$t_3,\,t_4$	$3.5265456\,\theta-6.1388016\,\theta^2+3.4830080\,\theta^3$	0.87075
$t_3,\,t_5$	$-3.3959328\,\theta+8.620448\,\theta^2-6.9660160\,\theta^3$	-1.74150
$t_4,\,t_4$	$2.3804183\,\theta-3.5265456\,\theta^2+1.7415040\,\theta^3$	0.59538
$t_4,\,t_5$	$-2.2922546\,\theta+5.2245120\,\theta^2-3.4830080\,\theta^3$	-0.55075
$t_5,\,t_5$	$2.2073563\,\theta-6.7918656\,\theta^2+6.9660160\,\theta^3$	2.38151

Table III-*b*-(iv)

t_i	$L(t\,;\,t_i,\,2)$
t_1	$(3-4t)/5$
t_2	$2(1-t)/5$
t_3	$1/5$
t_4	$2t/5$
t_5	$(-1+4t)/5$

Table III-*c*-(iv)

$t_i,\,t_j$	$L(t\,;\,t_i,\,2)\,L(t\,;\,t_j,\,2)$
$t_1,\,t_1$	$(9-24t+16t^2)/25$
$t_1,\,t_2$	$(6-14t+8t^2)/25$
$t_1,\,t_3$	$(3-4t)/25$
$t_1,\,t_4$	$(6-8t)/25$
$t_1,\,t_5$	$(-3+16t-16t^2)/25$
$t_2,\,t_2$	$(4-8t+4t^2)/25$
$t_2,\,t_3$	$(2-2t)/25$
$t_2,\,t_4$	$(4-4t)/25$
$t_2,\,t_5$	$(-2+10t-8t^2)/25$
$t_3,\,t_3$	$1/25$
$t_3,\,t_4$	$2t/25$
$t_3,\,t_5$	$(-1+4t)/25$
$t_4,\,t_4$	$4t^2/25$
$t_4,\,t_5$	$(-2t+8t^2)/25$
$t_5,\,t_5$	$(1-8t+16t^2)/25$

Table III-*d*-(iv)

$t_i,\,t_j$	$\int_0^\theta L(t\,;\,t_i,\,2)\,L(t\,;\,t_j,\,2)\,dt$	$\int_0^1 L(t\,:\,t_i,\,2)\,L(t\,;\,t_j,\,2)\,dt$
$t_1,\,t_1$	$0.36\theta-0.48\theta^2+0.213\theta^3$	0.093
$t_1,\,t_2$	$0.24\theta-0.28\theta^2+0.106\theta^4$	0.066
$t_1,\,t_3$	$0.12\theta-0.08\theta^2$	0.040
$t_1,\,t_4$	$0.24\theta-0.16\theta^2$	0.013
$t_1,\,t_5$	$-0.12\theta+0.32\theta^2-0.213\theta^3$	-0.013
$t_2,\,t_2$	$0.16\theta-0.16\theta^2+0.053\theta^3$	0.053
$t_2,\,t_3$	$0.08\theta-0.04\theta^2$	0.040
$t_2,\,t_4$	$0.16\theta-0.08\theta^2$	0.026
$t_2,\,t_5$	$-0.08\theta+0.2\theta^2-0.106\theta^3$	0.013
$t_3,\,t_3$	0.04θ	0.040
$t_3,\,t_4$	0.04θ	0.040
$t_3,\,t_5$	$-0.04\theta+0.08\theta^2$	0.040
$t_4,\,t_4$	$0.053\theta^3$	0.053
$t_4,\,t_5$	$-0.04\theta^2+0.106\theta^3$	0.066
$t_5,\,t_5$	$0.04\theta-0.16\theta^2+0.213\theta^3$	0.093

Table III-*e*-(iv)

t_i	$L'(t\,;\,t_i,\,2)$
t_1	-0.8
t_2	-0.4
t_3	0
t_4	0.4
t_5	0.8

Table III-*f*-(iv)

$t_i,\,t_j$	$\int_0^\theta L'(t\,;\,t_i,\,2)\,L'(t\,;\,t_j,\,2)\,dt$	$\int_0^1 L'(t\,;\,t_i,\,2)\,L'(t\,;\,t_j,\,2)\,dt$
$t_1,\,t_1$	$0.64\,\theta$	0.64
$t_1,\,t_2$	$0.32\,\theta$	0.32
$t_1,\,t_3$	0	0
$t_1,\,t_4$	$-0.32\,\theta$	-0.32
$t_1,\,t_5$	$-0.64\,\theta$	-0.64
$t_2,\,t_2$	$0.16\,\theta$	0.16
$t_2,\,t_3$	0	0
$t_2,\,t_4$	$-0.16\,\theta$	-0.16
$t_2,\,t_5$	$-0.32\,\theta$	-0.32
$t_3,\,t_3$	0	0
$t_3,\,t_4$	0	0
$t_3,\,t_5$	0	0
$t_4,\,t_4$	$0.16\,\theta$	0.16
$t_4,\,t_5$	$0.32\,\theta$	0.32
$t_5,\,t_5$	$0.64\,\theta$	0.64