Numerical verification of stationary solutions for Navier-Stokes problems

Nagatou, Kaori
Faculty of Mathematics, Kyushu University

Hashimoto, Kouji
Graduate School of Mathematics, Kyushu University

Nakao, Mitsuhiro T.
Faculty of Mathematics, Kyushu University

http://hdl.handle.net/2324/11856

Elsevier
バージョン：accepted
権利関係：
Numerical verification
of stationary solutions
for Navier-Stokes problems

K. Nagatou, K. Hashimoto
M.T. Nakao

MHF 2005-30
(Received September 15, 2005)
Numerical verification of stationary solutions for Navier-Stokes problems
K. Nagatou†, K. Hashimoto‡ and M.T. Nakao†
†Faculty of Mathematics, Kyushu University, Japan
‡Graduate School of Mathematics, Kyushu University, Japan

Abstract

We present a numerical method to enclose stationary solutions of the Navier-Stokes equations, especially 2-D driven cavity problem with regularized boundary condition. Our method is based on the infinite dimensional Newton’s method by estimating the inverse of the corresponding linearized operator. The method can be applied to the case for high Reynolds numbers and we show some numerical examples which confirm us the actual effectiveness.

Keywords: Numerical enclosure method, driven cavity flows, infinite dimensional Newton’s method

1 Introduction

We consider the following Navier-Stokes equations

\[
\begin{aligned}
-\Delta u + R \cdot (u \cdot \nabla) u + \nabla p &= f \quad \text{in } \Omega, \\
\text{div } u &= 0 \quad \text{in } \Omega, \\
u &= g \quad \text{on } \partial \Omega,
\end{aligned}
\]

where \(u, p\) and \(R\) are the velocity vector, pressure and the Reynolds number, respectively and the flow region \(\Omega\) is a convex polygonal domain in \(\mathbb{R}^2\). In what follows, for each rational number \(m\), let \(H^m(\Omega)\) denote the \(L^2\)-Sobolev space of order \(m\) on \(\Omega\). The function \(f = (f_1, f_2)\) means a density of body forces with \(f \in (H^1(\Omega))^2\) and \(g = (g_1, g_2) \in (H^{1/2}(\partial \Omega))^2\), where we assume that there exists a function \(\varphi \in H^2(\Omega)\) satisfying \((\varphi_y, -\varphi_x) = g\) on \(\partial \Omega\).

The above problem was discussed by Wiener [7] for low Reynolds numbers. The method proposed in it is based on Newton-Kantorovich theorem but it would not be able to apply to high Reynolds numbers, because the estimation for the inverse of the linearized operator directly depends on the Reynolds number. We also uses Newton type verification condition, but the method which verifies the invertibility of linearized operator is different from
the Wieners’ formulation. Our method has an advantage which enables us to verify the invertibility of the linearized operator, even for high Reynolds numbers, provided that the approximation subspace is sufficiently accurate and that the inverse operator actually exists in the rigorous sense. The numerical examples presented in Section 5 show this actual improvement.

2 Stream function and the linearized operator

We first introduce a stream function \( \psi \) satisfying
\[
\begin{aligned}
\psi &= (\psi_y, -\psi_x) \\
\text{by the incompressibility condition in (1.1), where subscripts } x \text{ and } y \text{ denote the partial derivative for } x \text{ and } y \text{ respectively. Using this function and newly denoting } u \text{ as } \psi - \varphi \text{ we can rewrite the equations (1.1) as}
\end{aligned}
\]
\[
\begin{aligned}
\Delta^2 u + \Delta^2 \varphi + R \cdot J(u + \varphi, \Delta (u + \varphi)) &= (f_2)_x - (f_1)_y \quad \text{in } \Omega, \\
u &= \frac{\partial u}{\partial n} = 0 \quad \text{on } \partial \Omega,
\end{aligned}
\]
where \( J \) is a bilinear form defined by
\[
J(u, v) = u_x v_y - u_y v_x
\]
and \( \frac{\partial}{\partial n} \) stands for the normal derivative. Our aim is to verify the existence of a weak solution \( u \in H^2_0(\Omega) \) of (2.1), where
\[
H^2_0(\Omega) \equiv \{ v \in H^2(\Omega) | v = \frac{\partial v}{\partial n} = 0 \text{ on } \partial \Omega \}
\]
with inner product \( <u, v>_{H^2_0(\Omega)} = (\Delta u, \Delta v)_{L^2} \) for \( u, v \in H^2_0(\Omega) \), and norm \( \|u\|_{H^2_0(\Omega)} = \|\Delta u\|_{L^2(\Omega)} \) for \( u \in H^2_0(\Omega) \).

Let \( S_h \) be a finite dimensional subspace of \( H^2_0(\Omega) \) that depends on \( h \) (0 < \( h < 1 \)). Usually \( S_h \) is taken to be a finite element subspace with mesh size \( h \). We calculate an approximate solution \( u_h \in C^1(\Omega) \) of (2.1) in the finite dimensional space, satisfying for all \( v_h \in S_h \)
\[
(\Delta u_h + \Delta \varphi, \Delta v_h)_{L^2} + (R \cdot J(u_h + \varphi, \Delta (u_h + \varphi)), v_h)_{L^2} = ((f_2)_x - (f_1)_y, v_h)_{L^2},
\]
and calculate \( u_s \in C^2(\Omega) \) by smoothing of \( u_h \). Then the linearized operator at \( u_s \) is represented as
\[
L u \equiv \Delta^2 u + R \cdot \{ J(u_s + \varphi, \Delta u) + J(u, \Delta (u_s + \varphi)) \},
\]
and \( L \) is considered as the operator from \( H^2_0(\Omega) \) to \( H^{-2}(\Omega) \) in weak sense.

We will verify the existence of the inverse \( L^{-1} : H^{-2}(\Omega) \rightarrow H^2_0(\Omega) \) and formulate the infinite dimensional Newton’s method.
3 Invertibility of the linearized operator

By direct computations, we find that for any \( q \in H^{-2}(\Omega) \) there exists a unique solution \( v \in H^2_0(\Omega) \) satisfying

\[
\begin{aligned}
\Delta^2 v &= q \quad \text{in } \Omega, \\
v = \frac{\partial v}{\partial n} &= 0 \quad \text{on } \partial \Omega.
\end{aligned}
\]  

(3.1)

For \( q \in H^{-2}(\Omega) \), let \( Kq \) be the unique solution \( v \in H^2_0(\Omega) \) of the equation (3.1) then \( K \) is a compact operator from \( H^{-1}(\Omega) \) to \( H^2_0(\Omega) \). Using the compact operator on \( H^2_0(\Omega) \)

\[
F_1(u) \equiv -R \cdot K \{ J(u_s + \varphi, \Delta u) + J(u, \Delta(u_s + \varphi)) \},
\]

the equation \( \mathcal{L}u = 0 \) is equivalent to the fixed point equation \( u = F_1(u) \). In order to show the invertibility of the linearized operator \( \mathcal{L} \), by the Fredholm alternative, we only have to show the uniqueness of the solution of the equation \( \mathcal{L}u = 0 \).

Let \( P_h : H^2_0(\Omega) \rightarrow S_h \) denote the \( H^2_0 \)-projection defined by

\[
(\Delta(u - P_h u), \Delta v_h)_{L^2} = 0 \quad \text{for all } v_h \in S_h,
\]

and we derive some error estimations for \( P_h \). In what follows, we restrict ourselves to that the domain \( \Omega \) is a unit square \((0,1) \times (0,1)\), and that \( S_h \) is the set of piecewise bicubic Hermite functions with uniform mesh on \( \Omega \) (e.g., [5]). However, our verification principle can also be applied to more general domains and approximation subspaces, when the appropriate a priori error estimates are obtained.

Concerning the error estimates for \( P_h \) we make use of the following lemma:

**Lemma 1.** For \( u \in H^4(\Omega) \cap H^2_0(\Omega) \) we have \( \|u - P_h u\|_{H^2_0(\Omega)} \leq (Ch)^2 \|\Delta^2 u\|_{L^2(\Omega)} \),

where \( C \) is a constant given in Table 1.
Remark. The constant $C$ in Lemma 1 was derived from the constructive error estimations with numerical computations for biharmonic problems, and it depends on each mesh size $h$ as seen in Table 1.

The basic idea for determination of the constant $C$ is similar to the methods in [1, 8]. We omit the proof of Lemma 1 here and will discuss it in the forthcoming paper [4] for details.

Now, as in [2] or [3], we decompose $u = F_1(u)$ into the finite and infinite dimensional parts:

\[
\begin{align*}
P_h u &= P_h F_1(u), \\
(I - P_h) u &= (I - P_h) F_1(u). 
\end{align*}
\] (3.2)

Since we apply a Newton-like method only for the former part of (3.2), we define the following operator:

\[ N^1_h (u) \equiv P_h u - [I - F_1]^{-1}_h (P_h u - P_h F_1(u)), \]

where $I$ is the identity map on $H_0^2(\Omega)$. And we assume that the restriction to $S_h$ of the operator $P_h[I - F_1] : S_h \to S_h$ has the inverse $[I - F_1]^{-1}_h$. The validity of this assumption can be numerically confirmed in actual computations.

We next define the operator $T_1 : H_0^2(\Omega) \rightarrow H_0^2(\Omega)$ by

\[ T_1(u) \equiv N^1_h (u) + (I - P_h) F_1(u). \]

Then $T_1$ becomes a compact map on $H_0^2(\Omega)$ and we have the following equivalence relation

\[ u = T_1(u) \iff u = F_1(u). \]

Our purpose is to find a unique fixed point of $T_1$ in a certain set $U \subset H_0^2(\Omega)$, which is called a ‘candidate set’. Given positive real numbers $\gamma$ and

<table>
<thead>
<tr>
<th>$1/h$</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>0.7377</td>
<td>0.7811</td>
<td>0.8091</td>
<td>0.8278</td>
<td>0.8418</td>
</tr>
</tbody>
</table>
α we define the corresponding candidate set $U$ by $U \equiv U_h \oplus [α]$, where $U_h \equiv \{ φ_h \in S_h \mid ∥φ_h∥_{H^2_0} \leq γ \}$, $[α] \equiv \{ φ_⊥ \in S_⊥ \mid ∥φ_⊥∥_{H^2_0} \leq α \}$ and $S_⊥$ means the orthogonal complement of $S_h$ in $H^2_0(Ω)$. If the relation $T_1(U) \subset \text{int}(U)$ holds, by Schauder’s fixed point theorem and the linearity of $T_1$, there exists a fixed point $u$ of $T_1$ in $U$ and the fixed point is unique, i.e., $u = 0$, which implies that the operator $L$ is invertible. Decomposing $T_1(U) \subset \text{int}(U)$ into finite and infinite dimensional parts we have a sufficient condition for it as follows:

$$\begin{align}
\sup_{u \in U} \|Λ^1_h(u)\|_{H^2_0(Ω)} &< γ \\
\sup_{u \in U} \|(I - P_h)F_1(u)\|_{H^2_0(Ω)} &< α.
\end{align}$$

(3.3)

We now derive the following theorem in which the verification condition (3.3) is numerically and simply described.

**Theorem 1.** Let $\{ φ_i \}$ be the basis of $S_h$ and define the following constants:

$$\begin{align}
C_0 &= Ch, \quad C_i^s = ∥∇(u_s + ϕ)∥_∞, \quad C_2^s = ∥∇(u_s + ϕ)∥_∞ + ∥∇(u_s + ϕ)∥_∞ \\
C_3^s &= ∥∇Δ(u_s + ϕ)∥_∞, \quad C_p = \frac{1}{2}, \quad M_1 = ∥L^T G^{-1} L∥_E,
\end{align}$$

$$\begin{align}
K_1 &= C_1^s + C_2^s C_3^s, \quad K_2 = C_1^s + C_0 C_3^s C_p, \quad K_3 = √2 C_1^s + C_p (C_2^s + C_0 C_3^s),
\end{align}$$

where $C$ is the same constant as in Lemma 1, $∥v∥_E$ denotes the matrix norm corresponding to the Euclidian vector norm,

$C_p$ is the Poincaré constant, the matrix $G = (G_{ij})$ is defined by $G_{ji} \equiv R(J(u_s + ϕ, Δφ_i) + J(ϕ_i, Δ(u_s + ϕ)), ϕ_j)_{L^2(Ω)} + (∆φ_i, ∆φ_j)_{L^2(Ω)}$, and $D = LL^T$ is a Cholesky decomposition of the matrix $D = (D_{ij})$ defined by $D_{ji} \equiv (∆φ_i, ∆φ_j)_{L^2(Ω)}$. For these constants, if the inequality

$$RC_0(K_1 + K_2 K_3 M_1 RC_0) < 1$$

(3.4)

holds then the operator $L$ is invertible.

**Proof.** We show sufficient conditions for (3.3). Denoting $u = u_1 + u_2$, $u_1 \in U_h$, $u_2 \in [α]$, by some simple calculations we have $Λ^1_h(u) = [I - F_1]_h^{-1} P_h F_1(u_2)$, and thus $∥Λ^1_h(u)∥_{H^2_0(Ω)} \leq M_1 ∥P_h F_1(u_2)∥_{H^2_0(Ω)}$ holds.
Using error estimation in Lemma 1, we have \( \| P_h F_1(u_2) \|_{H^0_0(\Omega)} \leq R C_0 K_3 \alpha \). Thus we derive a sufficient condition for the first inequality in (3.3) as

\[
M_1 R C_0 K_3 \alpha < \gamma. \tag{3.5}
\]

Now we estimate the left hand side of the second inequality in (3.3). Noting that

\[
\| (I - P_h) F_1(u) \|_{H^0_0(\Omega)} \leq R \| (I - P_h) K J(u_s + \varphi, \Delta u) \|_{H^0_0(\Omega)} \]

\[
+ \| (I - P_h) K J(u, \Delta(u + \varphi)) \|_{H^0_0(\Omega)} \]

\[
\leq R C_0 K_2 \gamma + R C_0 K_1 \alpha,
\]

we obtain the sufficient condition for the second inequality in (3.3) as

\[
R C_0 (K_1 \alpha + K_2 \gamma) < \alpha. \tag{3.6}
\]

Combining the conditions (3.5) and (3.6) we finally obtain the sufficient condition for (3.3) as \( R C_0 (K_1 + K_2 K_3 M_1 R C_0) < 1 \). □

4 Verification procedure for nonlinear problem

In what follows we assume that the invertibility of the linearized operator \( L \) is confirmed by the method described in the previous section. We will verify the existence of solutions for (2.1) in the neighborhood of \( u_X \in C^1(\Omega) \) satisfying \((\Delta u_X + \Delta \varphi, \Delta v_h)_{L^2(\Omega)} + (R \cdot J(u_s + \varphi, \Delta(u_s + \varphi)), v_h)_{L^2(\Omega)} = ((f_2)_x - (f_1)_y, v_h)_{L^2(\Omega)} \) for all \( v_h \in S_h \). Considering the function \( \bar{u} \) satisfying

\[
\begin{cases}
\Delta^2 \bar{u} = -\Delta^2 \varphi - R \cdot J(u_s + \varphi, \Delta(u_s + \varphi)) + (f_2)_x - (f_1)_y & \text{in } \Omega, \\
\frac{\partial \bar{u}}{\partial n} = 0 & \text{on } \partial \Omega,
\end{cases} \tag{4.1}
\]

and writing \( w \equiv u - \bar{u}, \ v_0 \equiv \bar{u} - u_X, \ u - u_X \) can be represented as \( w + v_0 \).

Noting that \( u_X = P_h \bar{u} \), we see that \( v_0 \in S_{\perp} \) and, by Lemma 1 the error estimate for \( v_0 \) can be derived:

\[
\| v_0 \|_{H^0_0(\Omega)} \leq (Ch)^2 \| -\Delta^2 \varphi - R \cdot J(u_s + \varphi, \Delta(u_s + \varphi)) + (f_2)_x - (f_1)_y \|_{L^2(\Omega)}.
\]

Now we can rewrite (2.1) as

\[
\begin{cases}
\Delta^2 w = -R \cdot J(w + u_X + v_0 + \varphi, \Delta(w + u_X + v_0 + \varphi)) + R \cdot J(u_s + \varphi, \Delta(u_s + \varphi)) & \text{in } \Omega, \\
w = \frac{\partial w}{\partial n} = 0 & \text{on } \partial \Omega.
\end{cases} \tag{4.2}
\]
Thus defining the compact map on $H_0^2(\Omega)$: $F_2(w) \equiv RK\{J(u_s + \varphi, \Delta(u_s + \varphi)) - J(w + u_X + v_0 + \varphi, \Delta(w + u_X + v_0 + \varphi))\}$, we have the fixed point equation $w = F_2(w)$ which is equivalent to (4.2). Now we formulate the infinite dimensional Newton’s method for this fixed point equation. Note that $w - [I - F_2^\prime(-v_0 - u_X + u_s)]^{-1}(I - F_2)(w)$ can be equivalently represented as $L^{-1}q(w)$, where $F_2^\prime(-v_0 - u_X + u_s)$ stands for Fréchet derivative of $F_2$ at $-v_0 - u_X + u_s$ and $q(w) \equiv R\{J(u_s + \varphi, \Delta(u_s + \varphi)) - J(w + u_X + v_0 + \varphi, \Delta(w + u_X + v_0 + \varphi)) + J(u_s + \varphi, \Delta w) + J(w, \Delta(u_s + \varphi))\}$. Then it is seen that $w = F_2(w) \iff w = T_2(w)$, where $T_2(w) \equiv L^{-1}q(w)$ is a compact map on $H_0^2(\Omega)$.

We intend to find a fixed point of $T_2$ in a set $W$ defined by $W \equiv \{w \in H_0^2(\Omega) \mid \|w\|_{H_0^2(\Omega)} \leq \alpha\}$, where $\alpha$ is a positive number. If the relation $T_2(W) \subset W$ holds, by Schauder’s fixed point theorem there exists a fixed point of $T_2$ in $W$. In order to derive a sufficient condition for $T_2(W) \subset W$, we first prepare for the following constants:

$$
\kappa \equiv C_0R(K_1 + K_2K_3M_1C_0R), \quad \tau_1 = \frac{C_0RM_1K_2}{1 - \kappa}, \quad \tau_2 = \frac{1}{1 - \kappa}, \quad \tau_3 = M_1(C_0RK_3\tau_1 + 1), \quad \tau_4 = M_1C_0RK_3\tau_2,
$$

where $C_4$ is an embedding constant satisfying $\|\nabla u\|_{L^4(\Omega)} \leq C_4\|u\|_{L^2(\Omega)}$ for $u \in H_0^2(\Omega)$ and we have used the optimal embedding estimates $C_4 = \frac{1}{\pi}$ which can be derived by the result in [6]. Moreover for a matrix $S = \left(\begin{array}{ccc} \tau_1^2 + \tau_3^2 & \tau_1\tau_2 + \tau_3\tau_4 & \tau_2^2 + \tau_4^2 \\
\tau_1\tau_2 + \tau_3\tau_4 & \tau_1^2 + \tau_3^2 & \tau_2^2 + \tau_4^2 \\
\tau_2^2 + \tau_4^2 & \tau_1\tau_2 + \tau_3\tau_4 & \tau_1^2 + \tau_3^2 \end{array}\right)$ and $M_2 \equiv \|S\|_{\bar{E}}^{\frac{1}{2}}$, define the following constants:

$$
C_1^X = \|\nabla (u_X + \varphi)\|_\infty, \quad C_2^X = \left\|\nabla \frac{\partial(u_X + \varphi)}{\partial x}\right\|_\infty + \left\|\nabla \frac{\partial(u_X + \varphi)}{\partial y}\right\|_\infty,
$$

$$
C_3^X = \|\Delta (u_X + \varphi)\|_\infty, \quad D_1^\delta = \|\nabla (u_X - u_s)\|_{L^2(\Omega)},
$$

$$
D_2^\delta = \|J(u_X - u_s, \Delta(u_s + \varphi))\|_{L^2(\Omega)}, \quad D_3^\delta = \|\Delta(u_X - u_s)\|_{L^2(\Omega)}.
$$

Since a sufficient condition for $T_2(W) \subset W$ is sup$_{w \in W} \|T_2(w)\|_{H_0^2(\Omega)} \leq \alpha$, by estimating the left hand side of this inequality, we can derive the following theorem.
Theorem 2. Assume that the invertibility condition (3.4) holds. Using
the same constants in Theorem 1, if there exists a real number \( \alpha > 0 \) satis-
ifying the quadratic inequality in \( \alpha \): 
\[
M_2 R \{ C_4^2 (\alpha + b)^2 + C_2^2 \alpha D_3^\delta + C_3^X C_0 b + \alpha D_1^\delta C_p + C_0 b (\sqrt{2} C_1^X + C_3^X C_0 b) + C_2^2 D_2^\delta + C_2^2 C_1^X D_3^\delta \} \leq \alpha,
\]
then there exists a
fixed point of \( T_2 \) in \( W \).

Proof. For \( q(w) \in H^{-2}(\Omega) \) consider the solution \( \phi \in H_0^2(\Omega) \) of the
problem
\[
\begin{align*}
\mathcal{L} \phi &= q(w) \quad \text{in } \Omega, \\
\phi &= \frac{\partial \phi}{\partial n} = 0 \quad \text{on } \partial \Omega.
\end{align*}
\]

Then writing \( \phi = \phi_h + \phi_{\perp} \), \( \phi_h \in S_h \), \( \phi_{\perp} \in S_{\perp} \), we have
\[
\begin{align*}
\| \phi_h \|_{H_0^2(\Omega)} &\leq M_1 R C_0 K_3 \| \phi \|_{H_0^2(\Omega)} + M_1 \| P_h K q(w) \|_{H_0^2(\Omega)}, \\
\| \phi_{\perp} \|_{H_0^2(\Omega)} &\leq R C_0 (K_1 \| \phi \|_{H_0^2(\Omega)} + K_2 \| P_h K q(w) \|_{H_0^2(\Omega)}) + \| (I - P_h) K q(w) \|_{H_0^2(\Omega)}.
\end{align*}
\]

Noting that \( \kappa < 1 \) holds because of the invertibility of \( \mathcal{L} \), we have
\[
\begin{align*}
\| \phi_h \|_{H_0^2(\Omega)} &\leq \tau_3 \| P_h K q(w) \|_{H_0^2(\Omega)} + \tau_4 \| (I - P_h) K q(w) \|_{H_0^2(\Omega)}, \\
\| \phi_{\perp} \|_{H_0^2(\Omega)} &\leq \tau_1 \| P_h K q(w) \|_{H_0^2(\Omega)} + \tau_2 \| (I - P_h) K q(w) \|_{H_0^2(\Omega)}.
\end{align*}
\]

Therefore by some simple calculations, using (4.4) and (5.5) we obtain
\[
\| \phi \|_{H_0^2(\Omega)} \leq M_2 \| K q(w) \|_{H_0^2(\Omega)} \leq M_2 \| q(w) \|_{H^{-2}}.
\]

Furthermore, we have the estimations
\[
\| q(w) \|_{H^{-2}} = \sup_{\theta \in H_0^2(\Omega), \| \theta \|_{H_0^2(\Omega)} = 1} \left| \langle q(w), \theta \rangle_{H^{-2}, H_0^2} \right|
\leq R \{ C_4^2 (\alpha + b)^2 + C_2^2 \alpha D_3^\delta + C_3^X C_0 b + \alpha D_1^\delta C_p + C_0 b (\sqrt{2} C_1^X + C_3^X C_0 b) + C_2^2 D_2^\delta + C_2^2 C_1^X D_3^\delta \}
\]
where \( < \cdot, \cdot >_{H^{-2}, H_0^2} \) means the canonical duality pairing. Thus we obtain
\[
\| \mathcal{L}^{-1} q(w) \|_{H_0^2(\Omega)} \leq \]
\[
M_2 R \{ C_4^2 (\alpha + b)^2 + C_2^2 \alpha D_3^\delta + C_3^X C_0 b + \alpha D_1^\delta C_p + C_0 b (\sqrt{2} C_1^X + C_3^X C_0 b) + C_2^2 D_2^\delta + C_2^2 C_1^X D_3^\delta \}
\]
and the desired assertion is proved.
5 Numerical examples

Particularly, we consider the two dimensional driven cavity problem with $f = 0$ and $(\varphi_y, -\varphi_x) = g$ in (1.1), where $\varphi(x, y) = x^2(1 - x)^2 y^2(1 - y)$.

Table 2: Verification Results for Driven Cavity Problem ($h = 1/75$)

<table>
<thead>
<tr>
<th>$R$</th>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$|v_0|_{H^2_0(\Omega)}$</th>
<th>$D_3$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1.1746</td>
<td>1.5841</td>
<td>6.9318e-4</td>
<td>3.8268e-6</td>
<td>5.9094e-4</td>
</tr>
<tr>
<td>200</td>
<td>1.1945</td>
<td>3.1510</td>
<td>6.9313e-4</td>
<td>4.8677e-6</td>
<td>3.2670e-3</td>
</tr>
</tbody>
</table>

The computations were carried out on the DELL Precision WorkStation 650 (Intel Xeon 3.2GHz) using MATLAB (Ver. 6.5.1). The verification results are shown in Table 2, and the solution $u$ in (2.1) is enclosed as

$$\|u - u_X\|_{H^2_0(\Omega)} \leq \|v_0\|_{H^2_0(\Omega)} + \alpha.$$

It seems that Wieners’ method would not be able to apply to the Reynolds number higher than 20 in [7]. On the other hand, we enclosed the stationary solution for the Reynolds number up to 200, and our method can be applied, in principle, to higher Reynolds numbers by using more accurate approximation subspaces, i.e., smaller mesh sizes.

References


MHF2003-1 Mitsuhiro T. NAKAO, Kouji HASHIMOTO & Yoshitaka WATANABE
A numerical method to verify the invertibility of linear elliptic operators with applications to nonlinear problems

MHF2003-2 Masahisa TABATA & Daisuke TAGAMI
Error estimates of finite element methods for nonstationary thermal convection problems with temperature-dependent coefficients

MHF2003-3 Tomohiro ANDO, Sadanori KONISHI & Seiya IMOTO
Adaptive learning machines for nonlinear classification and Bayesian information criteria

MHF2003-4 Kazuhiro YOKOYAMA
On systems of algebraic equations with parametric exponents

MHF2003-5 Masao ISHIKAWA & Masato WAKAYAMA
Applications of Minor Summation Formulas III, Plücker relations, Lattice paths and Pfaffian identities

MHF2003-6 Atsushi SUZUKI & Masahisa TABATA
Finite element matrices in congruent subdomains and their effective use for large-scale computations

MHF2003-7 Setsuo TANIGUCHI
Stochastic oscillatory integrals - asymptotic and exact expressions for quadratic phase functions -

MHF2003-8 Shoki MIYAMOTO & Atsushi YOSHIKAWA
Computable sequences in the Sobolev spaces

MHF2003-9 Toru FUJII & Takashi YANAGAWA
Wavelet based estimate for non-linear and non-stationary auto-regressive model

MHF2003-10 Atsushi YOSHIKAWA
Maple and wave-front tracking — an experiment

MHF2003-11 Masanobu KANEKO
On the local factor of the zeta function of quadratic orders

MHF2003-12 Hidefumi KAWASAKI
Conjugate-set game for a nonlinear programming problem
MHF2004-1 Koji YONEMOTO & Takashi YANAGAWA
  Estimating the Lyapunov exponent from chaotic time series with dynamic noise

MHF2004-2 Rui YAMAGUCHI, Eiko TSUCHIYA & Tomoyuki HIGUCHI
  State space modeling approach to decompose daily sales of a restaurant into time-dependent multi-factors

MHF2004-3 Kenji KAJIWARA, Tetsu MASUDA, Masatoshi NOUMI, Yasuhiro OHTA & Yasuhiko YAMADA
  Cubic pencils and Painlevé Hamiltonians

MHF2004-4 Atsushi KAWAGUCHI, Koji YONEMOTO & Takashi YANAGAWA
  Estimating the correlation dimension from a chaotic system with dynamic noise

MHF2004-5 Atsushi KAWAGUCHI, Kentarou KITAMURA, Koji YONEMOTO, Takashi YANAGAWA & Kiyofumi YUMOTO
  Detection of auroral breakups using the correlation dimension

MHF2004-6 Ryo IKOTA, Masayasu MIMURA & Tatsuyuki NAKAKI
  A methodology for numerical simulations to a singular limit

MHF2004-7 Ryo IKOTA & Eiji YANAGIDA
  Stability of stationary interfaces of binary-tree type

MHF2004-8 Yuko ARAKI, Sadanori KONISHI & Seiya IMOTO
  Functional discriminant analysis for gene expression data via radial basis expansion

MHF2004-9 Kenji KAJIWARA, Tetsu MASUDA, Masatoshi NOUMI, Yasuhiro OHTA & Yasuhiko YAMADA
  Hypergeometric solutions to the $q$-Painlevé equations

MHF2004-10 Raimundas VIDŪNAS
  Expressions for values of the gamma function

MHF2004-11 Raimundas VIDŪNAS
  Transformations of Gauss hypergeometric functions

MHF2004-12 Koji NAKAGAWA & Masakazu SUZUKI
  Mathematical knowledge browser

MHF2004-13 Ken-ichi MARUNO, Wen-Xiu MA & Masayuki OIKAWA
  Generalized Casorati determinant and Positron-Negaton-Type solutions of the Toda lattice equation

MHF2004-14 Nalini JOSHI, Kenji KAJIWARA & Marta MAZZOCCH
  Generating function associated with the determinant formula for the solutions of the Painlevé II equation
MHF2004-15 Kouji HASHIMOTO, Ryohei ABE, Mitsuhiro T. NAKAO & Yoshitaka WATANABE
Numerical verification methods of solutions for nonlinear singularly perturbed problem

MHF2004-16 Ken-ichi MARUNO & Gino BIONDINI
Resonance and web structure in discrete soliton systems: the two-dimensional Toda lattice and its fully discrete and ultra-discrete versions

MHF2004-17 Ryuei NISHII & Shinto EGUCHI
Supervised image classification in Markov random field models with Jeffreys divergence

MHF2004-18 Kouji HASHIMOTO, Kenta KOBAYASHI & Mitsuhiro T. NAKAO
Numerical verification methods of solutions for the free boundary problem

MHF2004-19 Hiroki MASUDA
Ergodicity and exponential $\beta$-mixing bounds for a strong solution of Lévy-driven stochastic differential equations

MHF2004-20 Setsuo TANIGUCHI
The Brownian sheet and the reflectionless potentials

MHF2004-21 Ryuei NISHII & Shinto EGUCHI
Supervised image classification based on AdaBoost with contextual weak classifiers

MHF2004-22 Hideki KOSAKI
On intersections of domains of unbounded positive operators

MHF2004-23 Masahisa TABATA & Shoichi FUJIMA
Robustness of a characteristic finite element scheme of second order in time increment

MHF2004-24 Ken-ichi MARUNO, Adrian ANKIEWICZ & Nail AKHMEDIEV
Dissipative solitons of the discrete complex cubic-quintic Ginzburg-Landau equation

MHF2004-25 Raimundas VIDŪNAS
Degenerate Gauss hypergeometric functions

MHF2004-26 Ryo IKOTA
The boundedness of propagation speeds of disturbances for reaction-diffusion systems

MHF2004-27 Ryusuke KON
Convex dominates concave: an exclusion principle in discrete-time Kolmogorov systems
MHF2004-28 Ryusuke KON
Multiple attractors in host-parasitoid interactions: coexistence and extinction

MHF2004-29 Kentaro IHARA, Masanobu KANEKO & Don ZAGIER
Derivation and double shuffle relations for multiple zeta values

MHF2004-30 Shuichi INOKUCHI & Yoshihiro MIZOGUCHI
Generalized partitioned quantum cellular automata and quantization of classical CA

MHF2005-1 Hideki KOSAKI
Matrix trace inequalities related to uncertainty principle

MHF2005-2 Masahisa TABATA
Discrepancy between theory and real computation on the stability of some finite element schemes

MHF2005-3 Yuko ARAKI & Sadanori KONISHI
Functional regression modeling via regularized basis expansions and model selection

MHF2005-4 Yuko ARAKI & Sadanori KONISHI
Functional discriminant analysis via regularized basis expansions

MHF2005-5 Kenji KAJIWARA, Tetsu MASUDA, Masatoshi NOUMI, Yasuhiro OHTA & Yasuhiko YAMADA
Point configurations, Cremona transformations and the elliptic difference Painlevé equations

MHF2005-6 Kenji KAJIWARA, Tetsu MASUDA, Masatoshi NOUMI, Yasuhiro OHTA & Yasuhiko YAMADA
Construction of hypergeometric solutions to the $q$-Painlevé equations

MHF2005-7 Hiroki MASUDA
Simple estimators for non-linear Markovian trend from sampled data: I. ergodic cases

MHF2005-8 Hiroki MASUDA & Nakahiro YOSHIDA
Edgeworth expansion for a class of Ornstein-Uhlenbeck-based models

MHF2005-9 Masayuki UCHIDA
Approximate martingale estimating functions under small perturbations of dynamical systems

MHF2005-10 Ryo MATSUZAKI & Masayuki UCHIDA
One-step estimators for diffusion processes with small dispersion parameters from discrete observations

MHF2005-11 Junichi MATSUKUBO, Ryo MATSUZAKI & Masayuki UCHIDA
Estimation for a discretely observed small diffusion process with a linear drift
MHF2005-12 Masayuki UCHIDA & Nakahiro YOSHIDA
AIC for ergodic diffusion processes from discrete observations

MHF2005-13 Hiromichi GOTO & Kenji KAJIWARA
Generating function related to the Okamoto polynomials for the Painlevé IV equation

MHF2005-14 Masato KIMURA & Shin-ichi NAGATA
Precise asymptotic behaviour of the first eigenvalue of Sturm-Liouville problems with large drift

MHF2005-15 Daisuke TAGAMI & Masahisa TABATA
Numerical computations of a melting glass convection in the furnace

MHF2005-16 Raimundas VIDŪNAS
Normalized Leonard pairs and Askey-Wilson relations

MHF2005-17 Raimundas VIDŪNAS
Askey-Wilson relations and Leonard pairs

MHF2005-18 Kenji KAJIWARA & Atsushi MUKAIHIRA
Soliton solutions for the non-autonomous discrete-time Toda lattice equation

MHF2005-19 Yuu HARIYA
Construction of Gibbs measures for 1-dimensional continuum fields

MHF2005-20 Yuu HARIYA
Integration by parts formulae for the Wiener measure restricted to subsets in $\mathbb{R}^d$

MHF2005-21 Yuu HARIYA
A time-change approach to Kotani’s extension of Yor’s formula

MHF2005-22 Tadahisa FUNAKI, Yuu HARIYA & Mark YOR
Wiener integrals for centered powers of Bessel processes, I

MHF2005-23 Masahisa TABATA & Satoshi KAIZU
Finite element schemes for two-fluids flow problems

MHF2005-24 Ken-ichi MARUNO & Yasuhiro OHTA
Determinant form of dark soliton solutions of the discrete nonlinear Schrödinger equation

MHF2005-25 Alexander V. KITAEV & Raimundas VIDŪNAS
Quadratic transformations of the sixth Painlevé equation

MHF2005-26 Toru FUJII & Sadanori KONISHI
Nonlinear regression modeling via regularized wavelets and smoothing parameter selection
MHF2005-27 Shuichi INOKUCHI, Kazumasa HONDA, Hyen Yeal LEE, Tatsuro SATO, Yoshihiro MIZOGUCHI & Yasuo KAWAHARA
On reversible cellular automata with finite cell array

MHF2005-28 Toru KOMATSU
Cyclic cubic field with explicit Artin symbols

MHF2005-29 Mitsuhiro T. NAKAO, Kouji HASHIMOTO & Kaori NAGATOU
A computational approach to constructive a priori and a posteriori error estimates for finite element approximations of bi-harmonic problems

MHF2005-30 Kaori NAGATOU, Kouji HASHIMOTO & Mitsuhiro T. NAKAO
Numerical verification of stationary solutions for Navier-Stokes problems