Auxiliary tables for the applications of n -dimensional t -distributions to certain class of empirical functions

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AUXILIARY TABLES FOR THE APPLICATIONS OF n -DIMENSIONAL t -DISTRIBUTIONS TO CERTAIN CLASS OF EMPIRICAL FUNCTIONS

By

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The importance of multi-dimensional t -distributions has been indicated in a series of the papers on successive process of statistical inferences by T. KITAGAWA [1]~[3] and some of numerical considerations have been given by C. W. DUNNETT and M. SOBEL [1] and [2].

In 1953 T. KITAGAWA [4] and [5] gave three different formulations of empirical functions and discussed statistical inferences concerning concurrence-functions and their derivatives.

Among others the uses of n -dimensional t -distributions were indicated in his formulation of the empirical functions of the type AD verifying to be fundamentally important. The object of the present paper is to provide for various auxiliary tables which are indispensable for the uses of n -dimensional t -distributions in this sense. We shall, however, restrict ourselves to the special case when the following three conditions are satisfied: (i) the n points of observations $0 \leq t_1 \leq t_2 \leq \dots \leq t_n \leq 1$ is equi-distributed; (ii) the numbers of observations are equal, i.e., $r_1 = r_2 = \dots = r_n = r$; (iii) $\{\varphi_\nu(t_i; n)\}$ ($\nu = 0, 1, 2, \dots, n-1$) is the orthogonal normalised polynomial system associated with $\{t_i\}$.

In view of Theorems 1.1 and 1.2 in T. KITAGAWA [4] our tasks of constructing auxiliary tables were scheduled in the following steps:

$$(1^\circ) \quad L(t; t_i, m) = \sum_{\nu=0}^{m-1} \varphi_\nu(t_i; n) \varphi_\nu(t; n)$$

$$(2^\circ) \quad a_{ij}(t) = L(t; t_i, m) L(t; t_j, m) \equiv a_{ij}(t; m)$$

$$(3^\circ) \quad A_{ij} = \int_0^1 L(t; t_i, m) L(t; t_j, m) dt \equiv A_{ij}(m)$$

$$(4^\circ) \quad A_{ij}^{(h)} = (\tau_h - \tau_{h-1})^{-1} \int_{\tau_{h-1}}^{\tau_h} L(t; t_i, m) L(t; t_j, m) dt \equiv A_{ij}^{(h)}(m)$$

$$(5^\circ) \quad D_{ij} = \int_0^1 L'(t; t_i, m) L'(t; t_j, m) dt$$

$$(6^\circ) \quad D_{ij}^{(h)} = (\tau_h - \tau_{h-1})^{-1} \int_{\tau_{h-1}}^{\tau_h} L'(t; t_i, m) L'(t; t_j, m) dt,$$

where $m \leq n$, $i, j = 1, 2, \dots, n$. Since there may be various systems of divisions $\{\tau_i\}$ our tables give the values of

$$(4^\circ)' \quad A_{ij}(\theta, m) \equiv \int_0^\theta L(t; t_i, m) L(t; t_j, m) dt$$

$$(6^\circ)' \quad D'_{ij}(\theta, m) = \int_0^\theta L'(t; t_i, m) L'(t; t_j, m) dt$$

instead of (4°) and (6°) respectively.

In this paper we give the case when $1 \leq m \leq n \leq 5$, and we are intending to extend our tables for larger numbers of observation-points n . For the expression of the rational numbers, we give their exact numbers by showing recurring decimals if they are infinite decimals, which yield us somewhat unified expressions from place to place, although they are convenient for practical uses. And we have used the gothic letters in order to show the cyclic parts of them, say 9.481 implies 9.481481....

The constructions of these tables were suggested to the author by Prof. T. KITAGAWA. Numerical calculations were due to the aids of Miss. T. NAGAMATSU. The author expresses here her hearty thanks to them. Although we have no available tables for n -dimensional t -distribution for $n \geq 2$, it is hoped that our auxiliary tables may serve to give us numerical pictures of the AD -type functions.

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Table I-a

$t_i \backslash \varphi_v$	$\varphi_0(t_i; 3)$	$\varphi_1(t_i; 3)$	$\varphi_2(t_i; 3)$
$t_1 (=0)$	$1/\sqrt{3}$	$-1/\sqrt{2}$	$1/\sqrt{6}$
$t_2 (=1/2)$	$1/\sqrt{3}$	0	$-2/\sqrt{6}$
$t_3 (=1)$	$1/\sqrt{3}$	$1/\sqrt{2}$	$1/\sqrt{6}$

$$\varphi_0(t; 3) = 1/\sqrt{3},$$
$$\varphi_1(t; 3) = (-1 + 2t)/\sqrt{2},$$
$$\varphi_2(t; 3) = (1 - 12t + 12t^2)/\sqrt{6}.$$

Table I-b-(i)

$t_i \backslash$	$\varphi_0(t_i; 3) \varphi_0(t; 3)$	$\varphi_1(t_i; 3) \varphi_1(t; 3)$	$\varphi_2(t_i; 3) \varphi_2(t; 3)$	$L(t; t_i, 3)$
t_1	1/3	$(1 - 2t)/2$	$(1 - 12t + 12t^2)/6$	$1 - 3t + 2t^2$
t_2	1/3	0	$(-1 + 12t - 12t^2)/3$	$4t - 4t^2$
t_3	1/3	$-(1 - 2t)/2$	$(1 - 12t + 12t^2)/6$	$-t + 2t^2$

Table I-c-(i)

t_i, t_j	$L(t; t_i, 3) L(t; t_j, 3)$
t_1, t_1	$1 - 6t + 13t^2 - 12t^3 + 4t^4$
t_1, t_2	$4(t - 4t^2 + 5t^3 - 2t^4)$
t_1, t_3	$-t + 5t^2 - 8t^3 + 4t^4$
t_2, t_2	$16(t^2 - 2t^3 + t^4)$
t_2, t_3	$4(-t^2 + 3t^3 - 2t^4)$
t_3, t_3	$t^2 - 4t^3 + 4t^4$

Table I-d-(i)

t_i, t_j	$\int_0^\theta L(t; t_i, 3) L(t; t_j, 3) dt$	$\int_0^1 L(t; t_i, 3) L(t; t_j, 3) dt$
t_1, t_1	$\theta - 3\theta^2 + 4.3\theta^3 - 3\theta^4 + 0.8\theta^5$	0.13
t_1, t_2	$2\theta^2 - 5.3\theta^3 + 5\theta^4 - 1.6\theta^5$	0.06
t_1, t_3	$-0.5\theta^2 + 1.6\theta^3 - 2\theta^4 + 0.8\theta^5$	-0.03
t_2, t_2	$5.3\theta^3 - 8\theta^4 + 3.2\theta^5$	0.53
t_2, t_3	$-1.3\theta^3 + 3\theta^4 - 1.6\theta^5$	0.06
t_3, t_3	$0.3\theta^3 - \theta^4 + 0.8\theta^5$	0.13

Table I-e-(i)

t_i	$L'(t; t_i, 3)$
t_1	$-3 + 4t$
t_2	$4 - 8t$
t_3	$-1 + 4t$

Table I-f-(i)

t_i, t_j	$\int_0^\theta L'(t; t_i, 3) L'(t; t_j, 3) dt$	$\int_0^1 L'(t; t_i, 3) L'(t; t_j, 3) dt$
t_1, t_1	$9\theta - 12\theta^2 + 5.3\theta^3$	2.3
t_1, t_2	$-12\theta + 20\theta^2 - 10.6\theta^3$	-2.6
t_1, t_3	$3\theta - 8\theta^2 + 5.3\theta^3$	0.3
t_2, t_2	$16\theta - 32\theta^2 + 21.3\theta^3$	5.3
t_2, t_3	$-4\theta + 12\theta^2 - 10.6\theta^3$	-2.6
t_3, t_3	$\theta - 4\theta^2 + 5.3\theta^3$	2.3

Table I-b-(ii)

$t_i \backslash$	$L(t; t_i, 2)$
t_1	$5/6 - t$
t_2	$1/3$
t_3	$-1/6 + t$

Table I-c-(ii)

$t_i, t_j \backslash$	$L(t; t_i, 2) L(t; t_j, 2)$
t_1, t_1	$25/36 - 5/3 t + t^2$
t_1, t_2	$5/18 - 1/3 t$
t_1, t_3	$-5/36 + t - t^2$
t_2, t_2	$1/9$
t_2, t_3	$-1/18 + 1/3 t$
t_3, t_3	$1/36 - 1/3 t + t^2$

Table I-d-(ii)

$t_i, t_j \backslash$	$\int_0^\theta L(t; t_i, 2) L(t; t_j, 2) dt$	$\int_0^1 L(t; t_i, 2) L(t; t_j, 2) dt$
t_1, t_1	$0.694 \theta - 0.83 \theta^2 + 0.3 \theta^3$	0.194
t_1, t_2	0.1θ	0.111
t_1, t_3	$0.139 \theta - 0.6 \theta^2$	0.027
t_2, t_2	$0.27 \theta - 0.16 \theta^2 + 0.3 \theta^3$	0.111
t_2, t_3	$-0.5 \theta + 0.16 \theta^2$	0.111
t_3, t_3	$0.027 \theta - 0.16 \theta^2 + 0.3 \theta^3$	0.194

Table I-e-(ii)

$t_i \backslash$	$L'(t; t_i, 2)$
t_1	4
t_2	-8
t_3	4

Table I-f-(ii)

$t_i, t_j \backslash$	$\int_0^\theta L'(t; t_i, 2) L'(t; t_j, 2) dt$	$\int_0^1 L'(t; t_i, 2) L'(t; t_j, 2) dt$
t_1, t_1	16θ	16
t_1, t_2	-32θ	-32
t_1, t_3	16θ	16
t_2, t_2	64θ	64
t_2, t_3	-32θ	-32
t_3, t_3	16θ	16

Table II-a

$t_i \backslash \varphi_v$	$\varphi_0(t_i; 4)$	$\varphi_1(t_i; 4)$	$\varphi_2(t_i; 4)$	$\varphi_3(t_i; 4)$
$t_1 (= 0)$	$1/2$	$-3/\sqrt{20}$	$1/2$	$-1/\sqrt{20}$
$t_2 (= 1/3)$	$1/2$	$-1/\sqrt{20}$	$-1/2$	$3/\sqrt{20}$
$t_3 (= 2/3)$	$1/2$	$1/\sqrt{20}$	$-1/2$	$-3/\sqrt{20}$
$t_4 (= 1)$	$1/2$	$3/\sqrt{20}$	$1/2$	$1/\sqrt{20}$

$$\varphi_0(t; 4) = 1/2$$

$$\varphi_1(t; 4) = (-3 + 6t) / \sqrt{20}$$

$$\varphi_2(t; 4) = (1 - 9t + 9t^2)/2$$

$$\varphi_3(t; 4) = (-1 + 47t - 135t^2 + 90t^3)/\sqrt{20}$$

Table II-b-(i)

$t_i \backslash$	$\varphi_0(t_i; 4) \varphi_0(t; 4)$	$\varphi_1(t_i; 4) \varphi_1(t; 4)$	$\varphi_2(t_i; 4) \varphi_2(t; 4)$	$\varphi_3(t_i; 4) \varphi_3(t; 4)$	$L(t; t_i, 4)$
t_1	$1/4$	$9(1 - 2t)/20$	$(1 - 9t + 9t^2)/4$	$(1 - 47t + 135t^2 - 90t^3)/20$	$1 - 5.5t + 9t^2 - 4.5t^3$
t_2	$1/4$	$3(1 - 2t)/20$	$-(1 - 9t + 9t^2)/4$	$-3(1 - 47t - 135t^2 - 90t^3)/20$	$9t - 22.5t^2 + 13.5t^3$
t_3	$1/4$	$-3(1 - 2t)/20$	$-(1 - 9t + 9t^2)/4$	$3(1 - 47t + 135t^2 - 90t^3)/20$	$4.5t + 18t^2 - 13.5t^3$
t_4	$1/4$	$-9(1 - 2t)/20$	$(1 - 9t + 9t^2)/4$	$-(1 - 47t + 135t^2 - 90t^3)/20$	$t - 4.5t^2 + 4.5t^3$

Table II-c-(i)

t_i, t_j	$L(t; t_i, 4) L(t; t_j, 4)$
t_1, t_1	$1 - 11t + 48.25t^2 - 108t^3 + 130.5t^4 - 81t^5 + 20.25t^6$
t_1, t_2	$9t - 72t^2 + 218.25t^3 - 317.25t^4 + 222.75t^5 - 60.75t^6$
t_1, t_3	$-4.5t + 42.75t^2 - 153t^3 + 256.5t^4 - 202.5t^5 + 60.75t^6$
t_1, t_4	$t - 10t^2 + 38.25t^3 - 69.75t^4 + 60.75t^5 - 20.25t^6$
t_2, t_2	$81t^2 - 405t^3 + 749.25t^4 - 607.5t^5 + 182.25t^6$
t_2, t_3	$-40.5t^2 + 263.25t^3 - 587.25t^4 + 546.75t^5 - 182.25t^6$
t_2, t_4	$9t^2 - 63t^3 + 155.25t^4 - 162t^5 + 60.75t^6$
t_3, t_3	$20.25t^2 - 162t^3 + 445.5t^4 - 486t^5 + 182.25t^6$
t_3, t_4	$-4.5t^2 + 38.25t^3 - 114.75t^4 + 141.75t^5 - 60.75t^6$
t_4, t_4	$t^2 - 9t^3 + 29.25t^4 - 40.5t^5 + 20.25t^6$

Table II-d-(i)

t_i, t_j	$\int_0^3 L(t; t_i, 4) L(t; t_j, 4) dt$	$\int_0^1 L(t; t_i, 4) L(t; t_j, 4) dt$
t_1, t_1	$0 - 5.5\theta^2 + 16.083\theta^3 - 27\theta^4 + 26.1\theta^5 - 13.5\theta^6 + 2.892857\theta^7$	0.07618
t_1, t_2	$4.5\theta^2 - 24\theta^3 + 54.5625\theta^4 - 63.45\theta^5 + 37.125\theta^6 - 8.678571\theta^7$	0.05893
t_1, t_3	$-2.25\theta^2 + 14.25\theta^3 - 38.25\theta^4 + 51.3\theta^5 - 33.76\theta^6 + 8.678571\theta^7$	-0.02183
t_1, t_4	$0.5\theta^2 - 3.3\theta^3 + 9.5625\theta^4 - 13.95\theta^5 + 10.125\theta^6 - 2.892857\theta^7$	0.00989
t_2, t_2	$27\theta^3 - 101.25\theta^4 + 149.85\theta^5 - 101.25\theta^6 + 26.035714\theta^7$	0.38571
t_2, t_3	$-13.5\theta^3 + 65.8125\theta^4 - 117.45\theta^5 + 91.125\theta^6 - 26.34285\theta^7$	-0.04821
t_2, t_4	$3\theta^3 - 15.75\theta^4 + 31.05\theta^5 - 27\theta^6 + 8.678571\theta^7$	-0.02143
t_3, t_3	$6.75\theta^3 - 40.5\theta^4 + 89.1\theta^5 - 81\theta^6 + 26.035714\theta^7$	0.38571
t_3, t_4	$-1.5\theta^3 + 9.5625\theta^4 - 22.95\theta^5 + 23.625\theta^6 - 8.678571\theta^7$	0.05893
t_4, t_4	$0.3\theta^3 - 2.25\theta^4 + 5.85\theta^5 - 6.75\theta^6 + 2.892857\theta^7$	0.07618

Table II-e-(i)

$t_i \backslash L'$	$L'(t; t_i, 4)$
t_1	$-5.5 + 18t - 13.5t^2$
t_2	$9 - 45t + 40.5t^2$
t_3	$-4.5 + 36t - 40.5t^2$
t_4	$1 - 9t + 13.5t^2$

Table II-f-(i)

$t_i, t_j \backslash$	$\int_0^\theta L'(t; t_i, 4) L'(t; t_j, 4) dt$	$\int_0^1 L'(t; t_i, 4) L'(t; t_j, 4) dt$
t_1, t_1	$30.25\theta - 99\theta^2 + 157.5\theta^3 - 121.5\theta^4 + 36.45\theta^5$	3.7000
t_1, t_2	$-49.5\theta + 93.375\theta^2 - 141.75\theta^3 + 197.4375\theta^4 - 109.35\theta^5$	-9.7865
t_1, t_3	$24.75\theta - 139.5\theta^2 + 310.5\theta^3 - 303.75\theta^4 + 109.35\theta^5$	1.3500
t_1, t_4	$-5.5\theta + 33.5\theta^2 - 83.25\theta^3 + 91.125\theta^4 - 36.45\theta^5$	-0.5750
t_2, t_2	$81\theta - 40.5\theta^2 + 249.75\theta^3 - 91.125\theta^4 + 328.05\theta^5$	527.1750
t_2, t_3	$-40.5\theta + 263.25\theta^2 - 722.25\theta^3 + 820.125\theta^4 - 328.05\theta^5$	-7.4250
t_2, t_4	$9\theta - 63\theta^2 + 189\theta^3 - 324\theta^4 + 109.35\theta^5$	-79.6500
t_3, t_3	$20.25\theta - 182\theta^2 + 553.5\theta^3 - 729\theta^4 + 328.05\theta^5$	-9.2000
t_3, t_4	$-4.5\theta + 38.25\theta^2 - 141.75\theta^3 + 212.625\theta^4 - 109.35\theta^5$	-4.7250
t_4, t_4	$\theta - 9\theta^2 + 36\theta^3 - 60.75\theta^4 + 36.45\theta^5$	3.7000

Table II-b-(ii)

$t_i \backslash$	$L(t; t_i, 3)$
t_1	$(19 - 63t + 45t^2)/20$
t_2	$(3 + 39t - 45t^2)/20$
t_3	$(-3 + 51t - 45t^2)/20$
t_4	$(1 - 27t + 45t^2)/20$

Table II-c-(ii)

$t_i, t_j \backslash$	$L(t; t_i, 3) L(t; t_j, 3)$
t_1, t_1	$(361 - 2394t + 5679t^2 - 5670t^3 + 2025t^4)/400$
t_1, t_2	$(57 + 552t - 3177t^2 + 4590t^3 - 2025t^4)/400$
t_1, t_3	$(-57 + 1158t - 4203t^2 + 5130t^3 - 2025t^4)/400$
t_1, t_4	$(19 - 576t + 2601t^2 - 4050t^3 + 2025t^4)/400$
t_2, t_2	$(9 - 234t + 1791t^2 - 3510t^3 + 2025t^4)/400$
t_2, t_3	$(-9 + 36t + 1989t^2 - 4050t^3 + 2025t^4)/400$
t_2, t_4	$(3 - 42t - 9600t^2 + 2970t^3 - 2025t^4)/400$
t_3, t_3	$(9 - 306t + 2871t^2 - 4590t^3 + 2025t^4)/400$
t_3, t_4	$(-3 + 132t - 1557t^2 + 3510t^3 - 2025t^4)/400$
t_4, t_4	$(1 - 54t + 819t^2 - 2430t^3 + 2025t^4)/400$

Table II-d-(ii)

$t_i, t_j \backslash$	$\int_0^{\theta} L(t; t_i, 3) L(t; t_j, 3) dt$	$\int_0^1 L(t; t_i, 3) L(t; t_j, 3) dt$
t_1, t_1	$(361\theta - 1297\theta^2 + 1559.6\theta^3 - 1417.5\theta^4 + 405\theta^5)/400$	0.11125
t_1, t_2	$(57\theta + 276\theta^2 - 1059\theta^3 + 1147.5\theta^4 - 405\theta^5)/400$	0.03875
t_1, t_3	$(-57\theta + 579\theta^2 - 1401\theta^3 + 1282.5\theta^4 - 405\theta^5)/400$	-0.00375
t_1, t_4	$(19\theta - 288\theta^2 + 867\theta^3 - 1012.5\theta^4 + 405\theta^5)/400$	-0.02375
t_2, t_2	$(9\theta - 117\theta^2 + 597\theta^3 - 877.5\theta^4 + 405\theta^5)/400$	0.04125
t_2, t_3	$(-9\theta + 18\theta^2 + 663\theta^3 - 1012.5\theta^4 + 405\theta^5)/400$	0.16125
t_2, t_4	$(3\theta - 21\theta^2 - 3200\theta^3 + 742.5\theta^4 - 405\theta^5)/400$	-0.00125
t_3, t_3	$(9\theta - 153\theta^2 + 957\theta^3 - 1147.5\theta^4 + 405\theta^5)/400$	0.17625
t_3, t_4	$(-3\theta + 66\theta^2 - 519\theta^3 + 877.5\theta^4 - 405\theta^5)/400$	0.04125
t_4, t_4	$(\theta - 27\theta^2 + 273\theta^3 - 607.5\theta^4 + 405\theta^5)/400$	0.11125

Table II-e-(ii)

L' t_i	$L'(t; t_i, 3)$
t_1	$-3.15 + 4.5 t$
t_2	$1.95 - 4.5 t$
t_3	$2.55 - 4.5 t$
t_4	$-1.35 + 4.5 t$

Table II-f-(ii)

t_i, t_j	$\int_0^9 L'(t; t_i, 3) L'(t; t_j, 3) dt$	$\int_0^1 L'(t; t_i, 3) L'(t; t_j, 3) dt$
t_1, t_1	$9.9225 \theta - 14.175 \theta^2 + 6.75 \theta^3$	2.4975
t_1, t_2	$-6.1425 \theta + 11.6375 \theta^2 - 6.75 \theta^3$	-1.2550
t_1, t_3	$-8.0325 \theta + 12.9825 \theta^2 - 6.75 \theta^3$	-1.8000
t_1, t_4	$4.2525 \theta - 10.2825 \theta^2 + 6.75 \theta^3$	0.7200
t_2, t_2	$3.8025 \theta - 8.775 \theta^2 + 6.75 \theta^3$	1.7775
t_2, t_3	$4.9725 \theta - 10.125 \theta^2 + 6.75 \theta^3$	1.5975
t_2, t_4	$-2.6325 \theta + 7.425 \theta^2 - 6.75 \theta^3$	-1.9575
t_3, t_3	$6.5025 \theta - 11.475 \theta^2 + 6.75 \theta^3$	1.7775
t_3, t_4	$-3.4425 \theta + 8.775 \theta^2 - 6.75 \theta^3$	-1.4175
t_4, t_4	$1.8225 \theta - 6.75 \theta^2 + 6.75 \theta^3$	1.8225

Table II-b-(iii)

$t_i \backslash L$	$L(t; t_i, 2)$
t_1	$(7 - 9t)/10$
t_2	$(4 - 3t)/10$
t_3	$(1 + 3t)/10$
t_4	$(-2 + 9t)/10$

Table II-c-(iii)

$t_i, t_j \backslash$	$L(t; t_i, 2) L(t; t_j, 2)$
t_1, t_1	$(49 - 126t + 81t^2)/100$
t_1, t_2	$(28 - 57t + 27t^2)/100$
t_1, t_3	$(7 + 12t - 27t^2)/100$
t_1, t_4	$(-14 + 81t - 81t^2)/100$
t_2, t_2	$(16 - 24t + 9t^2)/100$
t_2, t_3	$(4 + 9t - 9t^2)/100$
t_2, t_4	$(-8 + 42t - 27t^2)/100$
t_3, t_3	$(1 + 6t + 9t^2)/100$
t_3, t_4	$(-2 + 3t + 27t^2)/100$
t_4, t_4	$(4 - 36t + 81t^2)/100$

Table II-e-(iii)

$t_i \backslash$	$L'(t; t_i, 2)$
t_1	-0.9
t_2	-0.3
t_3	0.3
t_4	0.9

Table II-d-(iii)

$t_i, t_j \backslash$	$\int_0^{\theta} L(t; t_i, 2) L(t; t_j, 2) dt$	$\int_0^1 L(t; t_i, 2) L(t; t_j, 2) dt$
t_1, t_1	$(49\theta - 63\theta^2 + 27\theta^3)/100$	0.265
t_1, t_2	$(28\theta - 28.5\theta^2 + 9\theta^3)/100$	0.085
t_1, t_3	$(7\theta + 6\theta^2 - 9\theta^3)/100$	0.040
t_1, t_4	$(-14\theta + 40.5\theta^2 - 9\theta^3)/100$	-0.005
t_2, t_2	$(16\theta - 12\theta^2 + 3\theta^3)/100$	0.070
t_2, t_3	$(4\theta + 4.5\theta^2 - 3\theta^3)/100$	-0.055
t_2, t_4	$(-8\theta + 21\theta^2 - 9\theta^3)/100$	0.040
t_3, t_3	$(\theta + 3\theta^2 + 3\theta^3)/100$	0.070
t_3, t_4	$(-2\theta + 1.5\theta^2 + 9\theta^3)/100$	0.085
t_4, t_4	$(4\theta - 18\theta^2 + 27\theta^3)/100$	0.265

Table II-f-(iii)

$t_i, t_j \backslash$	$\int_0^{\theta} L(t; t_i, 2) L(t; t_j, 2) dt$
t_1, t_1	$(49\theta - 63\theta^2 + 27\theta^3)/100$
t_1, t_2	$(28\theta - 28.5\theta^2 + 9\theta^3)/100$
t_1, t_3	$(7\theta + 6\theta^2 - 9\theta^3)/100$
t_1, t_4	$(-14\theta + 40.5\theta^2 - 9\theta^3)/100$
t_2, t_2	$(16\theta - 12\theta^2 + 3\theta^3)/100$
t_2, t_3	$(4\theta + 4.5\theta^2 - 3\theta^3)/100$
t_2, t_4	$(-8\theta + 21\theta^2 - 9\theta^3)/100$
t_3, t_3	$(\theta + 3\theta^2 + 3\theta^3)/100$
t_3, t_4	$(-2\theta + 1.5\theta^2 + 9\theta^3)/100$
t_4, t_4	$(4\theta - 18\theta^2 + 27\theta^3)/100$

Table III-a

$t_i \backslash \varphi_v$	$\varphi_0(t_i; 5)$	$\varphi_1(t_i; 5)$	$\varphi_2(t_i; 5)$	$\varphi_3(t_i; 5)$	$\varphi_4(t_i; 5)$
$t_1(=0)$	$1/\sqrt{5}$	$-2/\sqrt{10}$	$2/\sqrt{14}$	$-1/\sqrt{10}$	$1/\sqrt{70}$
$t_2(=1/4)$	$1/\sqrt{5}$	$-1/\sqrt{10}$	$-1/\sqrt{14}$	$2/\sqrt{10}$	$-4/\sqrt{70}$
$t_3(=2/4)$	$1/\sqrt{5}$	0	$-2/\sqrt{14}$	0	$6/\sqrt{70}$
$t_4(=3/4)$	$1/\sqrt{5}$	$1/\sqrt{10}$	$-1/\sqrt{14}$	$-2/\sqrt{10}$	$-4/\sqrt{70}$
$t_5(=1)$	$1/\sqrt{5}$	$2/\sqrt{10}$	$2/\sqrt{14}$	$1/\sqrt{10}$	$1/\sqrt{70}$

$$\begin{aligned}\varphi_0(t; 5) &= 1/\sqrt{5}, \\ \varphi_1(t; 5) &= (-2 + 4t)/\sqrt{10}, \\ \varphi_2(t; 5) &= 16(1/8 - t + t^2)/\sqrt{14}, \\ \varphi_3(t; 5) &= (-3 + 86t - 240t^2 + 160t^3)/3\sqrt{10}, \\ \varphi_4(t; 5) &= (3 - 500t + 2740t^2 \\ &\quad - 4480t^3 + 2240t^4)/3\sqrt{70}.\end{aligned}$$

Table III-b-(i)

$t_i \backslash$	$\varphi_0(t_i; 5)\varphi_0(t; 5)$	$\varphi_1(t_i; 5)\varphi_1(t; 5)$	$\varphi_2(t_i; 5)\varphi_2(t; 5)$	$\varphi_3(t_i; 5)\varphi_3(t; 5)$
t_1	1/5	$4(1 - 2t)/10$	$4(1 - 8t + 8t^2)/14$	$(3 - 86t + 240t^2 - 160t^3)/30$
t_2	1/5	$2(1 - 2t)/10$	$2(-1 + 8t - 8t^2)/14$	$(-3 + 86t - 240t^2 + 160t^3)/15$
t_3	1/5	0	$4(-1 + 8t - 8t^2)/14$	0
t_4	1/5	$-2(1 - 2t)/10$	$2(-1 + 8t - 8t^2)/14$	$(3 - 86t + 240t^2 - 160t^3)/15$
t_5	1/5	$-4(1 - 2t)/10$	$4(1 - 8t + 8t^2)/14$	$(-3 + 86t - 240t^2 + 160t^3)/30$

$t_i \backslash$	$\varphi_4(t_i; 5)\varphi_4(t; 5)$	$L(t; t_i, 5)$
t_1	$(3 - 500t + 2740t^2 - 4480t^3 + 2240t^4)/210$	$1 - 25/3t + 70/3t^2 - 80/3t^3 + 32/3t^4$
t_2	$2(-3 + 500t - 2740t^2 + 4480t^3 - 2240t^4)/105$	$16t - 208/3t^2 + 96t^3 - 128/3t^4$
t_3	$(3 - 500t + 2740t^2 - 4480t^3 + 2240t^4)/35$	$-12t + 76t^2 - 128t^3 + 64t^4$
t_4	$2(-3 + 500t - 2740t^2 + 4480t^3 - 2240t^4)/105$	$16/3t - 112/3t^2 + 224/3t^3 - 128/3t^4$
t_5	$(3 - 500t + 2740t^2 - 4480t^3 + 2240t^4)/210$	$-t + 22/3t^2 - 16t^3 + 32/3t^4$

Table III-c-(i)

t_i, t_j	$L(t; t_i, 5) L(t; t_j, 5)$
t_1, t_1	$1 - 16.6t + 116.1t^2 - 442.2t^3 + 1010.2t^4 - 1422.2t^5 + 1208.8t^6 - 568.8t^7 + 113.7t^8$
t_1, t_2	$16t - 202.6t^2 + 1047.1t^3 - 2887.1t^4 + 4615.1t^5 - 4295.1t^6 + 2161.7t^7 - 455.1t^8$
t_1, t_3	$-12t + 176t^2 - 1041.3t^3 + 3224t^4 - 5674.6t^5 + 5717.3t^6 - 3072t^7 + 682.6t^8$
t_1, t_4	$16t - 512t^2 + 1530.6t^3 - 5034.6t^4 + 9450.6t^5 - 10153.6t^6 + 5802.6t^7 - 1365.3t^8$
t_1, t_5	$-t + 15.6t^2 - 100.4t^3 + 341.7t^4 - 668.4t^5 + 753.7t^6 - 455.1t^7 + 113.7t^8$
t_2, t_2	$256t^2 - 2218.6t^3 + 7879.1t^4 - 14677.3t^5 + 15132.4t^6 - 8192t^7 + 1820.4t^8$
t_2, t_3	$-192t^2 + 2048t^3 - 8469.3t^4 + 17706.6t^5 - 19968t^6 + 11605.3t^7 - 2730.6t^8$
t_2, t_4	$256t^2 - 2901.3t^3 + 12885.3t^4 - 29013.3t^5 + 35157.3t^6 - 21845.3t^7 + 541.3t^8$
t_2, t_5	$-16t^2 + 186.6t^3 - 860.4t^4 + 2026.6t^5 - 2588.4t^6 + 1706.6t^7 - 455.1t^8$
t_3, t_3	$144t^2 - 1824t^3 + 8848t^4 - 20992t^5 + 26112t^6 - 16384t^7 + 4096t^8$
t_3, t_4	$(-192t^2 + 2560t^3 - 13248t^4 + 3392t^5 - 45568t^6 + 30720t^7 - 8192t^8)/3$
t_3, t_5	$12t^2 - 164t^3 + 877.3t^4 - 2346.6t^5 + 3328t^6 - 2389.3t^7 + 682.6t^8$
t_4, t_4	$(256t^2 - 3584t^3 + 19712t^4 - 54272t^5 + 78848t^6 - 57344t^7 + 16384t^8)/9$
t_4, t_5	$-16t^2 + 229.3t^3 - 1301.3t^4 + 3733.3t^5 - 5717.3t^6 + 4437.3t^7 - 1365.3t^8$
t_5, t_5	$t^2 - 14.6t^3 + 85.7t^4 - 256t^5 + 412.4t^6 - 341.3t^7 + 113.7t^8$

Table III-d-(i)

t_i, t_j	$\int_0^{\theta} L(t; t_i, 5) L(t; t_j, 5) dt$	$\int_0^1 L(t; t_i, 5) L(t; t_j, 5) dt$
t_1, t_1	$\theta - 8.3\theta^2 + 38.7037\theta^3 - 110.5\theta^4 + 202.04\theta^5 - 237.037\theta^6 + 172.698412\theta^7 - 71.1\theta^8 + 12.530864\theta^9$	0.05150
t_1, t_2	$8\theta^2 - 67.5\theta^3 + 261.7\theta^4 - 577.42\theta^5 + 769.185\theta^6 - 613.587302\theta^7 + 270.2\theta^8 - 50.567901\theta^9$	0.05220
t_1, t_3	$-6\theta^2 + 58.6\theta^3 - 260.3\theta^4 + 655.8\theta^5 - 945.7\theta^6 + 816.761904\theta^7 - 384\theta^8 + 75.851\theta^9$	-0.03070
t_1, t_4	$8\theta^2 - 170.6\theta^3 + 382.6\theta^4 - 1006.93\theta^5 + 1575.1\theta^6 - 1450.523809\theta^7 + 725.3\theta^8 - 151.7\theta^9$	0.17250
t_1, t_5	$-0.5\theta^2 + 5.2\theta^3 - 25.1\theta^4 + 68.35\theta^5 - 111.4740\theta^6 + 107.682539\theta^7 - 56.8\theta^8 + 12.641975\theta^9$	-0.00512
t_2, t_2	$85.3\theta^3 - 554.6\theta^4 + 1575.82\theta^5 - 2446.2\theta^6 + 2161.7\theta^7 - 1024\theta^8 + 202.271605\theta^9$	0.31604
t_2, t_3	$-64\theta^3 + 512\theta^4 - 1693.86\theta^5 + 2951.1\theta^6 - 2852.571428\theta^7 + 1450.6\theta^8 - 303.407\theta^9$	-0.06773
t_2, t_4	$85.3\theta^3 - 725.3\theta^4 + 2577.06\theta^5 - 4835.5\theta^6 + 5022.482539\theta^7 - 2730.6\theta^8 + 606.814\theta^9$	0.13545
t_2, t_5	$-5.3\theta^3 + 46.6\theta^4 - 172.08\theta^5 + 337.6\theta^6 - 369.7\theta^7 + 213.3\theta^8 - 50.567901\theta^9$	0.00986
t_3, t_3	$48\theta^3 - 456\theta^4 + 1769.6\theta^5 - 3498.6\theta^6 + 3730.285714\theta^7 - 2048\theta^8 + 455.1\theta^9$	0.33016
t_3, t_4	$-24\theta^3 + 213.3\theta^4 - 883.2\theta^5 + 188.4\theta^6 - 2169.904762\theta^7 + 1280\theta^8 - 303.407\theta^9$	-0.06772
t_3, t_5	$4\theta^3 - 41\theta^4 + 175.46\theta^5 - 391.1\theta^6 + 475.428571\theta^7 - 298.6\theta^8 + 29.851\theta^9$	-0.03069
t_4, t_4	$9.481\theta^3 - 99.5\theta^4 + 438.4\theta^5 - 1005.037\theta^6 + 1251.5\theta^7 - 796.4\theta^8 + 22.271605\theta^9$	0.31604
t_4, t_5	$-5.3\theta^3 + 57.3\theta^4 - 260.26\theta^5 + 622.2\theta^6 - 816.761905\theta^7 + 554.6541\theta^8 - 151.7039\theta^9$	0.15662
t_5, t_5	$0.3\theta^3 - 3.6\theta^4 + 17.15\theta^5 - 42.5\theta^6 + 58.920635\theta^7 - 42.6\theta^8 + 12.641975\theta^9$	0.05150

Table III-e-(i)

t_i	$L'(t; t_i, 5)$
t_1	$-25/3 + 140/3 t - 80 t^2 + 128/3 t^3$
t_2	$16 - 416/3 t + 288 t^2 - 170.6 t^3$
t_3	$-12 + 152 t - 384 t^2 + 256 t^3$
t_4	$16/3 - 224/3 t + 224 t^2 - 512/3 t^3$
t_5	$-1 + 44/3 t - 48 t^2 + 128/3 t^3$

Table III-f-(i)

t_i, t_j	$\int_0^\theta L'(t; t_i, 5) L'(t; t_j, 5) dt$	$\int_0^1 L'(t; t_i, 5) L'(t; t_j, 5) dt$
t_1, t_1	$69.4 \theta - 388.8 \theta^2 + 1170.370 \theta^3 - 2044.4 \theta^4 + 1678.2 \theta^5 - 1137.7 \theta^6 + 260.0634914 \theta^7$	393.01058
t_1, t_2	$-133.3 \theta + 951.1 \theta^2 - 3383.703 \theta^3 + 6659.5 \theta^4 - 9950.75 \theta^5 + 4323.5 \theta^6 - 140.2539682 \theta^7$	-1673.82434
t_1, t_3	$100 \theta - 913.3 \theta^2 + 3751.1 \theta^3 - 8181.3 \theta^4 + 9830.4 \theta^5 - 6144 \theta^6 + 1560.3809523 \theta^7$	-3372.73016
t_1, t_4	$-44.4 \theta + 435.5 \theta^2 - 2259.259 \theta^3 + 4518.8 \theta^4 - 5814.04 \theta^5 + 3868.4 \theta^6 - 1040.2539682 \theta^7$	-335.11323
t_1, t_5	$8.3 \theta - 253.3 \theta^2 + 388.148 \theta^3 - 952.8 \theta^4 + 1219.5 \theta^5 - 910.2 \theta^6 + 260.0634914 \theta^7$	-240.34392
t_2, t_2	$256 \theta - 2218.6 \theta^2 + 12814.814 \theta^3 - 21333.3 \theta^4 + 25766.4 \theta^5 - 16384 \theta^6 + 4281.7 \theta^7$	3172.99259
t_2, t_3	$-192 \theta + 2048 \theta^2 - 10225.7 \theta^3 + 25792 \theta^4 - 34399.73 \theta^5 + 23210.6 \theta^6 - 6242.3809523 \theta^7$	-9.62540
t_2, t_4	$85.3 \theta - 967.1 \theta^2 + 3963.259 \theta^3 - 14051.5 \theta^4 + 20184.17 \theta^5 - 14565.2 \theta^6 + 4281.7 \theta^7$	-5351.11852
t_2, t_5	$-16 \theta + 186.6 \theta^2 - 1029.925 \theta^3 + 2933.3 \theta^4 - 4448.71 \theta^5 + 3413.3 \theta^6 - 140.2539682 \theta^7$	898.44233
t_3, t_3	$144 \theta - 1824 \theta^2 + 10773.3 \theta^3 - 30720 \theta^4 + 45056 \theta^5 - 32768 \theta^6 + 9362.5714285 \theta^7$	23.90496
t_3, t_4	$-64 \theta + 853.3 \theta^2 - 5361.7 \theta^3 + 16533.3 \theta^4 - 26214.4 \theta^5 + 20480 \theta^6 - 62415.1428571 \theta^7$	-56188.65397
t_3, t_5	$12 \theta - 164 \theta^2 + 1063.1 \theta^3 - 3424 \theta^4 + 5734.4 \theta^5 - 4778.6 \theta^6 + 1560.3809526 \theta^7$	3.22540
t_4, t_4	$28.4 \theta - 398.2 \theta^2 + 2654.814 \theta^3 - 8817.7 \theta^4 + 15132.4 \theta^5 - 12743.1 \theta^6 + 4281.7 \theta^7$	138.37037
t_4, t_5	$-5.3 \theta + 76.4 \theta^2 - 525.037 \theta^3 + 14316.8 \theta^4 - 3288.17 \theta^5 + 4171.851 \theta^6 - 140.2539682 \theta^7$	14606.38307
t_5, t_5	$0 - 14.6 \theta^2 + 103.703 \theta^3 - 373.3 \theta^4 + 911.1 \theta^5 - 227.5 \theta^6 + 260.0634920 \theta^7$	660.32275

Table III-b-(ii)

t_i	$L(t; t_i, 4)$
t_1	$69/70 - 125/21 t + 72/7 t^2 - 16/3 t^3$
t_2	$2/35 + 136/21 t - 120/7 t^2 + 32/3 t^3$
t_3	$-3/35 + 16/7 t - 16/7 t^2$
t_4	$2/35 - 88/21 t + 104/7 t^2 - 32/3 t^3$
t_5	$-1/70 + 29/21 t - 40/7 t^2 + 16/3 t^3$

Table III-c-(ii)

t_i, t_j	$L(t; t_i, 4) L(t; t_j, 4)$
t_1, t_1	$0.971633 - 11.734625 t + 55.708389 t^2 - 132.963265 t^3 + 169.287982 t^4 - 109.714286 t^5 + 28.4 t^6$
t_1, t_2	$0.056326 + 6.043537 t - 54.858965 t^2 + 178.862585 t^3 - 274.358277 t^4 + 201.142857 t^5 - 56.8 t^6$
t_1, t_3	$0.084490 + 2.763265 t - 16.740136 t^2 + 37.572989 t^3 - 35.700680 t^4 + 12.190476 t^5$
t_1, t_4	$0.056327 - 4.470748 t + 40.175964 t^2 - 142.356462 t^3 + 238.657596 t^4 - 188.952381 t^5 + 56.8 t^6$
t_1, t_5	$-0.014082 + 1.446259 t - 13.999546 t^2 + 53.551020 t^3 - 97.886621 t^4 + 85.3 t^5 - 28.4 t^6$
t_2, t_2	$0.003265 + 0.740136 t + 39.981960 t^2 - 220.821769 t^3 + 4320.36283 t^4 - 365.714285 t^5 + 113.7 t^6$
t_2, t_3	$-0.004897 - 0.424490 t + 1614.1496 t^2 - 54.900680 t^3 + 63.564625 t^4 - 24.380952 t^5$
t_2, t_4	$0.003265 + 0.130612 t - 27.268934 t^2 + 168.054421 t^3 - 368.471655 t^4 + 341.3 t^5 - 113.7 t^6$
t_2, t_5	$-0.000816 - 0.001361 t + 8.861678 t^2 - 60.527891 t^3 + 147.229024 t^4 - 152.380952 t^5 - 56.8 t^6$
t_3, t_3	$0.007347 - 0.391837 t - 5.616326 t^2 - 10.448979 t^3 + 5.224489 t^4$
t_3, t_4	$0.004898 + 0.489796 t - 10.982313 t^2 + 44.451701 t^3 - 58.340135 t^4 + 24.380952 t^5$
t_3, t_5	$0.001224 - 0.151020 t + 3.678911 t^2 - 16.674829 t^3 + 25.251700 t^4 - 12.190476 t^5$
t_4, t_4	$0.003265 - 0.478912 t + 19.258049 t^2 - 125.736048 t^3 + 310.131435 t^4 - 316.952381 t^5 + 113.7 t^6$
t_4, t_5	$0.000817 + 0.138775 t - 6.325623 t^2 + 44.919728 t^3 - 121.977324 t^4 + 140.190476 t^5 - 56.8 t^6$
t_5, t_5	$0.000204 - 0.039456 t + 2.070295 t^2 - 15.934693 t^3 + 47.383219 t^4 - 60.952381 t^5 + 28.4 t^6$

Table III-d-(ii)

t_i, t_j	$\int_0^{\theta} L(t; t_i, 4) L(t; t_j, 4) dt$	$\int_0^1 L(t; t_i, 4) L(t; t_j, 4) dt$
t_1, t_1	$0.971633 \theta - 5.867313 \theta^2 + 18.569463 \theta^3 - 33.240816 \theta^4 + 33.857596 \theta^5 - 18.252381 \theta^6 + 4.063492 \theta^7$	0.06831
t_1, t_2	$0.056326 \theta + 3.021769 \theta^2 - 18.286318 \theta^3 + 44.715646 \theta^4 - 54.871655 \theta^5 + 33.523761 \theta^6 - 8.126984 \theta^7$	0.03255
t_1, t_3	$0.084489 \theta + 1.381633 \theta^2 - 5.580045 \theta^3 + 9.393247 \theta^4 - 7.140136 \theta^5 + 2.031746 \theta^6$	0.17093
t_1, t_4	$0.056327 \theta - 2.235374 \theta^2 + 13.391988 \theta^3 - 35.589115 \theta^4 + 47.731519 \theta^5 - 31.492064 \theta^6 + 8.126984 \theta^7$	0.00974
t_1, t_5	$-0.014082 \theta + 0.723125 \theta^2 - 4.66515 \theta^3 + 13.387755 \theta^4 - 19.577320 \theta^5 + 14.2 \theta^6 - 4.063492 \theta^7$	0.00097
t_2, t_2	$0.003265 \theta + 0.370068 \theta^2 + 13.327287 \theta^3 - 55.205442 \theta^4 + 86.407257 \theta^5 - 60.952381 \theta^6 + 16.253968 \theta^7$	0.20402
t_2, t_3	$-0.004897 \theta - 0.212245 \theta^2 + 5.380498 \theta^3 - 13.725170 \theta^4 + 12.712925 \theta^5 - 4.063492 \theta^6$	0.09012
t_2, t_4	$0.003265 \theta + 0.065306 \theta^2 - 9.089644 \theta^3 + 42.013605 \theta^4 - 73.694331 \theta^5 + 56.8 \theta^6 - 16.282539 \theta^7$	-0.07232
t_2, t_5	$-0.000817 \theta - 0.000680 \theta^2 + 2.953893 \theta^3 - 15.131972 \theta^4 + 29.441805 \theta^5 - 25.396825 \theta^6 + 8.126984 \theta^7$	-0.00761
t_3, t_3	$0.007347 \theta - 0.195919 \theta^2 + 1.872109 \theta^3 + 2.612245 \theta^4 + 1.044898 \theta^5$	0.11619
t_3, t_4	$0.004898 \theta + 0.244898 \theta^2 - 3.660771 \theta^3 + 11.112925 \theta^4 - 11.668127 \theta^5 + 4.063492 \theta^6$	0.09742
t_3, t_5	$0.001224 \theta - 0.075510 \theta^2 + 1.226304 \theta^3 - 4.168707 \theta^4 + 5.050340 \theta^5 - 2.0317460 \theta^6$	0.00191
t_4, t_4	$0.003265 \theta - 0.239456 \theta^2 + 6.419349 \theta^3 - 31.434012 \theta^4 + 62.026287 \theta^5 - 52.8253970 \theta^6 + 16.282539 \theta^7$	0.23758
t_4, t_5	$0.000817 \theta + 0.069387 \theta^2 - 2.108541 \theta^3 + 11.229932 \theta^4 - 24.395464 \theta^5 + 23.365079 \theta^6 - 8.126984 \theta^7$	0.03423
t_5, t_5	$0.000204 \theta - 0.019728 \theta^2 + 0.690098 \theta^3 - 3.983673 \theta^4 + 9.476644 \theta^5 - 10.158730 \theta^6 + 4.063492 \theta^7$	0.06831

Table III-e-(ii)

t_i	$L'(t; t_i, 4)$
t_1	$-125/21 + 144/7 t - 16 t^2$
t_2	$136/21 - 240/7 t + 32 t^2$
t_3	$16/7 - 32/7 t$
t_4	$-88/21 + 208/7 t - 32 t^2$
t_5	$29/21 - 80/7 t + 16 t^2$

Table III-f-(ii)

t_i, t_j	$\int_0^\theta L'(t; t_i, 4) L'(t; t_j, 4) dt$	$\int_0^1 L'(t; t_i, 4) L'(t; t_j, 4) dt$
t_1, t_1	$35.4307813 \theta - 122.4489600 \theta^2 + 205.0608860 \theta^3 - 164.5714253 \theta^4 + 51.2 \theta^5$	4.67128
t_1, t_2	$-38.5486900 \theta + 115.3629077 \theta^2 - 333.1336389 \theta^3 + 301.7142892 \theta^4 - 102.4 \theta^5$	45.39487
t_1, t_3	$-13.6054400 \theta + 37.1156403 \theta^2 - 234.126403 \theta^3 + 18.2857139 \theta^4$	-192.33049
t_1, t_4	$24.9432700 \theta - 131.5373939 \theta^2 + 290.3295685 \theta^3 - 283.4285657 \theta^4 + 102.4 \theta^5$	2.70688
t_1, t_5	$-8.2199413 \theta + 48.2176794 \theta^2 - 117.7604571 \theta^3 + 127.99999975 \theta^4 - 51.2 \theta^5$	-0.96262
t_2, t_2	$41.9409747 \theta - 2.2204013 \theta^2 + 529.9971371 \theta^3 - 548.5714560 \theta^4 + 204.8 \theta^5$	-383.65375
t_2, t_3	$14.8027187 \theta - 53.9863667 \theta^2 + 24.9033005 \theta^3 - 36.5714279 \theta^4$	-50.55779
t_2, t_4	$-27.1382778 \theta + 96.9360368 \theta^2 - 117.1736996 \theta^3 + 240.4571382 \theta^4 - 204.8 \theta^5$	193.08120
t_2, t_5	$8.9432961 \theta - 60.68019286 \theta^2 + 179.8819904 \theta^3 - 238.5714336 \theta^4 + 102.4 \theta^5$	0.97366
t_3, t_3	$5.2432896 \theta - 10.4865792 \theta^2 + 6.9910526 \theta^3$	1.74776
t_3, t_4	$-9.5782300 \theta + 43.6596122 \theta^2 - 511.9999898 \theta^3 + 36.5714278 \theta^4$	-441.34718
t_3, t_5	$3.1564621 \theta - 16.2646861 \theta^2 + 29.6681079 \theta^3 - 18.2857139 \theta^4$	-1.72583
t_4, t_4	$17.5600621 \theta - 124.5169869 \theta^2 + 384.8323026 \theta^3 - 475.4285619 \theta^4 + 204.8 \theta^5$	7.24682
t_4, t_5	$-5.7868386 \theta + 44.4625779 \theta^2 - 150.6839749 \theta^3 + 210.2857101 \theta^4 - 102.4 \theta^5$	-4.12253
t_5, t_5	$1.9070264 \theta - 15.7823104 \theta^2 + 58.4242396 \theta^3 - 91.4285696 \theta^4 + 51.2 \theta^5$	-46.87961

Table III-b-(iii)

$t_i \backslash$	$L(t; t_i, 3)$
t_1	$(31 - 108 t + 80 t^2)/35$
t_2	$(9 + 26 t - 40 t^2)/35$
t_3	$(-3 + 80 t - 80 t^2)/35$
t_4	$(-5 + 54 t - 40 t^2)/35$
t_5	$(3 - 52 t + 80 t^2)/35$

Table III-c-(iii)

$t_i, t_j \backslash$	$L(t; t_i, 3) L(t; t_j, 3)$
t_1, t_1	$(961 - 6696 t + 16624 t^2 - 17280 t^3 + 6400 t^4)/1225$
t_1, t_2	$(279 - 166 t - 3268 t^2 + 6400 t^3 - 3200 t^4)/1225$
t_1, t_3	$(-93 + 2804 t - 11360 t^2 + 15040 t^3 - 6400 t^4)/1225$
t_1, t_4	$(-155 + 2214 t - 7472 t^2 + 8640 t^3 - 3200 t^4)/1225$
t_1, t_5	$(93 - 1936 t + 8336 t^2 - 12800 t^3 + 6400 t^4)/1225$
t_2, t_2	$(81 + 468 t - 44 t^2 - 2080 t^3 + 1600 t^4)/1225$
t_2, t_3	$(-27 + 642 t + 1480 t^2 - 5280 t^3 + 3200 t^4)/1225$
t_2, t_4	$(-45 + 356 t + 1244 t^2 - 3200 t^3 + 1600 t^4)/1225$
t_2, t_5	$(27 - 390 t - 752 t^2 + 4160 t^3 - 3200 t^4)/1225$
t_3, t_3	$(9 - 480 t + 6880 t^2 - 12800 t^3 + 6400 t^4)/1225$
t_3, t_4	$(15 - 562 t + 4840 t^2 - 7520 t^3 + 3200 t^4)/1225$
t_3, t_5	$(-9 + 396 t - 4640 t^2 + 10560 t^3 - 6400 t^4)/1225$
t_4, t_4	$(25 - 540 t + 3316 t^2 - 4320 t^3 + 1600 t^4)/1225$
t_4, t_5	$(-15 + 422 t - 3328 t^2 + 6400 t^3 - 3200 t^4)/1225$
t_5, t_5	$(9 - 312 t + 3184 t^2 - 8320 t^3 + 6400 t^4)/1225$

Table III-*d*-(iii)

t_i, t_j	$\int_0^{\theta} L(t; t_i, 3) L(t; t_j, 3) dt$	$\int_0^1 L(t; t_i, 3) L(t; t_j, 3) dt$
t_1, t_1	$(961\theta - 3348\theta^2 + 5541.3\theta^3 - 4320\theta^4 + 1280\theta^5)/1225$	0.09333
t_1, t_2	$(279\theta - 83\theta^2 - 1089.3\theta^3 + 1600\theta^4 - 640\theta^5)/1225$	0.05441
t_1, t_3	$(-93\theta + 1402\theta^2 - 3786.6\theta^3 + 3760\theta^4 - 1280\theta^5)/1225$	0.00190
t_1, t_4	$(-155\theta + 1107\theta^2 - 3736\theta^3 + 2160\theta^4 - 640\theta^5)/1225$	-0.01524
t_1, t_5	$(93\theta - 968\theta^2 + 2778.6\theta^3 - 3200\theta^4 + 1280\theta^5)/1225$	-0.01333
t_2, t_2	$(81\theta + 234\theta^2 - 14.6\theta^3 - 520\theta^4 + 320\theta^5)/1225$	0.08190
t_2, t_3	$(-27\theta + 321\theta^2 + 493.3\theta^3 - 1320\theta^4 + 640\theta^5)/1225$	0.00598
t_2, t_4	$(-45\theta + 178\theta^2 + 414.6\theta^3 - 800\theta^4 + 320\theta^5)/1225$	0.05524
t_2, t_5	$(27\theta - 195\theta^2 - 250.6\theta^3 + 1040\theta^4 - 640\theta^5)/1225$	-0.01524
t_3, t_3	$(9\theta + 240\theta^2 + 3440\theta^3 - 3200\theta^4 + 1280\theta^5)/1225$	0.11619
t_3, t_4	$(15\theta - 281\theta^2 + 1613.3\theta^3 - 1880\theta^4 + 640\theta^5)/1225$	0.08762
t_3, t_5	$(-9\theta + 198\theta^2 - 1546.6\theta^3 + 2640\theta^4 - 1280\theta^5)/1225$	0.01659
t_4, t_4	$(25\theta - 270\theta^2 + 1105.3\theta^3 - 1080\theta^4 + 320\theta^5)/1225$	0.08190
t_4, t_5	$(-15\theta + 211\theta^2 - 1109.3\theta^3 + 1600\theta^4 - 640\theta^5)/1225$	0.03808
t_5, t_5	$(9\theta - 156\theta^2 + 1061.3\theta^3 - 2080\theta^4 + 1280\theta^5)/1225$	0.09333

Table III-e-(iii)

t_i	$L'(t; t_i, 3)$
t_1	$(-108 + 160t)/35$
t_2	$(26 - 80t)/35$
t_3	$(80 - 160t)/35$
t_4	$(54 - 80t)/35$
t_5	$(-52 + 160t)/35$

Table III-f-(iii)

t_i, t_j	$\int_0^{\theta} L'(t; t_i, 3) L'(t; t_j, 3) dt$	$\int_0^1 L'(t; t_i, 3) L'(t; t_j, 3) dt$
t_1, t_1	$9.5216731 \theta - 14.1061824 \theta^2 + 6.9660160 \theta^3$	2.38151
t_1, t_2	$-2.2922546 \theta + 5.2245120 \theta^2 - 3.4830080 \theta^3$	-0.55075
t_1, t_3	$-7.0530912 \theta + 11.8694382 \theta^2 - 6.9660160 \theta^3$	-2.14966
t_1, t_4	$-4.7608366 \theta + 7.0530912 \theta^2 - 3.4830080 \theta^3$	-1.19075
t_1, t_5	$4.5845093 \theta - 10.4490240 \theta^2 + 6.9660160 \theta^3$	1.10150
t_2, t_2	$0.5518391 \theta - 1.6979664 \theta^2 + 1.7415040 \theta^3$	0.59554
t_2, t_3	$2.2922546 \theta - 4.3102224 \theta^2 + 3.4830080 \theta^3$	1.46504
t_2, t_4	$1.1461273 \theta - 2.6122560 \theta^2 + 1.7415040 \theta^3$	0.27538
t_2, t_5	$-1.1036782 \theta + 3.3959328 \theta^2 - 3.4830080 \theta^3$	-1.19075
t_3, t_3	$5.2245120 \theta - 10.4490240 \theta^2 + 6.9660160 \theta^3$	1.74150
t_3, t_4	$3.5265456 \theta - 6.1388016 \theta^2 + 3.4830080 \theta^3$	0.87075
t_3, t_5	$-3.3959328 \theta + 8.620448 \theta^2 - 6.9660160 \theta^3$	-1.74150
t_4, t_4	$2.3804183 \theta - 3.5265456 \theta^2 + 1.7415040 \theta^3$	0.59538
t_4, t_5	$-2.2922546 \theta + 5.2245120 \theta^2 - 3.4830080 \theta^3$	-0.55075
t_5, t_5	$2.2073563 \theta - 6.7918656 \theta^2 + 6.9660160 \theta^3$	2.38151

Table III-*b*-(iv)

t_i	$L(t; t_i, 2)$
t_1	$(3 - 4t)/5$
t_2	$2(1 - t)/5$
t_3	$1/5$
t_4	$2t/5$
t_5	$(-1 + 4t)/5$

Table III-*c*-(iv)

t_i, t_j	$L(t; t_i, 2) L(t; t_j, 2)$
t_1, t_1	$(9 - 24t + 16t^2)/25$
t_1, t_2	$(6 - 14t + 8t^2)/25$
t_1, t_3	$(3 - 4t)/25$
t_1, t_4	$(6 - 8t)/25$
t_1, t_5	$(-3 + 16t - 16t^2)/25$
t_2, t_2	$(4 - 8t + 4t^2)/25$
t_2, t_3	$(2 - 2t)/25$
t_2, t_4	$(4 - 4t)/25$
t_2, t_5	$(-2 + 10t - 8t^2)/25$
t_3, t_3	$1/25$
t_3, t_4	$2t/25$
t_3, t_5	$(-1 + 4t)/25$
t_4, t_4	$4t^2/25$
t_4, t_5	$(-2t + 8t^2)/25$
t_5, t_5	$(1 - 8t + 16t^2)/25$

Table III-*d*-(iv)

t_i, t_j	$\int_0^{\theta} L(t; t_i, 2) L(t; t_j, 2) dt$	$\int_0^1 L(t; t_i, 2) L(t; t_j, 2) dt$
t_1, t_1	$0.36\theta - 0.48\theta^2 + 0.213\theta^3$	0.093
t_1, t_2	$0.24\theta - 0.28\theta^2 + 0.106\theta^4$	0.066
t_1, t_3	$0.12\theta - 0.08\theta^2$	0.040
t_1, t_4	$0.24\theta - 0.16\theta^2$	0.013
t_1, t_5	$-0.12\theta + 0.32\theta^2 - 0.213\theta^3$	-0.013
t_2, t_2	$0.16\theta - 0.16\theta^2 + 0.053\theta^3$	0.053
t_2, t_3	$0.08\theta - 0.04\theta^2$	0.040
t_2, t_4	$0.16\theta - 0.08\theta^2$	0.026
t_2, t_5	$-0.08\theta + 0.2\theta^2 - 0.106\theta^3$	0.013
t_3, t_3	0.04θ	0.040
t_3, t_4	0.04θ	0.040
t_3, t_5	$-0.04\theta + 0.08\theta^2$	0.040
t_4, t_4	$0.053\theta^3$	0.053
t_4, t_5	$-0.04\theta^2 + 0.106\theta^3$	0.066
t_5, t_5	$0.04\theta - 0.16\theta^2 + 0.213\theta^3$	0.093

Table III-e-(iv)

$t_i \backslash$	$L'(t; t_i, 2)$
t_1	- 0.8
t_2	- 0.4
t_3	0
t_4	0.4
t_5	0.8

Table III-f-(iv)

$t_i, t_j \backslash$	$\int_0^{\theta} L'(t; t_i, 2) L'(t; t_j, 2) dt$	$\int_0^1 L'(t; t_i, 2) L'(t; t_j, 2) dt$
t_1, t_1	0.64 θ	0.64
t_1, t_2	0.32 θ	0.32
t_1, t_3	0	0
t_1, t_4	- 0.32 θ	- 0.32
t_1, t_5	- 0.64 θ	- 0.64
t_2, t_2	0.16 θ	0.16
t_2, t_3	0	0
t_2, t_4	- 0.16 θ	- 0.16
t_2, t_5	- 0.32 θ	- 0.32
t_3, t_3	0	0
t_3, t_4	0	0
t_3, t_5	0	0
t_4, t_4	0.16 θ	0.16
t_4, t_5	0.32 θ	0.32
t_5, t_5	0.64 θ	0.64