Development of an Intelligent Robot for an Agricultural Production Ecosystem (II): Modeling of the Competition between Rice Plants and Weeds

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INTRODUCTION

The development of a robot for an artificial ecosystem of agricultural production demands understanding and modeling an agricultural ecosystem. Damoto et al. (2003) reported the Lotka–Volterra equations for the competitive relation between crop and weeds. In this study, we will predict the populations of rice plants, and superior weeds (Tamagayatsuri (smallflower umbrella sedge), and Azena (smallflower umbrella sedge)) in paddy field by using models of interactions among three species on the agricultural ecosystem of the rice farming system in order to develop an intelligent robot which executes the tasks of control of weeds and snails in the paddy (Figure 1). Complex ecosystems with many species interacting with each other nonlinearly tend to exhibit chaotic dynamics (Keeling et al., 2002; Tuda and Shimada, 2005; Vano, et al., 2006).

SPECIES IN COMPETITION IN THE AGRICULTURAL PRODUCTION ECOSYSTEM

Two species in competition: rice plants and weed

The equations in our models for rice plants and weed are as follows

\[
\frac{dN_i}{dt} = r_i N_i \left(1 - \frac{N_i + \alpha_{ij} N_j}{K_i} \right) \tag{1}
\]

\[
\frac{dN_j}{dt} = r_j N_j \left(1 - \frac{N_j + \alpha_{ji} N_i}{K_j} \right) \tag{2}
\]

where, \(N_i\): density or biomass of rice plants, \(N_j\): density or biomass of weed, \(r_i\): intrinsic rate of rice plants, \(r_j\): intrinsic rate of weeds, \(\alpha_{ij}\): weeds to rice plants competition coefficient, \(\alpha_{ji}\): rice plants to weeds competition coefficient, \(K_i\): rice plants capacity and \(K_j\): weeds carrying capacity.

Models of competition based on the Lotka–Volterra equation are introduced in this researching in order to develop an intelligent robot for a rice production ecosystem. These prediction equations are useful to estimate the competition between populations or biomasses of rice plants and weeds in different phases of the rice crop season and using this information about the superior plants, the robot will make decision about the appropriate timing for removing the snails in excess in paddy field; therefore snails remaining in the field can eat weed and rice plants will grow up with less competition.

To understand the competition dynamics ecologically we examine solutions at equilibrium analytically. The way to accomplish this is to set the two equations equal to zero and solve both for \(N_i\) as a function of \(N_j\) (Gotelli, 1998). The results are two equations for straight lines. These straight lines are called isoclines (Equations 3 and 4). An isocline represents combinations of \(N_i\) and \(N_j\) for which there is no net increase or decrease in population growth for each species (because \(dN/dt=0\)). Where the lines cross, growth rates are zero for both species.

\[
N_i = K_i - \alpha_{ij} N_j \tag{3}
\]

\[
N_j = K_j - \alpha_{ji} N_i \tag{4}
\]

Case 1: The rice plants isocline is above the weed isocline. In the region below of both isoclines, the populations and biomasses of weed and rice plants both increase. In the area of the chart between the two iso-
clines, the population of weed isoclines decreases where as population of rice plants increases. The black circle at this point represents a stable equilibrium. The conclusion is that the population of weed declines to zero and rice plants increases to its carrying capacity \((K)\). In this case rice plants have competitively excluded weed (Fig. 2a).

Case 2: Weed isocline is above rice plants isocline. In the region below of both isoclines, the populations and biomasses of rice plants and weed both increase. In the area of the chart between the two isoclines, the populations and biomasses of rice plants decrease whereas the populations and biomasses of weed continue to increase. The result is that the populations and biomasses of rice plants decline to zero and weed increases to its carrying capacity \((K)\). This result is that the populations and biomasses of rice plants decrease whereas the populations and biomasses of weed continue to increase. In this case rice plants have competitively excluded weed (Fig. 2a).

Case 3: The isoclines of the rice plant and weed cross one another. In this case the carrying capacity of rice plants \((K)\) is higher than the carrying capacity of weed divided by the competition coefficient \((K/\alpha)\), and the carrying capacity of weed \((K)\) is higher than the carrying capacity of rice plants divided by the competition coefficient \((K/\alpha)\). In the area below both rice plants and weed isoclines and above both rice plants and weed isoclines the populations and biomasses increase or decrease as in the first two cases, and there is an unstable equilibrium point where the rice plants and weed isoclines intersect. For the populations and biomasses above the weed isocline and below the rice plants isocline the result becomes same as in the first case: competitive exclusion of weed by rice plants. In the area above the rice plants isocline and below the weed isocline, the result is the same as in the second case: competitive exclusion of weed by rice plants. In this case, the result will depend on the initial abundances (Fig. 2d).

### Jacobian Matrix for rice plants and weed

If \(J(N_1, N_2)\) is a fixed point, we can use the equations 1 and 2 when growth rates are zero and then construct a Jacobian matrix.

\[
\begin{align*}
\frac{dN_1}{dt} &= r_1N_1 \left(1 - \frac{N_1 + \alpha_1 N_2}{K_1}\right) \\
&= 0 \\
\frac{dN_2}{dt} &= r_2N_2 \left(1 - \frac{N_2 + \alpha_2 N_1}{K_2}\right) \\
&= 0
\end{align*}
\]

Then we define the system of differential equations using the equations 5 and 6.

\[
J(N_1, N_2) = \begin{bmatrix} \frac{\partial(Eq5)}{\partial N_1} & \frac{\partial(Eq5)}{\partial N_2} \\ \frac{\partial(Eq6)}{\partial N_1} & \frac{\partial(Eq6)}{\partial N_2} \end{bmatrix}
\]

And we do linearization in order to find the Jacobian of the vector function of the nonlinear system. We get the rendered general Jacobian matrix for rice plants and weed in competition as follows,

\[
J(N_1, N_2) = \begin{bmatrix} r_1 - 2 \frac{r_2}{K_1} N_1 - \frac{\alpha_1 r_1}{K_1} N_2 & \frac{\alpha_1 r_1}{K_1} N_1 \\ \frac{\alpha_2 r_2}{K_2} N_2 & r_2 - 2 \frac{r_1}{K_2} N_2 - \frac{\alpha_2 r_2}{K_2} N_1 \end{bmatrix}
\]

Using the isoclines of the equations 3 and 4, we can know the general stationary point \(P_1\) and \(P_2\)

\[
P_1 = (K_1 - \alpha_2 N_2, K_2 - \alpha_1 N_1)
\]

\[
P_2 = (K_1 - N_1, K_2 - N_2)
\]

We could analyze the stability of the system by through the evaluation of the Jacobian matrix in each
fixed points and find the eigenvalues and eigenvectors. An eigenvalue of a square matrix is a scalar ($\lambda$) and the points attracted are negative eigenvalue and the points repelled are positive eigenvalues. An eigenvector is an axis of attraction. If the eigenvalues have negative real parts, the fixed point is asymptotically stable (attractor). If at least one eigenvalue has positive real part, the fixed point is unstable (repeller). If eigenvalues are pure imaginary, the fixed point could be stable or unstable.

**Three species in competition: rice plants and weeds (Tamagayatsuri and Azena)**

Lotka–Volterra–type competition models that involve three superior species (rice plants, Tamagayatsuri and Azena) will have the following equations:

Equation for population growth of rice plants.

$$\frac{dN_1}{dt} = r_1 N_1 \left(1 - \frac{N_1 + \alpha_{12}N_2 + \alpha_{13}N_3}{K_1}\right)$$ (8)

Equation for population growth of Tamagayatsuri.

$$\frac{dN_2}{dt} = r_2 N_2 \left(1 - \frac{N_2 + \alpha_{21}N_1 + \alpha_{23}N_3}{K_2}\right)$$ (9)

Equation for population growth of Azena.

$$\frac{dN_3}{dt} = r_3 N_3 \left(1 - \frac{N_3 + \alpha_{31}N_1 + \alpha_{32}N_2}{K_3}\right)$$ (10)

where $N_i$: density or biomass of rice plant biomass, $N_j$: density or biomass of Tamagayatsuri, $N_k$: density or biomass of Azena, $r_i$: intrinsic rate of rice plants, $r_j$: intrinsic rate of Tamagayatsuri, $r_k$: intrinsic rate of Azena, $\alpha_{ij}$: Tamagayatsuri to rice plants competition coefficient, $\alpha_{ik}$: Azena to rice plants competition coefficient, $\alpha_{jk}$: rice plants to Tamagayatsuri competition coefficient, $\alpha_{ik}$: Azena to Tamagayatsuri competition coefficient, $\alpha_{jk}$: rice plants to Azena competition coefficient, $\alpha_{ik}$: Tamagayatsuri to Azena competition coefficient, $K_i$: rice plants carrying capacity, $K_j$: Tamagayatsuri carrying capacity and $K_k$: Azena carrying capacity. The populations of the superior weeds, Tamagayatsuri and Azena were 750 and 496, respectively in a lot of 50 m$^2$ with a population of 750 rice plants and practicing organic agriculture in Kyushu University Farm on August of 1996 (Table 1). A chart with the three species (rice plants, Tamagayatsuri and Azena) in competition after transplanting on June 20th, 2006, in an area of 50 m$^2$ is presented in Fig. 3. In order to make the chart, we coded and ran a program in Matlab and solved the ordinary differential equation system by the numerical method of Runge–Kutta. The data considered were: $r_1 = 0.15$, $r_2 = 0.20$, $r_3 = 0.15$, $\alpha_{12} = 0.06$, $\alpha_{21} = 0.06$, $\alpha_{13} = 0.07$, $\alpha_{31} = 0.08$, $\alpha_{23} = 0.07$, $K_1 = 750$, $K_2 = 500$ and $K_3 = 200$. The initial conditions for the three superior species in the agricultural production ecosystem were as follows $N_i = 750$, $N_j = 1$ and $N_k = 1$. We can also see from the Fig. 3 that after 20 days after transplanting of rice seedlings, the populations of Azena and Tamagayatsuri and rice plants are increasing. In the case of rice plants there is minor error due we used Lotka–Volterra to model the competition among them. The population of rice plants should be almost constant over the crop season. The populations of the three species in the same plot on July 30th, 2006 (forty days after transplanting) were as follows, $N_i = 742$, $N_j = 476$ and $N_k = 38$ and the populations of three superior species became stable after 60 days after transplanting. The general isoclines for three species in competition are as follows.

Isocline 1.

$$N_i = K_i - \alpha_{12}N_2 - \alpha_{13}N_3$$ (11)

Isocline 2.

$$N_j = K_j - \alpha_{21}N_1 - \alpha_{23}N_3$$ (12)

Isocline 3.

$$N_k = K_k - \alpha_{31}N_1 - \alpha_{32}N_2$$ (13)

**Table 1.** Results of researching on kind and population of superior weeds in lowland paddy field at Kyushu University farm on August 12th, 1996

<table>
<thead>
<tr>
<th>Experiment Site</th>
<th>Tamagayatsuri</th>
<th>Azena</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Organic Agriculture</td>
<td>496</td>
<td>38</td>
</tr>
<tr>
<td>b. Habitual Practice (Herbicide)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Tamagayatsuri: smallflower umbrella sedge (Cyperus difformis L.)

Azena: common false pimpernel (Lindernia procumbens (Krock.) Borbas = [Lindernia pyxidaria L.])

The weeds were researched on August 12th, 1996 in an experimental site of 50 square meters.

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**Fig. 3.** Populations of three superior species in competition (rice plants, Tamagayatsuri and Azena) after transplanting of rice seedlings in a lot of 50 m$^2$ in Kyushu University Farm on June 20th, 2006.
The Fig. 4 shows us the three isoclines of the competition model among three superior species (rice plants, Tamagayatsuri and Azena). In Lotka–Volterra model the three species have same populations and coexist in an equilibrium point, \( P_e \) in natural ecosystems. However, in the agricultural production ecosystem of paddy, the period of the crop season is much shorter and farmers do several farm works such as: irrigation, remove of weeds and snails, so the \( P_e \) is not reached. The Fig. 5 shows the directions of vectors fields of the three species in the competition model in the agricultural production ecosystem of paddy. A phase portrait between rice plants and Tamagayatsuri is showed in the Fig. 6, whereas the Fig. 7 shows us a phase portrait among three species in competition (rice plants, Tamagayatsuri and Azena).

**Jacobian Matrix for rice plants, Tamagayatsuri and Azena**

If \( J(N_1, N_2, N_3) \) is a fixed point, we can use the equations 8, 9, and 10 when growth rates are zero and then construct a Jacobian matrix.

\[
\begin{align*}
\frac{dN_1}{dt} &= r_1N_1\left(1 - \frac{N_1 + \alpha_{12}N_2 + \alpha_{13}N_3}{K_1}\right) \\
0 &= r_1N_1 - \frac{r_1N_1^2}{K_1} - \frac{\alpha_{12}r_1N_1N_2}{K_1} - \frac{\alpha_{13}r_1N_1N_3}{K_1} \quad (14) \\
\frac{dN_2}{dt} &= r_2N_2\left(1 - \frac{N_2 + \alpha_{21}N_1 + \alpha_{23}N_3}{K_2}\right) \\
0 &= r_2N_2 - \frac{r_2N_2^2}{K_2} - \frac{\alpha_{21}r_2N_1N_2}{K_2} - \frac{\alpha_{23}r_2N_2N_3}{K_2} \quad (15) \\
\frac{dN_3}{dt} &= r_3N_3\left(1 - \frac{N_3 + \alpha_{31}N_1 + \alpha_{32}N_2}{K_3}\right) \\
0 &= r_3N_3 - \frac{r_3N_3^2}{K_3} - \frac{\alpha_{31}r_3N_1N_3}{K_3} - \frac{\alpha_{32}r_3N_2N_3}{K_3} \quad (16)
\end{align*}
\]

Then we define the system of differential equations using the equations 14, 15 and 16.

\[
\begin{pmatrix}
\frac{\partial (Eq 14)}{\partial N_1} & \frac{\partial (Eq 14)}{\partial N_2} & \frac{\partial (Eq 14)}{\partial N_3} \\
\frac{\partial (Eq 15)}{\partial N_1} & \frac{\partial (Eq 15)}{\partial N_2} & \frac{\partial (Eq 15)}{\partial N_3} \\
\frac{\partial (Eq 16)}{\partial N_1} & \frac{\partial (Eq 16)}{\partial N_2} & \frac{\partial (Eq 16)}{\partial N_3}
\end{pmatrix}
\]

And we do linearization in order to find the Jacobian of the vector function of the nonlinear system. We get the rendered general Jacobian matrix for three species in competition as follows,

\[
J(N_1, N_2, N_3) =
\begin{pmatrix}
\alpha_{11} - \frac{\alpha_{12}r_1}{K_1}N_1 - \frac{\alpha_{13}r_1}{K_1}N_3 & -\frac{\alpha_{12}r_1}{K_1}N_2 & 0 \\
-\frac{\alpha_{21}r_2}{K_2}N_1 - \frac{\alpha_{23}r_2}{K_2}N_3 & b_{22} - \frac{\alpha_{23}r_2}{K_2}N_2 & 0 \\
-\frac{\alpha_{31}r_3}{K_3}N_1 - \frac{\alpha_{32}r_3}{K_3}N_2 & 0 & c_{33}
\end{pmatrix}
\]  
(17)
where

\[ a_{11} = r_1 - 2 \frac{\nu_i}{\kappa_1} N_i - \alpha_{12} \frac{\nu_i}{\kappa_1} N_i - \alpha_{13} \frac{\nu_i}{\kappa_1} N_i \]

\[ b_{23} = r_2 - 2 \frac{\nu_i}{\kappa_2} N_i - \alpha_{23} \frac{\nu_i}{\kappa_2} N_i - \alpha_{23} \frac{\nu_i}{\kappa_2} N_i \]

\[ c_{33} = r_3 - 2 \frac{\nu_i}{\kappa_3} N_i - \alpha_{33} \frac{\nu_i}{\kappa_3} N_i - \alpha_{33} \frac{\nu_i}{\kappa_3} N_i \]

Using the isoclines of the equations 11, 12 and 13 we can make the general equation of the stationary points \( P_i \) and \( P_j \)

\[ P_j (K_i - \alpha_{13} N_i - \alpha_{13} N_j, K_j - \alpha_{23} N_i - \alpha_{23} N_j, K_j - \alpha_{33} N_i - \alpha_{33} N_j) \]  \hspace{1cm} (18)

![Phase portrait between three species in competition (rice plants, Tamagayatsuri and Azena).](image)

To analyze the stability of the agricultural production ecosystem, we should evaluate the Jacobian matrix for rice plants, Azena, and Tamagayatsuri in each fixed point and obtain the eigenvalues and eigenvectors. The following is the analysis of the agricultural production ecosystem considering the stationary point \( P_i \) of the equation 18 and the Jacobian matrix of the equation 17

\[ J(P_i) = \begin{bmatrix}
  \alpha_{11} & -\alpha_{12} & -\alpha_{13} \\
  -b_{12} & b_{22} & -b_{23} \\
  -c_{31} & -c_{31} & c_{33}
\end{bmatrix} \]

where

\[ a_{12} = \alpha_{12} \frac{\nu_i}{\kappa_1} (K_i - \alpha_{13} N_i - \alpha_{13} N_j) \]

\[ a_{13} = \alpha_{13} \frac{\nu_i}{\kappa_1} (K_i - \alpha_{13} N_i - \alpha_{13} N_j) \]

\[ b_{23} = \alpha_{23} \frac{\nu_i}{\kappa_2} (K_i - \alpha_{23} N_i - \alpha_{23} N_j) \]

\[ c_{33} = \alpha_{33} \frac{\nu_i}{\kappa_3} (K_i - \alpha_{33} N_i - \alpha_{33} N_j) \]

The characteristic equation is given by

\[ \det([A] - \lambda[I]) = 0 \]

If \([A]\) is a \(n \times n\) matrix, then \([X] \neq 0\) is an eigenvector of \([A]\) if \([A][X] = \lambda[X]\) where \(\lambda\) is a scalar and \([X] \neq 0\). The scalar \(\lambda\) is called the eigenvalue of \([A]\) and \([X]\) is called the eigenvector corresponding to the eigenvalue \(\lambda\).
\[ k_3 = a_{11}b_{22}c_{33} - a_{11}b_{23}c_{31} - a_{12}b_{12}c_{33} - a_{12}b_{23}c_{31} - a_{13}b_{12}c_{32} - a_{13}b_{22}c_{31} \]

We get the following equation

\[ -\lambda^3 + (k_1)\lambda^2 - (k_2)\lambda + k_3 = 0 \quad (21) \]

The equation 21 has three cubic roots, which are the eigenvalues. If we also consider the following data: \( N_1 = 742, N_2 = 1, N_3 = 1, r_1 = 0.15, r_2 = 0.2, r_3 = 0.15, a_{11} = 0.06, a_{12} = 0.08, a_{13} = 0.05, a_{21} = 0.06, a_{22} = 0.07, a_{23} = 0.08, a_{31} = 0.07, a_{32} = 0.07, K_1 = 750, K_2 = 500 \) and \( K_3 = 200 \), we can get the Jacobian Matrix as follows,

\[
J(P_1) = \begin{bmatrix}
-2.85 & -0.009 & -0.012 \\
-0.00024 & 0.18 & -0.000028 \\
-0.00006 & -0.000052 & 0.089
\end{bmatrix}
\]

We evaluated the Jacobian matrix in the point \( P_1 (750, 455, 141) \) and got the following eigenvalues: \( \lambda_1 = -2.85, \lambda_2 = 0.18 \) and \( \lambda_3 = 0.89 \), therefore \( P_1 \) is unstable.

**DISCUSSION**

The farm works such as: tillage, paddling, transplanting and irrigations produce different initial conditions of the populations of rice plants and weeds such as Tamagayatsuri and Azena. The models, as an integral part of the development of an intelligent robot for an agricultural production ecosystem, estimate quantitatively the populations or biomasses of superior species over the time of the crop season. The prediction equations or models generated will be introduced into the Jacobian Matrix as follows,

\[
J(P_1) = \begin{bmatrix}
-2.85 & -0.009 & -0.012 \\
-0.00024 & 0.18 & -0.000028 \\
-0.00006 & -0.000052 & 0.089
\end{bmatrix}
\]

We evaluated the Jacobian matrix in the point \( P_1 (750, 455, 141) \) and got the following eigenvalues: \( \lambda_1 = -2.85, \lambda_2 = 0.18 \) and \( \lambda_3 = 0.89 \), therefore \( P_1 \) is unstable.

**CONCLUSION**

The stability of the competition among these three superior plants of rice production ecosystem is predicted by through of the eigenvalues of fixed points considering different farm works or phases of paddy. From our analysis of the competition among three superior plants (rice plants, Tamagayatsuri and Azena) without predation by golden apple snail, we can predict they coexist at a stable equilibrium point. It means the system is not chaotic but stable. The models generated will be introduced into the agricultural production ecosystem robot; therefore the robot can make decisions about the number of snails to remove from paddy. It is also necessary to consider factors, such as temperature, light and water depth dependency, which influences the snail’s activity.

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