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Tomiyama, Yoshiyuki
Faculty of Mathematics, Kyushu University

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Yoshiyuki Tomiyama

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Faculty of Mathematics
Kyushu University
Fukuoka, JAPAN

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Abstract

It is proved that every two-dimensional residual Galois representation of the absolute Galois group of an arbitrary number field lifts to a characteristic zero p -adic representation, if local lifting problems at places above p are unobstructed.

1 Introduction

Let \mathbf{k} be a finite field of characteristic $p \geq 3$. Let K be a number field of finite degree over \mathbb{Q} and G_K its absolute Galois group $\text{Gal}(\bar{K}/K)$. We consider continuous representations

$$\bar{\rho} : G_K \rightarrow \text{GL}_2(\mathbf{k}).$$

The central question that we study in this paper is the existence of a lift of $\bar{\rho}$ to $W(\mathbf{k})$, the ring of Witt vectors of \mathbf{k} . This question has been motivated by a conjecture of Serre ([S1]), that is, all odd absolutely irreducible continuous representations $\bar{\rho} : G_{\mathbb{Q}} \rightarrow \text{GL}_2(\mathbf{k})$ are modular of prescribed weight, level and character. This predicts the existence of a lift to characteristic zero. This conjecture was proved by Khare and Wintenberger in [KW1, KW2]. In [K], Khare proved the existence of lifts to $W(\mathbf{k})$ for any $\bar{\rho} : G_K \rightarrow \text{GL}_2(\mathbf{k})$ which are reducible. Ramakrishna proved under very general conditions on $\bar{\rho}$ that there exist lifts to $W(\mathbf{k})$ for $K = \mathbb{Q}$ in [R1, R2]. Gee's results ([G]) imply that there exist lifts to $W(\mathbf{k})$ for $p \geq 5$ and K satisfying $[K(\mu_p) : K] \geq 3$, where μ_p is the group of p -th roots of unity. Böckle and Khare have proved the general n -dimensional case for function field in [BK]. In this paper, we extend Theorem 1 of [R1] to arbitrary number fields. In particular, we will omit the condition $[K(\mu_p) : K] \geq 3$. Hence we can take the field K to be $\mathbb{Q}(\mu_p)^+$, the totally real subfield of $\mathbb{Q}(\mu_p)$.

For a place v of K , let K_v be the completion of K at v , and let G_v be its absolute Galois group $\text{Gal}(\bar{K}_v/K_v)$. Let $\text{Ad}^0 \bar{\rho}$ be the set of all trace zero two-by-two matrices over \mathbf{k} with Galois action through $\bar{\rho}$ by conjugation. Our main result is the following:

Theorem. *Let K be a number field, and let $\bar{\rho} : G_K \rightarrow \text{GL}_2(\mathbf{k})$ be a continuous representation with coefficients in a finite field \mathbf{k} of characteristic $p \geq 7$. Assume that $H^2(G_v, \text{Ad}^0 \bar{\rho}) = 0$ for each places $v \mid p$. Then $\bar{\rho}$ lifts to a continuous*

representation $\rho : G_K \rightarrow \mathrm{GL}_2(W(\mathbf{k}))$ which is unramified outside a finite set of places of K .

Our method used in the proof is essentially that of Ramakrishna [R1,R2]. In this paper, we follow the more axiomatic treatment presented in [T]. In Section 2, we recall a criterion of Ramakrishna [R2] and Taylor [T] for lifting problems. In Section 3, we define good local lifting problems at certain unramified places and ramified places not dividing p , which will be used in Section 4. In Section 4, we prove Theorem by using the criterion in Section 2 and local lifting problems in Section 3.

Throughout this paper, we assume that p is a prime ≥ 7 .

2 A criterion for lifting problems

In this section we recall a criterion of Ramakrishna [R2] and Taylor [T] for a lifting from a fixed residual Galois representation to a p -adic Galois representation.

Let \mathbf{k} be a finite field of characteristic p . Throughout this paper, we consider a continuous representation

$$\bar{\rho} : G_K \rightarrow \mathrm{GL}_2(\mathbf{k}).$$

Let S denote a finite set of places of K containing the places above p , the infinite places and the places at which $\bar{\rho}$ is ramified, and let K_S denote the maximal algebraic extension of K unramified outside S . Thus $\bar{\rho}$ factors through $\mathrm{Gal}(K_S/K)$. Put $G_{K,S} = \mathrm{Gal}(K_S/K)$. For each place v of K , we fix an embedding $\bar{K} \subset \bar{K}_v$. This gives a corresponding continuous homomorphism $G_v \rightarrow G_{K,S}$.

Let \mathcal{A} be the category of complete noetherian local rings (R, \mathfrak{m}_R) with residue field \mathbf{k} where the morphisms are homomorphisms that induce the identity map on the residue field.

Fix a continuous homomorphism $\delta : G_{K,S} \rightarrow W(\mathbf{k})^\times$, and for every $(R, \mathfrak{m}_R) \in \mathcal{A}$ let δ_R be the composition $\delta_R : G_{K,S} \rightarrow W(\mathbf{k})^\times \rightarrow R^\times$. Suppose $\bar{\rho} : G_{K,S} \rightarrow \mathrm{GL}_2(\mathbf{k})$ has $\det \bar{\rho} = \delta_{\mathbf{k}}$.

By a δ -lift (resp. $\delta|_{G_v}$ -lift) of $\bar{\rho}$ (resp. $\bar{\rho}|_{G_v}$) we mean a continuous representation $\rho : G_{K,S} \rightarrow \mathrm{GL}_2(R)$ (resp. $\rho_v : G_v \rightarrow \mathrm{GL}_2(R)$) for some $(R, \mathfrak{m}_R) \in \mathcal{A}$ such that $\rho \pmod{\mathfrak{m}_R} = \bar{\rho}$ (resp. $\rho_v \pmod{\mathfrak{m}_R} = \bar{\rho}|_{G_v}$) and $\det \rho = \delta_R$ (resp. $\det \rho_v = \delta_R|_{G_v}$). Let $\mathrm{Ad}^0 \bar{\rho}$ be the set of all trace zero two-by-two matrices over \mathbf{k} with Galois action through $\bar{\rho}$ by conjugation.

Definition 1. For a place v of K , we say that a pair (\mathcal{C}_v, L_v) , where \mathcal{C}_v is a collection of $\delta|_{G_v}$ -lifts of $\bar{\rho}|_{G_v}$ and L_v is a subspace of $H^1(G_v, \mathrm{Ad}^0 \bar{\rho})$, is *locally admissible* if it satisfies the following conditions:

- (P1) $(\mathbf{k}, \bar{\rho}|_{G_v}) \in \mathcal{C}_v$.
- (P2) The set of $\delta|_{G_v}$ -lifts in \mathcal{C}_v to a fixed ring $(R, \mathfrak{m}_R) \in \mathcal{A}$ is closed under conjugation by elements of $1 + \mathrm{M}_2(\mathfrak{m}_R)$.
- (P3) If $(R, \rho) \in \mathcal{C}_v$ and $f : R \rightarrow S$ is a morphism in \mathcal{A} then $(S, f \circ \rho) \in \mathcal{C}_v$.

- (P4) Suppose that (R_1, ρ_1) and $(R_2, \rho_2) \in \mathcal{C}_v$, and I_1 (resp. I_2) is an ideal of R_1 (resp. R_2) and that $\phi : R_1/I_1 \xrightarrow{\sim} R_2/I_2$ is an isomorphism such that $\phi(\rho_1 \pmod{I_1}) = \rho_2 \pmod{I_2}$. Let R_3 be the fiber product of R_1 and R_2 over $R_1/I_1 \xrightarrow{\sim} R_2/I_2$. Then $(R_3, \rho_1 \oplus \rho_2) \in \mathcal{C}_v$.
- (P5) If $((R, \mathfrak{m}_R), \rho)$ is a $\delta|_{G_v}$ -lift of $\bar{\rho}|_{G_v}$ such that each $(R/\mathfrak{m}_R^n, \rho \pmod{\mathfrak{m}_R^n}) \in \mathcal{C}_v$ then $(R, \rho) \in \mathcal{C}_v$.
- (P6) For $(R, \mathfrak{m}_R) \in \mathcal{A}$, suppose that I is an ideal of R with $\mathfrak{m}_R I = (0)$. If $(R/I, \rho) \in \mathcal{C}_v$ then there is a $\delta|_{G_v}$ -lift $\tilde{\rho}$ of $\bar{\rho}|_{G_v}$ to R such that $(R, \tilde{\rho}) \in \mathcal{C}_v$ and $\tilde{\rho} \pmod{I} = \rho$.
- (P7) Suppose that $((R, \mathfrak{m}_R), \rho_1)$ and (R, ρ_2) are $\delta|_{G_v}$ -lifts of $\bar{\rho}$ with $(R, \rho_1) \in \mathcal{C}_v$, and that I is an ideal of R with $\mathfrak{m}_R I = (0)$ and $\rho_1 \pmod{I} = \rho_2 \pmod{I}$. We shall denote by $[\rho_2 - \rho_1]$ an element of $H^1(G_v, \text{Ad}^0 \bar{\rho}) \otimes_{\mathbf{k}} I$ defined by $\sigma \mapsto \rho_2(\sigma)\rho_1(\sigma)^{-1} - 1$. Then $[\rho_2 - \rho_1] \in L_v \otimes_{\mathbf{k}} I$ if and only if $(R, \rho_2) \in \mathcal{C}_v$.

Remark 1. Note that we do regard \mathcal{C}_v as a functor from \mathcal{A} to the category of sets.

Let S_f be the subset of S consisting of finite places. Throughout this section, suppose that for each $v \in S_f$ a locally admissible pair (\mathcal{C}_v, L_v) is given.

Let $\bar{\chi}_p : G_K \rightarrow \mathbf{k}^\times$ be the mod p cyclotomic character. For the $\mathbf{k}[G_K]$ -module $\text{Ad}^0 \bar{\rho}$, by $\text{Ad}^0 \bar{\rho}(i)$ for $i \in \mathbb{Z}$ we denote the twist of $\text{Ad}^0 \bar{\rho}$ by the i th tensor power of $\bar{\chi}_p$, and by $\text{Ad}^0 \bar{\rho}^* := \text{Hom}(\text{Ad}^0 \bar{\rho}, \mathbf{k})$ we denote its dual representation. The G_K -equivariant trace pairing $\text{Ad}^0 \bar{\rho} \times \text{Ad}^0 \bar{\rho} \rightarrow \mathbf{k} : (A, B) \mapsto \text{Trace}(AB)$ is perfect. In particular, $\text{Ad}^0 \bar{\rho} \cong \text{Ad}^0 \bar{\rho}^*$ as representations. Thus $\text{Ad}^0 \bar{\rho}(1) \cong \text{Ad}^0 \bar{\rho}^*(1)$ as representations. By the Tate local duality this induces a perfect pairing

$$H^1(G_v, \text{Ad}^0 \bar{\rho}) \times H^1(G_v, \text{Ad}^0 \bar{\rho}(1)) \rightarrow H^2(G_v, \mathbf{k}(1)) \cong \mathbf{k}.$$

Definition 2. A δ -lift of type $(\mathcal{C}_v)_{v \in S_f}$ is a δ -lift such that $\rho|_{G_v} \in \mathcal{C}_v$ for all $v \in S_f$.

Definition 3. We define the *Selmer group* $H^1_{\{L_v\}}(G_{K,S}, \text{Ad}^0 \bar{\rho})$ to be the kernel of the map

$$H^1(G_{K,S}, \text{Ad}^0 \bar{\rho}) \rightarrow \bigoplus_{v \in S_f} H^1(G_v, \text{Ad}^0 \bar{\rho})/L_v$$

and the *dual Selmer group* $H^1_{\{L_v^\perp\}}(G_{K,S}, \text{Ad}^0 \bar{\rho}(1))$ to be the kernel of the map

$$H^1(G_{K,S}, \text{Ad}^0 \bar{\rho}(1)) \rightarrow \bigoplus_{v \in S_f} H^1(G_v, \text{Ad}^0 \bar{\rho}(1))/L_v^\perp$$

where $L_v^\perp \subset H^1(G_v, \text{Ad}^0 \bar{\rho}(1))$ is the annihilator of $L_v \subset H^1(G_v, \text{Ad}^0 \bar{\rho})$ under the above pairing.

Proposition 1. *Keep the above notation and assumptions. If*

$$H^1_{\{L_v^\perp\}}(G_{K,S}, \text{Ad}^0 \bar{\rho}(1)) = 0,$$

then there exists a δ -lift of $\bar{\rho}$ to $W(\mathbf{k})$ of type $(\mathcal{C}_v)_{v \in S_f}$.

Proof. By Theorem 4.50 of [H] we have the exact sequence

$$\begin{aligned} H^1(G_{K,S}, \text{Ad}^0 \bar{\rho}) &\xrightarrow{\alpha} \bigoplus_{v \in S_f} H^1(G_v, \text{Ad}^0 \bar{\rho})/L_v \rightarrow H^1_{\{L_v^\perp\}}(G_{K,S}, \text{Ad}^0 \bar{\rho}(1))^* \\ &\rightarrow H^2(G_{K,S}, \text{Ad}^0 \bar{\rho}) \xrightarrow{\beta} \bigoplus_{v \in S_f} H^2(G_v, \text{Ad}^0 \bar{\rho}). \end{aligned}$$

Consequently, we see that the map α is surjective and the map β is injective. Now we construct δ -lifts ρ_n of $\bar{\rho}$ to $W(\mathbf{k})/p^n$ of type $(\mathcal{C}_v)_{v \in S_f}$ inductively. By the condition (P1), there is nothing to prove for $n = 1$. Assume that there is a δ -lift ρ_{n-1} of $\bar{\rho}$ to $W(\mathbf{k})/p^{n-1}$ of type $(\mathcal{C}_v)_{v \in S_f}$. By the condition (P6), for each $v \in S_f$ we can lift $\rho_{n-1}|_{G_v}$ to a continuous homomorphism $\rho_v : G_v \rightarrow \text{GL}_2(W(\mathbf{k})/p^n)$ such that $(W(\mathbf{k})/p^n, \rho_v) \in \mathcal{C}_v$. Thus we can lift ρ_{n-1} to a continuous homomorphism $\rho : G_{K,S} \rightarrow \text{GL}_2(W(\mathbf{k})/p^n)$ by injectivity of the map β . By surjectivity of the map α we may find a class $\phi \in H^1(G_{K,S}, \text{Ad}^0 \bar{\rho})$ mapping to

$$([\rho_v - \rho|_{G_v}] \bmod L_v)_{v \in S_f} \in \bigoplus_{v \in S_f} H^1(G_v, \text{Ad}^0 \bar{\rho})/L_v.$$

We define $\rho_n := (1 + \phi)\rho$. By the condition (P7) the representation ρ_n is a δ -lift of $\bar{\rho}$ to $W(\mathbf{k})/p^n$ of type $(\mathcal{C}_v)_{v \in S_f}$. The induction is now complete. Then we have a δ -lift of $\bar{\rho}$ to $W(\mathbf{k})$ of type $(\mathcal{C}_v)_{v \in S_f}$ by the condition (P5) and the proposition is proved. \square

3 Local lifting problems

For a place v of K , consider a continuous homomorphism

$$\bar{\rho}_v : G_v \rightarrow \text{GL}_2(\mathbf{k}).$$

We denote by $\widehat{\varepsilon} : G_v \rightarrow W(\mathbf{k})^\times$ the Teichmüller lift for any character $\varepsilon : G_v \rightarrow \mathbf{k}^\times$ and $\widehat{\mu} \in W(\mathbf{k})$ the Teichmüller lift for any element μ of \mathbf{k} . Let χ_p be the p -adic cyclotomic character.

In this section, for ramified places not dividing p and certain unramified places, we construct a good locally admissible pairs (\mathcal{C}_v, L_v) with the $\delta_v := \widehat{\det \bar{\rho}_v} \widehat{\chi_p}^{-1} \chi_p$, which will be used in Section 4. Let I_v be the inertia subgroup of G_v . We distinguish following three cases.

3.1 Case I

Suppose $\bar{\rho}_v$ is unramified and $v \nmid p$. Suppose that

$$\bar{\rho}_v(s) = \begin{pmatrix} \lambda & \lambda \\ 0 & \lambda \end{pmatrix}$$

and $q_v \equiv 1 \pmod{p}$, where λ is an element of \mathbf{k}^\times and s is a lift of the Frobenius automorphism in G_v/I_v and q_v is the order of the residue field of K_v . Note that any δ_v -lift of $\bar{\rho}_v$ factors through the Galois group $\text{Gal}(K_v^t/K_v)$ of the maximal tamely ramified extension K_v^t of K_v . Let P_v be the wild inertia subgroup of

G_v . Let t be a topological generator of I_v/P_v . The Galois group $\text{Gal}(K_v^t/K_v)$ is generated topologically by s and t with the relation $sts^{-1} = t^{q_v}$. We now define a homomorphism $\rho_v : G_v \rightarrow \text{Gal}(K_v^t/K_v) \rightarrow \text{GL}_2(W(\mathbf{k})[[X]])$ by

$$s \mapsto \begin{pmatrix} \hat{\lambda}q_v & \hat{\lambda} \\ 0 & \hat{\lambda} \end{pmatrix}$$

and

$$t \mapsto \begin{pmatrix} 1 & X \\ 0 & 1 \end{pmatrix}.$$

The images of s and t satisfy the relation $sts^{-1} = t^{q_v}$. We define a pair (\mathcal{C}_v, L_v) . The functor $\mathcal{C}_v : \mathcal{A} \rightarrow \mathbf{Sets}$ is given by

$$\mathcal{C}_v(R) := \{\rho : G_v \rightarrow \text{GL}_2(R) \mid \text{there are } \alpha \in \text{Hom}_{\mathcal{A}}(W(\mathbf{k})[[X]], R) \text{ and } M \in 1 + M_2(\mathfrak{m}_R) \text{ such that } \rho = M(\alpha \circ \rho_v)M^{-1}\}.$$

Moreover, if $\rho_0 : G_v \rightarrow \text{GL}_2(\mathbf{k}[X]/(X^2))$ denotes the trivial lift of $\bar{\rho}_v$, we define a subspace $L_v \subset H^1(G_v, \text{Ad}^0 \bar{\rho}_v)$ to be the set

$$\{[c] \in H^1(G_v, \text{Ad}^0 \bar{\rho}_v) \mid (1 + Xc)\rho_0 \in \mathcal{C}_v(\mathbf{k}[X]/(X^2))\}.$$

Lemma 1. *We have*

- (i) $\dim_{\mathbf{k}} L_v = \dim_{\mathbf{k}} H^1(G_v/I_v, \text{Ad}^0 \bar{\rho}_v) = 1$.
- (ii) *The pair (\mathcal{C}_v, L_v) satisfies the conditions (P1)-(P7) of Definition 1.*

Proof. (i) First we prove that $\dim_{\mathbf{k}} H^1(G_v/I_v, \text{Ad}^0 \bar{\rho}_v) = 1$. By Proposition 18 of [S2] the dimension of $H^1(G_v/I_v, \text{Ad}^0 \bar{\rho}_v)$ is the same as that of $H^0(G_v, \text{Ad}^0 \bar{\rho}_v)$. Thus it suffices to show that $H^0(G_v, \text{Ad}^0 \bar{\rho}_v)$ is one-dimensional. This follows from

$$\begin{pmatrix} \lambda & \lambda \\ 0 & \lambda \end{pmatrix} \begin{pmatrix} a & b \\ c & -a \end{pmatrix} \begin{pmatrix} 1/\lambda & -1/\lambda \\ 0 & 1/\lambda \end{pmatrix} = \begin{pmatrix} a+c & -2a+b-c \\ c & -(a+c) \end{pmatrix},$$

where $a, b, c \in \mathbf{k}$.

Next we prove that $\dim_{\mathbf{k}} L_v = 1$. Let $f_1 : W[[X]] \rightarrow \mathbf{k}[X]/(X^2)$ be the morphism in \mathcal{A} determined by $f_1(X) = X$. We define $\rho_1 : G_v \rightarrow \text{GL}_2(\mathbf{k}[X]/(X^2))$ by the composition $f_1 \circ \rho_v$. The images of s and t satisfy the relation $sts^{-1} = t^{q_v}$. Let c_1 be the 1-cocycle corresponding to ρ_1 . The space L_v is spanned by the class of c_1 . Thus we have $\dim_{\mathbf{k}} L_v = 1$.

(ii) The conditions (P1), (P2), (P3), (P6) and (P7) follow from the definition of (\mathcal{C}_v, L_v) .

First we prove the condition (P4). Suppose that we have rings $(R_1, \mathfrak{m}_{R_1}), (R_2, \mathfrak{m}_{R_2}) \in \mathcal{A}$, lifts $\rho_i \in \mathcal{C}_v(R_i)$, ideals $I_i \subset R_i$, and an identification $\phi : R_1/I_1 \xrightarrow{\sim} R_2/I_2$ under which $\rho_1 \pmod{I_1} = \rho_2 \pmod{I_2}$. Take $\alpha_i \in \text{Hom}_{\mathcal{A}}(W(\mathbf{k})[[X]], R_i)$ and $M_i \in 1 + M_2(\mathfrak{m}_{R_i})$ such that $\rho_i = M_i(\alpha_i \circ \rho_v)M_i^{-1}$, $i = 1, 2$. We claim that there exist $\alpha \in \text{Hom}_{\mathcal{A}}(W(\mathbf{k})[[X]], R_3)$ and $M \in 1 + M_2(\mathfrak{m}_{R_3})$ such that $M(\alpha \circ \rho_v)M^{-1} = \rho_1 \oplus \rho_2$. By conjugating ρ_1 by some lift of $M_2 \pmod{I_2}$ to R_1 , we may assume that $M_2 = 1$. Since $\alpha_1 \circ \rho_v(s) = \alpha_2 \circ \rho_v(s)$, the matrix $M_1 \pmod{I_1}$ commutes with $(\alpha_1 \pmod{I_1}) \circ \rho_v(s)$. Let $\begin{pmatrix} 1+m_1 & m_2 \\ 0 & 1+m_3 \end{pmatrix} \in 1 + M_2(\mathfrak{m}_{R_1})$ be a lift of $M_1 \pmod{I_1}$. Put $M'_1 := \begin{pmatrix} 1+m_1 & m_2 \\ 0 & 1+m_3-x \end{pmatrix}$,

where $x := (q_v - 1)m_2 - m_1 + m_3$. Note that $x \in I_1$. Then $M'_1 \in 1 + M_2(\mathfrak{m}_{R_1})$ commutes with $\alpha_1 \circ \rho_v(s)$. We now replace M_1 by $\widetilde{M}_1 := M_1 M_1'^{-1}$ and α_1 by some $\widetilde{\alpha}_1 : W(\mathbf{k})[[X]] \rightarrow R_1$ such that $\widetilde{M}_1(\widetilde{\alpha}_1 \circ \rho_v)\widetilde{M}_1^{-1} = M_1(\alpha_1 \circ \rho_v)M_1^{-1}$. Defining $M := (\widetilde{M}_1, 1) \in 1 + M_2(\mathfrak{m}_{R_3})$ and $\alpha := (\widetilde{\alpha}_1, \alpha_2) : W(\mathbf{k})[[X]] \rightarrow R_3$, the condition (P4) is verified.

Next we prove the condition (P5). Suppose that we have a ring $R \in \mathcal{A}$ and a δ_v -lift ρ of $\bar{\rho}_v$ to R such that each $\rho \pmod{\mathfrak{m}_R^n} \in \mathcal{C}_v(R/\mathfrak{m}_R^n)$. Put $\rho_n := \rho \pmod{\mathfrak{m}_R^n}$. Take $\alpha_n \in \text{Hom}_{\mathcal{A}}(W(\mathbf{k})[[X]], R/\mathfrak{m}_R^n)$ and $M_n \in 1 + M_2(\mathfrak{m}_R/\mathfrak{m}_R^n)$ such that $\rho_n = M_n(\alpha_n \circ \rho_v)M_n^{-1}$. We claim that there exist $\alpha \in \text{Hom}_{\mathcal{A}}(R_v, R)$ and $M \in 1 + M_2(\mathfrak{m}_R)$ such that $M(\alpha \circ \rho_v)M^{-1} = \rho$. Put $S_n := \{(\alpha'_n, M'_n) \mid \rho_n = M'_n(\alpha'_n \circ \rho_v)M_n'^{-1}\}$. Since $\mathcal{C}_v(R/\mathfrak{m}_R^n)$ is finite, S_n is finite. For each n , S_n is not empty set. Thus $\varprojlim_n S_n$ is not empty set, the condition (P5) is verified. \square

3.2 Case II

Suppose $\bar{\rho}_v$ is ramified and $v \nmid p$. In addition, suppose $\bar{\rho}_v(I_v)$ is of order prime to p . Define the functor $\mathcal{C}_v : \mathcal{A} \rightarrow \mathbf{Sets}$ by

$$\mathcal{C}_v(R) := \{\rho : G_v \rightarrow \text{GL}_2(R) \mid \rho \pmod{\mathfrak{m}_R} = \bar{\rho}_v, \rho(I_v) \xrightarrow{\sim} \bar{\rho}_v(I_v), \det \rho = \delta_v\}.$$

Moreover, if $\rho_0 : G_v \rightarrow \text{GL}_2(\mathbf{k}[X]/(X^2))$ denotes the trivial lift of $\bar{\rho}_v$, we define a subspace $L_v \subset H^1(G_v, \text{Ad}^0 \bar{\rho}_v)$ to be the set

$$\{[c] \in H^1(G_v, \text{Ad}^0 \bar{\rho}_v) \mid (1 + Xc)\rho_0 \in \mathcal{C}_v(\mathbf{k}[X]/(X^2))\}.$$

Lemma 2. *We have*

- (i) $\dim_{\mathbf{k}} L_v = \dim_{\mathbf{k}} H^0(G_v, \text{Ad}^0 \bar{\rho}_v)$.
- (ii) *The pair (\mathcal{C}_v, L_v) satisfies the conditions (P1)-(P7) of Definition 1.*

Proof. This lemma follows from the definitions and the Schur-Zassenhaus theorem. \square

3.3 Case III

Suppose $\bar{\rho}_v$ is ramified and $v \nmid p$. In addition, suppose the order of $\bar{\rho}_v(I_v)$ is divisible by p . By Lemma 3.1 of [G], since $p \geq 7$, we may assume that $\bar{\rho}_v$ is given by the form

$$\bar{\rho}_v = \begin{pmatrix} \varphi \bar{\chi}_p & \gamma \\ 0 & \varphi \end{pmatrix},$$

for a character $\varphi : G_v \rightarrow \mathbf{k}^\times$ and a nonzero continuous function $\gamma : G_v \rightarrow \mathbf{k}$. The functor $\mathcal{C}_v : \mathcal{A} \rightarrow \mathbf{Sets}$ is given by

$$\mathcal{C}_v(R) := \{\rho : G_v \rightarrow \text{GL}_2(R) \mid \text{there are } \tilde{\gamma} \in \text{Map}(G_v, R) \text{ and } M \in 1 + M_2(\mathfrak{m}_R) \text{ such that } \rho = M \begin{pmatrix} \widehat{\varphi} \chi_p & \tilde{\gamma} \\ 0 & \widehat{\varphi} \end{pmatrix} M^{-1}, \tilde{\gamma} \pmod{\mathfrak{m}_R} = \gamma\}.$$

Moreover, if $\rho_0 : G_v \rightarrow \text{GL}_2(\mathbf{k}[X]/(X^2))$ denotes the trivial lift of $\bar{\rho}_v$, we define a subspace $L_v \subset H^1(G_v, \text{Ad}^0 \bar{\rho}_v)$ to be the set

$$\{[c] \in H^1(G_v, \text{Ad}^0 \bar{\rho}_v) \mid (1 + Xc)\rho_0 \in \mathcal{C}_v(\mathbf{k}[X]/(X^2))\}.$$

Lemma 3. *We have*

- (i) $\dim_{\mathbf{k}} L_v = \dim_{\mathbf{k}} H^0(G_v, \text{Ad}^0 \bar{\rho}_v)$.
- (ii) *The pair (\mathcal{C}_v, L_v) satisfies the conditions (P1)-(P7) of Definition 1.*

Proof. The proof of this lemma is almost identical argument as in [T, Section 1(E3)]. \square

4 Lifting theorem over arbitrary number fields

In this section, we give a generalization of Theorem 1 of [R1] to arbitrary number fields.

We define $\delta : G_{K,S} \rightarrow W(\mathbf{k})^\times$ by $\widehat{\det \bar{\rho}} \widehat{\chi_p}^{-1} \chi_p$. Throughout this section, we consider lifts of a fixed determinant δ and we always assume the following:

- The order of the image of $\bar{\rho}$ is divisible by p .

By the Schur-Zassenhaus theorem, if the order of the image of $\bar{\rho}$ is prime to p , we can find a lift to $W(\mathbf{k})$ of $\bar{\rho}$. Since $p \geq 7$ and the order of the image of $\bar{\rho}$ is divisible by p , we see from Section 260 of [D] that the image of $\bar{\rho}$ is contained in the Borel subgroup of $\text{GL}_2(\mathbf{k})$ or the projective image of $\bar{\rho}$ is conjugate to either $\text{PGL}_2(\mathbb{F}_{p^r})$ or $\text{PSL}_2(\mathbb{F}_{p^r})$ for some $r \in \mathbb{Z}_{>0}$. In the Borel case, by Theorem 2 of [K] we have a lift of $\bar{\rho}$ to $W(\mathbf{k})$. Thus we may assume that the projective image of $\bar{\rho}$ is equal to $\text{PSL}_2(\mathbb{F}_{p^r})$ or $\text{PGL}_2(\mathbb{F}_{p^r})$. Then, by Lemma 17 of [R1], $\text{Ad}^0 \bar{\rho}$ is an irreducible $G_{K,S}$ -module. (Note that one may replace the assumption that the image of $\bar{\rho}$ contains $\text{SL}_2(\mathbf{k})$ in [R1] with the assumption that the projective image of $\bar{\rho}$ contains $\text{PSL}_2(\mathbb{F}_p)$ without affecting the proof.) The irreducibility of $\text{Ad}^0 \bar{\rho}$ implies that of $\text{Ad}^0 \bar{\rho}(1)$.

Let $K(\text{Ad}^0 \bar{\rho})$ be the fixed field of $\text{Ker}(\text{Ad}^0 \bar{\rho})$. Put $E = K(\text{Ad}^0 \bar{\rho})K(\mu_p)$ and $D = K(\text{Ad}^0 \bar{\rho}) \cap K(\mu_p)$.

Lemma 4. *We have*

$$H^1(\text{Gal}(E/K), \text{Ad}^0 \bar{\rho}) = H^1(\text{Gal}(E/K), \text{Ad}^0 \bar{\rho}(1)) = 0.$$

Proof. First we prove that $H^1(\text{Gal}(E/K), \text{Ad}^0 \bar{\rho}) = 0$. It suffices to show that $H^1(\text{SL}_2(\mathbb{F}_{p^r}), \text{Ad}^0 \bar{\rho}) = 0$ and $H^1(\text{GL}_2(\mathbb{F}_{p^r}), \text{Ad}^0 \bar{\rho}) = 0$, where $\text{GL}_2(\mathbb{F}_{p^r})$ and $\text{SL}_2(\mathbb{F}_{p^r})$ act on $\text{Ad}^0 \bar{\rho}$ by conjugation. By Lemma 2.48 of [DDT], we see $H^1(\text{SL}_2(\mathbb{F}_{p^r}), \text{Ad}^0 \bar{\rho}) = 0$. Since the index of $\text{SL}_2(\mathbb{F}_{p^r})$ in $\text{GL}_2(\mathbb{F}_{p^r})$ is prime to p , we have $H^1(\text{GL}_2(\mathbb{F}_{p^r}), \text{Ad}^0 \bar{\rho}) = 0$.

Next we prove that $H^1(\text{Gal}(E/K), \text{Ad}^0 \bar{\rho}(1)) = 0$. As $D \subset K(\mu_p)$, we see $\text{Gal}(K(\text{Ad}^0 \bar{\rho})/D)$ contains the commutator subgroup of $\text{Gal}(K(\text{Ad}^0 \bar{\rho})/K)$. Since the projective image of $\bar{\rho}$ is equal to $\text{PSL}_2(\mathbb{F}_{p^r})$ or $\text{PGL}_2(\mathbb{F}_{p^r})$, we see this commutator subgroup is just $\text{PSL}_2(\mathbb{F}_{p^r})$. Thus $\text{Gal}(K(\text{Ad}^0 \bar{\rho})/K)/\text{PSL}_2(\mathbb{F}_{p^r}) \rightarrow \text{Gal}(D/K)$ is surjective, and so $[D : K] = 1$ or 2 . Assume that $[K(\mu_p) : K] = 1$, then $H^1(\text{Gal}(E/K), \text{Ad}^0 \bar{\rho}(1))$ is isomorphic to $H^1(\text{Gal}(E/K), \text{Ad}^0 \bar{\rho})$. Consequently $H^1(\text{Gal}(E/K), \text{Ad}^0 \bar{\rho}(1)) = 0$.

Assume that $[K(\mu_p) : K] \geq 3$, or $[K(\mu_p) : K] = 2$ and $[D : K] = 1$. We apply the inflation-restriction sequence to $\text{Gal}(E/K)$ and its normal subgroup $\text{Gal}(E/K(\text{Ad}^0 \bar{\rho}))$. Since $\text{Gal}(K_S/E)$ fixes $\text{Ad}^0 \bar{\rho}(1)$ we see $\text{Ad}^0 \bar{\rho}(1)^{\text{Gal}(E/K(\text{Ad}^0 \bar{\rho}))} = \text{Ad}^0 \bar{\rho}(1)^{\text{Gal}(K_S/K(\text{Ad}^0 \bar{\rho}))}$. We get the exact sequence

$$\begin{aligned} 0 \rightarrow H^1(\text{Gal}(K(\text{Ad}^0 \bar{\rho})/K), \text{Ad}^0 \bar{\rho}(1)^{\text{Gal}(K_S/K(\text{Ad}^0 \bar{\rho}))}) &\rightarrow H^1(\text{Gal}(E/K), \text{Ad}^0 \bar{\rho}(1)) \\ &\rightarrow H^1(\text{Gal}(E/K(\text{Ad}^0 \bar{\rho})), \text{Ad}^0 \bar{\rho}(1)^{\text{Gal}(K(\text{Ad}^0 \bar{\rho})/K)}). \end{aligned}$$

The last term is trivial as $\text{Gal}(E/K(\text{Ad}^0 \bar{\rho}))$ has order prime to p . As $\text{Gal}(K_S/K(\text{Ad}^0 \bar{\rho}))$ acts trivially on $\text{Ad}^0 \bar{\rho}$ we see the action of $\text{Gal}(K_S/K(\text{Ad}^0 \bar{\rho}))$ is $\chi_p|_{\text{Gal}(K_S/K(\text{Ad}^0 \bar{\rho}))}$, which is nontrivial, so $\text{Ad}^0 \bar{\rho}(1)^{\text{Gal}(K_S/K(\text{Ad}^0 \bar{\rho}))} = 0$. Thus the left term in the sequence is trivial, so $H^1(\text{Gal}(E/K), \text{Ad}^0 \bar{\rho}(1)) = 0$.

Assume that $[K(\mu_p) : K] = 2$ and $[D : K] = 2$, then we have $K(\mu_p) = D$. Note that $\text{PSL}_2(\mathbb{F}_{p^r})$ has no non-trivial abelian quotients. If the projective image of $\bar{\rho}$ is $\text{PSL}_2(\mathbb{F}_{p^r})$ for some $r \in \mathbb{Z}_{>0}$, then $\text{Gal}(E/K)$ has no non-trivial abelian quotients. This contradicts the assumption that $[K(\mu_p) : K] = 2$. Hence, we assume that the projective image of $\bar{\rho}$ is $\text{PGL}_2(\mathbb{F}_{p^r})$ for some $r \in \mathbb{Z}_{>0}$. Since the index of $\text{PSL}_2(\mathbb{F}_{p^r})$ in $\text{PGL}_2(\mathbb{F}_{p^r})$ is equal to the index of $\text{Gal}(E/K(\mu_p))$ in $\text{Gal}(E/K)$, $\text{Gal}(E/K(\mu_p))$ is isomorphic to $\text{PSL}_2(\mathbb{F}_{p^r})$. We have

$$H^1(\text{Gal}(E/K), \text{Ad}^0 \bar{\rho}(1)) \hookrightarrow H^1(\text{Gal}(E/K(\mu_p)), \text{Ad}^0 \bar{\rho}(1)).$$

Since $\text{Ad}^0 \bar{\rho}(1)$ is isomorphic to $\text{Ad}^0 \bar{\rho}$ as a $\text{Gal}(E/K(\mu_p))$ -module and the cohomology group $H^1(\text{Gal}(E/K(\mu_p)), \text{Ad}^0 \bar{\rho})$ is zero, the proof is complete. \square

Lemma 5. *If a pair (\mathcal{C}_v, L_v) which is locally admissible is given for each $v \in S_f$ and each elements $\phi \in H^1_{\{L_v^\perp\}}(G_{K,S}, \text{Ad}^0 \bar{\rho}(1))$ and $\psi \in H^1_{\{L_v\}}(G_{K,S}, \text{Ad}^0 \bar{\rho})$ are not zero, then we can find a prime $w \notin S$ and a locally admissible pair (\mathcal{C}_w, L_w) such that*

- (1) $\dim_{\mathbf{k}} H^1(G_w/I_w, \text{Ad}^0 \bar{\rho}) = \dim_{\mathbf{k}} L_w = 1$,
- (2) *the image of ψ in $H^1(G_w/I_w, \text{Ad}^0 \bar{\rho})$ is not zero,*
- (3) *the image of ϕ in $H^1(G_w, \text{Ad}^0 \bar{\rho}(1))/L_w^\perp$ is not zero.*

Proof. Note that Lemma 4 implies that the restrictions of the cocycles ψ and ϕ are non-zero homomorphisms $\phi : \text{Gal}(K_S/E) \rightarrow \text{Ad}^0 \bar{\rho}(1)$ and $\psi : \text{Gal}(K_S/E) \rightarrow \text{Ad}^0 \bar{\rho}$. Let E_ϕ and E_ψ be the fixed fields of the respective kernels. Then, $\text{Gal}(E_\phi/E) \rightarrow \text{Ad}^0 \bar{\rho}(1)$ and $\text{Gal}(E_\psi/E) \rightarrow \text{Ad}^0 \bar{\rho}$ are injective homomorphisms of $\mathbb{F}_p[G_{K,S}]$ -modules. Since $\text{Ad}^0 \bar{\rho}$ is irreducible $G_{K,S}$ -module, these morphisms are bijective, and we see $E_\phi \cap E_\psi = E_\psi (= E_\phi)$ or E . If the intersection is E , then $\text{Gal}(E_\phi E_\psi/E)$ is isomorphic to $\text{Gal}(E_\phi/E) \times \text{Gal}(E_\psi/E)$. If the intersection is E_ψ , then $\text{Gal}(E_\phi E_\psi/E)$ is isomorphic to $\text{Gal}(E_\psi/E)$ and $\text{Gal}(E_\phi/E)$. Therefore, $\text{Gal}(E_\phi E_\psi/E)$ may be regarded as a $\mathbf{k}[\text{Gal}(E/K)]$ -module, moreover, natural homomorphisms $\text{Gal}(E_\phi E_\psi/E) \rightarrow \text{Ad}^0 \bar{\rho}(1)$ and $\text{Gal}(E_\phi E_\psi/E) \rightarrow \text{Ad}^0 \bar{\rho}$ are surjective. Since $\text{PSL}_2(\mathbb{F}_{p^r})$ has no non-trivial abelian quotients, the image of the morphism $\tilde{\rho} \times \chi_p : G_{K,S} \rightarrow \text{PGL}_2(\mathbf{k}) \times \mathbf{k}^\times$ contains $\text{PSL}_2(\mathbb{F}_{p^r}) \times 1$, where $\tilde{\rho}$ is the projective image of $\bar{\rho}$ and χ_p is the mod p cyclotomic character of $G_{K,S}$. Thus there is an element $\sigma \in \text{Gal}(E/K)$ such that $\chi_p(\sigma) = 1$ and $\bar{\rho}(\sigma) = \begin{pmatrix} \lambda & \lambda \\ 0 & \lambda \end{pmatrix}$, for some element $\lambda \in \mathbf{k}^\times$. We denote by $\tilde{\sigma}$ a lift to $\text{Gal}(E_\phi E_\psi/K)$ of σ . Let L be the subset of $\text{Ad}^0 \bar{\rho}$ whose elements have the form $\begin{pmatrix} * & * \\ 0 & * \end{pmatrix}$ and let L' be the subset of $\text{Ad}^0 \bar{\rho}(1)$ whose elements have the form $\begin{pmatrix} * & * \\ 0 & * \end{pmatrix}$. Since L and L' are two-dimensional, there exists $\tau \in \text{Gal}(E_\phi E_\psi/E)$ such that $\psi(\tau) \notin -\psi(\tilde{\sigma}) + L$ and $\phi(\tau) \notin -\phi(\tilde{\sigma}) + L'$.

By the Čebotarev density theorem, we can choose a place $w \notin S$ which is unramified in $E_\phi E_\psi/K$ such that $\text{Frob}_w = \tau \tilde{\sigma}$. Take \mathcal{C}_w and L_w as in Case I. By Lemma 1 of this paper and Lemma 4.8 of [BK], it follows that (w, \mathcal{C}_w, L_w)

has the desired properties. (Note that one may replace function fields in [BK] with number fields without affecting the proof.) \square

Lemma 6. *Suppose that one is given locally admissible pairs $(\mathcal{C}_v, L_v)_{v \in S_f}$ such that*

$$\sum_{v \in S_f} \dim_{\mathbf{k}} L_v \geq \sum_{v \in S} \dim_{\mathbf{k}} H^0(G_v, \text{Ad}^0 \bar{\rho}).$$

Then we can find a finite set of places $T \supset S$ and locally admissible pairs $(\mathcal{C}_v, L_v)_{v \in T \setminus S}$ such that

$$H^1_{\{L_v^\perp\}}(G_{K,T}, \text{Ad}^0 \bar{\rho}(1)) = 0.$$

Proof. Suppose that $0 \neq \phi \in H^1_{\{L_v^\perp\}}(G_{K,S}, \text{Ad}^0 \bar{\rho}(1))$. By the assumption of the lemma and Theorem 4.50 of [H], we see that $\dim_{\mathbf{k}} H^1_{\{L_v\}}(G_{K,S}, \text{Ad}^0 \bar{\rho}) \geq \dim_{\mathbf{k}} H^1_{\{L_v^\perp\}}(G_{K,S}, \text{Ad}^0 \bar{\rho}(1))$. Then we can find $0 \neq \psi \in H^1_{\{L_v\}}(G_{K,S}, \text{Ad}^0 \bar{\rho})$. Thus we can find a place $w \notin S$ and a locally admissible pair (\mathcal{C}_w, L_w) such that

- (1) $\dim_{\mathbf{k}} H^1(G_w/I_w, \text{Ad}^0 \bar{\rho}) = \dim_{\mathbf{k}} L_w$,
- (2) $H^1_{\{L_v\}}(G_{K,S}, \text{Ad}^0 \bar{\rho}) \rightarrow H^1(G_w/I_w, \text{Ad}^0 \bar{\rho})$ is surjective,
- (3) the image of ϕ in $H^1(G_w, \text{Ad}^0 \bar{\rho}(1))/L_w^\perp$ is not zero,

by Lemma 5. We have an injection

$$H^1_{\{L_v^\perp\}}(G_{K,S}, \text{Ad}^0 \bar{\rho}(1)) \hookrightarrow H^1_{\{L_v^\perp\} \cup \{H^1(G_w, \text{Ad}^0 \bar{\rho}(1))\}}(G_{K,S \cup \{w\}}, \text{Ad}^0 \bar{\rho}(1))$$

and we see that its cokernel has order equal to

$$\#\text{Coker}(H^1_{\{L_v\}}(G_{K,S}, \text{Ad}^0 \bar{\rho}) \rightarrow H^1(G_w/I_w, \text{Ad}^0 \bar{\rho})),$$

by applying Theorem 4.50 of [H] to

$$H^1_{\{L_v^\perp\}}(G_{K,S}, \text{Ad}^0 \bar{\rho}(1))$$

and

$$H^1_{\{L_v^\perp\} \cup \{H^1(G_w, \text{Ad}^0 \bar{\rho}(1))\}}(G_{K,S \cup \{w\}}, \text{Ad}^0 \bar{\rho}(1)).$$

Thus

$$H^1_{\{L_v^\perp\}}(G_{K,S}, \text{Ad}^0 \bar{\rho}(1)) = H^1_{\{L_v^\perp\} \cup \{H^1(G_w, \text{Ad}^0 \bar{\rho}(1))\}}(G_{K,S \cup \{w\}}, \text{Ad}^0 \bar{\rho}(1)),$$

and we obtain an exact sequence

$$\begin{aligned} 0 \rightarrow H^1_{\{L_v^\perp\} \cup \{L_w^\perp\}}(G_{K,S \cup \{w\}}, \text{Ad}^0 \bar{\rho}(1)) &\rightarrow H^1_{\{L_v^\perp\}}(G_{K,S}, \text{Ad}^0 \bar{\rho}(1)) \\ &\rightarrow H^1(G_w, \text{Ad}^0 \bar{\rho}(1))/L_w^\perp. \end{aligned}$$

Hence $\phi \notin H^1_{\{L_v^\perp\} \cup \{L_w^\perp\}}(G_{K,S \cup \{w\}}, \text{Ad}^0 \bar{\rho}(1)) \subset H^1_{\{L_v^\perp\}}(G_{K,S}, \text{Ad}^0 \bar{\rho}(1))$. The lemma will follow by repeating such a computation. \square

Let S' denote the set of places of K consisting of the places above p , the infinite places and the places at which $\bar{\rho}$ is ramified.

Proof of Theorem. This follows almost at once from Proposition 1 and Lemma 6. For each places v satisfying $v \in S'_f$ and $v \nmid p$, take \mathcal{C}_v and L_v as in Case II or Case III. For places $v \mid p$, take \mathcal{C}_v and L_v as the collection of all $\delta|_{G_v}$ -lifts of $\bar{\rho}|_{G_v}$ and $H^1(G_v, \text{Ad}^0 \bar{\rho})$, respectively. By Theorem 4.52 of [H] and the assumption of Theorem, we have

$$\sum_{v|p} \dim_{\mathbf{k}} L_v = \sum_{v|p} \dim_{\mathbf{k}} H^0(G_v, \text{Ad}^0 \bar{\rho}) + \sum_{v|p} [K_v : \mathbb{Q}_p] \dim_{\mathbf{k}} \text{Ad}^0 \bar{\rho}$$

and thus we obtain

$$\sum_{v \in S'_f} \dim_{\mathbf{k}} L_v \geq \sum_{v \in S'} \dim_{\mathbf{k}} H^0(G_v, \text{Ad}^0 \bar{\rho}).$$

□

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