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Shimojo, Masataka
Laboratory of Animal Feed Science, Division of Animal Science, Department of Animal and Marine Bioresource Sciences, Faculty of Agriculture, Kyushu University

Shao, Tao

Masuda, Yasuhisa
Emeritus Professor, Kyushu University

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Introducing Viewpoints of Mechanics into Basic Growth Analysis – (Ⅶ) Mathematical Properties of Basic Growth Mechanics in Ruminant –

Masataka SHIMOJO*, Tao SHAO and Yasuhisa MASUDA[1]

Laboratory of Animal Feed Science, Division of Animal Science, Department of Animal and Marine Bioresource Sciences, Faculty of Agriculture, Kyushu University, Fukuoka 812–8581, Japan and Department of Grassland and Forage Science, College of Animal Sciences and Technology, Nanjing Agricultural University, Weigang 1, Nanjing 210095, the People's Republic of China

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This study was conducted to investigate mathematical properties of basic growth mechanics in ruminant by introducing newly developed viewpoints into mathematical operations of basic growth function. The negative sign, which appeared naturally by taking the square root of the differential equation based on basic growth mechanics, gave mathematical contradictions to the differential principle. In the process of correcting those contradictions, viewpoints of interest were newly introduced to give a deeper understanding of basic growth mechanics. These new viewpoints mathematically suggested the metabolic turnover of body components influencing increase or decrease of body weight and the dynamic equilibrium hidden behind the constant body weight. Conditional weak analogies between basic growth mechanics and Newton's three laws of motion were discussed more deeply, where problems with basic growth mechanics were shown. A conditional weak analogy to Newton's law of universal gravitation was also suggested by assuming a virtual force of attraction between the ruminant and the forage. However, the problem with this virtual force showed that there was not a concept of distance, nor was the effect on the virtual force of attracting even when there was a forced introduction of distance. It was suggested that investigating mathematical properties of basic growth mechanics gave conditional weak analogies to laws of motion developed by Newton.

INTRODUCTION

The growth analysis of a ruminant animal has a mathematical aspect of investigation, because integral and differential operations of growth functions are conducted in order to give analytic factors explaining how the ruminant animal grows. It was six years ago when we noticed the flavor of basic growth mechanics from symmetric properties of the basic growth function in its differentiation (Shimojo *et al.*, 2002b). This basic growth function described using the exponential function with base e had already originated from the basic growth analysis of the ruminant (Brody, 1945). By consulting a textbook on mechanics of motion (Kawabe, 2006), we have compared basic growth mechanics with Newton's three laws of motion (Shimojo *et al.*, 2006; Shimojo, 2007a; Shimojo *et al.*, 2007b, 2007c), which suggested an analogy between them. In November 2007 at Nanjing Agricultural University (the People's Republic of China), the first author of this paper talked about the suggested analogy on the translation into Chinese by the second author. In that talking, we were asked to give more detailed explanation to the analogy between basic growth mechanics and laws of motion. Our attempts so far show that there are things left unnoticed, lack of explanation, insufficient discussion and misunderstanding about mathematical operations of basic growth function.

[1] Emeritus Professor of Kyushu University
* Corresponding Author (E–mail: mshimojo@agr.kyushu–u.ac.jp)

The present study was designed to investigate mathematical properties of basic growth mechanics in ruminant by introducing newly developed viewpoints into mathematical operations of basic growth function.

MATHEMATICAL PROPERTIES OF BASIC GROWTH MECHANICS IN RUMINANT

In the present study mathematical operations are shown first, and then they are followed by their interpretation from the viewpoint of ruminant agriculture.

(A) Basic growth mechanics

Applying a series of calculations, mainly differentiation, to basic growth analysis leads to basic growth mechanics.

$$(1/W) \cdot (dW/dt) = \mathrm{RGR}, \tag{1}$$

where W = weight, t = time, RGR = relative growth rate. RGR in equation (1) takes positive, negative or zero value from the viewpoint of mathematics, unless what value RGR should take is designated.

The indefinite integral of equation (1) and determining the integration constant gives

$$W = W_0 \cdot \exp(\mathrm{RGR} \cdot t), \tag{2}$$

where W_0 = the weight at $t = 0$.
The derivative of W gives absolute growth rate (AGR),

$$\begin{aligned} dW/dt &= \mathrm{AGR} \\ &= \mathrm{RGR} \cdot W_0 \cdot \exp(\mathrm{RGR} \cdot t). \end{aligned} \tag{3}$$

The second derivative of W gives growth acceleration

(GA),

$$d^2W/dt^2 = \text{GA} = (\text{RGR})^2 \cdot W_0 \cdot \exp(\text{RGR} \cdot t). \quad (4)$$

Relating equations (2), (3) and (4) gives

$$\frac{dW/dt}{W} = \frac{d^2W/dt^2}{dW/dt} = \text{RGR}. \quad (5)$$

Thus,

$$(dW/dt)^2 = W \cdot (d^2W/dt^2). \quad (6)$$

Equation (6) takes a form of differential equation that is essential to basic growth mechanics (Shimojo *et al.*, 2007c).

The square root of equation (6) is given by

$$dW/dt = \pm\sqrt{W \cdot (d^2W/dt^2)}. \quad (7)$$

Inserting equations (2) and (4) into the right–hand side of equation (7) leads to

$$\begin{aligned} dW/dt &= \pm\sqrt{(W_0 \cdot \exp(\text{RGR} \cdot t)) \cdot ((\text{RGR})^2 \cdot W_0 \cdot \exp(\text{RGR} \cdot t))} \\ &= \pm\sqrt{W_0^2 \cdot (\text{RGR})^2 \cdot (\exp(\text{RGR} \cdot t))^2} \\ &= \pm(W_0) \cdot (\text{RGR}) \cdot (\exp(\text{RGR} \cdot t)). \end{aligned} \quad (8)$$

In our previous report (Shimojo *et al.*, 2006) positive and negative signs (±) were given to each of the first and second terms in the right–hand side of equation (8), because we assumed that the equation in which the third term was given signs was the same as the equation with the first term given signs. In this paper, however, we will also give signs to the third term in order to investigate what will occur mathematically.

(B) Giving positive and negative signs to each of three terms in equation (8)

Signs to the first term is given by

$$dW/dt = (\pm W_0) \cdot (\text{RGR}) \cdot (\exp(\text{RGR} \cdot t)). \quad (9)$$

If equation (9) exists, then this requires the following modification of equation (2) in order to conserve the differential principles,

$$W = (\pm W_0) \cdot \exp(\text{RGR} \cdot t). \quad (10)$$

Equation (10) shows that wherever there is a positive weight (W_0), there is a corresponding negative weight ($-W_0$), a concept of energy conservation.

Signs to the second term is given by

$$dW/dt = (W_0) \cdot (\pm\text{RGR}) \cdot (\exp(\text{RGR} \cdot t)). \quad (11)$$

The contradiction observed in equation (11) will be corrected by introducing equation (12),

$$W = W_0 \cdot \exp((\pm\text{RGR}) \cdot t), \quad (12)$$

and thus its derivative gives

$$dW/dt = (W_0) \cdot (\pm\text{RGR}) \cdot (\exp(\pm\text{RGR} \cdot t)). \quad (13)$$

Equation (12) shows that there is not only a positive rate of relative growth (RGR) but also a corresponding negative rate of relative growth (–RGR).

Signs to the third term is given by

$$dW/dt = (W_0) \cdot (\text{RGR}) \cdot (\pm\exp(\text{RGR} \cdot t)). \quad (14)$$

If equation (14) exists, then this requires modifying equation (2) as follows,

$$W = W_0 \cdot (\pm\exp(\text{RGR} \cdot t)). \quad (15)$$

Equation (15) shows that wherever there is a gain [$\exp(\text{RGR} \cdot t)$], there is a corresponding loss [$-\exp(\text{RGR} \cdot t)$]. In the present report, we will pay attention to the difference between equations (15) and (10).

(C) Equations derived from combining equations (10), (12) and (15)

Combining equations (10), (12) and (15) gives the following eight equations.

$$W_1 = W_0 \cdot \exp(\text{RGR} \cdot t), \quad (16\text{–}1)$$

$$-W_1 = (-W_0) \cdot \exp(\text{RGR} \cdot t). \quad (16\text{–}2)$$

$$W_2 = W_0 \cdot (-\exp(\text{RGR} \cdot t)), \quad (17\text{–}1)$$

$$-W_2 = (-W_0) \cdot (-\exp(\text{RGR} \cdot t)). \quad (17\text{–}2)$$

$$W_3 = W_0 \cdot \exp((-\text{RGR}) \cdot t), \quad (18\text{–}1)$$

$$-W_3 = (-W_0) \cdot \exp((-\text{RGR}) \cdot t). \quad (18\text{–}2)$$

$$W_4 = W_0 \cdot (-\exp((-\text{RGR}) \cdot t)), \quad (19\text{–}1)$$

$$-W_4 = (-W_0) \cdot (-\exp((-\text{RGR}) \cdot t)). \quad (19\text{–}2)$$

These include four more equations (17–1), (17–2), (19–1) and (19–2) in addition to those reported by Shimojo *et al.* (2006).

(D) Interpreting relationships between equations (16–1) and (16–2)

Equations (16–1) and (16–2) form a symmetric pair (W_1 versus $-W_1$), an inseparable relationship from the mathematical viewpoint. This phenomenon will be applied to the forage–based ruminant agriculture. If W_1 in equation (16–1) shows the ruminant body weight, then $-W_1$ in equation (16–2) is interpreted as the forage whose volume is expressed as the ruminant body weight (W_1), not the actual weight of the forage. The negative sign of $-W_1$ shows the situation of the forage that is harvested from the field for the consumption by the ruminant. The phenomena over the interval t_0 to t_1 are given by

$$W_1 - W_0 = \text{increase in ruminant body weight}, \quad (16\text{–}3)$$

$$-W_1 - (-W_0) = \text{forage consumption expressed as ruminant body weight increase}. \quad (16\text{–}4)$$

Introducing feed conversion ratio (FCR: the weight of the feed eaten divided by the body weight gain) into equation (16–4) gives the actual forage weight used to increase the ruminant body weight,

$$(-W_1 - (-W_0)) \cdot (\text{FCR}) = \text{actual forage weight used to increase ruminant body weight}. \quad (16\text{–}5)$$

Equation (16–3) and equation (16–4) are described mathematically on the positive side and the negative side of the same coordinate axes, respectively. Therefore, there will be an interpretation from the viewpoint of ruminant agriculture that both the ruminant and the forage are related with the same field. This is considered to be one of the mathematical proofs of field–forage–ruminant relationships, an issue of importance to the self–supporting ruminant agriculture. In Japan, however, there are many cases where cattle are fed feeds imported from foreign countries. Might this feed import, if mathematically described, be related to translations along coordinate axes or coordinate planes?

The analytic method taken in this paper to relate the ruminant with the forage is different from the two methods suggested by us previously, where the concept of intersection of ruminant production and forage production was introduced (Shimojo *et al.*, 2002a) and the complex representation using Euler's formula was introduced (Shimojo *et al.*, 2003a, 2003b) in order to show field–forage–ruminant relationships.

(E) Relationships between equations (17) and equations (16)

Although attention is paid to the difference between equations (17) and (16), equation (17–1) leads consequently to the same form as that of equation (16–2), and likewise equation (17–2) leads to equation (16–1). In these processes there are mathematical transformations: $W_0 \rightarrow -W_0$ in the former case, $-W_0 \rightarrow W_0$ in the latter case. They are mediated by

$$-\exp(\mathrm{RGR} \cdot t). \quad (20)$$

It is the negative sign of equation (20) that gives mutual transformations between W_0 and $-W_0$. In the forage–based ruminant agriculture, components of the ruminant body (W_0) are originally reduced to the forage ($-W_0$), and the forage ($-W_0$) is transformed into components of the ruminant body (W_0). Therefore, equation (20) plays an essential role in inseparable relationships between the ruminant and the forage.

(F) Interpretation of equations (18) and (19)

Equations (18) and (19) are the case of the decrease in ruminant body weight and the corresponding forage consumption expressed as the ruminant body weight decrease.

(G) Mathematical operation of time reversal problem

The main object of the present study is to investigate how the negative sign is interpreted when given to various factors analyzing growth phenomena. There is one more factor that will be given a negative sign, namely time (t). In this paper, the negative sign is given to t in both equations (16–1) and (18–1) in order to investigate mathematical operations of the time reversal problem.

$$W_1 = W_0 \cdot \exp(\mathrm{RGR} \cdot t), \quad (16\text{–}1)$$

$${}_1W = W_0 \cdot \exp(\mathrm{RGR} \cdot (-t)). \quad (16\text{–}1\text{–}1)$$

$$W_3 = W_0 \cdot \exp((-\mathrm{RGR}) \cdot t), \quad (18\text{–}1)$$

$${}_3W = W_0 \cdot \exp((-\mathrm{RGR}) \cdot (-t)). \quad (18\text{–}1\text{–}1)$$

The time reversal equations (16–1–1) and (18–1–1), since they are prohibited from appearing, should be corrected in order to recover equations (16–1) and (18–1), respectively. We will try to make RGR absorb the negative sign from $-t$ in order to correct the time reversal problem. Thus,

$$\begin{aligned} {}_1W &= W_0 \cdot \exp(\mathrm{RGR} \cdot (-t)) \\ &= W_0 \cdot \exp((-\mathrm{RGR}) \cdot t), \end{aligned}$$

and then, $-$RGR should also be corrected as follows to recover equation (16–1),

$$\begin{aligned} & W_0 \cdot \exp((-\mathrm{RGR}) \cdot t) \\ & \rightarrow W_0 \cdot \exp((-\mathrm{RGR}) \cdot t) \cdot \exp(2\mathrm{RGR} \cdot t) \\ & \qquad = W_0 \cdot \exp((-\mathrm{RGR} + 2\mathrm{RGR}) \cdot t) \\ & \qquad = W_0 \cdot \exp(\mathrm{RGR} \cdot t) \\ & \qquad = W_1. \quad (16\text{–}1) \end{aligned}$$

In the case of equation (18–1–1), likewise, recovering equation (18–1) requires the following mathematical operations,

$$\begin{aligned} {}_3W &= W_0 \cdot \exp((-\mathrm{RGR}) \cdot (-t)) \\ &= W_0 \cdot \exp((-(-\mathrm{RGR})) \cdot t) \\ &= W_0 \cdot \exp(\mathrm{RGR} \cdot t) \\ & \rightarrow W_0 \cdot \exp(\mathrm{RGR} \cdot t) \cdot \exp((-2\mathrm{RGR}) \cdot t) \\ & \qquad = W_0 \cdot \exp((\mathrm{RGR} - 2\mathrm{RGR}) \cdot t) \\ & \qquad = W_0 \cdot \exp((-\mathrm{RGR}) \cdot t) \\ & \qquad = W_3. \quad (18\text{–}1) \end{aligned}$$

These mathematical operations make us notice the following two phenomena that will occur. On the recovering way to equation (16–1) from equation (16–1–1) there occurs a calculation for correction ($-$RGR + 2RGR = RGR), a new introduction of 2RGR to absorb $-$RGR. This suggests a phenomenon that the ruminant body weight increases when the synthesis of body components (2RGR) exceeds the degradation of them ($-$RGR). On the recovery way to equation (18–1) from equation (18–1–1) there is a corrective calculation (RGR $-$ 2RGR = $-$RGR), where $-$2RGR is newly introduced in order to absorb RGR. This suggests that there is a decrease in ruminant body weight when the degradation of body components ($-$2RGR) exceeds the synthesis of them (RGR). However, these two estimates are far from the accuracy and might be slightly improved if modified as follows,

$$\begin{aligned} -\mathrm{RGR} + 2\mathrm{RGR} &= -\alpha \cdot \mathrm{RGR} + \beta \cdot \mathrm{RGR} \\ &= (-\alpha + \beta) \cdot \mathrm{RGR} \\ &= \mathrm{RGR}, \quad (16\text{–}1\text{–}2) \end{aligned}$$

where $-\alpha$ = degradation coefficient, β= synthesis coefficient, $-\alpha + \beta = 1$, $\alpha < \beta$.

$$\begin{aligned} \text{RGR} - 2\text{RGR} &= \gamma \cdot \text{RGR} - \delta \cdot \text{RGR} \\ &= (\gamma - \delta)\text{RGR} \\ &= -\text{RGR}, \end{aligned} \tag{18–1–2}$$

where γ = synthesis coefficient, $-\delta$ = degradation coefficient, $\gamma - \delta = -1$, $\gamma < \delta$.

From the mathematical viewpoint, these phenomena result from conserving the form of basic growth function by correcting the time reversal problem. However, these mathematical operations imply the metabolic turnover (Kleiber, 1975; Schmidt–Nielsen, 1984) of body components of the ruminant, despite inappropriate approaches from the viewpoint of ruminant nutrition.

(H) Dynamic equilibrium occurring in body weight when RGR = 0

In sections (A)～(G) we have investigated mathematical properties of basic growth mechanics in the case of RGR ≠ 0. In this section (H) we will investigate dynamic equilibrium that occurs in body weight when RGR = 0.

Equation (12) is used here instead of equation (10) for the simple investigation, and thus inserting RGR = 0 into equation (12) gives

$$\begin{aligned} W &= W_0 \cdot \exp((\pm\text{RGR}) \cdot t) \\ &= W_0 \cdot (\exp(0 \cdot t)) \\ &= W_0 \cdot 1 \\ &= W_0. \end{aligned} \tag{21}$$

Equation (21) shows that W_0 is kept constant with the passage of time owing to RGR = 0. However, this is an apparent phenomenon, because there is a feed consumption by the ruminant even when its body weight is kept constant. Therefore, what will occur when the ruminant eats nothing should be investigated. If there is no feed consumption, then there is a decrease in body weight. The description of body weight decrease is given, for example, by –RGR as shown in equation (18–1),

$$W_3 = W_0 \cdot \exp((-\text{RGR}) \cdot t). \tag{18–1}$$

Correcting equation (18–1) to recover equation (21) requires the following mathematical operation,

$$\begin{aligned} W_3 &= W_0 \cdot \exp((-\text{RGR}) \cdot t) \\ &\rightarrow W_0 \cdot \exp((-\text{RGR}) \cdot t) \cdot \exp(\text{RGR} \cdot t) \\ &= W_0 \cdot \exp((-\text{RGR} + \text{RGR}) \cdot t) \\ &= W_0 \cdot \exp(0 \cdot t) \\ &= W_0 \end{aligned} \tag{21}$$

On the recovering way to equation (21) from equation (18–1), there occurs a corrective calculation (–RGR + RGR = 0), where new RGR is introduced to absorb –RGR. This suggests that the body weigh is kept constant when the degradation of body components is compensated by the synthesis of them through feed consumption by the ruminant. This estimate might be slightly improved by the following modification,

$$\begin{aligned} -\text{RGR} + \text{RGR} &= -\varepsilon \cdot \text{RGR} + \varepsilon \cdot \text{RGR} \\ &= (-\varepsilon + \varepsilon) \cdot \text{RGR} \\ &= 0, \end{aligned} \tag{21-1}$$

where $-\varepsilon$ = degradation coefficient, ε = synthesis coefficient, $-\varepsilon + \varepsilon = 0$.

The present mathematical operation does not seem to be far from the nutritional approach, when compared with the case in section (G) in which correcting the time reversal problem implies relationships between degradation and synthesis of body components. Compensating the degradation of body components by consuming maintenance requirements keeps the body weight constant under a dynamic equilibrium. It is well known that the metabolic turnover of protein is of great importance to maintaining the homeostasis of animal body including its weight (Nagata, 2008).

(I) Suggested analogies to laws of motion

We suggested in our previous report (Shimojo *et al.*, 2006) that there were analogies between basic growth mechanics and Newton's three laws of motion. In the present study, we will take up this issue again, because there are newly develped viewpoints in order to investigate suggested analogies more deeply. We choose the following four equations for basic growth mechanics that have been taken up in this paper.

$$W = W_0. \tag{21}$$

$$(dW/dt)^2 = W \cdot (d^2W/dt^2). \tag{6}$$

$$dW/dt = \pm (W_0) \cdot (\text{RGR}) \cdot (\exp(\text{RGR} \cdot t)). \tag{8}$$

$$dW/dt = (\pm W_0) \cdot (\text{RGR}) \cdot (\exp(\text{RGR} \cdot t)). \tag{9}$$

(I–1) Suggested analogy to the law of inertia

Equation (21) shows that the ruminant body weight will be kept constant with the passage of time under a dynamic equilibrium, as shown in section (H). This apparent constant suggests an analogy to the law of inertia, Newton's first law of motion where an object will keep the linear motion with the constant velocity or will keep the state of rest if there is no external force acting on it (Kawabe, 2006). However, the trouble with growth mechanics is that there is a dynamic equilibrium that hides behind the apparent constant body weight. To tell the truth, therefore, this shows a conditional weak analogy to the law of inertia.

(I–2) Suggested analogy to Newton's equation of motion

Equation (6) shows that the product of body weight and growth acceleration gives the square of dW/dt. Mathematically speaking at the risk of making mistakes, the form of equation (6) seems to show a similarity to that of Newton's equation of motion that is described as follows (Kawabe, 2006),

$$dp/dt = m \cdot (d^2r/dt^2), \tag{22}$$

where m = mass of an object, r = position, p = momentum, t = time, d^2r/dt^2 = acceleration, dp/dt = force.

The comparison of equation (6) with equation (22) suggests weak analogies between the two terms, respectively. Thus,

$$W \quad \text{and} \quad m, \tag{23}$$

$$d^2W/dt^2 \quad \text{and} \quad d^2r/dt^2, \tag{24}$$

$$(dW/dt)^2 \quad \text{and} \quad dp/dt. \tag{25}$$

From comparing equations (6) and (22), $(dW/dt)^2$ may be interpreted as growth force (Shimojo *et al.*, 2006). Growth is absolutely different from motion, but the weak analogy between them suggested beyond the difference is a mystic phenomenon that is very difficult to understand and explain. However, this weak analogy is very important and the starting point for basic growth mechanics, from which other analogies are suggested.

(I–3) Suggested analogy to the law of action and reaction

Equation (8) shows that dW/dt is considered to be a quasi–force of growth, because $(dW/dt)^2$ is interpreted as growth force. There are two quasi–forces forming a pair in equation (8),

$$(dW/dt)_1 = W_0 \cdot \mathrm{RGR} \cdot \exp(\mathrm{RGR} \cdot t), \tag{8–1}$$

$$-(dW/dt)_1 = -(W_0 \cdot \mathrm{RGR} \cdot \exp(\mathrm{RGR} \cdot t)). \tag{8–2}$$

Therefore, the following equation is given by relating the two quasi–forces,

$$(dW/dt)_1 + (-(dW/dt)_1) = 0. \tag{26}$$

From comparing $(dW/dt)_1$ and $-(dW/dt)_1$, they show the same horizontal component of t, but show the opposite vertical component of W. The opposite vertical component in basic growth mechanics suggests a conditional weak analogy to the law of action ($F_{1\to2}$) and reaction ($F_{2\to1}$), Newton's third law of motion that is described as follows (Kawabe, 2006),

$$F_{1\to2} + F_{2\to1} = 0. \tag{27}$$

(I–4) Might a conditional weak analogy to Newton's law of universal gravitation be suggested?

Equation (9) is divided into two equations as follows,

$$(dW/dt)_\mathrm{R} = W_0 \cdot \mathrm{RGR} \cdot \exp(\mathrm{RGR} \cdot t), \tag{9–1}$$

$$(dW/dt)_\mathrm{F} = (-W_0) \cdot \mathrm{RGR} \cdot \exp(\mathrm{RGR} \cdot t), \tag{9–2}$$

where $(dW/dt)_\mathrm{R}$ = quasi–force related to the ruminant, $(dW/dt)_\mathrm{F}$ = quasi–force related to the forage.

Multiplying equation (9–1) by equation (9–2) gives the following equation (28), where the product of $(dW/dt)_\mathrm{R}$ and $(dW/dt)_\mathrm{F}$ may be interpreted as a force because of interpreting $(dW/dt)^2$ as a force.

$$\begin{aligned}
&(dW/dt)_\mathrm{R} \cdot (dW/dt)_\mathrm{F} \\
&= (W_0 \cdot \mathrm{RGR} \cdot \exp(\mathrm{RGR} \cdot t)) \cdot ((-W_0) \cdot \mathrm{RGR} \cdot \exp(\mathrm{RGR} \cdot t)) \\
&= \mathrm{RGR}^2 \cdot (W_0 \cdot \exp(\mathrm{RGR} \cdot t)) \cdot ((-W_0) \cdot \exp(\mathrm{RGR} \cdot t)) \\
&= -(\mathrm{RGR}^2) \cdot (W_0 \cdot \exp(\mathrm{RGR} \cdot t)) \cdot (W_0 \cdot \exp(\mathrm{RGR} \cdot t)).
\end{aligned} \tag{28}$$

The mean value over the given interval is usually used, and thus equation (28) is modified as

$$\begin{aligned}
&\overline{(dW/dt)_\mathrm{R}} \cdot \overline{(dW/dt)_\mathrm{F}} \\
&= -(\overline{\mathrm{RGR}})^2 \cdot (W_0 \cdot \exp(\overline{\mathrm{RGR}} \cdot t)) \cdot (W_0 \cdot \exp(\overline{\mathrm{RGR}} \cdot t)).
\end{aligned} \tag{29}$$

Three terms in the right–hand side of equation (29) are interpreted as follows: (i) the first term $(\overline{\mathrm{RGR}})^2$ takes a value that is kept constant over the given interval, (ii) the second term is the ruminant body weight, (iii) the third term shows the forage whose volume is expressed as the ruminant body weight. Mathematically speaking at the risk of making mistakes, the form of equation (29) seems to show a similarity to that of the numerator of Newton's law of universal gravitation that is described as follows (Kawabe, 2006),

$$F = -\frac{G \cdot M \cdot m}{r^2} = \frac{-G \cdot M \cdot m}{r^2}, \tag{30}$$

where G = gravitational constant, M = mass of one object, m = mass of the other object, r = the distance between the two objects, F = universal gravitation.

The comparison of equation (29) with the numerator of equation (30) suggests weak analogies between the two terms, respectively. Thus,

$$(\overline{\mathrm{RGR}})^2 \quad \text{and} \quad G, \tag{31}$$

$$W_0 \cdot \exp(\overline{\mathrm{RGR}} \cdot t) \quad \text{and} \quad M, \tag{32}$$

$$W_0 \cdot \exp(\overline{\mathrm{RGR}} \cdot t) \quad \text{and} \quad m. \tag{33}$$

$$\overline{(dW/dt)_\mathrm{R}} \cdot \overline{(dW/dt)_\mathrm{F}} \quad \text{and} \quad F. \tag{34}$$

From the suggested conditional weak analogy between equation (29) and the numerator of equation (30), equation (29) seems to show a virtual force of attracting that operates, with the intervention of constant $(\mathrm{RGR})^2$, between the ruminant and the forage. From the mathematical viewpoint as shown in equation (10), the ruminant and the forage are related to the same coordinate axes, therefore, equation (29) lacks the concept of distance between them. This is absolutely different from equation (30), where the attracting force of universal gravitation is inversely proportional to the square of distance between the two objects.

Based on the property of equation (29), both the ruminant and the forage belong to the same field or the farm. However, if applied to the case of importing them, equation (29) will be extended to include a distance by introducing the movement of them. This may be given mathematically by moving linearly the ruminant and the forage in order to make them meet each other at the same farm. If the linear distance from the farm to the ruminant is shown by D_R and the linear distance to the forage is shown by $D_{\mathbf{F}}$, then equation (29) is rewritten as,

$$\overline{(dW/dt)}_{\mathbf{R}} \cdot \overline{(dW/dt)}_{\mathbf{F}}$$
$$= -(\overline{\mathrm{RGR}})^2 \cdot (W_{\mathbf{0}} \cdot \exp(\overline{\mathrm{RGR}} \cdot t)) \cdot (W_{\mathbf{0}} \cdot \exp(\overline{\mathrm{RGR}} \cdot t))$$
$$= -\frac{(\overline{\mathrm{RGR}})^2 \cdot (D_{\mathbf{R}} \cdot W_{\mathbf{0}} \cdot \exp(\overline{\mathrm{RGR}} \cdot t)) \cdot (D_{\mathbf{F}} \cdot W_{\mathbf{0}} \cdot \exp(\overline{\mathrm{RGR}} \cdot t))}{D_{\mathbf{R}} \cdot D_{\mathbf{F}}} . \tag{35}$$

Comparing the denominator of equation (35) with that of equation (30) suggests a weak analogy between the two terms as follows,

$$D_{\mathbf{R}} \cdot D_{\mathbf{F}} \quad \text{and} \quad r^2. \tag{36}$$

However, the numerator is different between the two equations because of the inclusion of distance in equation (35) and of equation (30) whose numerator does not include distance. Equation (30) shows that the longer the distance between the two objects is, the weaker the force of universal gravitation is. Equation (35) shows that the distance does not affect the virtual force of attracting owing to the cancellation of the term of distance between the numerator and the denominator. If depended on imports in the ruminant agriculture, however, the farm is connected to the foreign country regardless of the length of the distance between them.

The distance, $D_{\mathbf{R}}$ and $D_{\mathbf{F}}$, in equation (35) may be replaced by the actual distance of transportation ($ADT_{\mathbf{R}}$ and $ADT_{\mathbf{F}}$). If this replacing is conducted, equation (35) is modified as,

$$\overline{(dW/dt)}_{\mathbf{R}} \cdot \overline{(dW/dt)}_{\mathbf{F}}$$
$$= -\frac{(\overline{\mathrm{RGR}})^2 \cdot (ADT_{\mathbf{R}} \cdot W_{\mathbf{0}} \cdot \exp(\overline{\mathrm{RGR}} \cdot t)) \cdot (ADT_{\mathbf{F}} \cdot W_{\mathbf{0}} \cdot \exp(\overline{\mathrm{RGR}} \cdot t))}{ADT_{\mathbf{R}} \cdot ADT_{\mathbf{F}}} . \tag{37}$$

In equation (37), the following two terms imply the concept of food–mileage that is given by the product of transported food volume and its transportation distance (Nakata, 2007). Thus,

$$ADT_{\mathbf{R}} \cdot (W_0 \cdot \exp(\overline{\mathrm{RGR}} \cdot t))$$
$$\text{implies food – mileage for the ruminant,} \tag{38}$$

$$ADT_{\mathbf{F}} \cdot (W_0 \cdot \exp(\overline{\mathrm{RGR}} \cdot t))$$
$$\text{implies food – mileage for the forage.} \tag{39}$$

(K) Conclusions

It is suggested from the present study that the investigation into mathematical properties of basic growth mechanics gives conditional weak analogies to laws of motion developed by Newton.

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