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AUXILIARY TABLES FOR THE APPLICATIONS OF n-DIMENSIONAL t-DISTRIBUTIONS TO CERTAIN CLASS OF EMPIRICAL FUNCTIONS

By

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The importance of multi-dimensional t-distributions has been indicated in a series of the papers on successive process of statistical inferences by T. KITAGAWA [1]~[3] and some of numerical considerations have been given by C. W. DUNNETT and M. SOBEL [1] and [2].

In 1953 T. KITAGAWA [4] and [5] gave three different formulations of empirical functions and discussed statistical inferences concerning concurrence-functions and their derivatives.

Among others the uses of n-dimensional t-distributions were indicated in his formulation of the empirical functions of the type AD verifying to be fundamentally important. The object of the present paper is to provide for various auxiliary tables which are indispensable for the uses of n-dimensional t-distributions in this sense. We shall, however, restrict ourselves to the special case when the following three conditions are satisfied: (i) the n points of observations $0 \leqq t_1 \leqq t_2 \leqq \cdots \leqq t_n \leqq 1$ is equi-distributed; (ii) the numbers of observations are equal, i.e., $r_1 = r_2 = \cdots = r_n = r$; (iii) $\{\varphi_\nu(t_i; n)\}$ $(\nu = 0, 1, 2, \cdots, n-1)$ is the orthogonal normalised polynomial system associated with $\{t_i\}$.

In view of Theorems 1.1 and 1.2 in T. KITAGAWA [4] our tasks of constructing auxiliary tables were scheduled in the following steps:

(1°) $$L(t; t_i, m) = \sum_{\nu=0}^{m-1} \varphi_\nu(t_i; n)\varphi_\nu(t; n)$$

(2°) $$a_{ij}(t) = L(t; t_i, m)L(t; t, m) \equiv a_{ij}(t; m)$$

(3°) $$A_{ij} = \int_0^1 L(t; t_i, m)L(t; t_j, m)dt \equiv A_{ij}(m)$$

(4°) $$A_{ij}^{(h)} = (\tau_h - \tau_{h-1})^{-1} \int_{\tau_{h-1}}^{\tau_h} L(t; t_i, m)L(t; t_j, m)dt \equiv A_{ij}^{(h)}(m)$$

(5°) $$D_{ij} = \int_0^1 L'(t; t_i, m)L'(t; t_j, m)dt$$

(6°) $$D_{ij}^{(h)} = (\tau_h - \tau_{h-1})^{-1} \int_{\tau_{h-1}}^{\tau_h} L'(t; t_i, m)L'(t; t_j, m)dt,$$

where $m \leqq n$, $i, j = 1, 2, \cdots, n$. Since there may be various systems of divisions $\{\tau_\nu\}$ our tables give the values of

$$(4^\circ)' \qquad A_{ij}(\theta, m) \equiv \int_0^\theta L(t\,;\, t_i, m) L(t\,;\, t_j, m)\, dt$$

$$(6^\circ)' \qquad D'_{ij}(\theta, m) = \int_0^\theta L'(t\,;\, t_i, m) L'(t\,;\, t_j, m)\, dt$$

instead of (4°) and (6°) respectively.

In this paper we give the case when $1 \leqq m \leqq n \leqq 5$, and we are intending to extend our tables for larger numbers of observation-points n. For the expression of the rational numbers, we give their exact numbers by showing recuring decimals if they are infinite decimals, which yield us somewhat unified expressions from place to place, although they are convenient for practical uses. And we have used the gothic letters in order to show the cyclic parts of them, say 9.**481** implies 9.481481 ⋯.

The constructions of these tables were suggested to the author by Prof. T. KITAGAWA. Numerical calculations were due to the aids of Miss. T. NAGAMATSU. The author expresses here her hearty thanks to them. Although we have no available tables for n-dimensional t-distribution for $n \geqq 2$, it is hoped that our auxiliary tables may serve to give us numerical pictures of the AD-type functions.

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Table I-*a*

t_i \ φ_ν	$\varphi_0(t_i\,;\,3)$	$\varphi_1(t_i\,;\,3)$	$\varphi_2(t_i\,;\,3)$
$t_1\ (=0)$	$1/\sqrt{3}$	$-1/\sqrt{2}$	$1/\sqrt{6}$
$t_2\ (=1/2)$	$1/\sqrt{3}$	0	$-2/\sqrt{6}$
$t_3\ (=1)$	$1/\sqrt{3}$	$1/\sqrt{2}$	$1/\sqrt{6}$

$$\varphi_0(t\,;\,3) = 1/\sqrt{3},$$
$$\varphi_1(t\,;\,3) = (-1+2t)/\sqrt{2},$$
$$\varphi_2(t\,;\,3) = (1-12t+12t^2)/\sqrt{6}.$$

Table I-*b*-(i)

t_i	$\varphi_0(t_i\,;\,3)\,\varphi_0(t\,;\,3)$	$\varphi_1(t_i\,;\,3)\,\varphi_1(t\,;\,3)$	$\varphi_2(t_i\,;\,3)\,\varphi_2(t\,;\,3)$	$L(t\,;\,t_i\,,\,3)$
t_1	$1/3$	$(1-2t)/2$	$(1-12t+12t^2)/6$	$1-3t+2t^2$
t_2	$1/3$	0	$(-1+12t-12t^2)/3$	$4t-4t^2$
t_3	$1/3$	$-(1-2t)/2$	$(1-12t+12t^2)/6$	$-t+2t^2$

Table I-c-(i)

t_i, t_j	$L(t; t_i, 3)\,L(t; t_j, 3)$
t_1, t_1	$1-6t+13t^2-12t^3+4t^4$
t_1, t_2	$4(t-4t^2+5t^3-2t^4)$
t_1, t_3	$-t+5t^2-8t^3+4t^4$
t_2, t_2	$16(t^2-2t^3+t^4)$
t_2, t_3	$4(-t^2+3t^3-2t^4)$
t_3, t_3	$t^2-4t^3+4t^4$

Table I-d-(i)

t_i, t_j	$\int_0^\theta L(t; t_i, 3)\,L(t; t_j, 3)\,dt$	$\int_0^1 L(t; t_i, 3)\,L(t; t_j, 3)\,dt$
t_1, t_1	$\theta-3\theta^2+4.3\theta^3-3\theta^4+0.8\theta^5$	0.13
t_1, t_2	$2\theta^2-5.3\theta^3+5\theta^4-1.6\theta^5$	0.06
t_1, t_3	$-0.5\theta^2+1.6\theta^3-2\theta^4+0.8\theta^5$	−0.03
t_2, t_2	$5.3\theta^3-8\theta^4+3.2\theta^5$	0.53
t_2, t_3	$-1.3\theta^3+3\theta^4-1.6\theta^5$	0.06
t_3, t_3	$0.3\theta^3-\theta^4+0.8\theta^5$	0.13

Table I-e-(i)

t_i	$L'(t; t_i, 3)$
t_1	$-3+4t$
t_2	$4-8t$
t_3	$-1+4t$

Table I-f-(i)

t_i, t_j	$\int_0^\theta L'(t; t_i, 3)\,L'(t; t_j, 3)\,dt$	$\int_0^1 L'(t; t_i, 3)\,L'(t; t_j, 3)\,dt$
t_1, t_1	$9\theta-12\theta^2+5.3\theta^3$	2.3
t_1, t_2	$-12\theta+20\theta^2-10.6\theta^3$	−2.6
t_1, t_3	$3\theta-8\theta^2+5.3\theta^3$	0.3
t_2, t_2	$16\theta-32\theta^2+21.3\theta^3$	5.3
t_2, t_3	$-4\theta+12\theta^2-10.6\theta^3$	−2.6
t_3, t_3	$\theta-4\theta^2+5.3\theta^3$	2.3

Table I-*b*-(ii)

t_i	$L(t\,;\,t_i,\,2)$
t_1	$5/6 - t$
t_2	$1/3$
t_3	$-1/6 + t$

Table I-*c*-(ii)

$t_i,\,t_j$	$L(t\,;t_i,2)\,L(t\,;t_j,2)$
$t_1,\,t_1$	$25/36 - 5/3t + t^2$
$t_1,\,t_2$	$5/18 - 1/3t$
$t_1,\,t_3$	$-5/36 \quad + t - t^2$
$t_2,\,t_2$	$1/9$
$t_2,\,t_3$	$-1/18 + 1/3t$
$t_3,\,t_3$	$1/36 - 1/3t + t^2$

Table I-*d*-(ii)

$t_i,\,t_j$	$\int_0^\theta L(t\,;\,t_i,2)\,L(t\,;\,t_j,2)\,dt$	$\int_0^1 L(t\,;\,t_i,2)\,L(t\,;\,t_j,2)\,dt$
$t_1,\,t_1$	$0.694\theta - 0.83\theta^2 + 0.3\theta^3$	0.194
$t_1,\,t_2$	0.1θ	0.111
$t_1,\,t_3$	$0.139\theta - 0.6\theta^2$	0.027
$t_2,\,t_2$	$0.27\theta - 0.16\theta^2 + 0.3\theta^3$	0.111
$t_2,\,t_3$	$-0.5\theta + 0.16\theta^2$	0.111
$t_3,\,t_3$	$0.027\theta - 0.16\theta^2 + 0.3\theta^3$	0.194

Table I-*e*-(ii)

t_i	$L'(t\,;\,t_i,\,2)$
t_1	4
t_2	−8
t_3	4

Table I-*f*-(ii)

$t_i,\,t_j$	$\int_0^\theta L'(t\,;t_i,2)\,L(t\,;\,t_j,2)\,dt$	$\int_0^1 L'(\,t;t_i,2)\,L'(t\,;t_j,2)\,dt$
$t_1,\,t_1$	$16\,\theta$	16
$t_1,\,t_2$	$-32\,\theta$	−32
$t_1,\,t_3$	$16\,\theta$	16
$t_2,\,t_2$	$64\,\theta$	64
$t_2,\,t_3$	$-32\,\theta$	−32
$t_3,\,t_3$	$16\,\theta$	16

Table II-*a*

t_i \ φ_ν	$\varphi_0(t_i;4)$	$\varphi_1(t_i;4)$	$\varphi_2(t_i;4)$	$\varphi_3(t_i;4)$
$t_1(=0)$	1/2	$-3/\sqrt{20}$	1/2	$-1/\sqrt{20}$
$t_2(=1/3)$	1/2	$-1/\sqrt{20}$	$-1/2$	$3/\sqrt{20}$
$t_3(=2/3)$	1/2	$1/\sqrt{20}$	$-1/2$	$-3/\sqrt{20}$
$t_4(=1)$	1/2	$3/\sqrt{20}$	1/2	$1/\sqrt{20}$

$$\varphi_0(t;4)=1/2$$
$$\varphi_1(t;4)=(-3+6t)/\sqrt{20}$$
$$\varphi_2(t;4)=(1-9t+9t^2)/2$$
$$\varphi_3(t;4)=(-1+47t-135t^2+90t^3)/\sqrt{20}$$

Table II-*b*-(i)

t_i	$\varphi_0(t_i;4)\varphi_0(t;4)$	$\varphi_1(t_i;4)\varphi_1(t;4)$	$\varphi_2(t_i;4)\varphi_2(t;4)$	$\varphi_3(t_i;4)\varphi_3(t;4)$	$L(t;t_i,4)$
t_1	1/4	$9(1-2t)/20$	$(1-9t+9t^2)/4$	$(1-47t+135t^2-90t^3)/20$	$1-5.5t+9t^2-4.5t^3$
t_2	1/4	$3(1-2t)/20$	$-(1-9t+9t^2)/4$	$-3(1-47t-135t^2-90t^3)/20$	$9t-22.5t^2+13.5t^3$
t_3	1/4	$-3(1-2t)/20$	$-(1-9t+9t^2)/4$	$3(1-47t+135t^2-90t^3)/20$	$4.5t+18t^2-13.5t^3$
t_4	1/4	$-9(1-2t)/20$	$(1-9t+9t^2)/4$	$-(1-47t+135t^2-90t^3)/20$	$t-4.5t^2+4.5t^3$

Table II-*c*-(i)

t_i, t_j	$L(t; t_i, 4)\, L(t; t_j, 4)$
t_1, t_1	$1 - 11t + 48.25t^2 - 108t^3 + 130.5t^4 - 81t^5 + 20.25t^6$
t_1, t_2	$9t - 72t^2 + 218.25t^3 - 317.25t^4 + 222.75t^5 - 60.75t^6$
t_1, t_3	$-4.5t + 42.75t^2 - 153t^3 + 256.5t^4 - 202.5t^5 + 60.75t^6$
t_1, t_4	$t - 10t^2 + 38.25t^3 - 69.75t^4 + 60.75t^5 - 20.25t^6$
t_2, t_2	$81t^2 - 405t^3 + 749.25t^4 - 607.5t^5 + 182.25t^6$
t_2, t_3	$-40.5t^2 + 263.25t^3 - 587.25t^4 + 546.75t^5 - 182.25t^6$
t_2, t_4	$9t^2 - 63t^3 + 155.25t^4 - 162t^5 + 60.75t^6$
t_3, t_3	$20.25t^2 - 162t^3 + 445.5t^4 - 486t^5 + 182.25t^6$
t_3, t_4	$-4.5t^2 + 38.25t^3 - 114.75t^4 + 141.75t^5 - 60.75t^6$
t_4, t_4	$t^2 - 9t^3 + 29.25t^4 - 40.5t^5 + 20.25t^6$

Table II-*d*-(i)

t_i, t_j	$\int_0^\vartheta L(t; t_i, 4)\, L(t; t_j, 4)\, dt$	$\int_0^1 L(t; t_i, 4) L(t; t_j, 4)\, dt$
t_1, t_1	$\vartheta - 5.5\vartheta^2 + 16.083\vartheta^3 - 27\vartheta^4 + 26.1\vartheta^5 - 13.5\vartheta^6 + 2.892857\vartheta^7$	0.07618
t_1, t_2	$4.5\vartheta^2 - 24\vartheta^3 + 54.5625\vartheta^4 - 63.45\vartheta^5 + 37.125\vartheta^6 - 8.678571\vartheta^7$	0.05893
t_1, t_3	$-2.25\vartheta^2 + 14.25\vartheta^3 - 38.25\vartheta^4 + 51.3\vartheta^5 - 33.76\vartheta^6 + 8.678571\vartheta^7$	−0.02183
t_1, t_4	$0.5\vartheta^2 - 3.3\vartheta^3 + 9.5625\vartheta^4 - 13.95\vartheta^5 + 10.125\vartheta^6 - 2.892857\vartheta^7$	0.00989
t_2, t_2	$27\vartheta^3 - 101.25\vartheta^4 + 149.85\vartheta^5 - 101.25\vartheta^6 + 26.035714\vartheta^7$	0.38571
t_2, t_3	$-13.5\vartheta^3 + 65.8125\vartheta^4 - 117.45\vartheta^5 + 91.125\vartheta^6 - 26.34285\vartheta^7$	−0.04821
t_2, t_4	$3\vartheta^3 - 15.75\vartheta^4 + 31.05\vartheta^5 - 27\vartheta^6 + 8.678571\vartheta^7$	−0.02143
t_3, t_3	$6.75\vartheta^3 - 40.5\vartheta^4 + 89.1\vartheta^5 - 81\vartheta^6 + 26.035714\vartheta^7$	0.38571
t_3, t_4	$-1.5\vartheta^3 + 9.5625\vartheta^4 - 22.95\vartheta^5 + 23.625\vartheta^6 - 8.678571\vartheta^7$	0.05893
t_4, t_4	$0.3\vartheta^3 - 2.25\vartheta^4 + 5.85\vartheta^5 - 6.75\vartheta^6 + 2.892857\vartheta^7$	0.07618

Table II-*e*-(i)

t_i \ L'	$L'(t\,;\,t_i,\,4)$
t_1	$-\ 5.5\ +\ 18\,t\ -\ 13.5\,t^2$
t_2	$9\ -\ 45\,t\ +\ 40.5\,t^2$
t_3	$-\ 4.5\ +\ 36\,t\ -\ 40.5\,t^2$
t_4	$1\ \ -9\,t\ +\ 13.5\,t^2$

Table II-*f*-(i)

$t_i,\ t_j$	$\int_0^\theta L'(t\,;\,t_i,\,4)\,L'(t\,;\,t_j,\,4)\,dt$	$\int_0^1 L'(t\,;\,t_i,\,4)L'(t\,;\,t_j,\,4)\,dt$
$t_1,\ t_1$	$30.25\theta\ \ -99\theta^2\ +157.5\theta^3\ \ -121.5\theta^4\ +36.45\theta^5$	3.7000
$t_1,\ t_2$	$-49.5\theta+93.375\theta^2-141.75\theta^3+197.4375\theta^4-109.35\theta^5$	−9.7865
$t_1,\ t_3$	$24.75\theta\ -139.5\theta^2\ +310.5\theta^3\ \ -303.75\theta^4+109.35\theta^5$	1.3500
$t_1,\ t_4$	$-5.5\theta\ +33.5\theta^2\ -83.25\theta^3\ +91.125\theta^4\ -36.45\theta^5$	−0.5750
$t_2,\ t_2$	$81\theta\ \ -40.5\theta^2+249.75\theta^3\ \ -91.125\theta^4+328.05\theta^5$	527.1750
$t_2,\ t_3$	$-40.5\theta+263.25\theta^2-722.25\theta^3\ +820.125\theta^4-328.05\theta^5$	−7.4250
$t_2,\ t_4$	$9\theta\ \ -63\theta^2\ \ +189\theta^3\ \ -324\theta^4+109.35\theta^5$	−79.6500
$t_3,\ t_3$	$20.25\theta\ -182\theta^2\ +553.5\theta^3\ \ -729\theta^4+328.05\theta^5$	−9.2000
$t_3,\ t_4$	$-4.5\theta\ +38.25\theta^2-141.75\theta^3\ +212.625\theta^4-109.35\theta^5$	−4.7250
$t_4,\ t_4$	$\theta\ \ -9\theta^2\ \ +36\theta^3\ \ -60.75\theta^4\ +36.45\theta^5$	3.7000

Table II-*b*-(ii)

t_i	$L(t\,;\,t_i,\,3)$
t_1	$(19-63t+45t^2)/20$
t_2	$(3+39t-45t^2)/20$
t_3	$(-3+51t-45t^2)/20$
t_4	$(1-27t+45t^2)/20$

Table II-*c*-(ii)

$t_i,\ t_j$	$L(t\,;\,t_i,\,3)\,L(t\,;\,t_j,\,3)$
$t_1,\ t_1$	$(361-2394t+5679t^2-5670t^3+2025t^4)/400$
$t_1,\ t_2$	$(57+552t-3177t^2+4590t^3-2025t^4)/400$
$t_1,\ t_3$	$(-57+1158t-4203t^2+5130t^3-2025t^4)/400$
$t_1,\ t_4$	$(19-576t+2601t^2-4050t^3+2025t^4)/400$
$t_2,\ t_2$	$(9-234t+1791t^2-3510t^3+2025t^4)/400$
$t_2,\ t_3$	$(-9+36t+1989t^2-4050t^3+2025t^4)/400$
$t_2,\ t_4$	$(3-42t-9600t^2+2970t^3-2025t^4)/400$
$t_3,\ t_3$	$(9-306t+2871t^2-4590t^3+2025t^4)/400$
$t_3,\ t_4$	$(-3+132t-1557t^2+3510t^3-2025t^4)/400$
$t_4,\ t_4$	$(1-54t+819t^2-2430t^3+2025t^4)/400$

Table II-*d*-(ii)

$t_i,\ t_j$	$\int_0^\theta L(t\,;\,t_i,\,3)\,L(t\,;\,t_j,\,3)\,dt$	$\int_0^1 L(t\,;\,t_i,\,3)\,L(t\,;\,t_j,\,3)\,dt$
$t_1,\ t_1$	$(361\theta-1297\theta^2+1559.6\theta^3-1417.5\theta^4+405\theta^5)/400$	0.11125
$t_1,\ t_2$	$(57\theta+276\theta^2-1059\theta^3+1147.5\theta^4-405\theta^5)/400$	0.03875
$t_1,\ t_3$	$(-57\theta+579\theta^2-1401\theta^3+1282.5\theta^4-405\theta^5)/400$	−0.00375
$t_1,\ t_4$	$(19\theta-288\theta^2+867\theta^3-1012.5\theta^4+405\theta^5)/400$	−0.02375
$t_2,\ t_2$	$(9\theta-117\theta^2+597\theta^3-877.5\theta^4+405\theta^5)/400$	0.04125
$t_2,\ t_3$	$(-9\theta+18\theta^2+663\theta^3-1012.5\theta^4+405\theta^5)/400$	0.16125
$t_2,\ t_4$	$(3\theta-21\theta^2-3200\theta^3+742.5\theta^4-405\theta^5)/400$	−0.00125
$t_3,\ t_3$	$(9\theta-153\theta^2+957\theta^3-1147.5\theta^4+405\theta^5)/400$	0.17625
$t_3,\ t_4$	$(-3\theta+66\theta^2-519\theta^3+877.5\theta^4-405\theta^5)/400$	0.04125
$t_4,\ t_4$	$(\theta-27\theta^2+273\theta^3-607.5\theta^4+405\theta^5)/400$	0.11125

Table II-*e*-(ii)

t_i \ L'	$L'(t\,;\,t_i,\,3)$
t_1	$-3.15\,+\,4.5\,t$
t_2	$1.95\,-\,4.5\,t$
t_3	$2.55\,-\,4.5\,t$
t_4	$-1.35\,+\,4.5\,t$

Table II-*f*-(ii)

$t_i,\ t_j$	$\int_0^\theta L'(t\,;\,t_i,3)\,L'(t\,;\,t_j,3)\,dt$	$\int_0^1 L'(t\,;\,t_i,3)\,L'(t\,;\,t_j,3)\,dt$
$t_1,\ t_1$	$9.9225\,\theta\ -14.175\,\theta^2+6.75\,\theta^3$	2.4975
$t_1,\ t_2$	$-6.1425\,\theta+11.6375\,\theta^2-6.75\,\theta^3$	-1.2550
$t_1,\ t_3$	$-8.0325\,\theta+12.9825\,\theta^2-6.75\,\theta^3$	-1.8000
$t_1,\ t_4$	$4.2525\,\theta-10.2825\,\theta^2+6.75\,\theta^3$	0.7200
$t_2,\ t_2$	$3.8025\,\theta\ -8.775\,\theta^2+6.75\theta^3$	1.7775
$t_2,\ t_3$	$4.9725\,\theta\ -10.125\,\theta^2+6.75\theta^3$	1.5975
$t_2,\ t_4$	$-2.6325\,\theta\ +7.425\,\theta^2-6.75\,\theta^3$	-1.9575
$t_3,\ t_3$	$6.5025\,\theta\ -11.475\,\theta^2+6.75\,\theta^3$	1.7775
$t_3,\ t_4$	$-3.4425\,\theta\ +8.775\,\theta^2-6.75\,\theta^3$	-1.4175
$t_4,\ t_4$	$1.8225\,\theta\ -6.75\,\theta^2+6.75\,\theta^3$	1.8225

Table II-*b*-(iii)

t_i \ L	$L(t\,;\,t_i,\,2)$
t_1	$(7\ -\ 9t)/10$
t_2	$(4\ -\ 3t)/10$
t_3	$(1\ +\ 3t)/10$
t_4	$(-\ 2\ +\ 9t)/10$

Table II-*c*-(iii)

$t_i,\ t_j$	$L(t\,;\,t_i,\,2)\,L(t\,;\,t_j,\,2)$
$t_1,\ t_1$	$(49-126t+81t^2)/100$
$t_1,\ t_2$	$(28\ -57t+27t^2)/100$
$t_1,\ t_3$	$(7\ +12t-27t^2)/100$
$t_1,\ t_4$	$(-\ 14\ +81t-81t^2)/100$
$t_2,\ t_2$	$(16\ -24t\ +9t^2)/100$
$t_2,\ t_3$	$(4\ +9t\ -9t^2)/100$
$t_2,\ t_4$	$(-\ 8\ +42t-27t^2)/100$
$t_3,\ t_3$	$(1\ +6t\ +9t^2)/100$
$t_3,\ t_4$	$(-\ 2\ +3t+27t^2)/100$
$t_4,\ t_4$	$(4\ -36t+81t^2)/100$

Table II-*e*-(iii)

t_i	$L'(t\,;\,t_i,\,2)$
t_1	$-\ 0.9$
t_2	$-\ 0.3$
t_3	0.3
t_4	0.9

Table II-*d*-(iii)

$t_i,\ t_j$	$\int_0^\theta L(t\,;\,t_i,\,2)\,L(t\,;\,t_j,\,2)\,dt$	$\int_0^1 L(t\,;\,t_i,\,2)\,L(t\,;\,t_j,\,2)\,dt$
$t_1,\ t_1$	$(49\theta\ -63\theta^2+27\theta^3)/100$	0.265
$t_1,\ t_2$	$(28\theta-28.5\theta^2\ +9\theta^3)/100$	0.085
$t_1,\ t_3$	$(7\theta\ +6\theta^2\ -9\theta^3)/100$	0.040
$t_1,\ t_4$	$(-14\theta+40.5\theta^2\ -9\theta^3)/100$	$-\ 0.005$
$t_2,\ t_2$	$(16\theta\ -12\theta^2\ +3\theta^3)/100$	0.070
$t_2,\ t_3$	$(4\theta\ +4.5\theta^2\ -3\theta^3)/100$	$-\ 0.055$
$t_2,\ t_4$	$(-8\theta\ +21\theta^2\ -9\theta^3)/100$	0.040
$t_3,\ t_3$	$(\theta\ +3\theta^2\ +3\theta^3)/100$	0.070
$t_3,\ t_4$	$(-2\theta\ +1.5\theta^2\ +9\theta^3)/100$	0.085
$t_4,\ t_4$	$(4\theta\ -18\theta^2+27\theta^3)/100$	0.265

Table II-*f*-(iii)

$t_i,\ t_j$	$\int_0^\theta L(t\,;\,t_i,\,2)\,L(t\,;\,t_j,\,2)\,dt$
$t_1,\ t_1$	$(49\theta\ -63\theta^2+27\theta^3)/100$
$t_1,\ t_2$	$(28\theta-28.5\theta^2\ +9\theta^3)/100$
$t_1,\ t_3$	$(7\theta\ +6\theta^2\ -9\theta^3)/100$
$t_1,\ t_4$	$(-14\theta+40.5\theta^2\ -9\theta^3)/100$
$t_2,\ t_2$	$(16\theta\ -12\theta^2\ +3\theta^3)/100$
$t_2,\ t_3$	$(4\theta\ +4.5\theta^2\ -3\theta^3)/100$
$t_2,\ t_4$	$(-8\theta\ +21\theta^2\ -9\theta^3)/100$
$t_3,\ t_3$	$(\theta\ +3\theta^2\ +3\theta^3)/100$
$t_3,\ t_4$	$(-2\theta\ +1.5\theta^2\ +9\theta^3)/100$
$t_4,\ t_4$	$(4\theta\ -18\theta^2+27\theta^3)/100$

Table III-*a*

t_i \ φ_ν	$\varphi_0(t_i;5)$	$\varphi_1(t_i;5)$	$\varphi_2(t_i;5)$	$\varphi_3(t_i;5)$	$\varphi_4(t_i;5)$
$t_1(=0)$	$1/\sqrt{5}$	$-2/\sqrt{10}$	$2/\sqrt{14}$	$-1/\sqrt{10}$	$1/\sqrt{70}$
$t_2(=1/4)$	$1/\sqrt{5}$	$-1/\sqrt{10}$	$-1/\sqrt{14}$	$2/\sqrt{10}$	$-4/\sqrt{70}$
$t_3(=2/4)$	$1/\sqrt{5}$	0	$-2/\sqrt{14}$	0	$6/\sqrt{70}$
$t_4(=3/4)$	$1/\sqrt{5}$	$1/\sqrt{10}$	$-1/\sqrt{14}$	$-2/\sqrt{10}$	$-4/\sqrt{70}$
$t_5(=1)$	$1/\sqrt{5}$	$2/\sqrt{10}$	$2/\sqrt{14}$	$1/\sqrt{10}$	$1/\sqrt{70}$

$$\varphi_0(t;5)=1/\sqrt{5},$$
$$\varphi_1(t;5)=(-2+4t)/\sqrt{10},$$
$$\varphi_2(t;5)=16(1/8-t+t^2)/\sqrt{14},$$
$$\varphi_3(t;5)=(-3+86t-240t^2+160t^3)/3\sqrt{10},$$
$$\varphi_4(t;5)=(3-500t+2740t^2-4480t^3+2240t^3)/3\sqrt{70}.$$

Table III-*b*-(i)

t_i	$\varphi_0(t_i;5)\varphi_0(t;5)$	$\varphi_1(t_i;5)\varphi_1(t;5)$	$\varphi_2(t_i;5)\varphi_2(t;5)$	$\varphi_3(t_i;5)\varphi_3(t;5)$
t_1	1/5	$4(1-2t)/10$	$4(1-8t+8t^2)/14$	$(3-86t+240t^2-160t^3)/30$
t_2	1/5	$2(1-2t)/10$	$2(-1+8t-8t^2)/14$	$(-3+86t-240t^2+160t^3)/15$
t_3	1/5	0	$4(-1+8t-8t^2)/14$	0
t_4	1/5	$-2(1-2t)/10$	$2(-1+8t-8t^2)/14$	$(3-86t+240t^2-160t^3)/15$
t_5	1/5	$-4(1-2t)/10$	$4(1-8t+8t^2)/14$	$(-3+86t-240t^2+160t^3)/30$

t_i	$\varphi_4(t_i;5)\varphi_4(t;5)$	$L(t;t_i,5)$
t_1	$(3-500t+2740t^2-4480t^3+2240t^4)/210$	$1-25/3t+70/3t^2-80/3t^3+32/3t^4$
t_2	$2(-3+500t-2740t^2+4480t^3-2240t^4)/105$	$16t-208/3t^2+96t^3-128/3t^4$
t_3	$(3-500t+2740t^2-4480t^3+2240t^4)/35$	$-12t+76t^2-128t^3+64t^4$
t_4	$2(-3+500t-2740t^2+4480t^3-2240t^4)/105$	$16/3t-112/3t^2+224/3t^3-128/3t^4$
t_5	$(3-500t+2740t^2-4480t^3+2240t^4)/210$	$-t+22/3t^2-16t^3+32/3t^4$

Table III-c-(i)

t_i, t_j	$L(t\,;\,t_i,\,5)\,L(t\,;\,t_j,\,5)$
t_1, t_1	$1 - 16.6t + 116.1t^2 - 442.2t^3 + 1010.2t^4 - 1422.2t^5 + 1208.8t^6 - 568.8t^7 + 113.7t^8$
t_1, t_2	$16t - 202.6t^2 + 1047.1t^3 - 2887.1t^4 + 4615.1t^5 - 4295.1t^6 + 2161.7t^7 - 455.1t^8$
t_1, t_3	$-12t + 176t^2 - 1041.3t^3 + 3224t^4 - 5674.6t^5 + 5717.3t^6 - 3072t^7 + 682.6t^8$
t_1, t_4	$16t - 512t^2 + 1530.6t^3 - 5034.6t^4 + 9450.6t^5 - 10153.6t^6 + 5802.6t^7 - 1365.3t^8$
t_1, t_5	$-t + 15.6t^2 - 100.4t^3 + 341.7t^4 - 668.4t^5 + 753.7t^6 - 455.1t^7 + 113.7t^8$
t_2, t_2	$256t^2 - 2218.6t^3 + 7879.1t^4 - 14677.3t^5 + 15132.4t^6 - 8192t^7 + 1820.4t^8$
t_2, t_3	$-192t^2 + 2048t^3 - 8469.3t^4 + 17706.6t^5 - 19968t^6 + 11605.3t^7 - 2730.6t^8$
t_2, t_4	$256t^2 - 2901.3t^3 + 12885.3t^4 - 29013.3t^5 + 35157.3t^6 - 21845.3t^7 + 541.3t^8$
t_2, t_5	$-16t^2 + 186.6t^3 - 860.4t^4 + 2026.6t^5 - 2588.4t^6 + 1706.6t^7 - 455.1t^8$
t_3, t_3	$144t^2 - 1824t^3 + 8848t^4 - 20992t^5 + 26112t^6 - 16384t^7 + 4096t^8$
t_3, t_4	$(-192t^2 + 2560t^3 - 13248t^4 + 3392t^5 - 45568t^6 + 30720t^7 - 8192t^8)/3$
t_3, t_5	$12t^2 - 164t^3 + 877.3t^4 - 2346.6t^5 + 3328t^6 - 2389.3t^7 + 682.6t^8$
t_4, t_4	$(256t^2 - 3584t^3 + 19712t^4 - 54272t^5 + 78848t^6 - 57344t^7 + 16384t^8)/9$
t_4, t_5	$-16t^2 + 229.3t^3 - 1301.3t^4 + 3733.3t^5 - 5717.3t^6 + 4437.3t^7 - 1365.3t^8$
t_5, t_5	$t^2 - 14.6t^3 + 85.7t^4 - 256t^5 + 412.4t^6 - 341.3t^7 + 113.7t^8$

Table III-*d*-(i)

t_i, t_j	$\int_0^\theta L(t; t_i, 5)\, L(t; t_j, 5)\, dt$	$\int_0^t L(t; t_i, 5)\, L(t; t_j, 5)\, dt$
t_1, t_1	$0-8.30^2+38.7037^3-110.5^4+202.04^5-237.037^6+172.6984120^7-71.1^8+12.530864^9$	0.05150
t_1, t_2	$80^2-67.5^3+261.7^4-577.42^5+769.185^6-613.5873020^7+270.2^8-50.567901^9$	0.05220
t_1, t_3	$-60^2+58.6^3-260.3^4+655.8^5-945.7^6+816.7619040^7-384^8+75.851^9$	−0.03070
t_1, t_4	$80^2-170.6^3+382.6^4-1006.93^5+1575.1^6-1450.5238090^7+725.3^8-151.7^9$	0.17250
t_1, t_5	$-0.50^2+5.2^3-25.1^4+68.35^5-111.4740^6+107.6825390^7-56.8^8+12.641975^9$	−0.00512
t_2, t_2	$85.3^3-554.6^4+1575.82^5-2446.2^6+2161.7^7-1024^8+202.271605^9$	0.31604
t_2, t_3	$-64^3+512^4-1693.86^5+2951.1^6-2852.5714280^7+1450.6^8-303.407^9$	−0.06773
t_2, t_4	$85.3^3-725.3^4+2577.06^5-4835.5^6+5022.4825390^7-2730.6^8+606.814^9$	0.13545
t_2, t_5	$-5.3^3+46.6^4-172.08^5+337.6^6-369.7^7+213.3^8-50.567901^9$	0.00986
t_3, t_3	$48^3-456^4+1769.6^5-3498.6^6+3730.285714^7-2048^8+455.1^9$	0.33016
t_3, t_4	$-24^3+213.3^4-883.2^5+188.4^6-2169.904762^7+1280^8-303.407^9$	−0.06772
t_3, t_5	$4^3-41^4+175.46^5-391.1^6+475.4285710^7-298.6^8+29.851^9$	−0.03069
t_4, t_4	$9.481^3-99.5^4+438.4^5-1005.037^6+1251.5^7-796.4^8+22.271605^9$	0.31604
t_4, t_5	$-5.3^3+57.3^4-260.26^5+622.2^6-816.761905^7+554.654^8-151.7039^9$	0.15662
t_5, t_5	$0.3^3-3.6^4+17.15^5-42.5^6+58.920635^7-42.6^8+12.641975^9$	0.05150

Table III-*e*-(i)

t_i	$L'(t\,;\,t_i,\,5)$
t_1	$-25/3+140/3\,t\;-80\,t^2+128/3\,t^3$
t_2	$16-416/3\,t+288\,t^2-170.6\,t^3$
t_3	$-12\;+152\,t-384\,t^2\;+256\,t^3$
t_4	$16/3-224/3\,t+224\,t^2-512/3\,t^3$
t_5	$-1\;+44/3\,t\;-48\,t^2+128/3\,t^3$

Table III-*f*-(i)

$t_i,\,t_j$	$\int_0^\theta L'(t\,;\,t_i,\,5)\;L'(t\,;\,t_j,\,5)\,dt$	$\int_0^1 L'(t\,;\,t_i,\,5)\,L'(t\,;\,t_j,\,5)\,dt$
$t_1,\,t_1$	$69.4\theta\;-388.8\theta^2\;+1170.370\theta^3\;-2044.4\theta^4\;+1678.2\theta^5\;-1137.7\theta^6\;+260.0634914\theta^7$	393.01058
$t_1,\,t_2$	$-133.3\theta\;+951.1\theta^2\;-3383.703\theta^3\;+6659.5\theta^4\;-9950.75\theta^5\;+4323.5\theta^6\;-140.2539682\theta^7$	−1673.82434
$t_1,\,t_3$	$100\theta\;-913.3\theta^2\;+3751.1\theta^3\;-8181.3\theta^4\;+9830.4\theta^5\;-6144\theta^6\;+1560.3809523\theta^7$	−3372.73016
$t_1,\,t_4$	$-44.4\theta\;+435.5\theta^2\;-2259.259\theta^3\;+4518.8\theta^4\;-5814.04\theta^5\;+3868.4\theta^6\;-1040.2539682\theta^7$	−335.11323
$t_1,\,t_5$	$8.3\theta\;-253.3\theta^2\;+388.148\theta^3\;-952.8\theta^4\;+1219.5\theta^5\;-910.2\theta^6\;+260.0634914\theta^7$	−240.34392
$t_2,\,t_2$	$256\theta-2218.6\theta^2+12814.814\theta^3-21333.3\theta^4+25766.4\theta^5-16384\theta^6+4281.7\theta^7$	3172.99259
$t_2,\,t_3$	$-192\theta\;+2048\theta^2\;-10225.7\theta^3\;+25792\theta^4-34399.73\theta^5\;+23210.6\theta^6\;-6242.3809523\theta^7$	−9.62540
$t_2,\,t_4$	$85.3\theta\;-967.1\theta^2\;+3963.259\theta^3-14051.5\theta^4+20184.17\theta^5\;-14565.2\theta^6\;+4281.7\theta^7$	−5351.11852
$t_2,\,t_5$	$-16\theta\;+186.6\theta^2\;-1029.925\theta^3\;+2933.3\theta^4\;-4448.71\theta^5\;+3413.3\theta^6\;-140.2539682\theta^7$	898.44233
$t_3,\,t_3$	$144\theta\;-1824\theta^2\;+10773.3\theta^3\;-30720\theta^4\;+45056\theta^5\;-32768\theta^6\;+9362.5714285\theta^7$	23.90496
$t_3,\,t_4$	$-64\theta\;+853.3\theta^2\;-5361.7\theta^3+16533.3\theta^4\;-26214.4\theta^5\;+20480\theta^6-62415.1428571\theta^7$	−56188.65397
$t_3,\,t_5$	$12\theta\;-164\theta^2\;+1063.1\theta^3\;-3424\theta^4\;+5734.4\theta^5\;-4778.6\theta^6\;+1560.3809526\theta^7$	3.22540
$t_4,\,t_4$	$28.4\theta\;-398.2\theta^2\;+2654.814\theta^3\;-8817.7\theta^4\;+15132.4\theta^5\;-12743.1\theta^6\;+4281.7\theta^7$	138.37037
$t_4,\,t_5$	$-5.3\theta\;+76.4\theta^2\;-525.037\theta^3+14316.8\theta^4\;-3288.17\theta^5+4171.851\theta^6\;-140.2539682\theta^7$	14606.38307
$t_5,\,t_5$	$0\;-14.6\theta^2\;+103.703\theta^3\;-373.3\theta^4\;+911.1\theta^5\;-227.5\theta^6\;+260.0634920\theta^7$	660.32275

Table III-*b*-(ii)

t_i	$L(t\,;\,t_i,\,4)$
t_1	$69/70 - 125/21\,t + 72/7\,t^2 - 16/3\,t^3$
t_2	$2/35 + 136/21\,t - 120/7\,t^2 + 32/3\,t^3$
t_3	$-3/35 + 16/7\,t - 16/7\,t^2$
t_4	$2/35 - 88/21\,t + 104/7\,t^2 - 32/3\,t^3$
t_5	$-1/70 + 29/21\,t - 40/7\,t^2 + 16/3\,t^3$

Table III-*c*-(ii)

$t_i,\ t_j$	$L(t\,;\,t_i,\,4)\,L(t\,;\,t_j,\,4)$
$t_1,\ t_1$	$0.971633 - 11.734625\,t + 55.708389\,t^2 - 132.963265\,t^3 + 169.287982\,t^4 - 109.714286\,t^5 + 28.4\,t^6$
$t_1,\ t_2$	$0.056326 + 6.043537\,t - 54.858965\,t^2 + 178.862585\,t^3 - 274.358277\,t^4 + 201.142857\,t^5 - 56.8\,t^6$
$t_1,\ t_3$	$0.084490 + 2.763265\,t - 16.740136\,t^2 + 37.572989\,t^3 - 35.700680\,t^4 + 12.190476\,t^5$
$t_1,\ t_4$	$0.056327 - 4.470748\,t + 40.175964\,t^2 - 142.356462\,t^3 + 238.657596\,t^4 - 188.952381\,t^5 + 56.8\,t^6$
$t_1,\ t_5$	$-0.014082 + 1.446259\,t - 13.999546\,t^2 + 53.551020\,t^3 - 97.886621\,t^4 + 85.3\,t^5 - 28.4\,t^6$
$t_2,\ t_2$	$0.003265 + 0.740136\,t + 39.981960\,t^2 - 220.821769\,t^3 + 432.036283\,t^4 - 365.714285\,t^5 + 113.7\,t^6$
$t_2,\ t_3$	$-0.004897 - 0.424490\,t + 1614.1496\,t^2 - 54.900680\,t^3 + 63.564625\,t^4 - 24.380952\,t^5$
$t_2,\ t_4$	$0.003265 + 0.130612\,t - 27.268934\,t^2 + 168.054421\,t^3 - 368.471655\,t^4 + 341.3\,t^5 - 113.7\,t^6$
$t_2,\ t_5$	$-0.000816 - 0.001361\,t + 8.861678\,t^2 - 60.527891\,t^3 + 147.229024\,t^4 - 152.380952\,t^5 - 56.8\,t^6$
$t_3,\ t_3$	$0.007347 - 0.391837\,t - 5.616326\,t^2 - 10.448979\,t^3 + 5.224489\,t^4$
$t_3,\ t_4$	$0.004898 + 0.489796\,t - 10.982313\,t^2 + 44.451701\,t^3 - 58.340135\,t^4 + 24.380952\,t^5$
$t_3,\ t_5$	$0.001224 - 0.151020\,t + 3.678911\,t^2 - 16.674829\,t^3 + 25.251700\,t^4 - 12.190476\,t^5$
$t_4,\ t_4$	$0.003265 - 0.478912\,t + 19.258049\,t^2 - 125.736048\,t^3 + 310.131435\,t^4 - 316.952381\,t^5 + 113.7\,t^6$
$t_4,\ t_5$	$0.000817 + 0.138775\,t - 6.325623\,t^2 + 44.919728\,t^3 - 121.977324\,t^4 + 140.190476\,t^5 - 56.8\,t^6$
$t_5,\ t_5$	$0.000204 - 0.039456\,t + 2.070295\,t^2 - 15.934693\,t^3 + 47.383219\,t^4 - 60.952381\,t^5 + 28.4\,t^6$

Table III-d-(ii)

t_i, t_j	$\int_0^\theta L(t\,;\,t_i,\,4)\,L(t\,;\,t_j,\,4)\,dt$	$\int_0^1 L(t\,;\,t_i,4)\,L(t\,;\,t_j,4)\,dt$
t_1, t_1	$0.971633\theta - 5.867313\theta^2 + 18.569463\theta^3 - 33.240816\theta^4 + 33.857596\theta^5 - 18.252381\theta^6 + 4.063492\theta^7$	0.06831
t_1, t_2	$0.056326\theta + 3.021769\theta^2 - 18.286318\theta^3 + 44.715646\theta^4 - 54.871655\theta^5 + 33.523761\theta^6 - 8.126984\theta^7$	0.03255
t_1, t_3	$0.084489\theta + 1.381633\theta^2 - 5.580045\theta^3 + 9.393247\theta^4 - 7.140136\theta^5 + 2.031746\theta^6$	0.17093
t_1, t_4	$0.056327\theta - 2.235374\theta^2 + 13.391988\theta^3 - 35.589115\theta^4 + 47.731519\theta^5 - 31.492064\theta^6 + 8.126984\theta^7$	0.00974
t_1, t_5	$-0.014082\theta + 0.723125\theta^2 - 4.66515\theta^3 + 13.387755\theta^4 - 19.577320\theta^5 + 14.2\theta^6 - 4.063492\theta^7$	0.00097
t_2, t_2	$0.003265\theta + 0.370068\theta^2 + 13.327287\theta^3 - 55.205442\theta^4 + 86.407257\theta^5 - 60.952381\theta^6 + 16.253968\theta^7$	0.20402
t_2, t_3	$-0.004897\theta - 0.212245\theta^2 + 5.380498\theta^3 - 13.725170\theta^4 + 12.712925\theta^5 - 4.063492\theta^6$	0.09012
t_2, t_4	$0.003265\theta + 0.065306\theta^2 - 9.089644\theta^3 + 42.013605\theta^4 - 73.694331\theta^5 + 56.8\theta^6 - 16.282539\theta^7$	− 0.07232
t_2, t_5	$-0.000817\theta - 0.000680\theta^2 + 2.953893\theta^3 - 15.131972\theta^4 + 29.441805\theta^5 - 25.396825\theta^6 + 8.126984\theta^7$	− 0.00761
t_3, t_3	$0.007347\theta - 0.195919\theta^2 + 1.872109\theta^3 + 2.612245\theta^4 + 1.044898\theta^5$	0.11619
t_3, t_4	$0.004898\theta + 0.244898\theta^2 - 3.660771\theta^3 + 11.112925\theta^4 - 11.668127\theta^5 + 4.063492\theta^6$	0.09742
t_3, t_5	$0.001224\theta - 0.075510\theta^2 + 1.226304\theta^3 - 4.168707\theta^4 + 5.050340\theta^5 - 2.0317460\theta^6$	0.00191
t_4, t_4	$0.003265\theta - 0.239456\theta^2 + 6.419349\theta^3 - 31.434012\theta^4 + 62.026287\theta^5 - 52.8253970\theta^6 + 16.282539\theta^7$	0.23758
t_4, t_5	$0.000817\theta + 0.069387\theta^2 - 2.108541\theta^3 + 11.229932\theta^4 - 24.395464\theta^5 + 23.365079\theta^6 - 8.126984\theta^7$	0 03423
t_5, t_5	$0.000204\theta - 0.019728\theta^2 + 0.690098\theta^3 - 3.983673\theta^4 + 9.476644\theta^5 - 10.158730\theta^6 + 4.063492\theta^7$	0.06831

Table III-*e*-(ii)

t_i	$L'(t\,;\,t_i,\,4)$
t_1	$-\,125/21\,+\,144/7\,t\,-\,16\,t^2$
t_2	$136/21\,-\,240/7\,t\,+\,32\,t^2$
t_3	$16/7\,-\,32/7\,t$
t_4	$-\,88/21\,+\,208/7\,t\,-\,32\,t^2$
t_5	$29/21\,-\,80/7\,t\,+\,16\,t^2$

Table III-*f*-(ii)

$t_i,\,t_j$	$\int_0^\theta L'(t\,;\,t_i,\,4)\,L'(t\,;\,t_j;\,4)\,dt$	$\int_0^1 L'(t\,;\,t_i,\,4)\,L'(t\,;\,t_j,\,4)\,dt$
$t_1,\,t_1$	$35.4307813\,\theta - 122.4489600\,\theta^2 + 205.0608860\,\theta^3 - 164.5714253\,\theta^4 + 51.2\,\theta^5$	4.67128
$t_1,\,t_2$	$-\,38.5486900\,\theta + 115.3629077\,\theta^2 - 333.1336389\,\theta^3 + 301.7142892\,\theta^4 - 102.4\,\theta^5$	45.39487
$t_1,\,t_3$	$-\,13.6054400\,\theta + 37.1156403\,\theta^2 - 234.126403\,\theta^3 + 18.2857139\,\theta^4$	$-\,192.33049$
$t_1,\,t_4$	$24.9432700\,\theta - 131.5373939\,\theta^2 + 290.3295685\,\theta^3 - 283.4285657\,\theta^4 + 102.4\,\theta^5$	2.70688
$t_1,\,t_5$	$-\,8.2199413\,\theta + 48.2176794\,\theta^2 - 117.7604571\,\theta^3 + 127.99999975\,\theta^4 - 51.2\,\theta^5$	$-\,0.96262$
$t_2,\,t_2$	$41.9409747\,\theta - 2.2204013\,\theta^2 + 529.9971371\,\theta^3 - 548.5714560\,\theta^4 + 204.8\,\theta^5$	$-\,383.65375$
$t_2,\,t_3$	$14.8027187\,\theta - 53.9863667\,\theta^2 + 24.9033005\,\theta^3 - 36.5714279\,\theta^4$	$-\,50.55779$
$t_2,\,t_4$	$-\,27.1382778\,\theta + 96.9360368\,\theta^2 - 117.1736996\,\theta^3 + 240.4571382\,\theta^4 - 204.8\,\theta^5$	193.08120
$t_2,\,t_5$	$8.9432961\,\theta - 60.68019286\,\theta^2 + 179.8819904\,\theta^3 - 238.5714336\,\theta^4 + 102.4\,\theta^5$	0.97366
$t_3,\,t_3$	$5.2432896\,\theta - 10.4865792\,\theta^2 + 6.9910526\,\theta^3$	1.74776
$t_3,\,t_4$	$-\,9.5782300\,\theta + 43.6596122\,\theta^2 - 511.9999898\,\theta^3 + 36.5714278\,\theta^4$	$-\,441.34718$
$t_3,\,t_5$	$3.1564621\,\theta - 16.2646861\,\theta^2 + 29.6681079\,\theta^3 - 18.2857139\,\theta^4$	$-\,1.72583$
$t_4,\,t_4$	$17.5600621\,\theta - 124.5169869\,\theta^2 + 384.8323026\,\theta^3 - 475.4285619\,\theta^4 + 204.8\,\theta^5$	7.24682
$t_4,\,t_5$	$-\,5.7868386\,\theta + 44.4625779\,\theta^2 - 150.6839749\,\theta^3 + 210.2857101\,\theta^4 - 102.4\,\theta^5$	$-\,4.12253$
$t_5,\,t_5$	$1.9070264\,\theta - 15.7823104\,\theta^2 + 58.4242396\,\theta^3 - 91.4285696\,\theta^4 + 51.2\,\theta^5$	$-\,46.87961$

Table III-*b*-(iii)

t_i	$L(t\,;\,t_i,\,3)$
t_1	$(31 - 108\,t + 80\,t^2)/35$
t_2	$(9 + 26\,t - 40\,t^2)/35$
t_3	$(-3 + 80\,t - 80\,t^2)/35$
t_4	$(-5 + 54\,t - 40\,t^2)/35$
t_5	$(3 - 52\,t + 80\,t^2)/35$

Table III-*c*-(iii)

$t_i,\ t_j$	$L(t\,;\,t_i,\,3)\,L(t\,;\,t_j,\,3)$
$t_1,\ t_1$	$(961 - 6696\,t + 16624\,t^2 - 17280\,t^3 + 6400\,t^4)/1225$
$t_1,\ t_2$	$(279 - 166\,t - 3268\,t^2 + 6400\,t^3 - 3200\,t^4)/1225$
$t_1,\ t_3$	$(-93 + 2804\,t - 11360\,t^2 + 15040\,t^3 - 6400\,t^4)/1225$
$t_1,\ t_4$	$(-155 + 2214\,t - 7472\,t^2 + 8640\,t^3 - 3200\,t^4)/1225$
$t_1,\ t_5$	$(93 - 1936\,t + 8336\,t^2 - 12800\,t^3 + 6400\,t^4)/1225$
$t_2,\ t_2$	$(81 + 468\,t - 44\,t^2 - 2080\,t^3 + 1600\,t^4)/1225$
$t_2,\ t_3$	$(-27 + 642\,t + 1480\,t^2 - 5280\,t^3 + 3200\,t^4)/1225$
$t_2,\ t_4$	$(-45 + 356\,t + 1244\,t^2 - 3200\,t^3 + 1600\,t^4)/1225$
$t_2,\ t_5$	$(27 - 390\,t - 752\,t^2 + 4160\,t^3 - 3200\,t^4)/1225$
$t_3,\ t_3$	$(9 - 480\,t + 6880\,t^2 - 12800\,t^3 + 6400\,t^4)/1225$
$t_3,\ t_4$	$(15 - 562\,t + 4840\,t^2 - 7520\,t^3 + 3200\,t^4)/1225$
$t_3,\ t_5$	$(-9 + 396\,t - 4640\,t^2 + 10560\,t^3 - 6400\,t^4)/1225$
$t_4,\ t_4$	$(25 - 540\,t + 3316\,t^2 - 4320\,t^3 + 1600\,t^4)/1225$
$t_4,\ t_5$	$(-15 + 422\,t - 3328\,t^2 + 6400\,t^3 - 3200\,t^4)/1225$
$t_5,\ t_5$	$(9 - 312\,t + 3184\,t^2 - 8320\,t^3 + 6400\,t^4)/1225$

Table III-*d*-(iii)

t_i, t_j	$\int_0^\theta L(t\,;\,t_i,\,3)\,L(t\,;\,t_j,\,3)\,dt$	$\int_0^1 L(t\,;\,t_i,\,3)\,L(t\,;\,t_j,\,3)\,dt$
t_1, t_1	$(961\theta - 3348\theta^2 + 5541.3\theta^3 - 4320\theta^4 + 1280\theta^5)/1225$	0.09333
t_1, t_2	$(279\theta - 83\theta^2 - 1089.3\theta^3 + 1600\theta^4 - 640\theta^5)/1225$	0.05441
t_1, t_3	$(-93\theta + 1402\theta^2 - 3786.6\theta^3 + 3760\theta^4 - 1280\theta^5)/1225$	0.00190
t_1, t_4	$(-155\theta + 1107\theta^2 - 3736\theta^3 + 2160\theta^4 - 640\theta^5)/1225$	− 0.01524
t_1, t_5	$(93\theta - 968\theta^2 + 2778.6\theta^3 - 3200\theta^4 + 1280\theta^5)/1225$	− 0.01333
t_2, t_2	$(81\theta + 234\theta^2 - 14.6\theta^3 - 520\theta^4 + 320\theta^5)/1225$	0.08190
t_2, t_3	$(-27\theta + 321\theta^2 + 493.3\theta^3 - 1320\theta^4 + 640\theta^5)/1225$	0.00598
t_2, t_4	$(-45\theta + 178\theta^2 + 414.6\theta^3 - 800\theta^4 + 320\theta^5)/1225$	0.05524
t_2, t_5	$(27\theta - 195\theta^2 - 250.6\theta^3 + 1040\theta^4 - 640\theta^5)/1225$	− 0.01524
t_3, t_3	$(9\theta + 240\theta^2 + 3440\theta^3 - 3200\theta^4 + 1280\theta^5)/1225$	0.11619
t_3, t_4	$(15\theta - 281\theta^2 + 1613.3\theta^3 - 1880\theta^4 + 640\theta^5)/1225$	0.08762
t_3, t_5	$(-9\theta + 198\theta^2 - 1546.6\theta^3 + 2640\theta^4 - 1280\theta^5)/1225$	0.01659
t_4, t_4	$(25\theta - 270\theta^2 + 1105.3\theta^3 - 1080\theta^4 + 320\theta^5)/1225$	0.08190
t_4, t_5	$(-15\theta + 211\theta^2 - 1109.3\theta^3 + 1600\theta^4 - 640\theta^5)/1225$	0.03808
t_5, t_5	$(9\theta - 156\theta^2 + 1061.3\theta^3 - 2080\theta^4 + 1280\theta^5)/1225$	0.09333

Table III-*e*-(iii)

t_i	$L'(t\,;\,t_i,\,3)$
t_1	$(-108+160t)/35$
t_2	$(26\ -80t)/35$
t_3	$(80-160t)/35$
t_4	$(54\ -80t)/35$
t_5	$(-52+160t)/35$

Table III-*f*-(iii)

$t_i,\,t_j$	$\int_0^\theta L'(t\,;\,t_i,\,3)\,L'(t\,;\,t_j,\,3)\,dt$	$\int_0^1 L'(t\,;\,t_i,\,3)\,L'(t\,;\,t_j,\,3)\,dt$
$t_1,\,t_1$	$9.5216731\,\theta - 14.1061824\,\theta^2 + 6.9660160\,\theta^3$	2.38151
$t_1,\,t_2$	$-2.2922546\,\theta + 5.2245120\,\theta^2 - 3.4830080\,\theta^3$	-0.55075
$t_1,\,t_3$	$-7.0530912\,\theta + 11.8694382\,\theta^2 - 6.9660160\,\theta^3$	-2.14966
$t_1,\,t_4$	$-4.7608366\,\theta + 7.0530912\,\theta^2 - 3.4830080\,\theta^3$	-1.19075
$t_1,\,t_5$	$4.5845093\,\theta - 10.4490240\,\theta^2 + 6.9660160\,\theta^3$	1.10150
$t_2,\,t_2$	$0.5518391\,\theta - 1.6979664\,\theta^2 + 1.7415040\,\theta^3$	0.59554
$t_2,\,t_3$	$2.2922546\,\theta - 4.3102224\,\theta^2 + 3.4830080\,\theta^3$	1.46504
$t_2,\,t_4$	$1.1461273\,\theta - 2.6122560\,\theta^2 + 1.7415040\,\theta^3$	0.27538
$t_2,\,t_5$	$-1.1036782\,\theta + 3.3959328\,\theta^2 - 3.4830080\,\theta^3$	-1.19075
$t_3,\,t_3$	$5.2245120\,\theta - 10.4490240\,\theta^2 + 6.9660160\,\theta^3$	1.74150
$t_3,\,t_4$	$3.5265456\,\theta - 6.1388016\,\theta^2 + 3.4830080\,\theta^3$	0.87075
$t_3,\,t_5$	$-3.3959328\,\theta + 8.620448\,\theta^2 - 6.9660160\,\theta^3$	-1.74150
$t_4,\,t_4$	$2.3804183\,\theta - 3.5265456\,\theta^2 + 1.7415040\,\theta^3$	0.59538
$t_4,\,t_5$	$-2.2922546\,\theta + 5.2245120\,\theta^2 - 3.4830080\,\theta^3$	-0.55075
$t_5,\,t_5$	$2.2073563\,\theta - 6.7918656\,\theta^2 + 6.9660160\,\theta^3$	2.38151

Table III-*b*-(iv)

t_i	$L(t\,;\,t_i,\,2)$
t_1	$(3-4t)/5$
t_2	$2(1-t)/5$
t_3	$1/5$
t_4	$2t/5$
t_5	$(-1+4t)/5$

Table III-*c*-(iv)

$t_i,\,t_j$	$L(t\,;\,t_i,\,2)\,L(t\,;\,t_j,\,2)$
$t_1,\,t_1$	$(9-24t+16t^2)/25$
$t_1,\,t_2$	$(6-14t+8t^2)/25$
$t_1,\,t_3$	$(3-4t)/25$
$t_1,\,t_4$	$(6-8t)/25$
$t_1,\,t_5$	$(-3+16t-16t^2)/25$
$t_2,\,t_2$	$(4-8t+4t^2)/25$
$t_2,\,t_3$	$(2-2t)/25$
$t_2,\,t_4$	$(4-4t)/25$
$t_2,\,t_5$	$(-2+10t-8t^2)/25$
$t_3,\,t_3$	$1/25$
$t_3,\,t_4$	$2t/25$
$t_3,\,t_5$	$(-1+4t)/25$
$t_4,\,t_4$	$4t^2/25$
$t_4,\,t_5$	$(-2t+8t^2)/25$
$t_5,\,t_5$	$(1-8t+16t^2)/25$

Table III-*d*-(iv)

$t_i,\,t_j$	$\int_0^\theta L(t\,;\,t_i,\,2)\,L(t\,;\,t_j,\,2)\,dt$	$\int_0^1 L(t\,:\,t_i,\,2)\,L(t\,;\,t_j,\,2)\,dt$
$t_1,\,t_1$	$0.36\theta-0.48\theta^2+0.213\theta^3$	0.093
$t_1,\,t_2$	$0.24\theta-0.28\theta^2+0.106\theta^4$	0.066
$t_1,\,t_3$	$0.12\theta-0.08\theta^2$	0.040
$t_1,\,t_4$	$0.24\theta-0.16\theta^2$	0.013
$t_1,\,t_5$	$-0.12\theta+0.32\theta^2-0.213\theta^3$	-0.013
$t_2,\,t_2$	$0.16\theta-0.16\theta^2+0.053\theta^3$	0.053
$t_2,\,t_3$	$0.08\theta-0.04\theta^2$	0.040
$t_2,\,t_4$	$0.16\theta-0.08\theta^2$	0.026
$t_2,\,t_5$	$-0.08\theta+0.2\theta^2-0.106\theta^3$	0.013
$t_3,\,t_3$	0.04θ	0.040
$t_3,\,t_4$	0.04θ	0.040
$t_3,\,t_5$	$-0.04\theta+0.08\theta^2$	0.040
$t_4,\,t_4$	$0.053\theta^3$	0.053
$t_4,\,t_5$	$-0.04\theta^2+0.106\theta^3$	0.066
$t_5,\,t_5$	$0.04\theta-0.16\theta^2+0.213\theta^3$	0.093

Table III-*e*-(iv)

t_i	$L'(t\,;\,t_i,\,2)$
t_1	-0.8
t_2	-0.4
t_3	0
t_4	0.4
t_5	0.8

Table III-*f*-(iv)

$t_i,\,t_j$	$\int_0^\theta L'(t\,;\,t_i,\,2)\,L'(t\,;\,t_j,\,2)\,dt$	$\int_0^1 L'(t\,;\,t_i,\,2)\,L'(t\,;\,t_j,\,2)\,dt$
$t_1,\,t_1$	0.64θ	0.64
$t_1,\,t_2$	0.32θ	0.32
$t_1,\,t_3$	0	0
$t_1,\,t_4$	-0.32θ	-0.32
$t_1,\,t_5$	-0.64θ	-0.64
$t_2,\,t_2$	0.16θ	0.16
$t_2,\,t_3$	0	0
$t_2,\,t_4$	-0.16θ	-0.16
$t_2,\,t_5$	-0.32θ	-0.32
$t_3,\,t_3$	0	0
$t_3,\,t_4$	0	0
$t_3,\,t_5$	0	0
$t_4,\,t_4$	0.16θ	0.16
$t_4,\,t_5$	0.32θ	0.32
$t_5,\,t_5$	0.64θ	0.64