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Calculation of Modulus T of the Anomalous Magnetic Vector over Large Areas

by

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Abstract

A differential technique was implemented for the calculation of the low sensitive to the sources magnetisation vector direction and showing high centricity T transform of the total anomalous magnetic field over large areas, where the changes in the values of the declination and inclination of the geomagnetic field are considerable. The technique was tested on model magnetic anomalies caused by three dipoles, possessing normal and reverse magnetisation and located at widely varying magnetic latitudes. Finally, the differential technique taking into account the changes in the geomagnetic field direction was applied on real data taken over the Japanese Islands and the surrounding areas. The pattern of the transformed field is considerably simplified and its extrema are better centred over the true position of the sources.

Keywords: Magnetic transform, Differential technique, Modulus T, Magnetisation vector direction

1. Introduction

Modulus T of the magnetic intensity vector $\mathbf{T}$ belongs to a group of magnetic field transforms, showing low sensitivity to the direction of source magnetisation vector and high centricity[1)]. It has non-negative values at all points of observation and extrema, located within the horizontal projections of the bodies, which makes easier the analysis of the original magnetic data ΔT. Compared to the T anomalies, the observed magnetic anomalies ΔT show complex pattern, consisting of several extrema, one positive or negative, and one or two peripheral extrema of the opposite sign. The extrema position is strongly influenced by the direction of the source magnetisation vector. The main one can be considerably displaced from the source epicentre, which together with the often occurring interference at the peripheries of the anomalies, causes difficulties in the analysis and correlation of the anomalous effects with their causing geologic and other bodies. One estimation of the sensitivity of modulus T to the direction of the source magnetisation vector is given by its

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relative integral deviation S[1]

$$S(I,D)=\frac{\int\int_{-\infty}^{+\infty}|T(I,D)-T(90^\circ,0)|\,dxdy}{\int\int_{-\infty}^{+\infty}|T(90^\circ,0)|\,dxdy}. \tag{1}$$

Transform T has ten times lower sensitivity to the direction of magnetisation vector, compared to the measured magnetic field. The centricity, on the other side, is defined by the location of the main extremum in relation to the source epicentre. Modulus T shows considerably increased centricity in comparison to the original field. The T transform of 2D fields is totally independent of the source magnetisation vector direction and exactly centred over the bodies.

Like reduction to the pole and the analytic signal, transform T alleviates the problems in magnetic interpretation due to the skewness of the total anomalous magnetic field when the geomagnetic field direction is not vertically down. But unlike reduction to the pole, it can produce correct results in areas where normal and reverse magnetisations are present, as often is the case with the total anomalous magnetic field over large areas.

It has also the advantage over the analytic signal, which also simplifies the pattern of the magnetic maps, of being a transform of the same order as the measured magnetic field. These features can make T transform a useful tool in regional mineral exploration and crustal structure studies.

The standard procedure for obtaining $T=\left(X^2+Y^2+Z^2\right)^{1/2}$ involves calculation of the components X, Y and Z of the total magnetic field intensity and is carried out in the frequency domain, using conventional phase transforms[2] with the assumption that the geomagnetic field direction does not change considerably over the studied region. This assumption is true for local magnetic anomalies but for regional magnetic anomalies it introduces significant errors in the transformed field. Therefore, a method taking into account the changing geomagnetic field direction is needed.

2. Method

The proposed method is similar to the technique, introduced by Arkani-Hamed[3] for differential reduction to pole of regional magnetic anomalies.

The transformation of ΔT to X, Y and Z could be implemented in the frequency domain in two stages: First, differential calculation of the potential U of the magnetic field ΔT; Second, calculation of the components X, Y and Z from U through differentiation, applying the transfer function $W(k)=-ik$ in the case of the horizontal components, where k is the radial frequency in the direction of differentiation, and $W(k_x,k_y)=-K$ in the case of the vertical component Z, where $K=\sqrt{k_x^2+k_y^2}$.

The calculation of the potential U from ΔT is usually implemented in the frequency domain, using the formula (see.[2])

$$F[U]=-\frac{F[\Delta T]}{\mathbf{g}\cdot\mathbf{l}}, \tag{2}$$

where

$$F[U]=\int\int_{-\infty}^{\infty}U(x,y)e^{-i(k_xx+k_yy)}dxdy,$$

and $\mathbf{l}$ is a unit vector along the direction of the geomagnetic field vector. Vector $\mathbf{g}$ is defined as $\mathbf{g} = (ik_x, ik_y, K)$. The magnetic anomaly ΔT, caused by a potential U, is represented with the expression

$$\Delta T = -\mathbf{l} \cdot \nabla U. \tag{3}$$

Vector $\mathbf{l}$ changes its direction over large areas and can be expressed as a sum of the mean value $\mathbf{l}_0$ of the geomagnetic field direction over the region and the deviation from this average value $\delta\mathbf{l}$

$$\mathbf{l} = \mathbf{l}_0 + \delta\mathbf{l}. \tag{4}$$

As a result of substitution of eqn.(4) in eqn.(3) and the transformation of the obtained equation in the spectral domain

$$\mathbf{l}_0 \cdot F[\nabla U] = -\left(F[\Delta T] + F[\delta\mathbf{l} \cdot \nabla U]\right), \tag{5}$$

and since $F[\nabla U] = \mathbf{g} \cdot F[U]$, finally we obtain

$$F[U] = -\frac{F[\Delta T] + F[\delta\mathbf{l} \cdot \nabla U]}{\mathbf{g} \cdot \mathbf{l}_0}. \tag{6}$$

Equation (6) is solved iteratively. First, the average values for I_0 and D_0 over the area are found. For these values, an initial approximation of the spectrum U of the anomaly ΔT is obtained in the frequency domain, applying formula (2). From the spectrum of U, the field components X, Y and Z, or as they are denoted in eqn.(3) $-\nabla U$, are calculated in the frequency domain. The components are transformed to the space domain and multiplied by the differences $\delta\mathbf{l}$ along the respective directions, calculated according to eqn.(4). The obtained matrix $-\delta\mathbf{l} \cdot \nabla U$ is transformed to the frequency domain, in order to calculate $-\left(F[\Delta T] + F[\delta\mathbf{l} \cdot \nabla U]\right)$. The latter spectrum is divided by $\mathbf{g} \cdot \mathbf{l}_0$, thus obtaining a new approximation of the spectrum of the potential. The procedure is repeated until a minimum difference between the values of U after two successive iterations in the space domain is achieved, i.e. until the expression

$$Error = \frac{\sum_{n=1}^{N_p} \left(U_{i+1}(n) - U_i(n)\right)^2}{N_p}, \tag{7}$$

where N_p is the grid points number, reaches a minimum value.

The procedure is unstable at low magnetic latitudes in a way, similar to the instability of the standard frequency filter for transformation of the measured field to its components, or to the reduced to the pole field.

3. Synthetic model

A simple example for calculation of transform T for the model magnetic field, caused by 3 isometric (intrusive) bodies with normal and reverse magnetisation, located at considerably different geographic latitudes and longitudes, demonstrates the applicability of the method and the advantages of the T transform. The inclination I_0 of the geomagnetic field varies from 10° to 90°, and that of the declination D_0, from −30° to 30°. **Figure 1** shows the cosine directions of the geomagnetic field in the model grid area.

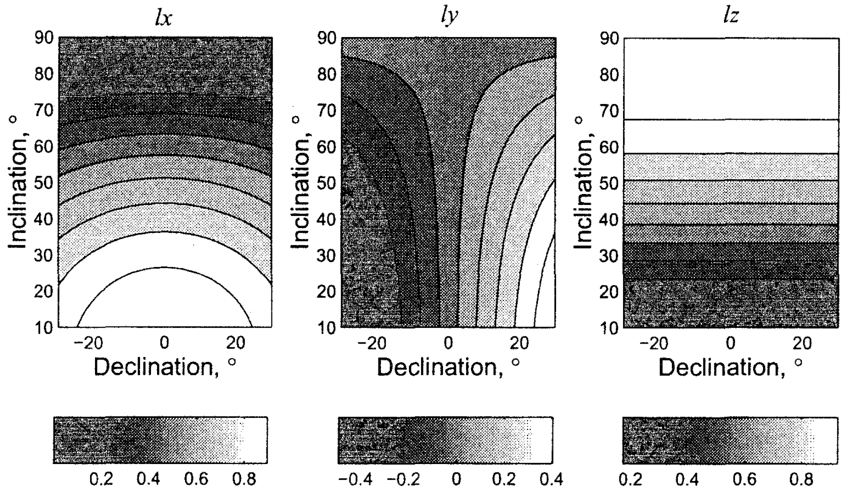

Fig. 1 Cosine directions of the geomagnetic field of the model area.

The model grid size is 128 × 64 points, and the grid spacing is 1 relative unit in the two perpendicular directions. The depths of the three spheres are the same and equal to 3 units. The sources were modelled with values of the inclination and declination of the magnetisation vector as follows: first sphere, $72.36°, 7.14°$ (normal magnetisation), second sphere, $-47.17°, -11.9°$ (reverse magnetisation), and third sphere, $31.42°, 12.86°$ (normal magnetisation). The horizontal locations of the centres of the sources are denoted with crosses in all maps. **Figure 2.a** shows the modelled field ΔT and **Fig.2.b** depicts the calculated with the differential method T transform.

The initial value for the direction of the normal geomagnetic field is assumed to be $I_0 = 50°$ and $D_0 = 0$, equal to the inclination and the declination of the field in the central point of the grid. The T field **Fig.2.b** is simplified compared to the original field and the extrema of the positive only anomalies are centred almost exactly over the sources centres. Highest deviation of the maximum of the T transform anomaly from the true horizontal projection of the centre of the sphere is observed for the third sphere. This is in agreement with the model studies on the dependence of the centricity of a spherical source of the magnetisation vector inclination, according to which the deviation from the projection of the true centre reaches a maximum at around $I = 10°$ and then decreases to become exactly centred at $I = 90°$[1).

Figure 3.a depicts the difference map between the theoretical and the calculated with the standard technique T transform. Since the area of the anomalies is small in relation to the total model grid area, the maps of the difference values help in estimating the size of the error, introduced by the use of a constant value for the direction of the geomagnetic field vector for the calculation of the field components, and consequently, the T transform. **Figure 3.b** shows the small differences between the calculated with the differential method transform and the theoretical one. The presence of some error in the map, obtained with the differential technique, is due to the use of the discrete Fourier transform and its accompanying edge effects. But this is an error, introduced during calculation of all linear transforms in the frequency domain.

Figure 4.a and **Fig.4.b** show the first approximation of the potential of the model anomalies and the potential values after the last iteration. Comparing the two figures, it could be noticed that larger changes are undergone by the anomalies of the bodies, located

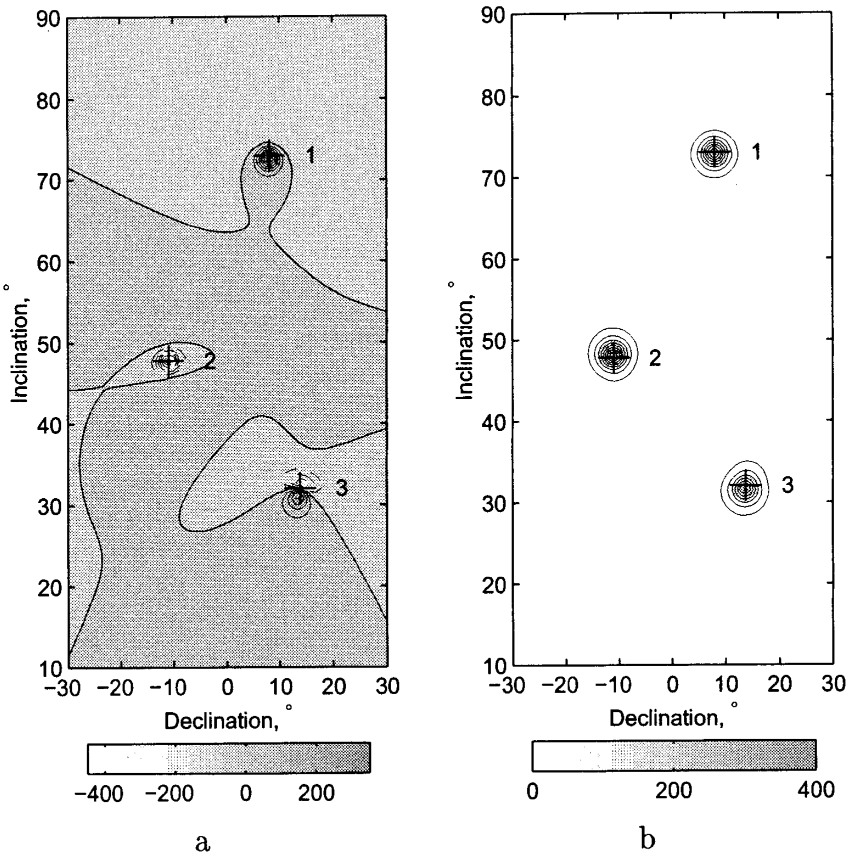

Fig. 2 a: Anomaly ΔT, nT, of the model; b: T transform, nT, calculated with the differential technique for obtaining the magnetic field components.

far from the central grid point, for which the initial approximation is calculated. These two figures demonstrate very descriptively the amount of the error, introduced by the use of a constant value for the normal field direction. The convergence for the synthetic example was achieved after the forth iteration (**Fig.5.a**).

3.1 Field example

The algorithm for calculation of the components of the magnetic field and the respective transform T from aeromagnetic data ΔT covering large areas was applied on magnetic anomalies from the Japanese Islands and the adjoining areas, presented as a grid in a CD-ROM, published by Geological Survey of Japan[4]. The area of the transformed data is limited between 129.14°E and 145.56°E geographic longitude and 28.36°N and 41.85°N geographic latitude. The aeromagnetic and marine data had been analitically continued to an altitude of 3200 m with the purpose of compiling the map of scale 1:4 000 000.

A considerable number of linear anomalies could be noticed, inspecting closely the magnetic map of Japan and the surrounding areas (**Fig.6**). The sources of the anomalies are magnetic layers of oceanic plates, ridges creating the oceanic plates, and igneous rocks, which were produced as a result of the island arc volcanic activities in overriding plates. Okubo *et al.*[5] interpreted the linear anomalies by drawing axes along the extrema of the positive parts of the anomalies, assuming that in the northern hemisphere the magnetic sources create dipolar anomalies, with a positive extremum to the south and a negative one

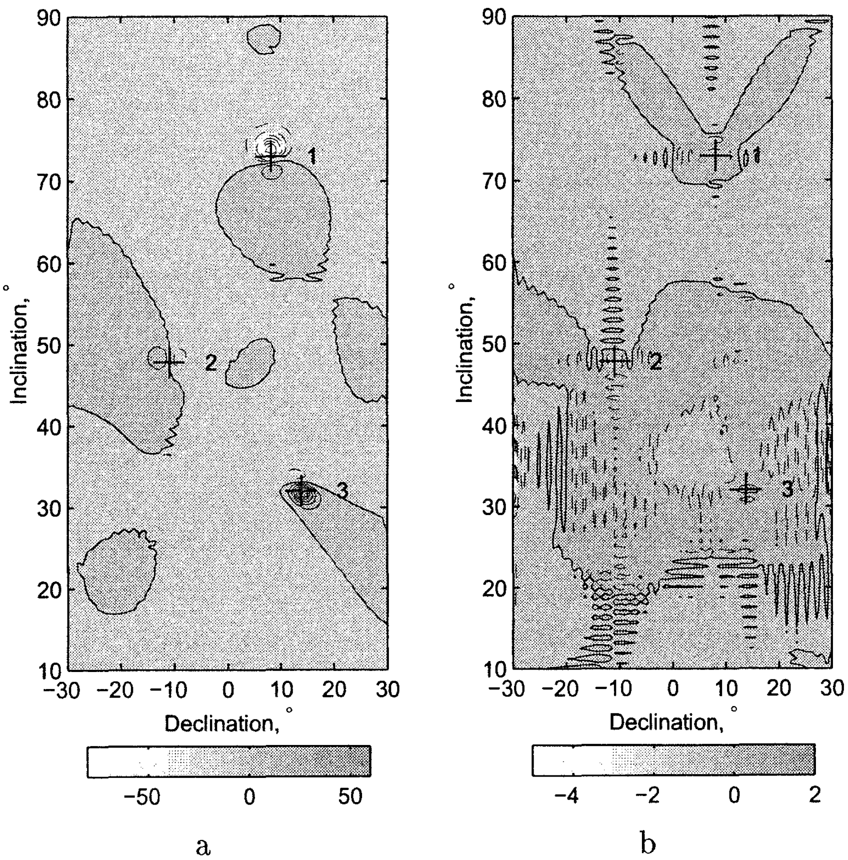

Fig. 3 a: Difference map, nT, between the theoretical values of T and T, calculated for constant values of I_0 and D_0. b: Difference map, nT, between the theoretical values of T and T, calculated with the differential technique for obtaining the magnetic field components.

to the north. They suggested that for that reason the true position of the linear sources may be several kilometers to the north of the drawn magnetic lineations. An interpretation of the linear anomalies could considerably be facilitated by the transformation of the measured magnetic field to a map of the modulus of the anomalous vector. Such a map would noticeably improve the centicity of the anomalies and would confirm the belonging of the different extrema to their anomalies. This is especially necessary in the case of magnetic rocks, formed in periods of successive reversal of the geomagnetic dipole.

The original magnetic grid data was transformed, using 1995 IGRF model for obtaining the cosine directions of the normal field over the area. These are shown in **Fig.8**. The size of the grid data for the anomalous magnetic field from **Fig.6** is 820 $\times$ 626 points, i.e. 1638 km $\times$ 1250 km. The algorithm was applied for initial values of the inclination and declination of the normal field 48° and $-7°$, respectively. The convergence of the algorithm was reached after the forth iteration (**Fig.5.b**). Comparing **Fig.5.a** with **Fig.5.b**, one can notice the higher convergence rate to the exact solution of the field example, due to the smaller size of the variations in the direction of the geomagnetic field, compared to those over the model area. **Figure 7** depicts the obtained final result for the T tranform. The field has only non-negative values and is significantly simplified. The prevailing linear trending anomalies are centred exactly over the sources and the tracing of their extrema can produce a precise

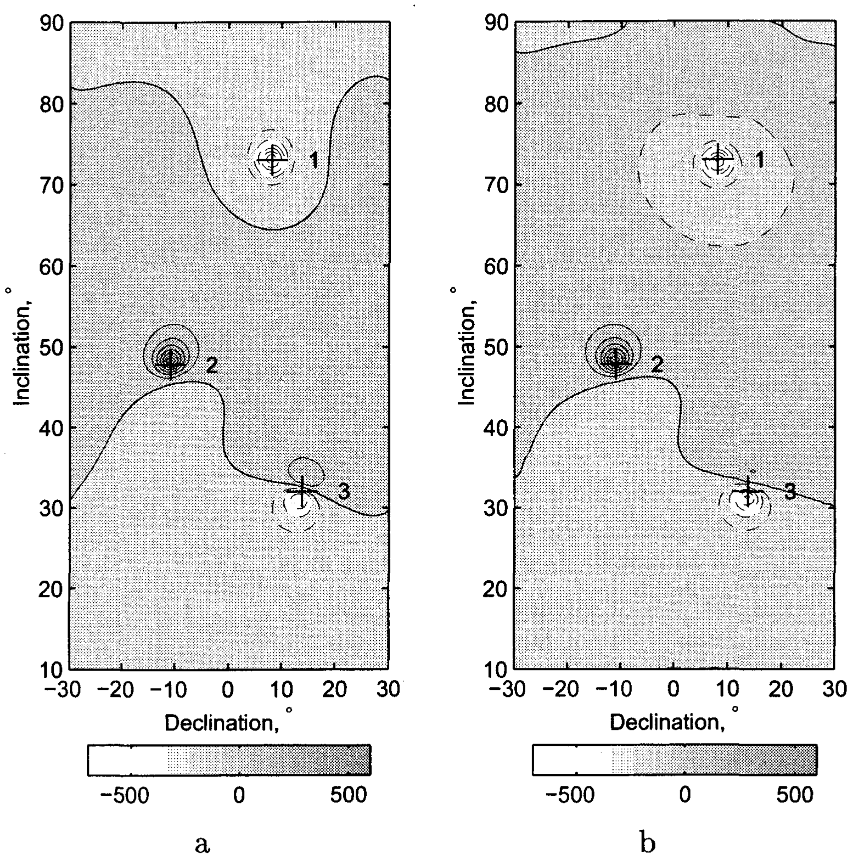

Fig. 4 a: Map of the values of the potential U as a result of the first approximation; b: Map of the final potential values U.

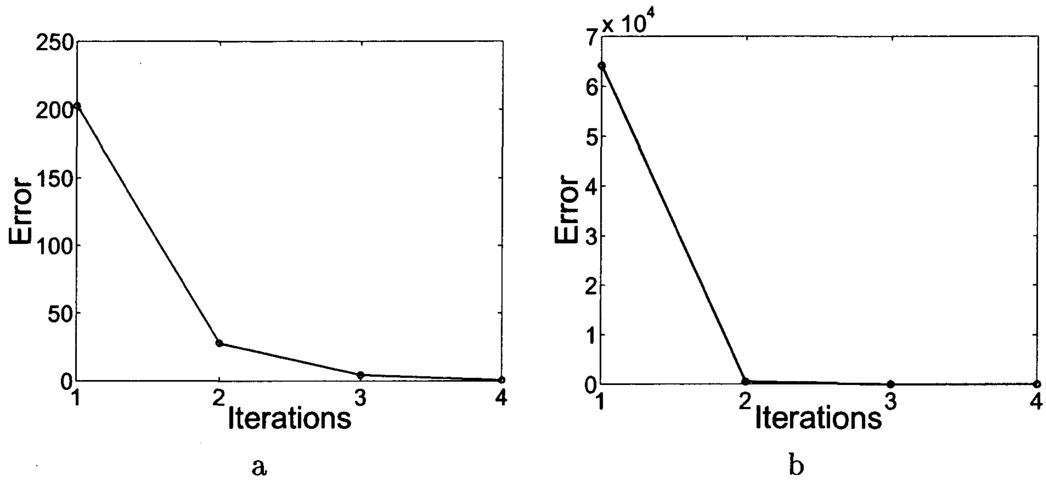

Fig. 5 The error as a function of the number of iterations. a: Model example with three isometric sources; b: Field example from the Japanese Islands and the surrounding areas.

map of the magnetic lineations of the area.

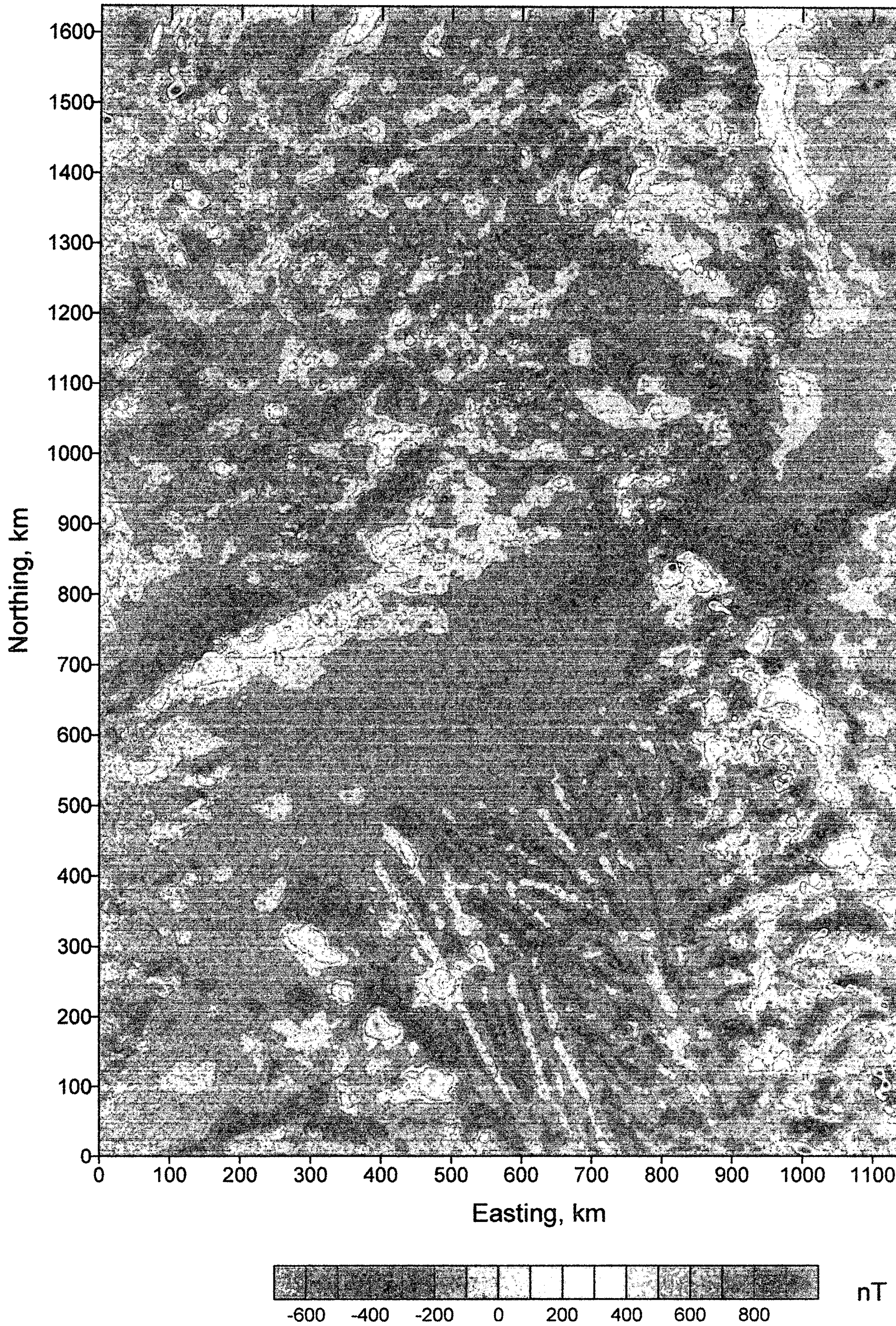

Fig. 6 Total anomalous magnetic field ΔT, Japan and surrounding areas, nT.

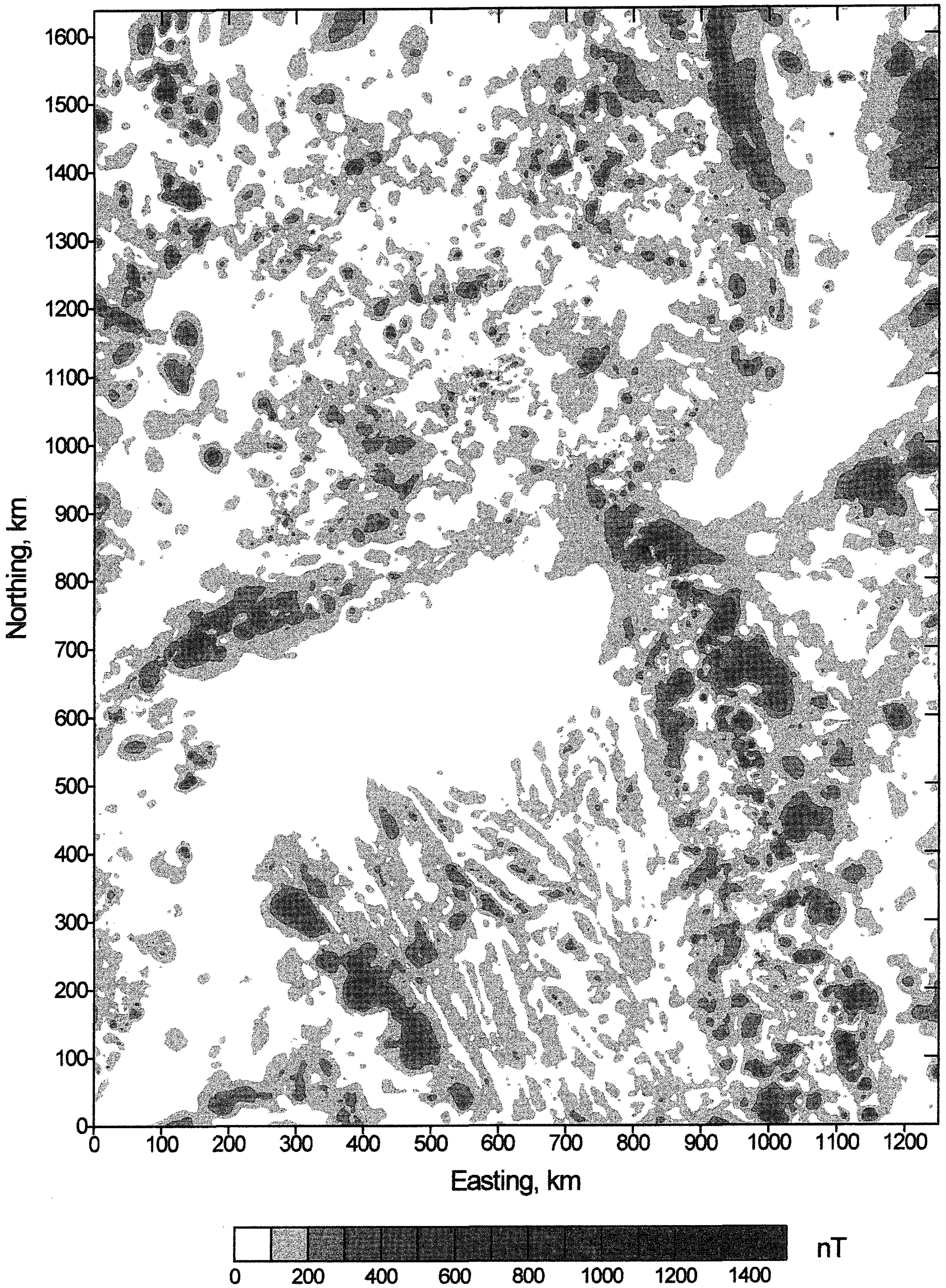

Fig. 7 Map of the calculated with the differential technique transformation T, Japan and surrounding areas, nT.

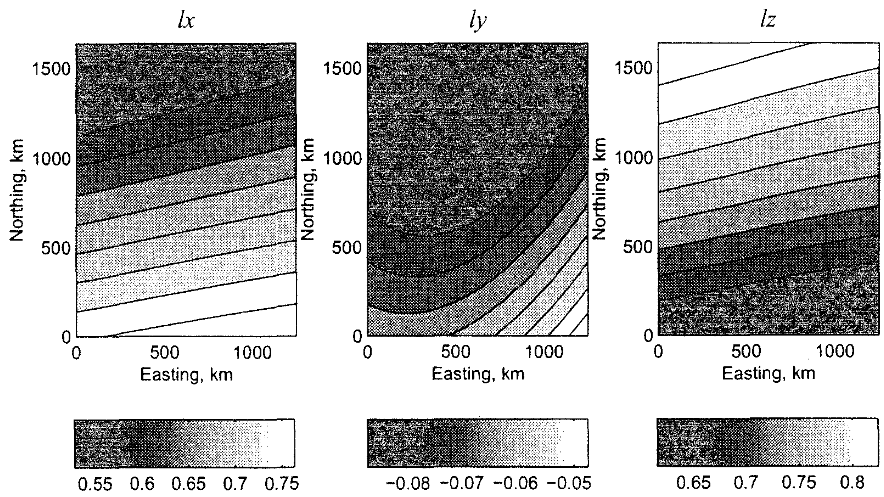

Fig. 8 Cosine directions of the geomagnetic field, Japan and surrounding areas.

4. Conclusion

The differential calculation of T transform makes possible a simple representation of the anomalous regional magnetic field. With the non-negative values of its anomalies and its improved centricity the regional map of the T transform could facilitate considerably a larger scale interpretation. The model example of application of the differential technique demonstrated that the method produced minute absolute errors, and at the same time showed the considerable error in the absolute values and in the shape of the anomalies when using the standard procedure. The applicability of the method on large grid data was proven through the transformation of the anomalous magnetic field over the Japanese Islands and the surrounding areas. The pattern of the obtained map is simplified and the large number of linear magnetic anomalies are exactly centred over their sources.

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