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ON THE DETERMINATION OF SAMPLE SIZE FROM THE TWO SAMPLE THEORTICAL FORMULATION

By

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§ 1. Two sample theoretical formulation.

Let us consider a normal population $N(m, \sigma^2)$ with unknown mean m and unknown variance σ^2, and let O_n: $(x_1, x_2, \cdots, x_n)$ be a random sample of size n from this population. Then, as well known, the confidence interval for the mean m with confidence coefficient $1-\alpha$ is given by

$$(1.01) \qquad (\bar{x} - t_{n-1}(\alpha)\, s\, n^{-1/2}, \qquad x + t_{n-1}(\alpha)\, s\, n^{-1/2}),$$

where $\bar{x}$ and s^2 are the sample mean and unbiassed variance estimate from obtained from the sample respectively, and $t_{n-1}(\alpha)$ is the α significant level of t-distribution with the $n-1$ degrees of freedom. The length of confidence interval obtained in this way is $2t_{n-1}(\alpha)\, s\, n^{-1/2}$, and the expectaion of this length is equal to $2t_{n-1}(\alpha)\, E\{s\}\, n^{-1/2} = 2t_{n-1}(\alpha)\, \sigma\, n^{-1/2} + O(n^{-3/2})$, which tends to zero as n increases infinitely and moreover we have $4t_{n-1}^2(\alpha)$ $n^{-1}\sigma^2\{s\} = 2t_{n-1}^2(\alpha)\, \sigma^2 n^{-2} + O(n^{-3})$. In view of these properties of length of the condidence interval, it will be suggested that there may be some possibilities that a length of confidence interval with any assigned confidence coefficient $1-\alpha$ can be smaller than any assigned length $2d$, by choosing the sample size sufficiently large. The real situations, however, should be more carefully formulated. There are two essential points which should be remarked. The first point is that the relation $t_{n-1}(\alpha)\, s\, n^{-1/2} \leq d$ cannot be established with the probability one, for any fixed n, while the second one is that since the population variance σ^2 is unknown to us, the equation such as $t_{n-1}(\alpha)\, s\, n^{-1/2} \leq d$ cannot give us any definite procedure for determining the sampl esize n. In view of these two facts, let us now appeal to the two sample theoretical approach, which will suggest following type of two sample procedure: (i) Let us at first draw a sample of size n_1 O_{n_1}: $(x_{1,1}, x_{1,2}, \cdots, x_{1,n_1})$ from our population, and let us make the undiassed estimate s_1^2 for σ^2:

$$(1.02) \qquad s_1^2 = (n_1 - 1)^{-1} \sum_{j=1}^{n_1} (x_{1j} - \bar{x}_1)^2 .$$

(ii) In view of this s_1, let us determine a size of second sample by some procedure, and draw a second sample of this size O_{n_2}: $(x_{21}, x_{22}, \cdots, x_{2n_2})$ from our population. Then our situation is to expect to satisfy the following relation:

(1.03) $$\Pr.\{t_{n_2-1}(\alpha)\, s_2 n_2^{-1/2} \leq d\} \geq 1-\beta,$$

where

(1.04) $$s_2^2 = (n_2-1)^{-1} \sum_{j=1}^{n_2} (x_{2j}-\bar{x}_2)^2 .$$

It is to be noted that the sample size n_2 is itself a random variable as a function of the stochastic variables. In what follows let us denote the stochastic variable which represents the sample size n_2 by a bold type $\boldsymbol{n}_2$ in order to distinguish from a constant number n_2. In order that we may realize this type of two sample procedure, what remines to us to determine is how to select the size of the second sample n_2. Here let us appeal to the F-distribution which gives us, for two independent unbiassed variance estimates s_1^2 and s_2^2 with the degress of freedom n_1-1 and n_2-1 respectively.

(1.05) $$\Pr.\{s_2/s_1 < (F_{n_1-1}^{n_2-1}(\beta))^{1/2}\} = 1-\beta .$$

In combination of (1.03) and (1.05) we have the relation

(1.06) $$\Pr.\left\{s_2 \leq \frac{dn_2^{1/2}}{t_{n_2-1}(\alpha)}\right\} \geq 1-\beta = \Pr.\{s_2 \leq s_1\sqrt{F_{n_1-1}^{n_2-1}(\beta)}\} .$$

Therefore we propose to determine the size of second sample n_2 such as the following relations shall hold simultaneously:

(1.07) $$\frac{dn_2^{1/2}}{t_{n_2-1}(\alpha)} \geq s_1\sqrt{F_{n_1-1}^{n_2-1}(\beta)},$$

and

(1.08) $$\frac{d(n_2-1)^{1/2}}{t_{n_2-2}(\alpha)} < s_1\sqrt{F_{n_1-1}^{n_2-2}(\beta)},$$

that is,

(1.09) $$\frac{d(n_2-1)^{1/2}}{t_{n_2-2}(\alpha)\sqrt{F_{n_1-1}^{n_2-2}(\beta)}} < s_1 \leq \frac{dn_2^{1/2}}{t_{n_2-1}(\alpha)\sqrt{F_{n_1-1}^{n_2-1}(\beta)}} .$$

Consequently we have following relations:

(1.10) $$\Pr.\left\{\frac{t_{n_2-1}(\alpha)}{\sqrt{\boldsymbol{n}_2}}\, s_2 < d\right\}$$

$$= \sum_{n_2=3}^{\infty} \Pr.\left\{\frac{t_{n_2-1}(\alpha)}{\sqrt{n_2}}\, s_2 < d/n_2\right\} \Pr.\{\boldsymbol{n}_2 = n_2\}$$

$$= \sum_{n_2=3}^{\infty} \Pr.\left\{\frac{d(n_2-1)^{1/2}}{t_{n_2-2}(\alpha)\sqrt{F_{n_1-1}^{n_2-2}(\beta)}} < s_1 \leq \frac{dn_2^{1/2}}{t_{n_2-1}(\alpha)\sqrt{F_{n_1-1}^{n_2-1}(\beta)}}\right\}$$

$$\cdot \Pr.\left\{\frac{t_{n_2-1}(\alpha)}{\sqrt{n}}\, s_2 < d/s_1\right\}$$

$$= \sum_{n_2=3}^{\infty} \int_{\frac{d(n_2-1)^{1/2}}{t_{n_2-2}(\alpha)\sqrt{F_{n_1-1}^{n_2-2}(\beta)}}}^{\frac{dn_2^{2/1}}{t_{n_2-1}(\alpha)\sqrt{F_{n_1-1}^{n_2-1}(\beta)}}} \varphi_{n_1}(s_1)\, ds_1 \int_0^{\frac{dn_2^{1/2}}{t_{n_2-1}(\alpha)}} \varphi_{n_2}(s_2)\, ds_2 .$$

§ 2. Numerical considerations.

Each term of the sums of the integrals in the last right hand side of (1.10) may be caluculated by means of the tables of the incomplete Γ-function [1]. We have constructed the distributoin functions of the stochastic variables n_2 with respect to the set of the values of α, β, $d^2\sigma^{-2}$ and n_1, where $\alpha = 0.01, 0.05$; $\beta = 0.01, 0.05$; $d^2\sigma^{-2} = 0.75, 0.25$; $n_1 = 5, 10, 15, 21, 25, 31$. The results are given in Tables I, II, III and IV. These tables suggest us the way how to choose a sample size n_2 in our two sample theoretical formulations; indeed this choose will be seen to depend upon the values of α, β and $d^2\sigma^{-2}$.

§ 3. General remarks.

There remaines the problem of two dependent samples. That is to say, in stead of taking consideration a new independent second sample O_2, we may consider a pooled sample of size $n_1 + n_2$, adding a new sample of size n_2 to that of size n_1. This problem was considered by MOOD [1], but, as he himself pointed out in MOOD [2], there was an error because he had been in failing to take account of the fact that n is a random variable.

In these problems there may be various procedures which will give us two sample formulations. The method due to STEIN [1] is specially worth while to mention, and KITAGAWA [1] gave some stochastic considerations about similar problems in Part III, and KITAGAWA [4] applies STEIN-BARNARD method to BEHRENS-FISHER's test. The two dependent sample formulations will be reconsidered in another occassion.

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Table of the Individual probabilities:

$$I(n_2\,;\ n_1/d^2\sigma^{-2},\ \alpha,\ \beta) = \int_{b(n_2-1)}^{b(n_2)} \varphi_{n_1-1}(s_1\,;\ \sigma)\,ds_1 \int_0^{c(n_2)} \varphi_{n_2}(s_2\,;\ \sigma)\,ds_2$$

$$b(n) \equiv dn^{1/2}\{t_{n-1}(\alpha)\}^{-1}\{F^{n-1}_{n_1-1}(\beta)\}^{-1/2}$$

$$c(n_2) \equiv dn_2^{1/2}\{t_{n_2-1}(\alpha)\}^{-1}$$

Table I — (1) $(\alpha = \beta = 1\%\,;\ d^2\sigma^{-2} = 0.75)$

n_1 \ n_1	11	12	13	14	15	16	17	18	19	20	C.S.
21	0	0.0002	0.0008	0.0017	0.0034	0.0067	0.0105	0.0155	0.0219	0.0292	0.0899
25	0	0.0001	0.0005	0.0014	0.0026	0.0073	0.0106	0.0181	0.0258	0.0350	0.1014
31	0	0.0001	0.0002	0.0009	0.0027	0.0061	0.0135	0.0199	0.0309	0.0438	0.1181

n_1 \ n_2	21	22	23	24	25	26	27	28	29	30	C.S.
21	0.0362	0.0416	0.0475	0.0524	0.0568	0.0568	0.0582	0.0572	0.0565	0.0551	0.6082
25	0.0456	0.0524	0.0577	0.0665	0.0680	0.0676	0.0670	0.0657	0.0593	0.0574	0.7086
31	0.0579	0.0610	0.0807	0.0818	0.0842	0.0790	0.0749	0.0682	0.0605	0.0525	0.8188

n_1 \ n_2	31	32	33	34	35	36	37	38	39	40	C.S.
21	0.0504	0.0439	0.0425	0.0354	0.0330	0.0297	0.0255	0.0224	0.0189	0.0162	0.9261
25	0.0495	0.0413	0.0359	0.0310	0.0263	0.0216	0.0179	0.0144	0.0114	0.0089	0.9668
31	0.0422	0.0322	0.0259	0.0203	0.0157	0.0116	0.0087	0.0062	0.0043	0.0030	0.9889

n_1 \ n_2	41	42	43	44	45	46	47	48	49	50	C.S.
21	0.0133	0.0100	0.0084	0.0068	0.0057	0.0045	0.0038	0.0029	0.0024	0.0019	0.9858
25	0.0099	0.0047	0.0037	0.0027	0.0021	0.0016	0.0012	0.0008	0.0007	0.0004	0.9946
31	0.0021	0.0012	0.0009	0.0005	0.0004	0.0003	0.0001	0.0001	0.0001	0	0.9946

n_1 \ n_2	51	52	53	54	55	56	57	58	59	60	T.S.
21	0.0014	0.0008	0.0009	0.0007	0.0005	0.0004	0.0003	0.0002	0.0002	0.0002	0.9914
25	0.0003	0.0001	0.0001	0.0001	0.0001	0.0001	0				0.9954
31	0										0.9946

Table I — (2) ($\alpha = \beta = 1\%$; $d^2\sigma^{-2} = 0.5$)

n_1 \ n_2	11	12	13	14	15	16	17	18	19	20	C.S.
25	0	0	0	0	0	0.0002	0.0004	0.0025	0.0015	0.0028	0.0074
31	0	0	0	0	0	0.0001	0.0002	0.0006	0.0011	0.0026	0.0046
n_1 \ n_2	21	22	23	24	25	26	27	28	29	30	C.S.
25	0.0047	0.0085	0.0091	0.0133	0.0171	0.0208	0.0244	0.0299	0.0339	0.0384	0.2075
31	0.0045	0.0067	0.0110	0.0154	0.0219	0.0262	0.0319	0.0379	0.0442	0.0499	0.2562
n_1 \ n_2	31	32	33	34	35	36	37	38	39	40	C.S.
25	0.0407	0.0410	0.0432	0.0451	0.0463	0.0466	0.0464	0.0459	0.0443	0.0427	0.6497
31	0.0526	0.0524	0.0543	0.0551	0.0550	0.0537	0.0521	0.0492	0.0454	0.0418	0.7658
n_1 \ n_2	41	42	43	44	45	46	47	48	49	50	C.S.
25	0.0406	0.0316	0.0338	0.0290	0.0267	0.0241	0.0219	0.0196	0.0173	0.0154	0.9097
31	0.0375	0.0296	0.0263	0.0228	0.0197	0.0170	0.0142	0.0120	0.0099	0.0072	0.9620
n_1 \ n_2	51	52	53	54	55	56	57	58	59	60	T.S.
25	0.0133	0.0113	0.0102	0.0087	0.0073	0.0063	0.0052	0.0043	0.0036	0.0031	0.9840
31	0.0065	0.0051	0.0044	0.0034	0.0026	0.0021	0.0016	0.0012	0.0009	0.0007	0.9905

Table II — (1) $(\alpha = 1\%,\ \beta = 5\%;\ d^2\sigma^{-2} = 0.75)$

n_1 \ n_2	7	8	9	10	11	12	13	14	15	16	17	C.S.
10	0	0.0002	0.0005	0.0014	0.0027	0.0048	0.0077	0.0111	0.0157	0.0203	0.0245	0.0889
15	0	0	0	0.0009	0.0018	0.0039	0.0074	0.0128	0.0196	0.0273	0.0342	0.1078
21	0	0	0	0.0001	0.0009	0.0027	0.0064	0.0117	0.0212	0.0340	0.0445	0.1215
25	0	0	0	0	0.0005	0.0019	0.0054	0.0116	0.0221	0.0362	0.0500	0.1277
31	0	0	0	0	0.0003	0.0012	0.0042	0.0105	0.0219	0.0396	0.0569	0.1346

n_1 \ n_2	18	19	20	21	22	23	24	25	26	27	28	C.S.
10	0.0280	0.0311	0.0337	0.0360	0.0368	0.0378	0.0386	0.0385	0.0377	0.0372	0.0362	0.4805
15	0.0419	0.0481	0.0520	0.0549	0.0567	0.0571	0.0564	0.0547	0.0505	0.0474	0.0438	0.0613
21	0.0554	0.0661	0.0734	0.0756	0.0740	0.0716	0.0684	0.0619	0.0536	0.0466	0.0395	0.8076
25	0.0646	0.0770	0.0851	0.0870	0.0830	0.0789	0.0715	0.0626	0.0507	0.0413	0.0330	0.8624
31	0.0767	0.0921	0.0998	0.1068	0.0935	0.0842	0.0713	0.0580	0.0366	0.0287	0.0236	0.9059

n_1 \ n_2	29	30	31	32	33	34	35	36	37	38	39	C.S.
10	0.0350	0.0330	0.0320	0.0296	0.0283	0.0270	0.0256	0.0242	0.0225	0.0210	0.0196	0.7783
15	0.0397	0.0350	0.0313	0.0268	0.0238	0.0207	0.0170	0.0147	0.0131	0.0112	0.0094	0.9140
21	0.0326	0.0272	0.0211	0.0160	0.0128	0.0099	0.0076	0.0058	0.0043	0.0032	0.0024	0.9505
25	0.0259	0.0199	0.0143	0.0100	0.0073	0.0052	0.0037	0.0025	0.0017	0.0011	0.0007	0.9547
31	0.0166	0.0166	0.0073	0.0044	0.0029	0.0017	0.0011	0.0006	0.0003	0.0001	0	0.9525

n_1 \ n_2	40	41	42	43	44	45	46	47	48	49	50	C S.
10	0.0181	0.0168	0.0142	0.0129	0.0120	0.0110	0.0100	0.0090	0.0083	0.0075	0.0062	0.9043
15	0.0078	0.0060	0.0051	0.0041	0.0033	0.0028	0.0022	0.0018	0.0015	0.0012	0.0009	0.9507
21	0.0017	0.0012	0.0007	0.0006	0.0004	0.0003	0.0002	0.0001	0.0001	0.0001	0	0.9559
25	0.0003	0.0003	0.0001	0								0.9554
31	0											0.9525

n_1 \ n_2	51	52	53	54	55	56	57	58	59	60		T. S.
10	0.0057	0.0053	0.0051	0.0044	0.0039	0.0035	0.0032	0.0028	0.0024	0.0022		0.9423
15	0.0008	0.0005	0.0005	0.0004	0.0003	0.0002	0.0002	0.0001	0.0001	0.0001		0.09539
21	0											0.9559
25	0											0.9554
31	0											0.9525

Table II — (2) ($\alpha = 1\%$, $\beta = 5\%$; $d^2 \sigma^{-2} = 0.5$)

n_1 \ n_2	12	13	14	15	16	17	18	19	20	21	22	C.S.
21	0.0001	0.0002	0.0004	0.0011	0.0024	0.0042	0.0069	0.0107	0.0157	0.0219	0.0269	0.0905
25	0	0	0.0002	0.0007	0.0018	0.0036	0.0063	0.0107	0.0164	0.0235	0.0302	0.0934
31	0	0	0.0001	0.0004	0.0012	0.0028	0.0056	0.0105	0.0170	0.0254	0.0357	0.0987

n_1 \ n_2	23	24	25	26	27	28	29	30	31	32	33	C.S.
21	0.0327	0.0383	0.0441	0.0465	0.0495	0.0512	0.0522	0.0520	0.0503	0.0480	0 0444	0.5992
25	0.0372	0.0454	0.0513	0.0543	0.0575	0.0594	0.0608	0.0592	0.0551	0.0492	0.0466	0.6694
31	0.0441	0.0534	0.0618	0.0652	0.0685	0.0694	0.0680	0.0660	0.0593	0.0489	0.0455	0.7488

n_1 \ n_2	34	35	36	37	38	39	40	41	42	43	44	C.S.
21	0.0421	0.0390	0.0357	0.0332	0.0302	0.0271	0.0230	0.0194	0.0166	0.0146	0.0124	0.8930
25	0.0419	0.0384	0.0343	0.0301	0.0262	0.0224	0.0188	0.0158	0.0118	0.0097	0.0080	0.9268
31	0.0400	0.0344	0.0287	0.0238	0.0193	0.0154	0.0120	0.0088	0.0066	0.0051	0.0036	0.9465

n_1 \ n_2	45	46	47	48	49	50	51	52	53	54	55	C.S.
21	0.0107	0.0091	0.0079	0.0064	0.0054	0.0046	0.0037	0.0030	0.0024	0.0020	0.0017	0.9499
25	0.0065	0.0052	0.0041	0.0033	0.0026	0.0020	0.0015	0.0012	0.0008	0.0006	0.0005	0.9551
31	0.0027	0.0020	0.0015	0.0010	0.0007	0.0004	0.0003	0.0002	0.0001	0		0.9554

n_1 \ u_2	56	57	58	59	60							T.S.
21	0.0014	0.0011	0.0008	0.0006	0.0004							0.9542
25	0.0004	0.0003	0.0001	0.0001	0							0.9560
31												0.9554

Table III — (1) ($\alpha = 5\%$, $\beta = 1\%$; $d^2\sigma^{-2} = 0.75$)

n_1 \ n_2	6	7	8	9	10	11	12	13	14	15	C.S.
15	0	0.0005	0.0011	0.0032	0.0077	0.0143	0.0231	0.0336	0.0424	0.0528	0.1787
21	0	0	0.0005	0.0022	0.0070	0.0171	0.0301	0.0489	0.0655	0.0821	0.2534
25	0	0	0.0002	0.0016	0.0058	0.0169	0.0336	0.0591	0.0803	0.1008	0.2983
31	0	0	0.0001	0.0010	0.0048	0.0160	0.0373	0.0697	0.0998	0.1262	0.3549

n_2 \ n_1	16	17	18	19	20	21	22	23	24	25	C.S.
15	0.0611	0.0642	0.0678	0.0695	0.0690	0.0667	0.0600	0.0548	0.0496	0.0443	0.7857
21	0.0950	0.0951	0.0949	0.0899	0.0807	0.0699	0.0543	0.0326	0.0449	0.0255	0.9362
25	0.1147	0.1105	0.1052	0.0935	0.0780	0.0613	0.0435	0.0314	0.0217	0.0146	0.9727
31	0.1393	0.1262	0.1106	0.0911	0.0654	0.0449	0.0269	0.0166	0.0095	0.0053	0.9907

n_2 \ n_1	26	27	28	29	30	31	32	33	34	35	C.S.
15	0.0368	0.0319	0.0271	0.0224	0.0188	0.0153	0.0116	0.0096	0.0077	0.0060	0.9729
21	0.0181	0.0128	0.0092	0.0062	0.0043	0.0028	0.0017	0.0011	0.0008	0.0005	0.9937
25	0.0090	0.0056	0.0035	0.0021	0.0012	0.0007	0.0004	0.0002	0.0001	0	0.9955
31	0.0026	0.0007	0.0007	0.0003	0.0002	0.0001	0	0	0	0	0.9953

n_2 \ n_1	36	37	38	39	40	41	42	43	44	45	C.S.
15	0.0048	0.0038	0.0029	0.0023	0.0017	0.0014	0.0008	0.0007	0.0006	0.0003	0.9922
21	0.0002	0.0002	0.0001	0.0001	0	0					0.9943
25	0										—
31	0										—

n_2 \ n_1	46	47	48	49	50	51	52	53	54	55	T.S.
15	0.0004	0.0002	0.0002	0.0001	0.0001	0					0.9932
21											0.9943
25											0.9955
31											0.9953

Table III — (2) ($\alpha = 5\%$, $\beta = 1\%$; $d^2\sigma^{-2} = 0.5$)

n_2 / n_1	9	10	11	12	13	14	15	16	17	18	C.S.
21	0	0.0003	0.0010	0.0024	0.0055	0.0091	0.0163	0.0242	0.0322	0.0405	0.1315
25	0	0.0002	0.0007	0.0019	0.0050	0.0095	0.0175	0.0282	0.0382	0.0502	0.1514
31	0	0.0001	0.0004	0.0031	0.0041	0.0091	0.0188	0.0334	0.0469	0.0631	0.1772
n_2 / n_1	19	20	21	22	23	24	25	26	27	28	C.S.
21	0.0498	0.0576	0.0641	0.0651	0.0667	0.0670	0.0652	0.0596	0.0551	0.0502	0.7319
25	0.0615	0.0715	0.0786	0.0777	0.0776	0.0751	0.0701	0.0608	0.0534	0.0458	0.8235
31	0.0774	0.0905	0.0954	0.0924	0.0875	0.0800	0.0697	0.0549	0.0445	0.0348	0.9043
n_2 / n_1	29	30	31	32	33	34	35	86	37	38	C.S.
21	0.0450	0.0392	0.0339	0.0272	0.0229	0.0193	0.0157	0.0128	0.0104	0.0081	0.9694
25	0.0382	0.0314	0.0251	0.0185	0.0145	0.0112	0.0084	0.0064	0.0047	0.0033	0.9852
31	0.0260	0.0192	0.0136	0.0088	0.0062	0.0041	0.0028	0.0018	0.0011	0.0007	0.9886
n_2 / n_1	39	40	41	42	43	44	45	46	47	48	C.S.
21	0.0065	0.0040	0.0039	0.0026	0.0021	0.0016	0.0012	0.0009	0.0007	0.0005	0.9904
25	0.0024	0.0019	0.0009	0.0007	0.0005	0.0004	0.0002	0.0002	0.0001	0.0001	0.9926
31	0.0004	0.0003	0.0001	0.0001	0	0	0	0	0	0	0.9895
n_2 / n_1	49	50	51	52	53	54	55	56	57	59	T.S.
21	0.0003	0.0003	0.0002	0.0002	0.0001	0					0.9915
25	0	0									0.9926
31	0	0									0.9895

Table III — (3) ($\alpha = 5\%$, $\beta = 1\%$; $d^2/\sigma^2 = 0.25$)

n_2 / n_1	16	17	18	19	20	21	22	23	24	25	C.S.
31	0	0.0001	0.0003	0.0006	0.0012	0.0023	0.0036	0.0055	0.0083	0.0114	0.0333
n_2 / n_1	26	27	28	29	30	31	32	33	34	35	C.S.
31	0.0154	0.0194	0.0239	0.0285	0.0337	0.0384	0.0415	0.0440	0.0462	0.0478	0.3721
n_2 / n_1	36	37	38	39	40	41	42	43	44	45	C.S.
31	0.0491	0.0498	0.0496	0.0489	0.0475	0.0447	0.0384	0.0359	0.0332	0.0305	0.7997
n_2 / n_1	46	47	48	49	50	51	52	53	54	55	C.S.
31	0.0272	0.0245	0.0218	0.0193	0.0170	0.0144	0.0124	0.0107	0.0090	0.0088	0.9648
n_2 / n_1	56	57	58	59	60	61	62	63	64	65	T.S.
31	0.0050	0.0051	0.0044	0.0037	0.0028	0.9858					

Table IV — (2) ($\alpha = \beta = 5\%$; $d^2\sigma^{-2} = 0.5$)

n_1 \ n_2	3	4	5	6	7	8	9	10	11	12	C.S.
10			0	0.0002	0.0008	0.0016	0.0034	0.0063	0.0110	0.0159	0.0392
15				0	0.0002	0.0006	0.0021	0.0051	0.0104	0.0176	0.0360
21					0	0.0002	0.0011	0.0034	0.0087	0.0153	0.0287

n_1 \ n_2	13	14	15	16	17	18	19	20	21	22	C.S.
10	0.0207	0.0253	0.0315	0.0362	0.0390	0.0413	0.0433	0.0443	0.0450	0.0448	0.4106
15	0.0274	0.0355	0.0469	0.0557	0.0602	0.0642	0.0660	0.0657	0.0646	0.0588	0.5810
21	0.0362	0.0390	0.0615	0.0730	0.0823	0.0865	0.0862	0.0820	0.0752	0.0633	0.7139

n_1 \ n_2	23	24	25	26	27	28	29	30	31	32	C.S.
10	0.0439	0.0418	0.0397	0.0377	0.0358	0.0338	0.0315	0.0295	0.0271	0.0243	0.7557
15	0.0546	0.0496	0.0449	0.0383	0.0335	0.0288	0.0244	0.0207	0.0170	0.0140	0.9068
21	0.0538	0.0445	0.0352	0.0282	0.0213	0.0161	0.0119	0.0087	0.0062	0.0043	0.9439

n_1 \ n_2	33	34	35	36	37	38	39	40	41		T.S.
10	0.0246	0.0205	0.0186	0.0169	0.0153	0.0139	0.0126	0.0117			0.8898
15	0.0111	0.0092	0.0074	0.0059	0.0048	0.0038	0.0028	0.0025	0.0017		0.9560
21	0.0030	0.0021	0.0014	0.0010	0.0007	0.0004	0.0003	0.0002	0.0001		0.9531

Table IV — (3) $(\alpha = \beta = 5\%;\ d^2\sigma^{-2} = 0.25)$

n_1 \ n_2	8	9	10	11	12	13	14	15	16	17	C.S.
10		0	0.0001	0.0002	0.0006	0.0005	0.0011	0.0017	0.0024	0.0033	0.0099
15			0	0.0001	0.0002	0.0004	0.0007	0.0013	0.0021	0.0031	0.0079
21				0	0.0001	0.0002	0.0005	0.0010	0.0017	0.0026	0.0061

n_1 \ n_2	18	19	20	21	22	23	24	25	26	27	C.S.
10	0.0045	0.0060	0.0073	0.0092	0.0110	0.0129	0.0140	0.0150	0.0163	0.0177	0.1238
15	0.0045	0.0069	0.0092	0.0118	0.0142	0.0170	0.0200	0.0233	0.0252	0.0273	0.1673
21	0.0042	0.0067	0.0098	0.0137	0.0176	0.0223	0.0274	0.0308	0.0356	0.0383	0.2125

n_1 \ n_2	28	29	30	31	32	33	34	35	36	37	C.S.
10	0.0185	0.0196	0.0202	0.0210	0.0214	0.0216	0.0218	0.0220	0.0223	0.0224	0.3346
15	0.0296	0.0305	0.0314	0.0317	0.0324	0.0330	0.0333	0.0331	0.0330	0.0326	0.4879
21	0.0418	0.0430	0.0452	0.0459	0.0448	0.0435	0.0428	0.0414	0.0397	0.0375	0.6381

n_1 \ n_2	38	39	40	41	42	43	44	45	46	47	C.S.
10	0.0225	0.0224	0.0223	0.0220	0.0213	0.0208	0.0203	0.0201	0.0196	0.0193	0.5452
15	0.0320	0.0312	0.0307	0.0291	0.0270	0.0257	0.0234	0.0223	0.0211	0.0199	0.7503
21	0.0354	0.0330	0.0300	0.0270	0.0241	0.0217	0.0196	0.0174	0.0154	0.0138	0.8755

n_1 \ n_2	48	49	50	51	52	53	54	55	56	57	C.S.
10	0.0188	00.185	0.0180	0.0177	0.0170	0.0165	0.0160	0.0156	0.0150	0.0144	0.7127
15	0.0183	0.0172	0.0161	0.0150	0.0138	0.0125	0.0116	0.0107	0.0098	0.0089	0.8842
21	0.0122	0.0106	0.0092	00.081	0.0070	0.0060	0.0051	0.0043	0.0036	0.0030	0.9446

n_1 \ n_2	58	59	60	Total
10	0.0136	0.0133	0.0120	0.7516
15	0.0080	0.0070	0.0061	0.9053
21	0.0026	0.0022	0.0016	0.9510